# Labor Market Pooling and Human Capital Investment Decisions*

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September 28, 2009

**Abstract**

Labor market pooling is considered one of the advantages of agglomerations. This paper presents a model of human capital formation in an imperfectly competitive, pooled local labor market with heterogeneous workers and firms. Firms produce with different technologies requiring diverse skills. Workers specialize into specific skills and accumulate general human capital. While labor market pooling provides static efficiency gains, our results also imply positive long-term effects: Under a diversified structure, firm-specific shocks increase workers' incentives to acquire both general and specific human capital. This not only raises productivity but also strengthens a region's capability to adapt to change.

JEL classification: I20, J24, J41

Keywords: Human capital, labor market pooling, unemployment

*For helpful comments and suggestions we thank Uwe Blien, Pramod N. Junankar, Annekatrin Niebuhr, Ulf Schiller, Konrad Stahl, Odd Rune Straume, Uwe Walz and Rüdiger Wapler. The views expressed here are those of the authors and do not necessarily reflect those of the authors' affiliated institutions.

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# 1 Introduction

Agglomerations grow faster and their labor productivity is higher compared to less agglomerated areas. Looking at US states, Ciccone and Hall (1996) report a 6 percent increase in labor productivity upon a doubling of employment density. Higher productivity is also evident in higher nominal wages: Glaeser and Mare (2001) confirm the existence of an urban wage premium of about 33 percent across US metropolitan areas; accounting for personal and job characteristics as well as for unobserved ability still leaves a wage premium of over 20 percent. These positive wage effects persist, even when workers leave an agglomeration. Hence, human capital appears to be one of the sources of agglomeration economies.

This paper employs a model of a pooled labor market in order to explain the productivity and wage premium typically found in agglomerations. Pooling creates an advantage for both workers and firms as it improves matching and facilitates adjustment to shocks, thus raising the return to human capital investment. Our paper adds to the existing literature by synthesizing the endogenous formation of heterogeneous skills and labor market pooling.

A labor market pooling advantage arises as the result of a portfolio effect and a matching effect. The matching effect of pooling refers to the complementarity of workers' specific skills and firms' skill requirements. As the average skill distance between workers and firms decreases in an agglomeration, so does the degree of mismatch. As regards the portfolio effect, a pooled labor market protects both firms and workers against demand uncertainty. The portfolio benefit will be higher the lower the correlation of demand shocks.

Our model builds upon the idea that human capital has both a general and a specific component, as expressed in Kim (1989).[1] In a series of papers, Kim (1989, 1990, 1991) studies the impact of local labor market size on wages and human capital formation. Adapting the Salop model of product differentiation to the labor market, Kim (1989) shows that wages rise and workers invest more in specific human capital as the market is enlarged. Following his approach, we model human capital formation in an imperfectly competitive and pooled local labor market. We then introduce price uncertainty on firms' product markets which influences the labor market outcome. In anticipation of firm behavior and

[1] Kim refers to specific human capital as intensive, and to general human capital as extensive.

price uncertainty, workers choose between specializing into a certain type of skills and accumulating general skills. We thus look at the endogenous determination of of both specific and general human capital under product market uncertainty.

The distinction between general and specific human capital allows us to capture two important aspects of labor market pooling: increased match quality and risk reduction. While investments into specific skills improve a worker's productivity if matched with the appropriate technology, general skills increase a worker's flexibility, i.e. her ability to retrain and adjust to a change in skill requirements.

Empirical support for this argument is provided by Glaeser and Saiz (2004): having first identified education as an important ingredient into agglomeration economies, they find evidence for a causation running from skills to growth, the mechanism being increased productivity (at the metropolitan area level). Their finding that skills matter most in declining cities supports the so called Reinvention City View, according to which skills are important for a city's adjustment to negative shocks. Human capital therefore increases the flexibility of the local work force and thus helps a region and its firms to better adjust to shocks in demand or technology. The adaptive nature of skills is also emphasized in Iyigun and Owen (2006): Within the framework of a growth model, education produces adaptive skills which increase the future adaptability of the workforce in an environment characterized by frequent technological change. This, in turn, increases the resources devoted to R&D and therefore the rate of technical change.

We will proceed as follows: Section 2 provides an overview of the related literature. Section 3 introduces the model. Section 4 derives labor market equilibria and analyzes workers' optimal investments into human capital. In section 5, we discuss a particular specification of our model where additional restrictions generate unemployment. We derive and discuss empirical predictions of our analysis in section 6. Section 7 concludes. All proofs are relegated to the appendix.

# 2 Related Literature

Empirical studies on the existence and extent of agglomeration economies, particularly those that estimate production functions, are surveyed in Rosenthal and Strange (2001, 2004). These studies typically focus on the industrial scope of agglomeration economies. Another strand of the literature focusses on the relationship between agglomeration and employment growth (e.g. Henderson, Kuncoro, and Turner, 1995; Rosenthal and Strange, 2003) or wages (e.g. Glaeser and Mare, 2001). All studies confirm the existence of positive agglomeration economies.

As regards the sources of agglomeration economies, most of the studies investigate the role of knowledge spillovers and the evidence suggests that knowledge spillovers do exist but vanish quickly with distance (Audretsch and Feldman, 2004; Jaffe, Trajtenberg, and Henderson, 1993). However, the strongest evidence is available for labor market pooling: Dumais, Ellison, and Glaeser (2002) analyze agglomeration economies at different levels of geographical aggregation. They find knowledge spillovers to occur only at the metropolitan level, input sharing to occur at the metropolitan and more so at the state level, and evidence in favor of labor market pooling at all levels of aggregation. Wheeler (2006) and Yankow (2006) confirm the existence of a wage growth effect following job changes, which supports a matching or coordination efficiency explanation of agglomeration economies.

Krugman (1991) presents an early formal treatment of labor market pooling and points out the advantage of a pooled labor market in the reduction of risk: workers benefit from lower income risk, firms from the availability of workers if they wish to respond to a positive demand shock. A different benefit of pooling, namely improved matching, is emphasized in Helsley and Strange (1990), who incorporate labor market heterogeneity into a general equilibrium model of a system of cities. With cities' population growth being determined endogenously over migration, the labor market can be shown to generate agglomeration economies as both workers and firms expect to be better matched in larger cities.

Strange, Hejazi, and Tang (2006) present a model that combines uncertainty and the matching problem: Firms with a particular but uncertain resource need find it easier to respond to the uncertainty if there is a wider range of resource suppliers in the local market. Conversely, resource suppliers prefer to locate where more downstream firms are active. Agglomeration increases the expected match quality in the face of uncertainty. Similarly, Wheeler (2001) argues that with heterogeneous skills and skill requirements, a larger market enables greater sorting. Consequently, urban markets exhibit both higher productivity and higher wages.

Other models of labor market pooling analyze the location decision of firms, see for example Matouschek and Robert-Nicoud (2005) and Combes and Duranton (2006). The latter emphasize the close relationship between pooling and knowledge spillovers as joint determinants of the location decision. Agglomeration is the result of a trade-off between the benefits and costs of pooling: the former arise from the knowledge that a poached worker brings to her new employer, the latter consist of higher wages that a firm has to pay in order to retain and attract workers. As the product market approaches perfect competition, firms eventually choose to locate in separate labor markets as the costs of higher wages outweigh the benefits of spillovers. Hence, Combes and Duranton (2006) demonstrate how the intensity of competition on product markets can be linked to labor market outcomes.

Kim (1990), Rotemberg and Saloner (2000), and Picard and Toulemonde (2004) consider human capital investments in pooled labor markets, focussing on the benefits of larger markets. In Kim (1990), the average skill distance between workers and firms decreases as cities grow. With lower costs of mismatch workers face a stronger incentive to invest in human capital. Rotemberg and Saloner (2000) demonstrate that competition among firms for trained individuals ensures workers with the appropriate return on their human capital investment. Picard and Toulemonde (2004) model a pooled labor market with endogenous human capital formation. With more firms in the local market, the range of abilities in demand is extended and so a worker's probability of finding a perfect match increases. An implication of the model is that more general education, by expanding the range of abilities of a worker, improves the matching and thus fosters spatial dispersion of industry.

Finally, Kim and Mohtadi (1992) formalize a dynamic effect of pooling: Economic growth is determined from ever increasing specialization under the assumption of constant population growth. With more workers and firms in the market, matching is improved. This encourages further specialization by workers and generates ongoing growth.

# 3 The Model

We consider a local labor market composed of a mass of workers normalized to unity and two firms competing for these workers. Firms produce for a market that is subject to random price fluctuations. Before entering the labor market, workers decide on their human capital investment in anticipation of future labor market outcomes. Both workers and firms are risk-neutral and maximize expected income and profits, respectively.

## 3.1 Workers

A worker's human capital is characterized by a specific, worker-innate skill, $s \geq 0$, as well as a general ($x > 0$) and a specific ($y > 0$) component. In her investment decision, a worker only chooses the latter two components, while her skill is predetermined and private information. Let $y$ be the amount of output produced by a worker. Hence, specific human capital directly determines productivity in the production process. In contrast, general human capital helps the worker to adapt her skills to the specific needs of the employer. If her skill $s$ differs from the employer's skill requirement $s^*$ a worker has to bear adjustment costs $c^a$ which general human capital helps to reduce. We assume the following specification of adjustment costs: $c^a(s, s^*) = |s - s^*|/x$. Human capital formation is costly, too. For simplicity we use the functional form $c^h(x, y) = \alpha(x^2 + y^2)/2$, where $\alpha > 0$ will be assumed large enough to ensure the existence of optimal solutions.

Workers are confronted with two kinds of uncertainty: First, when making their investment decision, workers do not know how far in terms of skills they are apart from their prospective employers. We assume that skills $s$ are distributed uniformly over $[0, S]$ with density $S$. Only after human capital decisions have been made will the ordering of skills relative to prospective employers be revealed to workers. This kind of uncertainty captures the effect of technological developments which affect production structures.[2] The average distance between

[2]For example, a student in IT might specialize in a specific programming language, not knowing whether this or some other language will be demanded by his future employer. Ex ante, various programming languages might be equally important. Similarly, law students have to specialize at some point of their studies, without knowing for sure what they are going to need most in their future job.

a worker and a firm can be interpreted as the degree of mismatch in the local labor market. The second uncertainty is with respect to firms' labor demand, as product market conditions are changing (see below). Expectations over mismatch and demand will influence workers' investment decisions which in turn will affect labor market outcomes.

## 3.2 Product Market

When selling their products to the markets, the two firms face stochastic price realizations. We assume that the output price for firm $i \in \{1, 2\}$, $\tilde{p}_i > 0$ fluctuates around the expected value $E[\tilde{p}_i] = p$ with variance $\sigma_p^2$, the distribution of both prices being identical. Although the two firms produce for different markets, we allow for some dependence in demand which is captured by the correlation coefficient $\rho \in [-1, 1]$ of the two prices. Price realizations are observed by all agents. Prices determine productivity and firms compete in the labor market.

The introduction of stochastic prices allows us to analyze the link between product market uncertainty, labor demand and human capital investment decisions. In the model, the correlation coefficient indicates to what extent firms depend on common factors. Higher correlations might be due to firms being in similar industries. However, often different industries depend on common factors (such as energy prices or prices of raw materials). Additionally, firms within the same industry may be affected by idiosyncratic shocks to different degrees. We therefore use a general notion of local diversification (specialization): a market can be considered diversified (specialized) the lower (higher) the correlation of firms' product market prices.

Apart from the demand shock interpretation, our analysis can also be applied to markets where firms differ in their innovation rates. If process innovation is an important feature of local industries, then a firm may turn a successful innovation into a productive advantage. Higher productivity temporarily increases the firm's labor demand and its market power, until rival innovations, knowledge spillovers, or the expiry of related patents level out productivity differences between competitors. In a low-technology industrial environment with little innovative activity, the probability that external factors affect firms' productivity jointly is much higher. In this sense, higher (lower) correlation of prices may also be interpreted as a local environment with lower (higher) innovation rates.

### 3.3 Labor Market

We consider two firms which differ in their production technology in the sense that they require workers of different skills. The two firms' technologies are characterized by maximum differentiation. We order worker skills according to their closeness to firm 1's required skill: $s = 0$ denotes the skill that matches firm 1's (firm 2's) demand perfectly (least), whereas $s = S$ is the worst (perfect) skill-match for firm 1 (firm 2). By definition of skill space and ordering, the two firms are located at the endpoints of line $S$ which represents the skill space. The length of that line can then be interpreted as the degree of worker heterogeneity. A smaller $S$ means that firms become more similar in terms of their technologies and skill requirements, and that the average distance (and hence mismatch) between a worker and the nearest firm decreases. The fixed mass of workers is then distributed over a smaller interval.

The two firms are unable to observe a worker's innate skill. Given that workers do not know their relative skill position in advance, they all form the same expectations and hence choose the same level of human capital. As a result, firms cannot use human capital as a signal of skill.[3] This reduces firms' strategies to offering the same wage to all workers and let them incur the (unobservable) adjustment costs. Workers will then sort themselves to the firm with the highest wage offer net of adjustment costs. Our model of wage competition is thus equivalent to price competition within the well-known Hotelling model.[4] Similar to the reservation price assumption in the product market version, workers require a non-negative wage.

Our assumption about firms' locations at the endpoints of line $S$ and the a-priori non-observability of their relative positions along $S$ for workers is less restrictive than it seems. One could imagine the set of skills to be a circle of circumference $2S$ where the two firms' location choice follows workers' human capital decisions. In the spirit of the analysis by Kats (1995), one can show that there exist sufficient conditions for the existence of an equilibrium with maximum differentiation between the two firms. Since we are looking at an application of

[3]Similarly, workers are unable to reveal their skills otherwise, as they will all be tempted to announce the skill which promises them the highest wage, as long as skills are non-verifiable.

[4]For example, see Hotelling (1929), Anderson, De Palma, and Thisse (1992) or Bester, De Palma, Leininger, Thomas, and Von Thadden (1996).

this type of model, we take locations as given. However, in order to ensure our market equilibrium comprises both firms, we impose the following parameter restriction:

$$\max[\tilde{p}_i - \tilde{p}_j] < \frac{3S}{xy} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j \; . \tag{1}$$

Technically, this ensures a labor market equilibrium where both firms employ a positive mass of workers even in the case where the price difference reaches its maximum value ($\max[\tilde{p}_i - \tilde{p}_j]$). Furthermore, the condition ensures that no firm will find it profitable to drive its competitor out of the market.

Another parameter restriction is given by the participation constraint of workers, as we will first consider only full employment equilibria. As long as

$$\min \tilde{p}_i \geq \frac{3S}{2xy} \quad \text{for } i \in \{1, 2\} \tag{2}$$

is fulfilled, all workers are employed in equilibrium even when both firms realize their lowest possible prices. Figure 1 summarizes the timing of our model. In


Figure 1: Time line

the following section, we will solve the model, starting with the derivation of the labor market equilibrium.

# 4 Full Employment Analysis

## 4.1 Labor Market Equilibrium

An equilibrium in the local labor market consists of a pair of wages, $w_1$ and $w_2$, and the allocation of workers to firms. With full employment in equilibrium, the marginal worker of skill $\hat{s}$ who is indifferent between working for firm 1 and firm 2 can be derived from $w_1 - \frac{\hat{s}}{x} = w_2 - \frac{S-\hat{s}}{x}$:

$$\hat{s}(w_1, w_2) = \frac{S}{2} + \frac{x}{2}(w_1 - w_2) \; . \tag{3}$$

Workers within $[0, \hat{s}]$ then constitute firm 1's labor supply, those within $[\hat{s}, S]$ choose to work for firm 2. Then, profits of the two firms can be given depending on prices and wages:

$$\pi_1(w_1, w_2) = \frac{\hat{s}(w_1, w_2)}{S}(\tilde{p}_1 y - w_1) \quad \text{and} \tag{4}$$

$$\pi_2(w_1, w_2) = \frac{S - \hat{s}(w_1, w_2)}{S}(\tilde{p}_2 y - w_2) \; . \tag{5}$$

Partial differentiation with respect to own wages results in the individual reaction function of firm $i$

$$w_i(w_j) = \frac{\tilde{p}_i y + w_j}{2} - \frac{S}{2x} \tag{6}$$

which then leads to the equilibrium (*Bertrand*) wage of firm $i$

$$w_i^B = \frac{2\tilde{p}_i + \tilde{p}_j}{3} y - \frac{S}{x} \tag{7}$$

and the equilibrium marginal worker

$$\hat{s}^B = \frac{xy}{6}(\tilde{p}_1 - \tilde{p}_2) + \frac{S}{2} \; . \tag{8}$$

Two conditions have to be fulfilled for this equilibrium to exist for all price realizations: First, the marginal worker has to be within $[0, S]$; this condition is ensured by restriction (1). Second, the marginal worker has to receive a non-negative net wage even when price realizations for both firms are at their lowest level; condition (2) ensures this.

## 4.2 Labor Market Pooling

Our set-up allows a worker's productivity to differ between firms, hence equilibrium wages and employment can be asymmetric. We now show how our model thus captures advantages from labor market pooling for both firms and workers.

Consider workers first. Wages are uncertain in two respects: One kind of uncertainty refers to a worker's position in skill space relative to the two firms. The other concerns her productivity as prices have not been realized. Hence, the expected net wage of a worker is:

$$E[w^T] = E_{\tilde{p}} \left[ \frac{\hat{s}}{S}(w_1 - E_{s \leq \hat{s}}[c_1^a(s)]) + \frac{S - \hat{s}}{S}(w_2 - E_{s > \hat{s}}[c_2^a(s)]) \right] \; . \tag{9}$$

The outer expectations are taken with respect to prices. The first term within brackets captures the expected net wage of a worker at firm 1, weighted by the probability of employment there. As the worker's position within $[0, \hat{s}]$ is uncertain, expectations with respect to $s$, too, have to be formed (while $\hat{s}$ still depends on price realizations). The second term captures the same probability weighted payoff to the worker in case she is employed at firm 2.

By symmetry, expected profits of the two firms are equal and only depend on the realization of prices. They can thus be derived for example from firm 1 profits

$$E[\pi_1] = E_{\tilde{p}} \left[ \frac{\hat{s}}{S} (\tilde{p}_1 y - w_1) \right] . \tag{10}$$

Inserting equilibrium wages and employment from (7) and (8) into (9) and (10) yields our first set of results.

**Proposition 1** *The equilibrium in the labor market is characterized as follows:*

1. *the expected payoffs of workers and firms and the expected total surplus are, respectively:*

$$\begin{aligned} E[w^T] &= py - \frac{5S}{4x} + \frac{xy^2}{18S} \sigma_p^2 (1-\rho) && (11) \\ E[\pi_i] &= \frac{S}{2x} + \frac{xy^2}{9S} \sigma_p^2 (1-\rho) && (12) \\ E[TS] &= py - \frac{S}{4x} + \frac{5xy^2}{18S} \sigma_p^2 (1-\rho) \, . && (13) \end{aligned}$$

2. *all expected payoffs decrease in price correlation, $\rho$, and (weakly) increase in price uncertainty, $\sigma_p^2$ (portfolio effect);*

3. *expected wages and total surplus decrease in skill differentiation, $S$ (matching effect).*

In equilibrium, the labor market features two pooling effects, a portfolio and a matching effect. The latter is also present in other models of local labor markets such as Thisse and Zenou (2000) or Hamilton, Thisse, and Zenou (2000). Mismatch arises from worker-firm pairs with differing skill supply and requirement and is reflected in the adjustment costs. In a more agglomerated area, there exists a greater number of firms and a finer division of labor among them. In our setting, this is captured by firms being closer in terms of their skill requirements (smaller $S$). This implies that the costs of worker-firm mismatch decrease, thus raising overall productivity in the market. However, a finer division of labor also increases the competitive pressure on the labor market, as firms compete for less heterogeneous workers. Thus, workers reap the benefits in the form of higher wages, while firms' profits may decrease.[5]

The portfolio effect arises even under our assumption of risk neutrality. It is generated by the combination of labor market flexibility and asymmetries in firms' productivity. The ability of firms to adjust employment and wages allows them to shift employment from the less to the more productive firm. This improves employment efficiency and benefits firms. Part of this gain, however, is transferred to the workers as firms compete in wages. Hence, all market participants gain from (expected) employment adjustments following productivity shocks. These advantages only occur for less-than-perfect price correlations ($\rho < 1$). They decrease with correlation and increase with uncertainty (higher $\sigma_p^2$).

## 4.3 Human Capital Formation

We now turn to analyzing workers' choice of human capital investment in the first stage of the model. The maximization problem of an individual worker is

$$\max_{x,y} E[w^T] - c^h \ . \tag{14}$$

Focussing on interior solutions to this problem, we can state our next result.

**Proposition 2** *Workers' optimal human capital investments, $x^*$ and $y^*$, have the following properties:*

1. *general and specific human capital are complements, as long as $\rho < 1$;*

2. *both types of human capital increase in the expected product price, $p$, as long as $\rho < 1$;*

[5]The overall effect of $S$ on profits is ambiguous: Firms also benefit from a smaller segment size as it increases the relative importance of asymmetric labor market outcomes in firm profits. For sufficiently low correlation and sufficiently high uncertainty, profits may increase when $S$ falls.

3. *both types of human capital decrease in price correlation, $\rho$;*

4. *both types of human capital increase in price uncertainty, $\sigma_p^2$, as long as $\rho < 1$;*

5. *specific human capital decreases in skill differentiation, $S$.*

The first interesting result in proposition 2 is that the two types of human capital complement each other even tough they are independent of each other in both the productivity and the costs of investment. This complementarity is due to the portfolio effect: for perfectly correlated prices ($\rho = 1$), both types of human capital would be independent. As a consequence, a region and its workforce can therefore be both highly productive and flexible. This complementarity of human capital types induced by the portfolio effect also drives the subsequent results in proposition 2. Higher expected productivity ($p$) directly raises the value of specific human capital. Complementarity then raises the returns to both types of human capital even further, such that the overall effect on both types is positive. Price uncertainty and price correlation directly influence the degree of employment adjustment in response to asymmetric prices. As this turnover positively affects the expected wage, both types of human capital are higher in a more diversified and uncertain market.

The effect of skill differentiation $S$ on human capital levels is more complex as the matching and portfolio effects interact. Without the portfolio effect ($\rho = 1$), an increase in worker heterogeneity would exacerbate the degree of mismatch in the labor market. As general human capital can alleviate this effect, workers would be expected to increase their general human capital. At the same time, the benefit of diversification and turnover decreases with greater skill differentiation. Hence, the benefit of both types of human capital captured in the portfolio effect decreases, inducing lower investments into human capital. While the latter effect is unambiguous with respect to specific human capital, the overall effect on general human capital is indeterminate.

In sum, our results suggest two channels for higher human capital in a local labor market. First, a more diverse composition of firms raises the return to both types of human capital. Second, more agglomerated markets with higher division of labor among firms increase the return to specific human capital.

# 5 The Impact of Unemployment

So far we have assumed full employment. However, unemployment is a persistent phenomenon in today's industrialized economies. This section shows how the possibility of unemployment affects human capital formation. Specifically, we will allow for some price realizations to be sufficiently low to induce unemployment (see Thisse and Zenou, 2000; Jellal, Thisse, and Zenou, 2005). Hence, we now consider a market where (temporary) drops in productivity may be quite severe.

## 5.1 Model Adjustments

In order to simplify the analysis, we now assume a specific shock structure. A firm faces either of two price realizations, $\tilde{p}_i \in \{p-\varepsilon; p+\varepsilon\}$, $\varepsilon \in [0; p)$, which occur with equal probability (hence, $\sigma_p^2 = \varepsilon^2$). Consequently, there are only four possible combinations of the two firms' prices. Given our symmetry assumptions, we can define the probabilities for these realizations as follows:

$$\begin{array}{rcl} \text{Prob}[\tilde{p}_1 = \tilde{p}_2 = p+\varepsilon] = & \frac{1+\rho}{4} & = \text{Prob}[\tilde{p}_1 = \tilde{p}_2 = p-\varepsilon] \\ \text{Prob}[\tilde{p}_1 = p+\varepsilon, \tilde{p}_2 = p-\varepsilon] = & \frac{1-\rho}{4} & = \text{Prob}[\tilde{p}_1 = p-\varepsilon, \tilde{p}_2 = p+\varepsilon]\ . \end{array} \tag{15}$$

As before, $\rho$ captures the correlation between the two prices and thus determines the probability of symmetric versus asymmetric prices.

We introduce unemployment by assuming that for low price realizations, the skill distance between the two firms is sufficiently large to let distant workers' productivity fall below their reservation wage:

$$(p-\varepsilon)y - \frac{S}{x} < 0\ . \tag{16}$$

This restriction ensures that in case of low price realizations at both firms, wages will be depressed to the extent that some workers in the middle of the skill space $[0, S]$ would earn a negative net wage. At the same time, we continue to assume that both firms are active and that there is full employment in the other three price combinations (high prices for both or asymmetric prices). The former assumption still holds by condition (1), the latter requires the following restriction instead of (2):

$$p \geq \frac{3S}{2xy}\ . \tag{17}$$

Overall, these parameter restrictions constrain both the extent of price fluctuation and the level of expected prices from above and below:[6]

$$\varepsilon \in \left(p - \frac{S}{xy}, \frac{3S}{2xy}\right) \quad \text{and} \tag{18}$$

$$p \in \left[\frac{3S}{2xy}, \frac{5S}{2xy}\right) . \tag{19}$$

With the above adjustments in place, we now reconsider the labor market equilibria.

## 5.2 Labor Market Equilibria

By conditions (1) and (17) our analysis from the previous section holds as long as at least one firm's price realization is high. Hence, for three of our four possible cases, the results for equilibrium wages and employment are given by (7) and (8), respectively. Hence, it suffices to focus on the case of low price realizations for both firms. In this case, the two firms act as monopsonists in their segment of the labor market as workers' alternative is the (zero) reservation wage.

Consider firm 1's wage setting (firm 2's decision is symmetric). The marginal worker accepting a wage offer $w_1$ is indifferent between the net wage and the reservation wage: $w_1 - \frac{\hat{s}_1}{x} = 0$. Firm 1 thus faces the labor supply function

$$\hat{s}_1(w_1) = w_1 x . \tag{20}$$

Firm 1's profits are then $\pi_1(w_1) = \frac{\hat{s}(w_1)}{S}((p - \varepsilon)y - w_1)$ and maximizing them yields the following (*monopsony*) wage offer

$$w^M = \frac{y}{2}(p - \varepsilon) \tag{21}$$

and its marginal worker at

$$\hat{s}_1^M = \frac{y}{2}x(p - \varepsilon) . \tag{22}$$

Restriction (16) implies that some workers in the middle of the labor market segment do not accept the wage offer. It ensures that the two firms can act monopsonistically and that workers with $s \in [\hat{s}_1^M, S - \hat{s}_1^M]$ remain unemployed.

[6]These restrictions are necessary to exclude a case of intermediate productivity that leads to a rather peculiar equilibrium, as already noted by Salop (1979).

Finally, the expected wage of a worker (with respect to his skill position $s$) in the case of low price realizations for both firms equals

$$E[w^M] = \frac{1}{4S}(p - \varepsilon)^2 y^2 x \ . \tag{23}$$

Before reconsidering human capital investments, it is worth pointing out two aspects of the scenario considered here: First, the combination of low productivity and the existence of outside options (for example due to unemployment benefits) causes unemployment in this version of the model. Second, an outside option has ambiguous effects for the average worker as it provides a safety net at the cost of granting firms monopsony power. It is only for very low levels of productivity that the overall effect on expected wages is positive in comparison with a market where no outside option exists.

### 5.3 Human Capital Formation Reconsidered

To analyze human capital formation, we need to re-calculate the expected wage of a worker, taking now into consideration the four possible cases. This yields

$$\begin{aligned} E[w^T] \ &= \ \frac{1+\rho}{4}\left((p+\varepsilon)y - \frac{5S}{4x} + \frac{1}{4S}(p-\varepsilon)^2 y^2 x)\right) \\ &\quad + \frac{1-\rho}{2}\left(py - \frac{5S}{4x} + \frac{1}{9S}\varepsilon^2 y^2 x\right) \end{aligned} \tag{24}$$

which is the probability-weighted sum of the expected wage for symmetric price realizations plus the expected wage for asymmetric realizations. While this expression features some of the terms familiar from the expected wage equation in the full employment setting, unemployment now introduces an additional complementarity between the two types of human capital: even for perfectly aligned prices ($\rho = 1$), the two types reinforce their positive effect on wages by the reduction of unemployment.

As before, a worker solves the optimization problem

$$\max_{x,y} E[w^T] - c^h \tag{25}$$

in order to determine her human capital investments.

**Proposition 3** *In the model with unemployment, workers' optimal human capital investments, $x^*$ and $y^*$, have the following properties:*

1. *general and specific human capital are complements;*
2. *both types of human capital increase in the expected product price, $p$;*
3. *both types of human capital decrease in price correlation, $\rho$;*
4. *specific human capital decreases in skill differentiation, $S$.*

These results are very similar to those of the basic model without unemployment. The main difference is that the effect of uncertainty (here captured by the extent of the price fluctuation $\varepsilon$) is now ambiguous. While uncertainty in the base model simply increases the scope of employment adjustments, it now also raises the level of unemployment in case of negative shocks for both firms. Hence, uncertainty now also carries a cost as unemployment destroys the value of any human capital investment. This cost of uncertainty is an important feature of the extension as it stresses a negative effect of uncertainty not captured in the model with full employment.

Additionally, the results on human capital complementarity and the (related) effect of the product price are reinforced in proposition 3. Unemployment thus provides an additional mechanism to link the different types of human capital. In sum, the introduction of unemployment into the base model strengthens our key results.

# 6 Discussion

## 6.1 Empirical Implications

Our analysis highlights the role of labor market pooling in generating agglomeration economies via human capital formation. The pooling mechanism is based on the interaction between the local labor market and product market characteristics. This setting is captured by two exogenous variables: less than perfectly correlated demand fluctuations which give rise to the portfolio effect, and the degree of skill differentiation which determines the extent of mismatch in the labor market. In the following, we derive some empirical predictions that arise from our model.

H1: Aggregate productivity, wages and firm profits increase the less correlated the demand fluctuations.

H2: Wages increase the lower the degree of skill differentiation.

Pooling raises productivity, profits and wages via the portfolio effect (H1): it improves the allocation of labor such that specific human capital is employed more effectively. As specific skills become more similar and the average degree of mismatch is reduced, net wages will rise in the face of lower adjustment costs (H2); the impact of mismatch on firms' profits, however, is ambiguous.

H1 and H2 capture short-run, static effects of pooling. The following two hypotheses consider the longer run where pooling also affects human capital accumulation.

H3: General and specific human capital of a local workforce are higher the less correlated the demand fluctuations.

H4: General and specific human capital are complements; complementarity rises the less correlated the demand fluctuations.

As wages increase due to the pooling advantage this raises the return to both types of human capital, so incentives to invest in specific and in general skills improve (H3). As a worker becomes more productive, it pays to invest in general skills which help to adapt her specific skills to a new technology. Conversely, if flexibility is high, it pays to become more productive for flexibility to be rewarded (H4).

The results for H1 and H2 imply that in a pooled local labor market, we would expect to see higher wages and higher productivity in those industries that experience asymmetric demand shocks more often and at the same time have similar skill requirements. To test for the mismatch effect (H2), the degree of skill differentiation in a labor market may be proxied by the degree of occupational diversification. Wheaton and Lewis (2002) analyze the role of labor markets in generating increasing returns in agglomerations. Using two different measures of localization, the specialization of employment in terms of occupation and industry, they test for the idea that externalities based on human capital should be linked to "own" industry and occupational employment. Occupational specialization for a sample of manufacturing employment in US SMAs is found to yield a wage premium of 23 percent. This finding supports a thick market interpretation corresponding to H2. Duranton and Puga (2005) arrive at a similar result: They report increased employment in managerial and executive functions relative to production employment in urban areas as evidence of functional urban specialization.

Testing for the portfolio effect (H1) requires capturing demand fluctuations, and in particular their correlation. If we assume that demand shocks are mainly industry-specific, a measure of industrial diversity can serve as a proxy for the correlation of demand shocks. Wheaton and Lewis (2002) consider industrial diversity and find that wage increases due to industry specialization are even more pronounced than those due to occupational specialization. At first, this finding seems in contrast with the role we attach to (industry) diversification. However, our approach requires that firms be able to share their labor input, so they should not be too far apart in terms of skill requirements. Perfectly uncorrelated demand fluctuations will not produce a portfolio effect if skill requirements are so different that labor cannot easily move between firms of different industries. Consequently, a simple specialization index as is typically used when testing for the existence of localization versus urbanization economies might not suffice in our context. Additionally, the assumption of industry-specific shocks may not hold if local firms, even though belonging to different industries, are vertically linked and thus subject to the same demand shocks. This would then result in nearly perfect correlation of shocks. Our approach thus calls for a more detailed look into the nature of shocks, possibly a decomposition of demand fluctuations into industry- and firm-specific components.

Overman and Puga (2009) provide an example for a more detailed analysis of fluctuations. The authors consider absolute deviations in plant-level employment changes from the industry employment change. Averaging across time and plants in an industry yields a measure of "idiosyncratic volatility" for firms within a given sector. Adapting the approach to cover local idiosyncratic volatility, ideally at the occupational level, should capture the nature of fluctuations which form the basis for our results.

Testing for hypotheses H3 and H4 requires more detailed information about workers' human capital. So far, we are not aware of any study linking human capital investment decisions to labor market pooling.

Labor mobility within a local labor market and across related sectors and occupations might provide a further road along which our model can be indirectly tested, since such mobility indicates a pooling mechanism at work. For example, Bleakley and Lin (2006) analyze whether thick markets increase the returns to matching and therefore provide additional incentives for workers to invest in occupation and/or industry-specific skills. Although the focus of their analysis is not on human capital formation, their finding of an increase in sectoral transitions with higher education is consistent with the mechanisms underlying hypothesis H3.

Finally, our results may be used for further economic conjectures. In our model, a lower correlation in demand fluctuations implies higher human capital levels. Provided human capital serves as a major source of economic growth (Lucas, 1988), an increase in human capital can be expected to have dynamic, long-term effects: We would expect to see higher growth in local wages, productivity and profits in markets where demand fluctuations are less correlated. Lower correlations in demand fluctuations thus permanently affect labor market outcomes and increase local differences over time. This is consistent with Glaeser and Mare (2001) who explain the urban wage premium by both a a level as well as a growth effect. Closer analyses linking the wage growth effect to employment or product market fluctuations should yield more information about the nature of labor market pooling effects.

# 7 Conclusion

Agglomerations with a pooled labor market enjoy two advantages: protection against (asymmetric) shocks and lower mismatch. These advantages raise productivity and are higher if there is a diverse set of firms requiring a similar set of skills. A more efficient allocation of labor and reduced mismatch raise firms' profits as well as wages. With higher wages workers find it worthwhile to accumulate more human capital. Our analysis also predicts that directly productive human capital and adaptive skills are complements. In the long-run, therefore, the workforce in a pooled labor market will not only be more productive but also more flexible in adapting to fluctuations and technological change. Moreover, human capital will augment the pooling advantage: the more skilled the local work force, the more pronounced are the benefits of employment adjustments.

We show that pooling provides a tool that protects regions against imperfectly correlated shocks to labor demand. As the paper by Magnani (2001) reveals, workers are able to anticipate the risk of demand shocks to their own industry and to respond by changing to jobs in other industries. Such inter-sectoral mobility is then shown to be rising in education. Haskel, Kersley, and Martin (1997) also show that firms, if given the ability to deploy their workforce as they wish, respond to changes in demand by adjusting employment, rather than by adjusting hours or prices or by labor hoarding. We therefore suggest policy measures which rely upon and strengthen such individual responses by removing obstacles to occupational and sectoral labor mobility and by improving education.

# A Appendix

## A.1 Proof of proposition 1

1. Inserting equilibrium wages and employment from (7) and (8) into (9) and (10) yields:

   - For the expected wage:

$$\begin{aligned} E[w^T] &= E\left[\frac{\hat{s}^B}{S}(w_1^B - \frac{\hat{s}^B}{2x}) + \frac{S-\hat{s}^B}{S}(w_2^B - \frac{S-\hat{s}^B}{2x})\right] && (26)\\ &= E[\tilde{p}_1]y - \frac{5S}{4x} + \frac{xy^2}{18S}\left(E[\tilde{p}_1^2] - E[\tilde{p}_1\tilde{p}_2]\right) && (27)\\ &= py - \frac{5S}{4x} + \frac{xy^2}{18S}\sigma_p^2(1-\rho) && (28) \end{aligned}$$

     where use is made of $E[\tilde{p}_1] = E[\tilde{p}_2] = p$, $\sigma_p^2 = Var[\tilde{p}_i] = E[\tilde{p}_i^2] - (E[\tilde{p}_i])^2$, $Cov[\tilde{p}_1, \tilde{p}_2] = E[\tilde{p}_1\tilde{p}_2] - (E[\tilde{p}_i])^2$ and $\rho = \frac{Cov[\tilde{p}_1,\tilde{p}_2]}{Var[\tilde{p}_i]}$.

   - For expected profits:

$$\begin{aligned} E[\pi_1] &= E\left[\frac{\hat{s}^B}{S}(\tilde{p}_1 y - w_1^B)\right] && (29)\\ &= \frac{S}{2x} + \frac{xy^2}{9S}\left(E[\tilde{p}_1^2] - E[\tilde{p}_1\tilde{p}_2]\right) && (30)\\ &= \frac{S}{2x} + \frac{xy^2}{9S}\sigma_p^2(1-\rho) && (31) \end{aligned}$$

   - For expected total surplus, by adding up:

$$\begin{aligned} E[TS] &= E[w^T] + 2E[\pi_i] && (32)\\ &= py - \frac{S}{4x} + \frac{5xy^2}{18S}\sigma_p^2(1-\rho) && (33) \end{aligned}$$

2. Inspection of the above results yields $\frac{dE[w^T]}{d\rho} < 0$, $\frac{dE[\pi_i]}{d\rho} < 0$ and $\frac{dE[TS]}{d\rho} < 0$, as well as $\frac{dE[w^T]}{d\sigma_p^2} \geq 0$, $\frac{dE[\pi_i]}{d\sigma_p^2} \geq 0$ and $\frac{dE[TS]}{d\sigma_p^2} \geq 0$.

3. Inspection of the above results yields $\frac{dE[w^T]}{dS} < 0$ and $\frac{dE[TS]}{dS} < 0$.

## A.2 Proof of proposition 2

The worker maximizes:

$$\phi(x,y) \equiv E[w^T] - c^h = py - \frac{5S}{4x} + \frac{xy^2}{18S}\sigma_p^2(1-\rho) - \frac{\alpha}{2}(x^2+y^2) \qquad (34)$$

Let $\phi_i$ denote the partial derivative of $\phi$ with respect to $i$, and $\phi_{ij}$ its cross-partial derivative with respect to $i$ and $j$. Then, the first order conditions for the optimum human capital investments are

$$\phi_y \equiv p + \frac{xy}{9S}\sigma_p^2(1-\rho) - \alpha y = 0 \tag{35}$$

and

$$\phi_x \equiv \frac{5S}{4x^2} + \frac{y^2}{18S}\sigma_p^2(1-\rho) - \alpha x = 0 \tag{36}$$

Additionally, the following second order conditions have to be satisfied:

$$\phi_{yy} \equiv \frac{x}{9S}\sigma_p^2(1-\rho) - \alpha < 0 \ , \tag{37}$$

$$\phi_{xx} \equiv -\frac{5S}{2x^3} - \alpha < 0 \tag{38}$$

and

$$\phi_{yy}\phi_{xx} - (\phi_{yx})^2 > 0 \tag{39}$$

where $\phi_{yx} \equiv \frac{y}{9S}\sigma_p^2(1-\rho) \geq 0$.

1. Complementarity of $x$ and $y$ follows from $\phi_{yx} > 0$ for $\rho < 1$.

2. Total differentiation of (35) and (36) with respect to $x$, $y$ and $p$ yields

$$\frac{dy}{dp} = \frac{-\phi_{yp}\phi_{xx} + \phi_{xp}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} > 0 \tag{40}$$

   and

$$\frac{dx}{dp} = \frac{-\phi_{xp}\phi_{yy} + \phi_{yp}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \geq 0 \tag{41}$$

   where $\phi_{yp} = 1$ and $\phi_{xp} = 0$. The signs follow immediately.

3. Total differentiation of (35) and (36) with respect to $x$, $y$ and $\rho$ yields

$$\frac{dy}{d\rho} = \frac{-\phi_{y\rho}\phi_{xx} + \phi_{x\rho}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} < 0 \tag{42}$$

   and

$$\frac{dx}{d\rho} = \frac{-\phi_{x\rho}\phi_{yy} + \phi_{y\rho}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} < 0 \tag{43}$$

   where $\phi_{y\rho} = -\frac{xy}{9S}\sigma_p^2 < 0$ and $\phi_{x\rho} = -\frac{y^2}{18S}\sigma_p^2 < 0$. The signs follow immediately.

4. Total differentiation of (35) and (36) with respect to $x$, $y$ and $\sigma_p$ yields

$$\frac{dy}{d\sigma_p} = \frac{-\phi_{y\sigma_p}\phi_{xx} + \phi_{x\sigma_p}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \geq 0 \tag{44}$$

and

$$\frac{dx}{d\sigma_p} = \frac{-\phi_{x\sigma_p}\phi_{yy} + \phi_{y\sigma_p}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \geq 0 \tag{45}$$

where $\phi_{y\sigma_p} = \frac{2xy}{9S}\sigma_p(1-\rho) \geq 0$ and $\phi_{x\sigma_p} = \frac{y}{9S}\sigma_p(1-\rho) \geq 0$. The signs follow immediately and inequalities are strict for $\rho < 1$.

5. Total differentiation of (35) and (36) with respect to $x$, $y$ and $S$ yields

$$\frac{dy}{dS} = \frac{-\phi_{yS}\phi_{xx} + \phi_{xS}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \leq 0 \tag{46}$$

where $\phi_{yS} = -\frac{xy}{9S^2}\sigma_p^2(1-\rho) \leq 0$ and $\phi_{xS} = \frac{5}{4x^2} - \frac{y^2}{18S^2}\sigma_p^2(1-\rho)$. Now the sign of $\frac{dy}{dS}$ is not obvious. However, rearranging the numerator yields

$$\begin{aligned} -\phi_{yS}\phi_{xx} + \phi_{xS}\phi_{yx} = -\sigma_p^2(1-\rho)\Bigg(\alpha\frac{xy}{9S^2} + \frac{5y}{36Sx^2} \\ + \frac{y^3}{162S^3}\sigma_p^2(1-\rho)\Bigg) \quad \leq 0 \end{aligned} \tag{47}$$

such that $\frac{dy}{dS} \leq 0$ is confirmed.

## A.3 Proof of proposition 3

The proof is structurally similar to the proof of proposition 2. Now, the worker maximizes:

$$\begin{aligned} \psi(x,y) \equiv E[w^T] - c^h &= \frac{1+\rho}{4}\left((p+\varepsilon)y - \frac{5S}{4x} + \frac{1}{4S}(p-\varepsilon)^2y^2x\right) \\ &+ \frac{1-\rho}{2}\left(py - \frac{5S}{4x} + \frac{1}{9S}\varepsilon^2y^2x\right) - \frac{\alpha}{2}(x^2+y^2) \end{aligned} \tag{48}$$

Let $\psi_i$ denote the partial derivative of $\psi$ with respect to $i$, and $\psi_{ij}$ its cross-partial derivative with respect to $i$ and $j$. Then, the first order conditions for the optimum human capital investments are

$$\psi_y \equiv \frac{1+\rho}{4}\left(p+\varepsilon+\frac{1}{2S}(p-\varepsilon)^2yx\right) + \frac{1-\rho}{2}\left(p+\frac{2}{9S}\varepsilon^2yx\right) - \alpha y = 0 \tag{49}$$

and

$$\psi_x \equiv \frac{1+\rho}{4}\left(\frac{5S}{4x^2}+\frac{1}{4S}(p-\varepsilon)^2y^2\right)+\frac{1-\rho}{2}\left(\frac{5S}{4x^2}+\frac{1}{9S}\varepsilon^2y^2\right)-\alpha x=0 \quad (50)$$

Additionally, the following second order conditions have to be satisfied:

$$\psi_{yy} \equiv \frac{1+\rho}{8S}(p-\varepsilon)^2x+\frac{1-\rho}{9S}\varepsilon^2x-\alpha<0 \quad (51)$$

$$\psi_{xx} \equiv -\frac{3-\rho}{4}\frac{5S}{2x^3}-\alpha<0 \quad (52)$$

$$\psi_{yy}\psi_{xx}-(\psi_{yx})^2>0 \quad (53)$$

where $\psi_{yx} \equiv \frac{1+\rho}{8S}(p-\varepsilon)^2y+\frac{1-\rho}{9S}\varepsilon^2y>0$.

Total differentiation then yields the following system of equations:

$$\begin{pmatrix} \psi_{yy} & \psi_{yx} \\ \psi_{yx} & \psi_{xx} \end{pmatrix}\begin{pmatrix} dy \\ dx \end{pmatrix}+\begin{pmatrix} \psi_{yp} & \psi_{y\rho} & \psi_{yS} \\ \psi_{xp} & \psi_{x\rho} & \psi_{xS} \end{pmatrix}\begin{pmatrix} dp \\ d\rho \\ dS \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (54)$$

where

$$\psi_{yp} \equiv \frac{1+\rho}{4}\left(1+\frac{1}{S}(p-\varepsilon)yx\right)+\frac{1-\rho}{2}>0 \quad (55)$$

$$\psi_{xp} \equiv \frac{1+\rho}{8S}(p-\varepsilon)y^2>0 \quad (56)$$

$$\psi_{y\rho} \equiv \frac{1}{4}\left(p+\varepsilon+\frac{1}{2S}(p-\varepsilon)^2yx\right)-\frac{1}{2}\left(p+\frac{2}{9S}\varepsilon^2yx\right)<0 \quad (57)$$

$$\psi_{x\rho} \equiv \frac{1}{4}\left(\frac{5S}{4x^2}+\frac{1}{4S}(p-\varepsilon)^2y^2\right)-\frac{1}{2}\left(\frac{5S}{4x^2}+\frac{1}{9S}\varepsilon^2y^2\right)<0 \quad (58)$$

$$\psi_{yS} \equiv -\frac{1+\rho}{8S^2}(p-\varepsilon)^2yx-\frac{1-\rho}{9S^2}\varepsilon^2yx<0 \quad (59)$$

$$\psi_{xS} \equiv \frac{1+\rho}{4}\left(\frac{5}{4x^2}-\frac{1}{4S^2}(p-\varepsilon)^2y^2\right)+\frac{1-\rho}{2}\left(\frac{5}{4x^2}-\frac{1}{9S^2}\varepsilon^2y^2\right) \quad (60)$$

With the exception of expressions (57) and (58), the above signs can be inferred directly. To confirm the other two signs, rearrange (57) to

$$\psi_{y\rho} \equiv \frac{p-\varepsilon}{8S}\left[(p-\varepsilon)yx-S\right]-\frac{1}{8}(p-\varepsilon)-\frac{1}{9S}\varepsilon^2yx<0 \quad (61)$$

By restriction (16), the term in brackets is negative, confirming the sign. Similarly, rearranging (58) yields

$$\psi_{x\rho} \equiv \frac{(p-\varepsilon)yx+S}{16Sx^2}\left[(p-\varepsilon)yx-S\right]-\frac{S}{4x^2}-\frac{1}{18S}\varepsilon^2y^2<0 \quad (62)$$

With these results, we can proof:

1. Complementarity of $x$ and $y$ follows from $\psi_{yx} > 0$.

2.

$$\frac{dy}{dp} = \frac{-\psi_{yp}\psi_{xx} + \psi_{xp}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} > 0 \tag{63}$$

and

$$\frac{dx}{dp} = \frac{-\psi_{xp}\psi_{yy} + \psi_{yp}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} > 0 \tag{64}$$

3.

$$\frac{dy}{d\rho} = \frac{-\psi_{y\rho}\psi_{xx} + \psi_{x\rho}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} < 0 \tag{65}$$

and

$$\frac{dx}{d\rho} = \frac{-\psi_{x\rho}\psi_{yy} + \psi_{y\rho}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} < 0 \tag{66}$$

4.

$$\frac{dy}{dS} = \frac{-\psi_{yS}\psi_{xx} + \psi_{xS}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} < 0 \tag{67}$$

Here, the sign of $\frac{dy}{dS}$ is not obvious. However, rearranging the numerator yields

$$\begin{aligned} -\psi_{yS}\psi_{xx} + \psi_{xS}\psi_{yx} \quad = \quad & \psi_{yS}\alpha \\ & -\psi_{yx}\left(\frac{1+\rho}{16S^2}(p-\varepsilon)^2y^2 + \frac{1-\rho}{18S^2}\varepsilon^2y^2\right) \\ & -\frac{3-\rho}{4}\frac{5}{4x^2}\left(\frac{1+\rho}{8S}(p-\varepsilon)^2y + \frac{1-\rho}{9S}\varepsilon^2y\right) \end{aligned} \tag{68}$$

such that $\frac{dy}{dS} < 0$ is confirmed.

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