Bolzano's definition of analytic propositions

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Abstract

We begin by drawing attention to some drawbacks of what we shall call the Frege-Quine definition of analytic truth. With this we contrast the definition of analytic propositions given by Bolzano in his *Wissenschaftslehre*. If Bolzano's definition is viewed, as Bolzano himself almost certainly did not view it, as attempting to capture the notion of analyticity as truth-in-virtue-of-meaning which occupied centre stage during the first half of the last century and which, Quine's influential assault on it notwithstanding, continues to attract philosophical attention, it runs into some very serious problems. We argue that Bolzano's central idea can, nevertheless, be used as the basis of a new definition which avoids these problems and possesses definite advantages over the Frege-Quine approach. Our title notwithstanding, we make no claim to contribute to the exegesis of Bolzano's thought and works, which we must leave to those more expert in these matters than we are. Naturally, we have done our best not to misrepresent Bolzano's views, and believe we have avoided doing so. But it bears emphasis that it is no part of our intention to suggest that the modifications to his definition which we propose would have had any appeal for him, or that he had, or would have had, any sympathy with the project which motivates them.

1 Frege's definition

A noteworthy feature of Frege's explanation of the distinction between analytic and synthetic judgements is that he views the distinction as an *epistemological* one, in parallel with the obviously epistemological distinction between a priori and a posteriori judgements:

> Now these distinctions between a priori and a posteriori, synthetic and analytic, concern ... not the *content* of the judgement

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> but the *justification* for making the judgement.... When a proposition is called a posteriori or analytic, in my sense, this is not a judgement about the conditions, psychological, physiological and physical, which have made it possible to form the content of the proposition in our consciousness; nor is it a judgement about the way in which some other man has come, perhaps erroneously, to believe it true; rather, it is a judgement about the ultimate ground upon which rests the justification for holding it to be true.([Frege(1884)], §3)

Clearly Frege's main concern here is to distance himself from any sort of psychological account of the distinctions he is about to explain, and from any suggestion that they relate to different ways in which judgements are to be causally explained. But it is worth emphasizing that in holding that the distinctions concern justification, he is also distancing himself from, or at least avoiding commitment to, any view on which the distinctions concern the grounds of truth – what *makes* the judgement true – in the way that is suggested by, for example, subsequent characterizations of analyticity in terms of 'truth-in-virtue-of-meaning'. We shall return to this point, and its significance, much later. Frege continues:

> This means that the question is removed from the sphere of psychology, and assigned, if the truth concerned is a mathematical one, to the sphere of mathematics. It now becomes a problem of finding the proof of the proposition, and of following it back to the primitive truths. If in the course of doing so, we come only only general logical laws and definitions, then the truth is an analytic one, bearing in mind that we must take account also of any propositions on which the admissibility of any definition depends.

Thus according to Frege, a judgement is analytic iff the proposition judged true can be proved from using only general logical laws, together with definitions.

There is an obvious similarity between Frege's definition and Quine's subsequent characterization of what he terms the 'second class' of statements generally held to be analytic. The 'first class' of such statements are those, such as 'No unmarried man is married', which, he says, 'may be called *logically true*, where a logical truth is a statement which is true and remains true under all reinterpretations of its components other than the logical particles'. But, he continues

> ... there is also a second class of analytic statements, typified by:
>
> (2) No bachelor is married
>
> The characteristic of such a statement is that it can be turned into a logical truth by putting synonyms for synonyms.([Quine(1953)]pp.22-3)

We might, then, define:

> A statement S is *broadly analytic* iff (i) S is logically true, or (ii) for some logically true statement S^*, S is transformable into S^* by substituting synonymous expressions

Statements which qualify as broadly analytic by clause (i) may be said to be *narrowly analytic*. Although, for reasons too well-known to require restatement here, Quine himself does not regard this as an acceptable definition, no harm need result from labelling it as the *Frege-Quine definition* (of broad analyticity).

The Frege-Quine definition has two notable drawbacks. The first concerns logical truths. Such truths compose the base class in terms of which the remainder of the class of broadly analytic truths is defined. But while it is clear that statements in the remainder are supposed to count as analytic because reducible to logical truths, the status of logical truths themselves as analytic is left entirely without explanation. The point is not that the choice of logical truths to compose the base class is arbitrary – just about anyone who has any use for the notion of analyticity would classify them as analytic. And everyone would agree that it would be absurd to take instead, say, the laws of thermodynamics, or the truths recorded in Mrs. Beaton's Manual of Cookery and Household Management, as the base class. The point is just that the definition gives no hint why logical truths should themselves be regarded as analytic.[1]

A second drawback concerns the extensional correctness of the definition. If we think of it, not as a straightforward stipulation, but as intended to codify an already accepted notion, then it seems clearly to fail. For there appears to be a significant class of statements which those who think they understand the notion would wish to see classified as 'true-purely-in-virtue-of-meaning', so as analytic in the intended spirit of the notion, whose members are neither logical truths nor reducible to logical truths by substitution of synonyms for synonyms. Well known candidates are such statements as 'Anything red is coloured', 'If one event precedes another, and the second precedes a third, then the first precedes the third' – the reader will surely be able to think of many others. Perhaps some candidates are more controversial than others – witness 'Nothing can be red and green all over' – but that there is a substantial class of statements which fall under the intuitive extension of 'analytic', yet elude classification as analytic by the Frege-Quine definition because they essentially involve terms which do not admit of the definitional paraphrases which would permit their reduction to logical truths, seems beyond serious question.[2]

[1] We are not suggesting that this drawback is one which Quine would, or should, have worried about. It is a drawback only for someone who is trying, as Quine was not, to give an acceptable definition which does not merely circumscribe the extension of the term 'analytic' but captures the essence of the concept. Quine thought there was no essence to capture, and was merely trying to characterize, for critical purposes, the class of statements commonly taken to be analytic. Whether he should have been worried by the second shortcoming to which we draw attention is another question entirely.

[2] The second of these drawbacks, and something close to the first, are pointed out by Paul Boghossian in a couple of places (see [Boghossian(1997)], pp.338-9, [Boghossian(1996)], p.368).

2 Bolzano's definition

In his *Wissenschaftslehre*[Bolzano(1837)], volume II, section 148, Bernard Bolzano gives a definition of analytic propositions which holds out some promise of addressing the last point. His definition of what he calls narrowly or logically analytic propositions is of some historical interest, because it anticipates by about 100 years the definition of logical truth given by Quine mentioned above.[3] Bolzano takes being true and being false, being analytic and being synthetic to be properties, in the primary sense, of what he calls *propositions in themselves* [Sätze an sich], which he distinguishes both from verbal and mental propositions. He takes propositions to be structured entities composed of ideas or concepts. In this chapter, he considers the effects of varying some of the ideas that make up a proposition, whilst keeping the other ideas involved in it fixed. What he means by varying an idea here is replacing it uniformly throughout a proposition by another idea. He notices that some propositions are such that if we keep only the logical ideas or concepts occurring in them fixed, we may vary any of the remaining ideas without changing the truth-value of the proposition.[4] It is these propositions which he defines to be logically analytic, or analytic in the narrower sense. Schematic examples he actually gives are: 'A is A', 'An A which is a B is an A', 'An A which is a B is a B', and 'Every object is either B or non-B'.

If we say, in accordance with a well-established terminology, that an expression occurs *essentially* in a statement if and only if uniformly replacing it throughout that statement may result in a statement that differs in truth-value from the original one, and give a parallel explanation of an idea's occurring essentially in a proposition, then we can see that Bolzano's definition of logical analyticity is virtually the same as Quine's definition of logical truth: for Bolzano, a proposition is logically analytically true if and only if it is true and only logical ideas or concepts occur in it essentially; while for Quine, a statement is logically true iff it is true and contains only logical expressions essentially. The interest of Bolzano's definition is not, however, confined to its being a forerunner of Quine's. Like Quine, Bolzano makes a distinction between broader and narrower analytic truths. But whereas for Quine the broader notion is to be explained, if it can be explained at all, on the basis of the narrower one, Bolzano

[3] As Quine acknowledges – see [Quine(1966)], fn.2, p.103. In saying the Bolzano's definition anticipates Quine's, we are claiming only that the central idea of Quine's definition is already present in Bolzano's, not that the two are equivalent. They are not. Most importantly, Quine's has the unfortunate consequence that such sentences as $\exists x \exists y x \neq y$ qualify as logical truths, whereas thay are not logically analytic by Bolzano's.

[4] More precisely, the result of varying these ideas will be a proposition having the same truth-value, if it has *denotation* at all. By saying that an idea is *denotative*, Bolzano means that it 'has an object falling under it' (see [Berg(1973)], p.82. Bolzano's word is *gegenständlich*). In the case of propositions, the result of substituting of one idea for another may be a proposition which fails to have the same truth-value because it lacks denotation altogether. Bolzano gives the example 'The man Caius is mortal', telling us that while every replacement for the idea of Caius must yield a true proposition if it yields a proposition with denotation at all, it may be that an idea is substituted – such as the idea of a rose or a triangle – which results in a proposition lacking denotation altogether. See [Berg(1973)], pp.188-9)

reverses the direction of explanation – for him, it is the broader notion which is basic, and logically or narrowly analytic truth is merely a special case of it.[5] To understand how this comes about, we need to look more closely at his explanation.

Bolzano's general concern (see especially §147) is with the effects of varying one or other of the ideas in a proposition on its truth-value. Let p be any proposition, and let $i_1, \ldots, i_n$ be the ideas of which it is composed. Take one of these ideas, i_k. Then in general, some of the results of varying i_k by putting another idea in its place will be true propositions, and some will be false. Roughly speaking, Bolzano defines the degree of validity[6]of p with respect to i_k to be the ratio of true propositions that result from varying i_k to the total number of propositions that are obtainable by varying i_k. In the limiting case when every proposition that so results is true – so that the validity of p with respect to i_k $= 1$ – Bolzano says that the proposition is universally valid with respect to i_k (or universally invalid, if every resulting proposition is false). We could express this by saying that the idea i_k occurs inessentially in p. Bolzano then, in effect, defines a proposition to be analytic, in his broad sense, if it contains at least one idea inessentially.

This proposal contrasts with Frege's, and with the Frege-Quine definition, in several respects.

First, whereas Frege and Frege-Quine seek to define analytic *truth*, Bolzano's definiens is analyticity – i.e. analytic *truth-or-falsehood.* For him, analytically *true* and analytically *false* propositions are simply propositions which are both analytic and true, or analytic and false, respectively. This, as we shall see in due course, is a source of some difficulty; but for now, we simply note the point.[7]

Second, the definitions diverge over the *bearers* of analyticity. For Frege, analyticity is a property of *judgements*, and for Frege-Quine, of *statements*, while for Bolzano, it is a property of *propositions-in-themselves.* This difference may be of some significance for the detailed exegesis of Bolzano's own view, but that is not our business here. It is straightforward enough to transpose Bolzano's definition so as to apply to statements, and while we shall respect his usage when reporting or commenting on his actual views, we shall switch, without special notice, to taking statements as the bearers of analyticity, when

[5]It is, of course, no accident that Quine privileges the narrow notion. For he believes that while the broader notion cannot be satisfactorily explained, because an explanation requires appeal to the problematic notion of synonymy, or some equally problematic alternative, the narrow notion can be explained, drawing only upon the unproblematic notions of truth and uniform substitution. Whether he is right so to believe is not our concern here. For an early statement of the case against, see [Strawson(1957)]

[6]Bolzano's term is *Gültigkeit*, which Rolf George [George(1972)] translates as 'satisfiability'; Jan Berg's translation ([Berg(1973)], p.187) has 'validity'; 'degree of validity', which seems to us more accurate, was suggested by Wolfgang Künne. Our formulation omits some restrictions Bolzano introduces, but which do not affect our discussion.

[7]Curiously, Bolzano's examples of logically analytic propositions are all examples of true propositions; but he does give as examples of analytic propositions some which he clearly takes to be false. See e.g. [Berg(1973)], p.192, where he cites 'A morally evil man nevertheless enjoys eternal happiness' as an analytic proposition which remains false under any substitution for the idea of *man.*

we come to consider modifications of his definition.

There is a third, far more important point of contrast, at least between Bolzano's definition and Frege's: while, as we have noted, Frege takes being analytic, like being a priori or a posteriori, to be fundamentally an *epistemological* notion, there is no whiff of epistemology in Bolzano's account of it. His concern is simply with the effects of varying certain of the ideas composing a proposition upon its truth-value. Once again, this is a point to which we shall return in the sequel.

3 Potential advantages of Bolzano's definition

Well and good. The question arising now is what, if any, may be the advantages of Bolzano's definition over that of Frege-Quine.[8] One apparent such advantage may speedily be seen to be illusory. It is clear that logically analytic propositions are, for Bolzano, a special case of analytic propositions in his broader sense. For logically analytic truths will be true propositions in which *all* but logical ideas occur inessentially. It may now appear that the primary advantage of Bolzano's definition is that it captures a broader notion of analytic truth, corresponding to Quine's second class, whilst deploying only the relatively modest resources – viz. the notions of truth and uniform substitution – which Quine thinks sufficient to characterize the narrower class of logical truths. It may thus appear that Bolzano provides a way of bypassing the difficulties Quine raises about the explanation of the broader notion – that he succeeds in defining it without reliance upon the notion of synonymy or any of the other notions Quine regards as equally suspect. It is important to see that this apparent advantage is merely apparent.

The reason why this is so becomes clear as soon as we ask whether the proposition expressed by, for example, 'Vixens are female' qualifies as (broadly) analytic in Bolzano's sense. At first sight, it fails to do so, since – to put the difficulty in Bolzano's terminology – it appears to contain no idea that can be varied at will without variation in truth-value. If the written proposition is assumed accurately to reflect the composition out of ideas of the proposition in itself that the sentence 'Vixens are female' expresses, then that proposition must be reckoned synthetic in Bolzano's sense; for it will contain no idea inessentially. And so it will be with indefinitely many further examples of propositions which would be classified as analytic, at least by anyone who has any use for the (broader) notion at all.

Of course, Bolzano would regard the proposition expressed by 'Vixens are female' as analytic, even though it appears at first to fail to qualify as such by

[8]Our focus here is entirely on the potential advantages of Bolzano's definition, when it is viewed as an alternative to the more familiar Frege-Quine definition. As we say in our abstract, we make no claim concerning what may have been Bolzano's own purposes in defining analyticity, what role his definition may have been intended to play in his overall philosophy, or what relation he may have taken it to bear to Kant's definition(s). For interesting discussions of these and other questions about Bolzano's actual views, see [Künne(2008a)], reprinted in [Künne(2008b)], which contains several other relevant papers, and [Lapointe(2011)], chs. 4,5

his definition. In a note on his definition, he says:

> In order to determine whether a proposition which is given a certain linguistic expression is analytic or synthetic, more is required than a cursory inspection of its words. A proposition may be analytic, perhaps logically analytic, or even identical, though its literal phrasing does not make this immediately apparent. ... Thus it may not be immediately obvious that the proposition 'Every effect has a cause' is in fact identical, or at any rate analytic; for by 'effect' we always mean something which is brought about by something else, and the phrase 'to have a cause' means as much as 'to be brought about by something else'; thus the above proposition merely means 'Whatever is brought about by something else is brought about by something else'[9]

If we say, as Bolzano would presumably have been happy to say, that a spoken proposition is analytic if the proposition-in-itself expressed by it is so, then the point he is making here could be put by saying that the proposition-in-itself (see page 4) that is expressed by a given spoken proposition is that proposition-in-itself that results from the given spoken proposition by fully expanding it accordance with definitions of its ingredient words. But this means, of course, that to justify the acknowledgement of the proposition expressed by 'Vixens are female' as analytic, Bolzano has after all to rely upon claims about synonymy, and so has not after all provided a way of explaining broad analyticity that both gives it the intuitively correct extension and bypasses Quine's objections to the notion.[10]

A genuine advantage of Bolzano's definition, assuming it to be acceptable, lies elsewhere – in its greater generality. Specifically, it promises to accommodate as analytic examples of the 'third kind' which fail to be so classified by the Frege-Quine definition. Putative examples, again, are:

> If Mozart's stockings are yellow, then they are coloured
> If Vivaldi's birthday precedes Handel's, and Handel's precedes Bach's, then Vivaldi's precedes Bach's

For in these propositions, the ideas of Mozart's stockings, and Vivaldi's, Handel's and Bach's birthdays all occur inessentially. And with a small refinement of Bolzano's definition, we can ensure that the more general propositions such as:

> Anything yellow is coloured
> If one event precedes a second, and the second precedes a third, the first precedes the third

[9] [Bolzano(1837)], §148. By an 'identical' proposition Bolzano means an instance of the Law of Identity 'A is A'. A quite different explanation how Bolzano can count the propositions expressed by 'No bachelor is married' and 'Vixens are female' as analytic is suggested by Lapointe (see [Lapointe(2011)], pp.64-6). We see no good reason not to adopt the simpler one suggested in the this passage.

[10] The point we have been emphasizing is made very clearly by Wolfgang Künne (see [Künne(2008a)], pp.298-300)

also qualify. Of course, they do not qualify on his definition as it is, because neither contains any idea inessentially. But it would not be unreasonable to claim that a generalization is analytic iff all its instances are, and to modify the definition accordingly. Under the modified definition, these and similar general statements *would* qualify. Thus Bolzano's definition together with our modest emendation appears to have the very desirable consequence that just the kind of true statement which we previously claimed ought to count as analytic – but fails to do so on the Frege-Quine definition – gets correctly classified. So although Bolzano's definition does not dispense with reliance on the notion of synonymy, it does allow us to recognize as analytic many statements which are not reducible to logical truths by synonymous substitution.

Further, there is at least some progress with the other drawback of the Frege-Quine definition – the unexplained status of logical truths as analytic. For since, on Bolzano's definition, logically analytic propositions are just a special case of analytic propositions in general, there is no *special* problem about explaining why they are analytic. But only partial progress – for obviously, assuming the definition to be otherwise in good standing, there would still be a good question why it should be thought to capture whatever intuitive idea informs our application of the notion of analyticity. But before pursuing that question, we should face up to the fact that the definition is open to a seemingly fatal line of objection.[11]

4 Over-extension (1)

We have taken one of the advantages of Bolzano's definition to consist in its capturing a broader range of analytic truths than the Frege-Quine definition. But if the objection we are going to consider is sound, the definition is *too* broad – because it has the consequence that many propositions are to be reckoned analytically true that are not so, but are very plainly at best statements of contingent empirical fact.

Consider any contingently true generalization – this can be either some true statement of natural law, or equally some true accidental generalization. For simplicity, and without loss of generality, we may suppose it to have the form: $\forall x(Fx \rightarrow Gx)$. Now consider any one of its instances: $Fa \rightarrow Ga$. Then under the supposition we are making, this statement is not merely true, but remains so under any uniform substitution on a. Accordingly, whilst the parent generalization no doubt comes out as synthetic under Bolzano's definition – there being, we may assume, (uniform) substitutions on F or G (or both) which yield a falsehood – the instance qualifies as analytic. Thus it is true – though

[11] The objection we are about to consider is, of course, an objection to the definition when it is viewed as attempting to capture the notion at which Frege-Quine is aimed – a notion on which analytically true propositions will be invariably necessary and knowable a priori. This perspective is assumed for the remainder of the paper, and in particular, by our claims about the potential advantages of Bolzano's definition and of the modifications of it we consider. Whether the Kneales' and the other main problem we consider are problems for Bolzano's own project is not our concern here. See also note 15 below.

presumably not in consequence of any natural law – that no eighteenth century philosopher died on the anniversary of his birth. Thus whatever substitution is made for 'Kant' in 'Kant was not an eighteenth century philosopher who died on the anniversary of his birth', a true statement results. Hence our proposition about Kant must be reckoned *analytically* true, on Bolzano's account.

This objection, noted by William and Martha Kneale[12], appears quite devastating – for it appears that the very feature of Bolzano's definition in virtue of which it promises to capture a wider notion than either that of narrow or logical analyticity (i.e. logical truth, as defined by Quine) or broad analyticity as explained in terms of reducibility to logical truth via definitional expansion, is precisely what is responsible for the disaster.[13] If we view Bolzano's definition as an attempt to generalize Quine's definition of logical truth, the generalization amounts to this: whereas Quine requires for a statement to be logically true that *all* the non-logical expressions occurring in it should do so inessentially, Bolzano requires (for a statement to be analytic) only that *some* of the non-logical expressions occurring in it should do so. But in any instance of a true general statement, the singular terms will occur inessentially, so that any such statement will count as analytic. The resulting unwanted expansion of the class of analytic truths thus appears as the inevitable, and clearly unacceptable, price of seeking to define a broader notion in terms of the incapacity of uniform substitution to change truth value.

Is there any way to meet this difficulty? Can we find a revision of Bolzano's definition which retains its advantages whilst avoiding this consequence?

[12]See [Kneale(1962)], p.366-7; the Kneales are also responsible for the nice example. They clearly assume the perspective on Bolzano's definition described in the note 12. A kind of obverse of their example may be got by considering false existential generalizations: if $\exists x A(x)$ is false, then a will occur inessentially in $A(a)$, so that each and every instance of the generalization will count as analytic in Bolzano's sense, regardless of the status of the parent existential generalization. Clearly there will be further anomalies. Thus consider and statement $\forall x(Fx \vee (\neg Fx \wedge p))$, where p is some contingent truth. Any instance $Fa \vee (\neg Fa \wedge p)$ will rank as analytic. Of course, were p false rather than true, a might well fail to occur inessentially, since Fa might be true but Fb false. a's inessential occurrence is contingent on the truth-value of p. This – contingently inessential occurrence – is what the Kneales' and similar examples exploit.

[13]It appears so, but is it so? In fact, an analogue, or at least a close relative, of the Kneales' problem afflicts Quine's definition of logical truth itself, independently of Bolzano's generalization. As is well-known, for any natural number n we can express that there exist at least n objects in the language of first-order quantification theory with identity, for example by writing $\forall x \exists_{n-1} y\, y \neq x$ (where $\exists_n x$, meaning 'There are at least n x', is recursively definable in the usual way). A nominalist who thinks that there are only concrete objects, but that there are at least 17 of them, will take $\forall x \exists_{16} y\, y \neq x$ to be true, but it is surely not a *logical* truth. Further, each of its instances $\exists_{16} y\, y \neq a$ will contain a inessentially, and so will qualify as a logical truth by Quine's definition, just as it qualifies as logically analytic under Bolzano's. To be sure, a philosopher of a very different persuasion (but probably not Quine!) might argue that these are no *contingent*, *empirical* truths, but are *necessary*. But that brings no respite, since it leaves untouched the central point, which is that they are surely not *logically* necessary or *logically* true – so that Quine's definition, and hence the Frege-Quine definition of analyticity which rests upon it, is in as bad a shape as Bolzano's.

5 Blind alleys

5.1 A two-part definition?

Whilst Bolzano's definition misclassifies as analytically true any instance of a synthetically true generalization, it appears to yield the right verdict when applied to the parent generalization itself. Thus there is, for example, no idea for which we may freely substitute any other idea in the proposition expressed by 'No eighteenth century philosopher died on his birthday'. More generally, Bolzano's definition appears to yield intuitively correct results when its application is restricted to propositions expressed by sentences devoid of singular terms. Thus it may seem that we could secure a base class of analytic propositions, avoiding the Kneales' objection, by restricting the application of Bolzano's definition to statements free of singular terms. We might then, it seems, take care the remaining 'good' candidates, including analytic propositions whose expression involves the use of singular terms, by adding that a statement is analytically true if it is deducible from some statement(s) belonging to the base class. In short, the proposal is for two part definition:

> (1) A purely general statement is analytically true if it is true and contains at least one expression inessentially
>
> (2) Any statement is analytically true if it is a logical consequence of some statement(s) analytically true by (1)

This proposal makes the status as analytically true of statements involving reference to particular objects derivative from that of analytic general statements, and so goes flat against our earlier proposal to secure the analyticity of statements like 'Whatever is yellow is coloured' (analyticities of the third kind) by taking a generalization to be analytic iff all its instances are. But since we have not shown that that is the only way to accommodate analyticities of the third kind, the present proposal remains, so far, a live option, and it is therefore worth considering whether, should it prove possible to accommodate analyticities of the third kind in some other way, it would be a viable option.

It would not be fair to object that the proposal is merely ad hoc. There is a well-established tradition of thought which has it that necessary truth has its source in relations among general concepts. The treatment of singular statements as analytic only when they are logically derivable from analytic general statements might be seen as a reflection of what is right in that admittedly somewhat sketchy thought.

It might also be objected that the proposal makes an unexplained use of the notion of logical consequence, and that when this is explained, the definition will turn out to be viciously circular. As against this, we may note that if this were a good objection, it would tell equally against the Frege-Quine definition. But in fact, it is unclear that an explanation of logical consequence need involve any appeal to the notion of analyticity. Standard explanations, to be sure, invoke the notion of necessary truth-preservation, or logical necessity, but neither is usually explained in terms of analyticity, and there is no compelling reason to think they must be. There is, however, a more serious objection.

Even if we *can* exclude counterexamples to the original definition by the emendation proposed, this does not dispose of the problem, because we can reduplicate the difficulty at the next level up. That is, just as we obtain counter-examples to Bolzano's original definition by exploiting synthetically true *first*-level generalizations to locate statements featuring singular terms that ought not to be, but are, counted as analytic by Bolzano's definition, so we can find synthetically true *second*-level generalizations whose truth ensures that uniform replacement of *first-level predicates* will not alter truth value – with the result that certain first-level generalizations that instantiate them rank as analytic under our revised definition, when they ought to come out as synthetic.

In fact, we can give an effective procedure for generating such higher-level counter-examples. We may assume that there are some merely synthetically true first-level generalizations. Let $\forall x Qx$ be any such. Then the first-level predicate Qx is true of every object. But then the second-level generalization $\forall F \forall x(Fx \rightarrow Qx)$ is likewise synthetically true. Take any instance, say $\forall x(Px \rightarrow Qx)$. Then this will rank as analytic by clause (1) – for however we vary P, the resulting statement will be true, just because $\forall F \forall x(Fx \rightarrow Qx)$ is.

5.2 Necessitated inessentiality?

It may be suggested that once we see *why* Bolzano's original definition is vulnerable to the kind of counter-examples we have discussed, it is not too difficult to see how his definition needs to be modified so as to exclude them. We can reformulate Bolzano's original definition in this way:

> S is analytic iff there is an expression u occurring in S such that where v is any other expression of the same grammatical type as u, the statement that results from S by substituting v for u throughout S is materially equivalent to S

or more concisely:

$$S \text{ is analytic iff } \exists u(u\, occurs\, in\, S \wedge \forall v(S[^u_v] \leftrightarrow S))$$

The present problem is that whenever S is an instance of some contingently true general statement, not only S, but also every other instance of that general statement will be true, as it happens, with the result that however we vary the names or singular terms occurring in S, the resulting statements will always be alike in truth-value with S. So S will count as analytic. If, on the other hand, S is an instance of a contingently false general statement, S will not count as analytic, even if it happens to be true, because there will be some other instance of the general statement which is false, and so some singular term that can be substituted for a singular term occurring in S to yield a statement different in truth-value from S. Clearly, however, whether a statement is or is not analytic ought not to depend in this way on what merely *happens* to be the case. What determines whether or not S is analytic should be not whether substitutions of the kind in question *do as a matter of fact* lead to a change in truth-value, but whether or not they *could* do so. This suggests that we should strengthen Bolzano's definition in the following way :

$$S \text{ is analytic iff } \exists u(u\, occurs\, in\, S \wedge \Box\forall v(S[^u_v] \leftrightarrow S))$$

This small adjustment clearly suffices to block unwanted candidates such as instances of true, but only contingently true, generalizations, since while the parent generalization's truth ensures that uniform substitution on singular terms *will* preserve truth-value, its contingency means that it *need not* do so. Of course, anyone who sympathizes with Quine's scepticism about the intelligibility of intensional idioms (such as the necessity operator) as opposed to supposedly purely extensional ones (such as truth and uniform substitution) will find this strengthening unacceptable. But we have already seen that the hope that Bolzano's approach would enable us to give an account of analytic truth in purely extensional terms is doomed to frustration. So we may set aside that objection here. There is, however, a much more serious problem.

Consider the proposition:

> If *this ring* is pure gold, *it* is entirely composed of a substance whose atomic number is 79

We may substitute any singular term we wish for the italicized words and the resulting proposition not only will, but must, be true – assuming, as we certainly may, that the generalization of which we have taken an instance is not only true, but true as a matter of metaphysical necessity. But while the generalization, and so each of its instances, is metaphysically *necessary*, none of these propositions is *analytically* true. In short, the proposed emendation, as it stands, precipitates a collapse of the distinction between metaphysical necessity and analyticity. The trouble lies with the unqualified or indiscriminate use of the necessity operator. In order to get the extensionally correct result, we would need somehow to specify that $\Box$ is to express the right kind of necessity – one grounded purely in senses, or concepts – and it is quite unclear how we could do so without using the very notion we are trying to explain.

6 Over-extension (2) – the embedding problem

We should now take note of a further serious problem with Bolzano's original definition, when it is viewed as an attempt to capture the traditional conception of analyticity as truth-in-virtue-of-meaning and as a potential improvement on the Frege-Quine definition. Let p be any proposition which qualifies as analytic by Bolzano's definition in virtue of containing the idea i inessentially, and consider its conjunction with q, where q is any intuitively synthetic proposition having the same truth-value as p, but not containing the idea i at all.[14] By

[14] The point of the restriction is that if q contains i, then uniform replacement of i throughout $p \wedge q$ by another idea i' may result in a proposition $p \wedge q'$ which differs in truth-value from $p \wedge q$, just because i does not occur inessentially in q.

hypothesis, p contains at least one idea which may be varied at will without yielding a proposition differing in truth-value from p. But then clearly the same must go for $p \wedge q$, given that q and p are alike in truth-value, and that i does not occur in q, so that varying i does not disturb q's truth-value. Then $p \wedge q$ will likewise qualify as analytic. Yet it clearly should not do so. It is true enough that if p is analytically *false*, so will be any conjunction of which it is a conjunct, so that Bolzano's definition gives the right result here. But suppose instead that p is true, and so analytically true according to the obvious way of defining analytic truth in Bolzano's terms. If q is true but synthetic, it seems clear that their conjunction should count as at best synthetically true. (cf. 'Married men are men and Handel outlived Bach').[15]

Essentially the same problem arises over disjunction. Let p be analytic, with i occurring inessentially, and let q be synthetic, materially equivalent to p, and i-free. Then $p \vee q$ will likewise be analytic. The problem, this time, arises when p is analytically false. Similar difficulties will arise with other embeddings of any proposition that is analytic by Bolzano's lights. In general terms, the problem is that if a statement A is analytic by Bolzano's definition, so will be any statement B which incorporates A, provided that the expressions in A in virtue of which it qualifies as analytic do not occur in B other than as parts of A, and A does not occur within a referentially opaque context in B.

It is easy to see that this embedding problem applies equally to each of our two attempts to rescue Bolzano's definition from the Kneales' objection. For if containing at least one idea inessentially is a sufficient condition for analyticity, as on the first proposal, then any conjunction one of whose conjuncts is analytic must be so also. And requiring, as on the second proposal, that uniform replacement of at least one idea should necessarily leave truth-value undisturbed equally clearly does nothing to alleviate the problem.

7 *Post mortem* – and a better proposal?

7.1 Epistemologizing Bolzano

Let's take stock. We have at this point two outstanding objections to Bolzano's proposal: the first, due to the Kneales, allows of a response only at the cost of the apparent circularity of invoking a notion of necessity in the explanans which itself promises to require explanation in terms of analyticity; the second

[15] We should emphasize that we are not claiming that the problem is a problem for Bolzano, given his own purposes in giving his definition. That it is a problem for Bolzano is suggested by Jan Berg. Although Berg presents the problem as 'an objection from a modern viewpoint', conceding that 'Bolzano would probably not have considered [this] objection serious', he thinks it serious enough to add 'At any rate, this consequence of [his definition of broad analyticity] makes us concentrate our interest on the notion of logical analyticity' (see [Berg(1962)], p.101, also his editorial introductions to [Bolzano(1987)], p.18, and to [Berg(1973)], p.18). We are grateful to Wolfgang Künne for the first two references. As Künne emphasizes (see [Künne(2008a)], p.248ff, esp. fn.48), while it is true that under Bolzano's definition, analytic propositions may be contingent and knowable only by empirical investigation, it is by no means clear that Bolzano would have found this consequence unwelcome or disturbing.

– the embedding problem – seems to impose a disconnection between inessential occurrence and analyticity prima facie fatal to Bolzano's account.

Let us focus on the first of these difficulties. There is, as we have already emphasized (see page 6), a major difference between Bolzano's definition and Frege's: for Frege, the distinction between analytic and synthetic judgements, in line with that between a priori and a posteriori, is an *epistemological* one – the claim that a judgement is analytic is a claim about how it may be *justified*. By contrast, Bolzano defines the analyticity of a proposition simply in terms of the effect of varying some of its ingredient ideas upon its truth-value. But as the Kneales' objection brings out, inessential occurrence is no sure guide to analytic status, for it may have its source in some background contingencies. An obvious corollary is that the fact that an idea (or expression) occurs inessentially in a proposition (or statement) may itself be something recognizable a posteriori, via independent knowledge of the relevant contingencies. A further unwanted consequence is thus that Bolzano's definition threatens the traditional connection between analyticity and a priori knowability. In the light of all this, a natural and plausible response to the first of our problems is to 'epistemologize' Bolzano's definition : a proposition is logically analytic if it not only contains at least one idea inessentially and only logical ideas essentially, but is such that the fact that it does so can be recognized *simply* by relying upon one's grasp of those ideas or concepts involved in the proposition which cannot be varied freely; and, generalizing this, a proposition is analytic if it not only contains at least one idea inessentially, but is such that that fact can be recognized simply by relying upon one's grasp of those ideas or concepts involved in the proposition which cannot be varied freely. [16]

Accordingly, we may – as a first approximation – consider the following definition:

E-Bolzano 1 A statement A is analytic iff (i) A contains at least one expression which can be freely varied without change of truth-value (ii)

[16] Interestingly, although his definition remains resolutely non-epistemological, Bolzano was sensitive to the kind of connection on which the proposed revision focuses. Commenting on his examples of logically analytic propositions (See page 4), Bolzano writes:

> The examples of analytic propositions I have just cited are differentiated ... by the fact that nothing is necessary for judging the[ir] analytic nature ... besides logical knowledge, because the concepts that make up the invariant part of these propositions all belong to logic. (*Wissenschaftslehre* §148, [Berg(1973)], p.193)

What is especially interesting here is Bolzano's saying that only logical knowledge is needed to recognize the *analytic nature* (rather than, as one might expect, the *truth*) of such propositions as those expressed by 'A is A', 'An A which is B is an A', etc. Recognizing the analytic nature consists, in his view, in seeing that certain ideas involved in the proposition can be varied in any way we please, and the result will be a proposition having the same truth-value as the original. It is this idea that the epistemologized version of Bolzano's definition we are about to consider takes up and generalizes. We are not, of course, suggesting that Bolzano himself harboured any thought that his original definition might be modified along these lines. On the contrary, he is firmly opposed to the introduction of any kind of epistemological considerations in defining analyticity. The non-epistemic character of Bolzano's definition is emphasized by Michael Dummett in [Dummett(1991)], pp.28-30

> that fact can be recognized by anyone who understands the remaining, non-variable expressions composing A, and grasps the semantic significance of its syntax[17]

This modified definition avoids the Kneales' objection, and finesses the need to modalize in response to it. The objection exploits *empirical inessentiality* – the fact that an expression may indeed occur inessentially in a statement, but only courtesy of that statement's being an instance of some true empirical generalization. Where a statement does contain an expression which can be varied without change of truth value, but *only* because that statement is an instance of a true empirical generalization, grasp of the remaining expressions composing the statement precisely does *not* suffice to enable one to recognize that there is an expression which can be varied without disturbing truth-value. To know that, in such a case, one would need to know that the parent generalization is true, and mastery of the expressions involved in the candidate statement, though necessary, is insufficient for such knowledge.

However, while this modification escapes the Kneales' objection and preserves the principal potential advantage of Bolzano's original definition – of enabling us to see logical analyticity as a special case of a more general phenomenon, thereby avoiding the necessity of viewing the analyticity of logically true statements as a matter of direct stipulation, as on the Frege-Quine definition – it does nothing to alleviate the other major difficulty we found with Bolzano's original, viz. the embedding problem. Epistemologizing Bolzano's definition in the way indicated does not help. For if A is analytic in virtue of containing an expression e which can be varied freely without altering the truth-value of A, and B is any longer statement incorporating A but containing no additional occurrences of e, anyone who understands B will be able to recognize that it contains A as a part, and contains no additional occurrences of e, and so will be able to recognize that B contains e inessentially.

7.2 The embedding problem solved

The embedding problem shows that even if Bolzano's definition leads us to count relatively simple statements as analytically true, or analytically false, just when they would be so classified in accordance with the traditional conception, it is liable to go badly astray when applied to more complex statements embedding them. Why is this? An obvious thought is that the problem reflects an important discrepancy between analyticity in Bolzano's sense and the traditional conception associated with the notions of truth/falsehood-in-virtue-of-meaning. The former is, as we might put it, *upwards-hereditary*, in the sense that the result of incorporating a Bolzano-analytic statement as part of a more complex

[17] Here and subsequently we treat analyticity as a property of *statements* (interpreted sentences), rather than *propositions*. When this is done, it is crucial to emphasize that the basis on which inessential occurrence is to be recognizable includes not only understanding of the statement's remaining, non-variable expressions, but also grasp of its syntax. We have included this last requirement here, but, in the interests of brevity, we will often leave it to be understood in the sequel.

statement must likewise be Bolzano-analytic, provided only that the remainder of the containing statement is free of further occurrences of the expressions occurring inessentially in the Bolzano-analytic part. But the traditional notion of analyticity clearly lacks this property. To take the simplest and most obvious examples, while the analytic falsehood of one conjunct suffices for that of the conjunction as a whole, a conjunction is analytically true only if both conjuncts are so; and while the analytic truth of one disjunct suffices for that of any disjunction incorporating it as a disjunct, a disjunction is analytically false only if both disjuncts are.

This initial diagnosis suggests that we might solve the problem by giving a recursive definition, using Bolzano-analyticity (or rather, our epistemologized version of it) to characterize a suitable base class, and using the recursive clauses to impose suitable requirements on the components of complex statements. Such a recursive definition can indeed be given, and in an appendix, we illustrate how this may be done for a first-order language. There is, however, another shortcoming which reflection on the embedding problem discloses, and this suggests a rather different remedy, making no essential play with recursion.

As previously observed (see p.5), Bolzano's definition of analytic propositions covers both analytically true and analytically false propositions, making no distinction between them. By contrast, such a distinction is central to the traditional conception, which explains analytic truth as truth-in-virtue-of-meaning and analytic falsehood as falsehood-in-virtue-of-meaning. Of course, one could define notions of analytic truth and falsehood in terms of Bolzano's notion of analyticity together with the notions of truth and falsehood, and one could define a general notion of analytic proposition in terms of the traditional notions of analytic truth and analytic falsehood. But there remains a crucial difference. Starting from Bolzano's definition, we obtain:

> A is *analytically true* iff A is analytic and A is true, and A is *analytically false* iff A is analytic and A is false

Starting from the traditional notions, we obtain:

> A is *analytic* iff A is analytically true or A is analytically false

But the resultant notions of analytic truth and analytic falsehood under the first definition are plainly not equivalent to analytic truth and analytic falsehood as traditionally understood. Indeed, they are not even co-extensive, since

> Haydn outlived Mozart and if Bartok and Kodaly were compatriots, Bartok and Kodaly were compatriots

counts as analytically true in Bolzano's sense, whereas it is clearly not so according to the traditional conception.

It is a consequence of precisely this divergence between Bolzano's notion and the traditional one that epistemologizing Bolzano's definition, as suggested in the preceding sub-section, does nothing to solve the embedding problem. Recognizing that a statement is analytic in the sense that it contains at least one expression which may be varied without change of truth-value is consistent with total ignorance of the statement's truth-value. In particular, someone who can recognize that the right conjunct in our last example is Bolzano-analytic is in position to recognize that the whole conjunction is so. Of course, this ability consists with total ignorance of the fact that the conjunction is, as it happens, true; but that is of no matter, since knowledge of truth-value is not required for knowledge of Bolzano-analyticity.

This suggests a quite different way of dealing with the embedding problem: emend *E-Bolzano 1* to deal separately with analytic truth and analytic falsehood, and require recognition of truth-value as well as recognition of inessential occurrence. This yields

E-Bolzano 2.1 A statement A is *analytically true* iff (i) A is true, (ii) A contains at least one expression which can be freely varied without change of truth-value, and (iii) that (i) and (ii) both hold can be recognized by anyone who understands the remaining, non-variable expressions composing A

E-Bolzano 2.2 A statement A is *analytically false* iff (i) A is false, (ii) A contains at least one expression which can be freely varied without change of truth-value, and (iii) that (i) and (ii) both hold can be recognized by anyone who understands the remaining, non-variable expressions composing A

Clearly, this adjustment is by itself enough to dispose of the embedding problem, without need for the complications of a recursive definition. Suppose, for example, that A is analytically true (i.e. meets conditions (i)-(iii) above), and consider its conjunction with any synthetic statement, B. Even if B is true, the fact that it is, and hence the fact that $A \wedge B$ is true, will *not* be recognizable solely on the basis of competence with the remaining, essentially occurring, expressions in the conjunction. To be sure, should A be analytically *false*, then anyone competent in the use of the relevant expressions will be able to recognize that conditions (i)-(iii) are met with respect to its conjunction with any other statement, so that the conjunction will qualify as analytically false – but that is as it should be. Oppositely, A's analytic *truth* will suffice for that of any disjunction $A \vee B$, but rightly so; while should A be analytically false, this will *not* suffice to force analytic falsehood on its disjunction with arbitrary B, for B may well be true. Other sentential compounds likewise raise no problem.

It remains to modify the foregoing proposal to accommodate analyticities of the third kind. These may be captured by modifying our definition in the way previously envisaged with Bolzano's own definition (See p.3). We propose:

E-Bolzano 3.1 A statement A is *analytically true* iff (a) (i) A is true, (ii) A contains at least one expression which can be freely varied without change of truth-value, and (iii) that (i) and (ii) both hold can be recognized by anyone who understands the remaining, non-variable expressions composing A, or (b) A is a universal generalization whose

> instances are all analytically true or an existential generalization at least one of whose instances is so

with a similar adjustment for the definition of analytic falsehood.

8 Analytic knowledge, epistemic and 'metaphysical' analyticity

8.1 An obvious complaint answered

The foregoing proposal is, however, open to a very immediate complaint[18]: that the requirements that A be true (false) and that it should contain at least one expression inessentially, feature within it as *separate*, so far quite unconnected conditions. This was forced, given that – in contrast to Bolzano – we are distinguishing analytic truth and falsehood in our definitions. All the same, if any kind of account of the *nature* of analyticity is to be attempted, as opposed to a mere putative characterization of the extension of the notion, it is imperative to say more about how the two conditions are supposed to interact. As things stand, there is nothing to forestall the impression that two distinct acts of recognition are implicated in the recognition of analyticity – recognition of truth-value, on the one hand, and recognition of inessential occurrence, on the other. Worse, indeed, once the first component— recognition of truth-value— is written into the definitions in the fashion illustrated, does not the additional clause requiring inessential occurrence – the distinctive feature of Bolzano's original definition — become a mere curlicue?[19] What does the requirement that understanding the target sentence should enable recognition that one or more expressions occur inessentially within it add to the notion of epistemic analyticity, proposed by Paul Boghossian over the last couple of decades[20], according to which analytic sentences are those whose truth-value can be recognized purely on the basis of understanding them?

At first blush, it must be admitted, little or nothing of significance. But the appearance is arguably deceptive. The recognition that, say, 'If Bolzano is in Prague, then Bolzano is in Prague' is true, and the recognition that it contains the sentence 'Bolzano is in Prague' inessentially, are *not* two separate feats of recognition. Rather, someone who understands the conditional sentence can recognize that it expresses a truth (assuming, of course, that the proper names do not shift reference between antecedent and consequent) precisely because they know, in virtue of their understanding of the conditional construction, that provided the same sentence figures as antecedent and consequent, the conditional will be true, no matter what sentence that is. One recognizes truth, in such a case, by way of recognizing inessential occurrence. The point is no peculiarity

[18] The same complaint applies equally to the recursive definition described in the appendix, as the reader may easily verify

[19] In its English sense; apparently in mid-19th century American English it means a caper.

[20] See [Boghossian(1996)], [Boghossian(1997)], and [Boghossian(2003)])

of logically analytic truths. The same goes for any other minimally analytic truth. Consider, say, the proposition that if George is a brother, he is a sibling. No one whose working vocabulary includes both ‘brother’ and ‘sibling’ needs to know anything about George in order to know that this proposition is true. Nor need they recognize that ‘If George is a brother, he is a sibling’ may be transformed into the logical truth, ‘If George is a male sibling, he is a sibling’. The ability to recognize the truth of the proposition can be entirely parallel to that of someone competent to recognize the truth of the proposition that if Mozart’s socks are yellow, they are coloured – where there can be no question of recognition proceeding through transformation into a logical truth, because there is no such transformation. One can recognize the truth of this particular proposition because one knows, simply in virtue of a competence with the terms, that no matter what term fills both gaps in the schematic sentence ‘if . . . is a brother, . . . is a sibling’, the resulting sentence will express a truth. And so, mutatis mutandis, for the proposition about my socks. And of course, the same goes for recognition of minimal analytic falsehood. Anyone competent in the use of sentential negation and conjunction can recognize that ‘Cats are mammals and cats are not mammals’ must (assuming, of course, no relevant ambiguity) be false because they know, courtesy of their competence with the terms, that no matter what declarative sentence, true or false, occupies both gaps in the schema, ‘. . . and it is not the case that . . . ’, the result will be false.[21]

What these examples illustrate, we claim, is indeed the essentially *schematic* character of knowledge of analyticity – analytic truth or falsehood – in the basic (non-inferential) case. This is the insight that Bolzano’s definition – or at least his emphasis on inessential occurrence – contains. To recognize that the proper names occur inessentially in ‘If Haydn’s birth preceded Mozart’s, then Mozart’s followed Haydn’s’ *is*, in effect, to recognize that, no matter which terms fill the gaps in the schema ‘If __’s birth preceded . . . ’s, then . . . ’s followed __’s’, provided the same term fills the first and last, and the same term the second and third, the resulting statement will, and indeed must, have the same truth-value as our statement about Haydn and Mozart. The same goes for recognition of inessential occurrence in analytic falsehoods, such as ‘Mozart’s last symphony was composed before Haydn’s, and Haydn’s last before Beethoven’s, but Beethoven’s last was composed before Mozart’s’. Recognition of analytic truth is, or centrally involves, recognition that a certain schema always yields a truth, on uniform insertion of suitable expressions in its gaps; and so, mutatis mutandis, for recognition of analytic falsehood.

Care is needed, though, in expressing the point. One does not – at least in general, if not invariably – *first* recognize that a sentence contains inessential occurrences of one or more expressions and *then*, purely on that basis, move

[21]No question is begged here against dialetheists. They do not deny that contradictions are always false. It is just that they think, and presumably claim to know, that some are true as well. They can avail themselves of this explanation of our recognition of their falsehood. That leaves the task of explaining putative knowledge of their truth. But that is none of our business.

to recognition that the results will be invariantly true (or invariably false) no matter what expressions replace them. It is not that one recognizes analytic truth, or falsity, merely *by* recognizing inessential occurrence. To recognize that a sentence contains certain expressions inessentially need afford one no more than the knowledge that its truth-value will not depend on the semantic value of those expressions; exactly that was the gist of the embedding problem. The schematic character of analyticity is rather this: that in recognizing that a sentence is analytic, one recognizes that, such are the meanings of some (essentially occurring) expressions in it, and such is the semantic composition of the sentence as a whole, the sentence will—indeed, must—invariantly express a truth (or invariantly express a falsehood) no matter what the semantic values of the remaining (inessentially occurring) expressions it contains. In effect, the proposal is that the root of the notion of analyticity is a property not of truth-apt sentences in general but of *open* sentences: a property (the *Bolzano property*) which holds in virtue of the syntax and the semantic values of the expressions they contain and which ensures invariance of truth-value no matter whether we close them by instantiation or by universal generalization.[22] Thus neither 'If Mozart's stockings are yellow, they are coloured' nor 'Everything yellow is coloured' is prior, in point of analyticity, to the other; rather each is posterior to the schema, 'If ... is yellow, it is coloured'. The ground of our recognition of the analyticity of both the former, it is proposed, is the schematic knowledge expressed by the latter.[23]

We thus arrive at the following modification of *E-Bolzano 3*:

E-Bolzano 4-Schematic A schema $S(\eta)$ is analytic iff (i) where $S(e)$ results from uniform replacement of η throughout $S(\eta)$ by any expression e of the type of η, $S(e)$ is always true, or always false, and (ii) that (i) holds is recognizable by anyone who understands $S(\eta)$

E-Bolzano 4.1 A statement A is *analytically true* iff (a) A is an instance of an analytic schema whose instances are always true or (b) A is a universal generalization whose instances are all analytically true or an existential generalization at least one of whose instances is so

E-Bolzano 4.2 A statement A is *analytically false* iff (a) A is an instance of an analytic schema whose instances are always false or (b) A is a universal generalization one of whose instances is analytically false or an existential generalization all of whose instances are so

[22] The idea that for Bolzano, analyticity is a property of propositional forms is suggested in [Lapointe(2011)] , see p.62-4; but this is not easily squared with what Bolzano himself says and conflicts with a more orthodox interpretation – see [Künne(2008a)], p.233 ff. We take no stand on this exegetical issue.

[23] There is some delicacy with the point, since we are not saying, of course that no one can recognize analyticity whose language does not contain the resources for the expression of schemata. The claim is that what is recognized, when someone recognizes the analyticity of 'If George is a brother, he is a sibling', or 'Anything yellow is coloured', is something which, had they the appropriate expressive resources, could be formulated by means of a suitable claim about an open sentence. This should not seem uncomfortable unless one takes it that a subject's knowledge is everywhere bounded by the resources they have for its expression.

It would, we think, be unwise to claim that this modification takes care of all statements which might plausibly be reckoned analytically true or analytically false – in effect, that any analytically true statement is either an instance of an analytic schema, or is obtainable from such schemata by universal or existential generalization. Indeed, as far as English and other natural languages are concerned, it seems clear that this is not so. An interesting class of exceptions can be illustrated by examples such as:

> Red is a colour
> Red is different from green
> Temporal precedence is a transitive relation

These examples exploit what we might call higher-order singular terms corresponding to first-level predicates – the nouns 'red' and 'green' corresponding to the predicates '. . . is red' and '. . . is green', and the abstract noun phrase 'temporal precedence' corresponding to the relational predicate '. . . temporally precedes__'. How such examples are to be handled is a matter of some interest. It would distract us too much from our central line of argument to pursue this question here. We discuss it briefly in an appendix.

8.2 Non-epistemic analyticity

It is noteworthy that this proposal immediately provides resources sufficient to respond to Boghossian's recently influential critique of what he termed the "metaphysical conception" of analyticity – the notion encapsulated in the idea of truth-purely-in-virtue-of-meaning. Boghossian complains that, taken at face value, the latter notion is incoherent: that no sentence can be true purely in virtue of its meaning. For any sentence S, if S is true, it will be because for some proposition p, S expresses p and it is a fact that p. A contribution from the world, or the facts, is always required even if the contribution is assured.[24] It is natural, however, to feel some discomfort with Boghossian's own response to his point: the proposal to scrap the metaphysical notion altogether, in favour of an epistemic one, whereby a sentence ranks as epistemically analytic just in case an understanding of it provides a sufficient basis for recognition of its truth (or falsity).[25] For bracketing any scepticism whether that there are indeed such sentences, it could hardly be the last word about them to characterize them in that – purely epistemic – way. If grasp of a sentence's meaning puts a subject in position to recognize its truth, there has to be something about its meaning in virtue of which that is so.[26] The proper conclusion is therefore only that,

[24] See, for example, [Boghossian(1997)], p.335: "How could the *mere* fact that S means that p make it the case that S is true? Doesn't it also have to be the case that p?"

[25] cf. [Boghossian(1997)], p.334: "On this [the epistemic] understanding, then, 'analyticity' is an overtly *epistemological* notion: a statement is 'true by virtue of its meaning' provided that grasp of its meaning alone suffices for justified belief in its truth".

[26] Acknowledging that analyticity cannot satisfactorily be conceived purely epistemically carries no commitment to any particular view about its source, much less a commitment to a realist or 'metaphysical' view. Hence our preference for the colourless term 'non-epistemic analyticity' over Boghossian's more florid 'metaphysical analyticity'

whatever that something is, it cannot be happily captioned as that the meaning of the sentence is such as to ensure its truth (falsity) with no contribution from the world.

E-Bolzano-Schematic now supplies a first-pass description of what the 'something' is: it is the property a sentence has when, such are its syntax and the meanings of the expressions essentially occurring in it, the open sentence resulting from the deletion of all inessentially occurring expressions and/or quantifiers is such as to generate a truth (falsehood) no matter how it is completed. This account finesses any threat of 'marginalisation' of the world in the process of the determination of the truth-values of analytic sentences, since analyticity is not now, in the first instance, a property of truth-apt sentences at all. We may of course extend the scope of the epithet, 'analytic', to encompass sentences resulting from analytic matrices by substitution or quantification into their argument places, by means of such further definitions as *E-Bolzano 4.1* and *E-Bolzano 4*.2. But then the truth-value of an analytic sentence is determined, just as it should be, *both* by meaning – the meaning of the open sentence from which it results, *and* by the world – in delivering the semantic values, necessary if it is to have a truth-value at all, of the particular inessentially occurring expressions it contains.[27]

8.3 Concluding remarks

So, for a theorist who wishes to salvage a metaphysical – better: non-epistemic – notion of analyticity, that may seem like progress. A caveat is immediately needed, however, since the characterization just offered over-extends to embrace, 'If x is composed of water, x is composed of H_2O' as well as 'If x is yellow, x is coloured'. And now it is tempting to think that the needed distinction can only be that, in the latter case, grasp of meaning supplies a complete basis for recognition of the invariance of truth-value of instances while in the former it does not. To exclude the unwanted cases, then, capturing just the traditionally analytic and excluding the necessary a posteriori, it seems that we must still characterize analyticity epistemically, as a property of the meanings of open sentences in virtue of which, unsupplemented by other information, it can be recognized . . . etc. And this, it may well seem, still cannot be the last word; it cannot be that *all* there is to say about the property is that it sustains the relevant epistemic feat; there has to be an explanation of how it is sustained, of what it is about the matrices in question that enables one who understands them to recognize that their instances are invariant in truth value.

Accordingly, a properly non-epistemic notion of analyticity must, it seems to us, find use for the notion of *grounding*: specifically, for different ways in which the possession by an open sentence of the Bolzano property may be underwritten. The invited distinction is very much along traditional lines: analytic sentences are instances, or generalizations, of matrices whose possession of the

[27] A rather different response to Boghossian's *two-factor* argument, as she labels it, is advocated by Gillian Russell (in [Russell(2008)], pp.31-7). For a brief discussion of it, see Appendix 2.

Bolzano-property is grounded purely in the *senses* of the expressions they contain, and in their *syntax*; other cases, like the water-H_2O example, possess the Bolzano property in virtue of aspects of the essential nature of the semantic values of the expressions they contain essentially.

So in the end we arrive at a well-visited staging post on the road to vindication, or repudiation, of the notion of analyticity. Further progress from here, if possible at all, will require four things: consolidation of the notion of sense, explication of the notion of ground, an explanation of how the Bolzano-property can indeed be grounded in sense, and an explanation of how that fact can be non-inferentially recognized. Misgivings about any of these projects will continue to fuel scepticism about the notion. But the utterly convincing intellectual phenomenology of the usual stock of basic examples will continue to fuel resistance to that scepticism. We do not attribute to Bolzano any special insight into how the deadlock might be broken. But we do think that his ideas contain a contribution to the proper formulation of the problem that later discussion lost sight of. That is what we have tried to outline here.

Appendix 1: a recursive solution to the embedding problem

As observed in 6.2, it is plausible to think that the embedding problem might be avoided by recasting our epistemologized version of Bolzano's definition as a recursive definition. Such a definition must, of course, assume a quite detailed analysis of the structure of the language to which it is to be applied, taking into account all the ways in which complex sentences may be constructed out of simpler ones. Although we know of no convincing reason to doubt that such an analysis may be given for natural languages such as English, we are certainly not in a position to provide one. We shall therefore address ourselves to a much more modest task – describing how a suitable recursive definition may be constructed for a schematic first-order language.

We assume, then, a first-order language comprising the usual truth-functional sentential operators together with universal and existential quantifiers binding individual variables. The language will have a stock of first-level predicates of varying adicity, along with a stock of singular terms, from which the simplest sentences of the language may be formed.

Our first task in implementing this suggestion is to circumscribe a suitable base class of analytic statements. This is less straightforward than might be anticipated. We cannot take the base class to comprise just atomic or logically simple statements, since there are complex statements – e.g. the statement that if my socks are yellow, they are coloured – which we wish to count as analytic but which do not inherit their analyticity from that of their components. Indeed, it is far from obvious that there are *any* logically simple analytic statements. But if any complex statements are to be included in the base class, we must take especial care to block inclusion of any which simply re-introduce the embedding problem. This can be accomplished by taking the base class to comprise just those statements which, in addition to satisfying the epistemological condition previously proposed (see *E-Bolzano 1*), meet the further condition that they contain no proper part which does so. Clearly this will exclude such monsters as 'If p then p and grass is green', whilst admitting such as 'If my socks are yellow, they are coloured'.

However, there is a more serious snag. As we have observed, Bolzano sought to define what it is for a statement to be analytic, without differentiating between analytic truth and analytic falsehood; and *E-Bolzano 1* follows him in this regard. This poses no direct obstacle to devising a suitable clause for *negation* – clearly $\neg A$ will be analytic iff A is. But with the binary connectives we are stymied. What clause should we adopt for conjunction, for example? We can't say that $A \wedge B$ is analytic iff A and B *both* are – for analytic *falsehood* of either conjunct alone suffices for that of the conjunction, regardless of the status and truth-value of the other. But we can't say that $A \wedge B$ is analytic iff *one of* A and B is – for if one conjunct is analytically *true*, the conjunction is surely so only if the other is so as well. It is easily verified that similar difficulties preclude any satisfactory clauses for the other connectives. The moral is clear. We must, after all, define analytic truth and falsehood separately.

Accordingly, we give a two part characterization of our base class. We abbreviate 'analytically true' and 'analytically false' to 'a-true' and 'a-false' respectively. We then define:

> A is *minimally a-true* iff (i) A is true, (ii) A contains at least one expression inessentially, (iii) the fact that (i) and (ii) are met can be recognized by anyone who understands A and grasps the semantic significance of its syntax, and (iv) no proper subformula of A meets (i),(ii) and (iii)
>
> A is *minimally a-false* iff (i) A is false, etc., [as for *a-true*, with 'false' replacing 'true']

The full definition of analytic truth and analytic falsehood may then be given as follows:

> If A is minimally a-true, A is a-true
>
> If A is minimally a-false, A is a-false
>
> If A is $\neg B$, A is a-true iff B is a-false, and a-false iff B is a-true
>
> If A is $B \wedge C$, then A is a-true iff B and C are, and a-false if B or C is
>
> If A is $B \vee C$, then A is a-true if B or C is, and a-false iff both are
>
> If A is $B \rightarrow C$, then A is a-true if $\neg B$ or C is, and a-false iff B is a-true and C a-false
>
> If A is $B \leftrightarrow C$, then A is a-true iff $B \rightarrow C$ and $C \rightarrow B$ are, and a-false one of them is a-true and the other a-false
>
> If A is $\forall v B(v)$, then A is a-true iff for every t, $B\binom{v}{t}$ is, and a-false iff for some t, $B\binom{v}{t}$ is

If A is $\exists v B(v)$, then A is a-true iff for some t, $B(^v_t)$ is, and a-false iff for every t, $B(^v_t)$ is

Otherwise A is neither a-true nor a-false

We have conditionals *only*, not biconditionals, in the clauses for a-truth for $\vee$ and $, \rightarrow$, and the clause for a-falsehood for $\wedge$, because statements with these operators as principal may qualify as minimally analytic; for example: $Fa \vee \neg Fa, Fa \rightarrow Fa, Fa \wedge \neg Fa$, as well as more interesting examples which are not logically analytic, such as 'a is red $\rightarrow$ a is coloured', etc. Instances of analyticities of the third kind qualify in precisely this way, while their parent general analyticities qualify by the clauses for the quantifiers.

There is no obvious obstacle to extending a definition along these lines to richer and expressively more powerful languages, involving higher-order quantification, or modal and perhaps other non-truthfunctional operators. However, we shall not pursue such extensions here.

Appendix 2: Gillian Russell's response to the Two-Factor objection

Boghossian asked ([Boghossian(1997)], p.335): "How could the mere fact that S means that p make it the case that S is true? Doesn't it also have to be the case that p?" The two-factor objection draws on the platitude that when a sentence is true, its being so is a function both of what the sentence means and how the world stands in relevant respects. According to Gillian Russell ([Russell(2008)], ch.1), one may coherently respond to it thus: to maintain that the truth-values of some sentences are fully determined by their meanings is not to be committed to denying that the world plays a part in determining their truth-value. Of course, that claim is incoherent, unless there are different kinds of determination, or senses of 'determine', in play. But so, she claims, there are.

To give her strategy some independent plausibility, Russell draws an analogy with multiplication. When one of the factors, a and b, is 0, their product $a \times b$ is likewise 0, no matter what the value of the other factor. Supposing $a = 0$, we might say that the product is wholly determined by a. But this does not oblige us to deny that it results from multiplying *two* numbers – after all, without the other factor, b, there would be no product at all!

So it is, Russell argues, with the determination of a sentence's truth-value by the two factors of sentence-meaning and the state of the world. In general, neither factor wholly determines truth-value. But in the case of analytic sentences, just as with multiplication by 0, one factor – the sentence's meaning – by itself wholly determines the sentence's truth-value. However, this does not oblige us to deny that the other, worldly, factor plays a part, any more than we are obliged to deny that the other factor plays a part in multiplication by 0. It is just that, whatever the other factor is, we get the same result. Still, the result is the product of *two* factors, not one.

To underwrite this response, Russell offers some distinctions. Let f be an n-ary function, and let $x_i...x_k$ be some or all of the n-tuple of arguments $x_1...x_n$. Then, first, $x_i...x_k$ *fully determine* the value $y = f(x_1...x_n)$ if, for any n-tuple of arguments $x'_1...x'_n$ which coincide with $x_1...x_n$ over $x_i...x_k$, f has y as value, regardless of the remaining arguments, if any. Second, an *argument-place* i in the sequence of argument-places $\langle 1, ...n\rangle$ *partially determines* the *value-place* of the function, if there are sequences of arguments $x_1...x_n$ and $x'_1...x'_n$ which differ in exactly their i^{th} place, such that $f(x_1...x_i...x_n) \neq f(x_1...x'_i...x_n)$. Finally, an argument x_i *redundantly determines* the value, y, of the function if f's i^{th} argument-place partially determines its value-place, but there is no sequence of arguments $x'_1...x'_n$ which differs from $x_1...x_n$ in and only in the i^{th} place, and delivers a value $y' \neq y$. These definitions ensure, as of course they are precisely designed to do, that some of the arguments to a function may fully determine its value while another of its arguments redundantly determines that value.[28]

Applying these distinctions to the case in hand, Russell's proposal is that just as the binary function, multiplication, maps pairs of numbers to numbers, so there is a binary function – she labels it $\mathcal{M}$ – which maps pairs whose first-member is a sentence-meaning (or proposition) and whose second is a state of the world to truth-values (see [Russell(2008)], p.35). In the case of an analytic sentence S, the truth-value is *fully* determined by the first factor, S's meaning – that is, $\mathcal{M}$'s value for the pair $\langle m, w\rangle$, where m is S's meaning, will be the truth-value True, no matter what the value of w, the state of the world, may be. But this does not mean that the state of the world plays no part, for it does 'redundantly' determine the truth-value. Thus, she claims, the two-factor objection can be answered.

As the discussion in the main body of this paper will have made clear, we are sympathetic to the spirit of Russell's proposal to preserve a non-epistemic notion of analyticity from Boghossian's objection. But we are doubtful about the specific tack she takes. The most immediate complaint is that it fails properly to address the central point of the two factor objection, viz. the claim that the world not merely invariably plays *some* part in determining truth-value, but that when any sentence S is true, what makes it so is the fact that for some p, S says that p, and – specifically – *that it is the case that* p. That is, it is – on the worldly side – not just any old fact, but the particular fact that p which combines with S's meaning to deliver the truth-value. In the case of a multiplication for the form, $0 \times a$, the a-argument is there simply to make up the numbers, as it were – since, as remarked, without it or something in its place, there would be no output value. But in the case of the determination of a sentence's truth-value by its meaning and the 'state of the world', the latter has to have a specific character – specifically, as demanded by the meaning of S, it has to incorporate the fact that p – if the value, True, is to result. So Russell's analogy breaks down: the suggestion that states of the world and the non-

[28] Note that while full and redundant determination are relations on *arguments* to functions and their values, partial determination relates argument- and value-*places*. If partial determination were defined on arguments instead of argument-places, the definition of redundant determination would be flatly inconsistent.

zero factors in multiplications by zero are alike in 'redundantly determining' the truth-values of analytic sentences and the product of the multiplications respectively, masks a crucial difference in the roles they play.

To sharpen the complaint, let's take the worldly argument of the $\mathcal{M}$-function to be a specific state of affairs— a state of the world in the sense in which, as we write, that Obama is the US President, or that there is currently an intensification of hostilities in Gaza, are states of the world. Let's write the curly-bracketed $\{$*Vivaldi died in Venice*$\}$ to denote the state of the world consisting in Vivaldi's dying in Venice, and let $\{S\}$ in general denote both actually obtaining state of affairs and merely possible ones (as in our example, Dear Reader – Vivaldi actually died in Vienna). Let the square-bracketed [Vivaldi died in Venice] denote the meaning of "Vivaldi died in Venice"; and let's write $\mathcal{M}$([S],$\{S\}) = v$ to express that Russell's $\mathcal{M}$-function has value v for a certain sentence-meaning and state of affairs as its arguments. Clearly, we should have $\mathcal{M}$([S],$\{S\}) =$ True and $\mathcal{M}$([S],$\{\neg S\}) =$ False. But the crucial question is: What should be the value of our function for an arbitrary pair $\langle$[S],$\{T\}\rangle$ where S and T are different sentences. What, for example, is $\mathcal{M}$([Vixens are female],$\{$*Haydn invented the string quartet*$\}$)? The question poses a dilemma. Either $\mathcal{M}$ is defined for such ill-matched argument pairs, or it is not. Since Haydn's invention of the string quartet has no bearing upon the sex of vixens, it is natural to declare $\mathcal{M}$ undefined in this case. But if we say this here, we must say it everywhere except when first and second arguments are specified using the same sentence or its negation. And that would be hopeless for Russell's purpose, since the only argument pairs for which an *analytic* sentence A will be defined will be $\langle [A], \{A\}\rangle$ and $\langle [A], \{\neg A\}\rangle$ – or perhaps, slightly less restrictively, for pairs in which the sentence used to specify the second argument expresses the same proposition as that used to specify the first. The result is that the desired contrast with non-analytic sentences is lost – since for exactly the same reason, we should deny that $\mathcal{M}$ is defined for all other argument pairs, such as $\langle$[Vivaldi died in Venice],$\{$*Haydn was born in Rohrau*$\}\rangle$ in which the state of affairs figuring as the second argument has no bearing on the truth-value of the proposition which figures as the first.

If, on the other horn, we insist that $\mathcal{M}$ *is* defined for such pairs, and claim that analytic statements are precisely distinguished by the fact that when a statement A is analytically true, $\mathcal{M}$([A],$\{B\}$) has the value True regardless of the state of affairs serving as its second argument, we thereby surrender all grip on the idea, encapsulated in the truth-meaning platitude underpinning the two-factor objection, that even in the case of an analytic statement A, there is a *particular* worldly factor that is distinctively relevant to A's truth, viz. the fact that A.

It is true that Russell herself seems to have in mind that the worldly factors which redundantly determine $\mathcal{M}$'s value in the case of analytic sentences are not states of the world understood in this particularistic way but are something more like possible worlds in the usual sense, i.e. *complete* ways for the world to be.[29] But this makes no difference. The objection does not go away. Rather, it resurfaces as the requirement that in order for a state of the world, globally so interpreted, to redundantly determine A's truth-value as True, it has to be the case that any possible state of the world – any possible argument for the second place in the $\mathcal{M}$-function – will incorporate the fact that A. This requirement has no counterpart in Russell's prototype of multiplication by zero – there is, as it were, no *zero-specific* requirement placed on the second factor in the multiplication if zero is to be the product, in the way that there is an A-specific requirement placed on the global worldly states if True is always to be the value. Moreover there now seems to be an imminent danger that, so far from making space for something like the traditional idea of truth-purely-in-virtue-of-meaning, the resulting proposal has the order of determination the wrong way round. It is down to the nature of zero that the result of multiplying it by any number is zero. Correspondingly, for a defender of the traditional idea, it ought to be down to the nature – the meaning – of an analytic truth A that the value $\mathcal{M}$ gives for the pair consisting of it and any possible state of the world is True. But on Russell's proposal, with states of the world now globally understood, matters seem to run the other – wrong – way around: that is, it is only because every possible state of the word incorporates the fact that A that $\mathcal{M}([\text{A}], \{\ldots\})$ yields True whatever replaces the dots.[30]

[29] Her term 'state of the world' for the second argument could, of course, be interpreted either way. But in giving particular examples (see [Russell(2008)], p.35, fn.4), she uses w_α and w_β, explaining that 'w_α denotes the actual world, and w_β denotes a possible world in which snow is black'.

[30] And of course it does not matter, for all that has so far been said, *why* every possible state of the world incorporates the fact that A – why there are no non-A worlds. This is in effect why, in his review of Russell's book, Boghossian was able justly to complain that her proposed solution "still leaves us with the problem of distinguishing the merely necessary from the analytic, because we can equally well say that in the case of a necessarily true sentence, the truth-value is 'fully determined' by the meaning factor alone" (see [Boghossian(2011)], p.371).

Russell is, of course, fully alive to the danger that her attempt to rescue the metaphysical notion of analyticity will lead to all necessary truths counting as analytic, and devotes a large chunk of her book (chapters 2 and 3) to addressing it. We have no space pursue that question here, beyond observing that the point we have pressed does not depend upon its resolution. As far as we can see, the complications and revisions she introduces in these chapters are entirely driven by the need to avert the threatened collapse of analyticity into necessity, and have no bearing on the two-factor objection, which she appears to take to have been adequately answered in the preceding chapter. The ensuing complications do not materially affect that answer. True enough, the binary function $\mathcal{M}$ from meaning-world pairs to truth-values is replaced (see [Russell(2008)], pp.53-7) by a quaternary function $\mathcal{M}'$ whose arguments are a context of introduction, a context of utterance and a context of evaluation for an expression, along with what she calls a 'reference determiner' associated with it. In case the expression is a sentence, the $\mathcal{M}'$-function takes quadruples of such arguments to a truth-value. Crucially, however, the third argument – the context of evaluation – will still be, in effect, just as with the simpler $\mathcal{M}$-function, a 'state of the world'. And that is enough to set up the complaint of the Appendix.

Appendix 3: Higher-order singular analyticities

As noted in the main text (see p.21), there are candidate analyticities, deploying what we called higher-order singular terms, which are not – at least not obviously – obtainable from analytic open sentences by instantiation or quantification, and which, therefore, pose a challenge to the essentially schematic conception of analyticity we have presented as a development of Bolzano's central idea. As illustrative examples, consider:

> Red is a colour
> Red is a property
> Red is different from green
> Crimson is a determinate of red
> Temporal precedence is a relation
> Temporal precedence is transitive
> Addition is a function from numbers to numbers
> The natural numbers are closed under addition

These are all, intuitively, as good a range of candidates for epistemic analyticity as are any sentences. They are all, that is, such that it is tempting to say that someone who fully understands them is thereby put in position to know that they express truths. Yet how might they be accommodated by the schematic conception?

A comprehensive treatment of such examples is beyond the scope of the present discussion.[31] Here we merely outline what in our view (from the standpoint of a non-sceptic about the notion of analyticity) should be made of them.

The natural first thought, if such cases are to be brought within range of the schematic approach, is to try to translate them into sentences whose analyticity is straightforwardly treatable in terms of that approach. The use of abstract nouns to express, in compressed form, what may more compendiously be expressed as a first-level generalization about concrete entities is common enough in natural languages. Thus we can say 'Wisdom is a virtue' when we might have expressed ourselves less concisely by saying 'Anyone who is wise is, to that extent, virtuous', or some such. No doubt many of the kind of examples illustrated might be brought within reach of the schematic conception by paraphrasing them as generalizations in this kind of way, which would then be covered by *E-Bolzano 4.1* or *E-Bolzano 4.2*. For instance, it might be proposed that our first and sixth examples above might be paraphrased as:

> Anything which is red is, as such, coloured

and

[31] The use of higher-order singular terms seems to us to merit systematic study. It is, arguably, no mere an isolated curiosity; on the contrary, a case can be made that the introduction of such singular terms corresponding to predicates, relational, and functional expressions, often by more or less explicit kinds of nominalization, plays an indispensable rôle in semantic and ontological theorizing. But it raises some hard problems. For a discussion of some of them, see [Hale and Linnebo()].

If one event precedes another, and the second precedes a third, then the first precedes the third

We doubt, however, that the reductive paraphrase strategy can give an adequate account of all the problematic higher-order cases. Even if some of the examples may seem to admit of fairly natural and plausible transformation into lower-level generalizations, it is very doubtful that all do so – how, for example, should we paraphrase 'Red is a property', or 'Addition is a function from numbers to numbers'. But more importantly, what exactly is paraphrase supposed to accomplish? The initial explanandum, remember, is the apparent *epistemic* analyticity of the kind of example illustrated. So is the idea that the *epistemic route* by which the (putative) analyticity of such examples is recognized *goes through* the suggested paraphrase? That seems to us quite implausible, even in the case of the examples like 'Red is a colour', where, on the contrary, it seems that once a speaker has acquired the use of the nouns 'red' and 'colour', she can directly recognize the truth of 'Red is a colour', without the need for any detour through the proposed paraphrase.

The right approach, we believe, is to take the singularity of the examples seriously. Rather than essay to see them as some kind of idiomatic variants on lower-level generalisations, we should attempt to account for their distinctive epistemic status in a way that connects directly with their overt syntactic structure. 'Red is a colour' says something about the kind of property that the particular property, red, is; 'Temporal precedence is transitive' likewise says something about the character of the relation of temporal precedence, and so on. These are higher-order singular necessary truths, and a satisfactory account of their distinctive epistemic status should acknowledge them as such.

Singular necessary truths in general are not a rare bird. Since *Naming and Necessity*, philosophers have been very mindful of the kind of singular necessities typified by 'Water is H_2O, 'Heat is molecular motion', 'Saul Kripke is the son of Dorothy and Myer', and so on. These are necessities of *essence*, broadly construed: propositions whose truth is determined by the essential nature of the referents of their subject terms. We suggest it is no different with the examples in the list above. 'Red is a colour' is true in virtue of the essential nature of the property, red; 'Temporal precedence is transitive' is true in virtue of the essential nature of the relation of temporal precedence. Yet the Kripkean examples are of necessities known *a posteriori*, while the higher-order singular statements on our list present, rather, as knowable *a priori* – hence their appearance as, intuitively, epistemically analytic. How then can there be any close comparison between the two kinds of case?

Well, the comparison can be saved while acknowledging the epistemic contrast, provided that the relevant essences, in the case of the higher-order singular statements, and unlike the Kripkean cases, are themselves knowable a priori: more specifically, provided that those aspects of the nature of the associated properties in virtue of which one who understands statements of the relevant kind incorporating higher-order singular terms is empowered to recognise them as true, are given with an understanding of the relevant singular terms.

An account pursuing that thought may draw on what is standardly called an *abundant* conception of properties and relations.[32] According to the abundant conception, it suffices for a predicate to express, or stand for, a property or relation that it be associated with a well-determined satisfaction condition, and the nature of the property or relation involved is, moreover, fully manifest in that satisfaction-condition. Thus the essence or nature of an abundant property or abundant relation does not lie beneath the surface, as it were, awaiting discovery by painstaking scientific investigation, but is open to view, and directly available to anyone who grasps the relevant satisfaction condition (see [Hale(2013)], §11.2).

In such cases, there is an a priori route to knowledge of essence, and this extends to support knowledge a priori of the higher-order analyticities illustrated by our list. A priori recognition that red is a colour rests, not on registering a relation of inclusion between the senses of the nouns 'red' and 'colour', still less on appreciating that it is equivalent to some analytic generalisation featuring the predicates, "red" and "coloured", but on knowing, as competence in its use requires, that the noun 'red' stands for the property attributed by the use of the corresponding adjective – the property whose essence is fully encapsulated in the satisfaction condition associated with the predicate '. . . is red', along with knowledge of a parallel fact about what the noun 'colour' stands for.

If this suggestion is broadly correct, we should recognise that we have to deal with two quite different kinds of analyticity. There is the phenomenon of analytic schematic generality, discerned by Bolzano, on which the main body of our discussion has concentrated; and there is a phenomenon of abundant property essence, illustrated by the present examples. Still, in view of the semantic relationship between (n-place) predicates and the corresponding higher-order terms – which of course needs a proper account – and supposing that an abundant conception of the referents of the latter, and the transparency that enjoins, is accepted, it remains that both phenomena are rooted *au fond* in the satisfaction-conditions of predicates, and thus in meaning.

Which is as it ought to be.[33]

[32] The terminology of abundant and sparse properties originates in [Lewis(1986)]. The general distinction is in [Armstrong(1979)]. See also [Bealer(1982)], [Swoyer(1996)], and [Shaffer(2004)]. For a useful overview, see [Mellor and Oliver(1997)]. Our own discussions include [Hale and Wright(2009)], pp.197-9, 207-9, [Hale(2013)], §§1.12, 7.2, and [Hale(2010)], §§3,9.

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