Essays on financial volatility forecasting

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Abstract

The accurate estimation and forecasting of volatility is of utmost importance for anyone who participates in the financial market as it affects the whole financial system and, consequently, the whole economy. It has been a popular subject of research with no general conclusion as to which model provides the most accurate forecasts. This thesis enters the ongoing debate by assessing and comparing the forecasting performance of popular volatility models. Moreover, the role of key parameters of volatility is evaluated in improving the forecast accuracy of the models. For these purposes a number of US and European stock indices is used. The main contributions are four. First, I find that implied volatility can be per se forecasted and combining the information of implied volatility and GARCH models predict better the future volatility. Second, the GARCH class of models are superior to the stochastic volatility models in forecasting the one-, five- and twenty two-days ahead volatility. Third, when the realised volatility is modelled and forecast directly using time series, I find that the HAR model performs better than the ARFIMA. Finally, I find that the leverage effect and implied volatility significantly improve the fit and forecasting performance of all the models.

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1 Introduction and Research Focus

1.1 Introduction

Stock market volatility has been one of the most attractive and successful areas of research in time series econometrics and financial economics over the last few years. Indeed, as Campbell et al. (1997) noted: *"...what distinguishes financial economics is the central role that uncertainty plays in both financial theory and its empirical implementation..."* (p. 3). Volatility has become a crucial issue not only for investors, but also for almost anyone who is involved in the financial markets, even as a spectator.

To many among the general public, the term volatility refers to the fluctuations in asset prices within a short period of time. To them, volatility is synonymous with risk and the quantity of volatility they have to face is a key input in order to take decisions about their investments and portfolio creations. Market participants are willing to bear a certain level of risk. For this reason there is the need of a good forecast of the behaviour of stock market volatility. In the economic sense, Andersen et al. (2006) define volatility as *"the variability of the random (unforeseen) component of a time series. More precisely, or narrowly, in financial economics, volatility is often defined as the (instantaneous) standard deviation (or "sigma") of the random Wiener-driven component in a continuous-time diffusion model"* (p. 780).

The main incentive for the vast empirical and theoretical investigation focusing on the estimation and forecasting of the stock return volatility was the worldwide stock market collapse of 1987. There is an extensive body of research in the US stock market, which examines the changes in stock return volatility because of the 1987 crash. Schwert (1990) examined the influence of the 20.4% decrease in stock prices of the Standard&Poor's (S&P) composite portfolio because of the 1987 crash using daily data from 1885 to 1988. Baillie & DeGennaro (1990) investigated the volatility in the period of the 1987 crash providing evidence that the relationship between stock returns and their volatility is weak.

Volatility is a measure of the dispersion of an asset price about its mean over a specific period of time. This means that volatility is associated with the variance of the asset price. A volatile stock means that the price of the stock has a sizable variation over time, something that makes the stock riskier and can be thought of as a symptom of market disruption.

For those who deal with derivative securities, the need of understanding volatility is mandatory, because it is the key element which permeates most financial instruments. It determines the fair value of an option or any other financial security with these characteristics. The breakthrough in option pricing occured when Black & Scholes (1973) and Merton (1973) developed an analytical model which is known as the Black-Scholes option pricing formula for determining the theoretical value of a European call option. The importance of volatility in their model is determinative, as it is the only parameter that cannot be directly observed from the market opposite to all the other parameters - current stock price, strike price, maturity time and risk-free interest rate - that are all known or can be observed from the market. Except for the valuation of option prices, volatility is significantly essential for asset pricing models and hedging strategies.

Thus, it is evident that the need of estimating and forecasting volatility is of utmost importance for anyone who participates in the financial market as it affects the whole financial system and, consequently, the whole economy. Modelling and forecasting volatility is an important task in financial markets and over the last three decades there is an extensive research that reflects the important role of volatility in investment, option pricing and risk management. Although a plethora of models has been proposed for volatility no conclusion has been reached yet as to which model produces the most accurate volatility estimates and forecasts. The aim of this thesis is to analyse the predictive ability of alternative volatility models and assess the role of key parameters in improving the forecasting performance of these models.

The remainder of this chapter is organized as follows. Section 1.2 introduces the related literature on volatility modelling and forecasting. It also discusses the stylized facts of financial volatility and the proxies have been developed to measure the latent 'true' volatility. In Section 1.3 the outline of the thesis is provided.

1.2 Modelling volatility

Volatility is inherently latent and over the last years several models have been developed in order to estimate and forecast volatility. In the next subsections a variety of alternative procedures for modelling volatility is presented. But first, I introduce some notation useful for the discussion of the different models.

1.2.1 Basic notation and notions of volatility

Based on the work of Andersen et al. (2006) and Black & Scholes (1973), consider an asset whose discrete-time return process is described by the following equation

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}} = \mu_t + \varepsilon_t, \qquad \varepsilon_t \overset{i.i.d.}{\sim} (0, \sigma_t^2) \tag{1}$$

The return at time t, r_t, is the percentage change in the asset price S over the period from $t-1$ to t. This is equal to the decomposition of the return process into the deterministic mean return,μ_t and the random component ε_t. By definition ε_t is a zero mean random disturbance term, serially correlated, and its conditional variance equals σ_t^2, which may be changing over time.

ε_t can be expressed as

$$\varepsilon_t = z_t \sigma_t, \qquad z_t \sim N(0, 1) \tag{2}$$

where z_t is a white noise process and σ_t is the volatility process should be estimated and forecasted. So,

$$r_t = \mu_t + \sigma_t z_t \tag{3}$$

It is, also, useful to think of the return process as evolving in continuous time. The return process may be written in standard differential equation (sde) form as

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{4}$$

where dS is the change in asset price over the time interval dt, μ denotes the drift, σ refers to the spot volatility and dz is a standard Brownian motion process. It is the limiting process of equation (1) as time goes to zero and the result is this lognormal diffusion model. Modern option pricing theory and the Black-Scholes model based on equation (4) in deriving the option pricing formula.

1.2.2 Simple volatility models

The term simple for the models denoted below pertains to the feature of these models not to require parameter estimation.

Historical volatility

The most straightforward way to measure and forecast volatility from asset prices is to measure the historical volatility. Historical volatility can be defined as the variance (or standard deviation) of the return provided by the stock over some historical period and then this becomes the volatility forecast for all future periods (Brooks, 2008). When the return is expressed as the percentage change in the market variable over a specified period, like in equation (1), and assuming that the mean of the return process, $\bar{r}$, is zero, the variance rate, a measure of volatility, is estimated by

$$\sigma_t^2 = \frac{1}{T}\sum_{i=1}^{T} r_{t-i}^2 \tag{5}$$

Exponential Weighted Moving Average

The exponential weighted moving average (EWMA) is an extension of the historical volatility introduced by Riskmetrics. The EWMA approach has the attractive feature that allows more recent observations to affect more the forecast of volatility that the events belonging further to the past.

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + r_{t-1}^2 \tag{6}$$

where λ is the decay factor which governs the weight is given to all lagged observations. Riskmetrics has set the decay factor at $\lambda = 0.94$ for data sampled at a daily frequency and $\lambda = 0.97$ for montly data.

1.2.3 Characteristics of volatility

It is well known that there are several salient characteristics about financial volatility. Athough volatility is inherently latent, its features are well documented through theory and empirical analysis. Many volatility models have been developed in order to incorporate some of these stylized facts. This section highlights and briefly discusses some of these characteristics.

Volatility clustering

It is first observed by Mandelbrot (1963b) who wrote that *"Large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes"* (p. 418). Later on, Fama (1965) stressed that large price changes are followed by large price changes, but of unpredictable sign. From such observations, one can conclude that volatility is not constant, but is varying through time and serially correlated, something that gives motivations to GARCH and stochastic volatility models (see below sections 1.2.4 and 1.2.5, respectively).

Leptokurtosis

Asset prices tend to have fat tails as it has been noted by Mandelbrot (1963b): *"The empirical distributions of price changes are usually too "peaked" to be relative to samples from Gaussian populations ... the histograms of price changes are indeed unimodal and their central bells remind the Gaussian ogive. But, there are typically so many outliers that ogives fitted to the mean square of price changes are much lower and flatter than the distribution of the data themselves."* (pp. 394-395). Fama (1965) found evidence of excess kurtosis in the distribution of stock returns. This characteristic led to a literature where stock returns are modeled as independently and identically distributed random variables having some thick-tailed distribution (Degiannakis & Xekalaki, 2004).[1]

[1] See, for example, Mandelbrot (1963a,b), Clark (1973), Hagerman (1978)

Leverage effect

Black (1976) was the first one observed that changes in stock returns seem to be negatively correlated with changes in stock volatility. The phenomenon of the asymmetric response of volatility to negative and positive returns of the same size is the so-called leverage effect. Fixed costs, like financial and operating leverage can partially interpret it. (see, e.g. Black, 1976 and Christie, 1982) Leverage effect is noticeable by plotting the market price and its volatility. Schwert (1989) shows evidence that periods of market recession are characterized by higher volatility.

Long memory

While stock returns are uncorrelated or exhibit a weak autocorrelation, they are dependent. Stock returns are not independently and identically distributed (Ding et al., 1993a). There is slow decay autcorrelation in absolute and squared returns. This is interpreted as a sign of long memory in volatility.

Non-trading periods

Financial markets seem to be affected by the information accumulated during non-trading periods. This reflects in the prices when the markets reopen, causing an increase in the volatility which is not proportional to the period the market was close. As Fama (1965) and French & Roll (1986) found, information accumulates slower when markets are closed than when they are open. Also, as French & Roll (1986) and Baillie & Bollerslev (1989) demonstrated, volatility tends to be higher following weekends and holidays, but not as much as it would be under a constant rate of information.

Forecastable events

Forecastable announcement of important information is connected with high ex ante volatility. For example, Cornell (1978) and Patell & Wolfson (1979, 1981) show that volatility is higher when earning announcements are expected. Also, across a trading day, there are forecastable events that increase volatility. For instance, volatility is usually higher in the beginning and end of a trading day. (see, for example, Harris, 1986, Baillie & Bollerslev, 1991)

Co-movements in volatility

Another characteristic of volatility is that changes in market volatility tend to change stock volatilities in the same direction as noted by Black (1976). As it has been documented later, this commonality in volatility changes also applies across different markets.[2]

Obviously, volatility has many features that financial economists and econometricians should guide in their choice of models and model builders should consider when developing a model. Of course, not all of these characteristics should be included in order a forecasting volatility model to be successful.

1.2.4 ARCH/GARCH Models

While it has been long recognized that the assumption of constant volatility is inefficient and that volatility clusters (see, Bollerslev et al., 1992 and Bera & Higgins, 1993), it is only since the introduction of ARCH/GARCH model (Engle, 1982; Bollerslev, 1986) that these temporal dependencies have been modelled using econometrics techniques. Since then, there is a voluminous literature that evaluates the predictive power of GARCH models against the simple statistical models.

ARCH

The current interest in modelling and forecasting asset return volatility has been spurred by the pioneering work of Engle (1982), in which he introduced one of the most prominent tools that has emerged for characterizing time-varying volatility, the Autoregressive Conditional Heteroskedasticity (ARCH) model. In the ARCH model, conditional variance varies over time and is a linear

[2] For example, Engle et al. (1990), Hamao et al. (1990) and King et al. (1994) investigated the inks between volatility changes across international markets.

function of past squared error terms.

Consider that returns follow the process as shown in equations (1) and (2), which for convenience is repeated here.

$$r_t = \mu_t + \varepsilon_t \tag{7}$$

where

$$\varepsilon_t = z_t h_t, \qquad z_t \overset{i.i.d.}{\sim} (0,1) \tag{8}$$

where h_t is the conditional variance.

The ARCH model characterizes the distribution of the stochastic error term, ε_t, conditional on all relevant information through time $t-1$. So, it assumes that

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t^2)$$

where

$$h_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 \tag{9}$$

with $a_0 > 0$ and $a_i \geqslant 0,\ i = 1,...,q$ in order to be sure that conditional variance will be positive. This process is referred to as *ARCH(q) process*.

GARCH

The generalized ARCH (GARCH) model, which has been developed by Bollerslev (1986), provides a parsimonious parameterization for the conditional variance

$$h_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}^2 \tag{10}$$

with $a_0 > 0, \quad a_i \geqslant 0\ for\ i = 1,...,q\ and \quad \beta \geqslant 0\ for\ j = 1,...,p$. This process is referred to as *GARCH(p,q) process*. It generates a one-period ahead estimate for the variance as a weighted long run average variance (a_0), information about previous volatility ($\sum_{i=1}^{q} a_i \varepsilon_{t-i}^2$) and the previous estimated variances ($\sum_{j=1}^{p} \beta_j h_{t-j}^2$). The model is covariance stationary if and only if $\sum_{i=1}^{q} a_i + \sum_{j=1}^{p} \beta_j < 1$. Its unconditional variance is constant and equal to

$$h = \frac{a_o}{1 - \sum_{i=1}^{q} a_i - \sum_{j=1}^{p} \beta_j}$$

The GARCH(p,q) models successfully captures some of the characteristics of asset returns, like volatility clustering and leptokurtosis and can be readily modified to capture features such as non-trading periods and forecastable events. However, its structure enforces important restrictions. For this reason, numerous extensions of the GARCH model have been developed.

The empirical success of the GARCH models triggered the development of other more sophisticated GARCH models. For example, models that exploit the long memory characteristic of volatility have been developed such as the FIGARCH models of Baillie et al. (1996) and the FIEGARCH of Baillie et al. (1996). The component GARCH model of Engle & Lee (1993) and the related development in Gallant et al. (1999) and Muller et al. (1997) as well the multifractal model of Calvet & Fisher (2004) are alternative ways of capturing long memory volatility dynamics. Moreover, the presence of the leverage effect, i.e. the strong negative relationship between the stock returns and volatility, is a robust empirical finding and many papers have been written looking at modelling leverage effect in stock returns. Foe this reason, Nelson (1991) and Glosten et al. (1993) have been proposed two of the most popular extensions of GARCH, the exponential GARCH (EGARCH) and the GJR-GARCH, respectively. These models seem to provide more accurate forecasts than the simple GARCH. For example, Cao & Tsay (1992) favor the EGARCH model for stock indices and exchange rates, while Brailsford & Faff (1996) find GJR better than GARCH for stock indices.

1.2.5 Stochastic volatility models

Another class of time-varying volatility models is known as stochastic volatility (SV) models. As its name implies, SV models differ from the GARCH class of models in the assumption of the latter that volatility is a deterministic function of observable variables given all information available. In SV models, volatility is a random latent variable. According to the work of Clark (1973), SV models postulate that volatility is a function of random information arrival that may be unobservable. Thus, volatility will have some unpredictable component.

Consider, again, returns follow the process as shown in equation (7). Assuming that the drift is negligible for small time horizons, the basic log-normal AR(1)-SV model of Taylor (1986) is defined as

$$r_t = z_t exp(0.5h_t), \qquad z_t \overset{i.i.d}{\sim} N(0,1) \tag{11}$$

where

$$h_t = a_0 + \sum_{j=1}^{p} \beta_j h_{t-j} + \eta_t, \qquad \eta_t \overset{i.i.d.}{\sim} N(0, \sigma_\eta^2) \tag{12}$$

where η_t is an innovation term which could be correlated with z_t. This additional innovation in the dynamics of the conditional variance allows SV model to be more flexible in describing stylized facts than the GARCH models (Poon & Granger, 2003).

The fact that the SV model allows the logarithm of the volatility to evolve, it is ensured the positivity of the conditional variance of the process without the need of further constraints. Unlike the SV models, in GARCH models constraints imposed on the parameters in order to ensure that the volatility remains always positive are often violated during the process of estimation. The process h_t and $\sum_{j=1}^{p} \beta_j$ in (19) can be interpreted as the random process of new information arrivals in financial markets and the persistence in the volatility, respectively. There are also different specifications of the SV models. For example, Jacquier et al. (1994) model the log of h_t as an AR(1) process, so that $r_t = \sqrt{h_t} z_t$ and $logh_t = a_o + \sum_{j=1}^{p} \beta_j logh_{t-j} + \eta_t$, which is clearly equivalent to equations (18) and (19), respectively.

The effect of the leptokyrtosis that many financial series exhibit, can be incorporated in SV models. By allowing z_t in equation (18) to have a standardized student t-distribution as used in Harvey et al. (1994a), Chib et al. (2002) and Jacquier et al. (2004). With regard to accounting for the leverage effect, several extensions of the SV model exist. Harvey & Shephard (1996), Jacquier et al. (2004) and Yu (2005) allow a negative contemporaneous correlation between the innovation

z_t and η_t to allow for asymmetry.

The flexibility of the SV models to describe stylized facts has drawn the attention of the academics.[3] Another advantage of SV models is their theoretical background that is, SV models are closer to theoretical models in finance particularly those in option pricing. On the other hand, one of the most important limitations of the SV models, unlike the GARCH models, is their analytical intractability, because they have no closed form solutions. As a result, it is hard the likelihood function to be evaluated. However, last years advances in research provided various powerful methods for estimating and forecasting SV models, such as the Method of Moments (MM) approach, variations of the Generalized Method of Moments (GMM) approach through simulations, analytical solutions and the likelihood approach through numerical integration.[4]

Although the SV models were developed in parallel with the GARCH models, they have received much less attention in the volatility forecasting literature, because of their estimation complexity. The few studies that evaluate and compare the forecasting performance of the discrete-time SV model with the GARCH have not reach a conclusion as to which model class performs best, see Yu (2002), Bluhm & Yu (2000), Pederzoli (2006), Chortareas et al. (2011) among others.

1.2.6 Implied volatility

An alternative option for modelling volatility for cases in which traded options exist is the use of implied volatility. Implied volatility is based on the Black-Scholes model and various generalizations. As previously mentioned in the introduction, the Black-Scholes option pricing formula gives the fair value of a call option c as a function of

$$c = f(S, K, \sigma, r, T) \tag{13}$$

where S is the price of the underlying asset, K is the strike price, σ is the volatility, r is the risk-free interest rate and T is the time to maturity. All the independent variables are directly observables except for the volatility, σ, that must be estimated. Since the market price of an option

[3] See the review papers byTaylor (1994), Ghysels et al. (1996).
[4] For a review of the estimation of the SV models see the surveys of Shephard (1996) and Broto & Ruiz (2004).

is observable, it is possible to solve the Black-Scholes model backwards from the observed price to derive or imply what the market volatility should be. This measure of volatility is called implied volatility and it is often used as a market's expectation of volatility over the options' maturity.

Over the last decades, there is a vast academic research about implied volatility. In particular, initially, academic interest focused one the issues concerning the estimation difficulties of implied volatility. In efficient markets, by definition, each asset has only one volatility. Difficulties arise when option traded on that asset with the same expiry, but with different strike price produce different implied volatility estimates. The implied volatility obtained by Black-Scholes option pricing model varies with respect to the strike price, so as to deep-in-the-money and deep-out-of-the-money options exhibit higher volatility than at-the-money options. Volatility smile, skew and smirk are names given to non-linear shapes of implied volatility plots against the strike price (Poon & Granger, 2003). Starting with Latane & Rendleman (1976) and Chiras & Manaster (1978), various different weighting schemes have been proposed. Another research category focused on the implied volatility's information content regarding future realised volatility and its ability to predict the latter. For example, Engle & Ng (1993) found that historical volatility provides significant superior information compare with implied volatility.[5]

Although early studies of option implied volatility suffered many estimation deficiencies, a good number of more recent studies, such as Christensen & Prabhala (1998), Pong et al. (2004) and Jiang & Tian (2005), found that implied volatility contains a significant amount of information about future volatility and it sometimes is better than volatility forecast is produced by more sophisticated time series models.

The importance of the implied volatility can be seen from the fact that the Chicago Board Option Exchange (CBOE), in 1993, became the first organized exchange that introduced implied volatility indices. In 2003 the construction of VIX changed and the popular VIX uses the current prices of the S&P500 index options to represent the expected future market volatility over the next 30 calendar days. Following the successful example of CBOE, many other exchanges across the world have developed their own indices. Thereafter, there is a large amount of literature that assess

[5]For a review of forecasting volatility see Figlewski (1997) and Poon & Granger (2003).

the information content of implied volatility in the context of forecasting volatility (Blair et al., 2001; Koopman et al., 2005; Giot, 2003). Moreover, the forecastability of implie volatility per se is a more recent relatively underresearched area, see Konstantinidi et al. (2008), Fernandes et al. (2014) among others.

1.2.7 Realised volatility

An important element in the context of accurately estimating and forecasting volatility is the measure of the 'true' volatility. As volatility is latent, a proxy is necessary. For several years the ex-post daily squared returns have been used to evaluate volatility forecasts. However, the last 15 years the availability of high-frequency data have evolved the literature on measuring and forecasting. Andersen & Bollerslev (1998) first used the high-frequency data to construct a new volatility measure. They showed that the so-called realised variance (RV), computed by the sum of squared intraday returns, is a more precise measure of volatility than the ex-post daily squared returns.

The study of Andersen & Bollerslev (1998) was an answer on the critique about GARCH models. Until then, several papers had noted that while GARCH models were successful in modelling volatility, they were explaining little of the variability in ex-post squared returns(Figlewski, 1997; Jorion, 1995). However, Andersen & Bollerslev (1998) found that the poor perfomance of the GARCH model is not a failure of the model itself, but a failure to correctly specify the measure of the true volatility. Although daily squared returns is an unbiased estimate of volatility it is a noisy measure. More specifically, consider the returns r_t such that $r_t = \sigma_t z_t$, where σ_t is the time-varying volatility and $z_t \sim i.i.d.(0,1)$. The volatility proxy using squared returns is $r_t^2 = \sigma_t^2 z_t^2$ and if σ_t is correctly specified then $E(r_t^2) = \sigma_t^2$. However, the r_t^2 is a noisy estimate of σ_t^2 due to the noisy component z_t^2. Thus, Andersen & Bollerslev (1998) suggest that the measure of the true volatility should be based on cumulative intraday returns, because the noisy component is diminished.

$$RV_t = \sqrt{\sum_{i=1}^{N} r_{t,i}^2} \tag{14}$$

where $r_{t,i}$ is the ith intraday returns on day i. Andersen et al. (2001a,b) and Barndorff-Nielsen & Shephard (2002a,b) show that RV is a precise estimator of the latent integrated volatility.

Since then RV is the dominant proxy in the literature. A large part of the literature focuses on determining the best possible way for measuring daily volatility using intraday data. Several alternatives to the standard RV measure have been proposed to alleviate microstructure noise (Barndorff-Nielsen et al., 2008; Hansen & Lunde, 2006; Zhang et al., 2005) or to detect jumps, see Barndorff-Nielsen & Shephard (2004) among others.

RV was primarly used as an estimator of the actual volatility to assess the forecasting performance of the volatility models. The availability of high frequency data has also inspired research into the potential vlue of RV as an information source to improve existing volatility models (Blair et al., 2001; Engle, 2002; Hol & Koopman, 2002). These studies indicate that intraday return series contain incremental information for future volatility beyond that contained in GARCH and SV models.

Alternatively, as Andersen et al. (2003) noted, the intraday volatility process modelled directly strongly outperforms the popular GARCH and SV models. They proposed to model the logarithm of RV using a Autoregressive Fractionally Integrated Moving Average (ARFIMA) model in order to capture the long memory feature of volatility.

$$\phi(L)(1-L)^d(log(RV)-\mu) = \theta(L)\varepsilon_t \tag{15}$$

where $\phi(L)$ and $\theta(L)$ is the lag operator that defines the autoregressive and moving average components, respectively, and ε_t is a Gaussian white noise with mean zero and variance σ_t^2. Following Andersen et al. (2003), a number of studies evaluates the forecasting performance of the ARFIMA model over the GARCH and SV models (Koopman et al., 2005; Hol & Koopman, 2002; Pong et al., 2004; Martens et al., 2009).

However, Corsi (2009) pointed out that the ARFIMA model is a convenieant math trick, but without a clear economic interpretation. Corsi (2009), based on the Heterogeneous Market Hypothesis, proposed the Heterogeneous Autoregressive (HAR) model, an additive cascade model of different volatility components over different time horizons. The HAR model is

$$RV_{t+1} = \alpha_0 + \alpha_d RV_t + \alpha_w RV_{t-5,5} + \alpha_m RV_{t-22,22} + u_t \tag{16}$$

So the HAR model predicts future volatility using three volatility components, the daily, weekly and monthly. Although its simple structure the HAR model can successfully forecast volatility and Corsi (2009) using three series, the S&P500, USD/CHF and T-Bond found that the HAR steadily performs better than short-memory models and is comparable to the ARFIMA. Following the work of Corsi, several papers evaluate the forecasting performance of the HAR model and many extensions have been examined in order to account for different stylized facts of volatility, see Andersen et al. (2007), Corsi et al. (2008), Corsi & Renò (2012), Bollerslev et al. (2009) among others.

1.3 Outline of Thesis

The accurate estimation and forecasting of volatility in financial market is an issue of crucial importance and has been a popular subject of research with no general conclusion as to which model provides the most accurate forecasts. This thesis aims to determine the model that best forecast future volatility. In particular, this research looks into the role of key parameters in improving the fit and forecasting performance of various volatility models. For the purposes of my analysis an extensive dataset of US and European stock market indices is used assessing whether the results may different across countries.

Chapters 2 and 3 evaluate the predictive ability of GARCH and implied volatility models using US and European indices, respectively. More specifically, the goal of these chapters is to assess whether IV forecast is a better predictor of stock return volatility than the GARRCH. These chapters bring together two dinstict strand of literature in order to assess the model that produces the most accurate forecast. First, I investigate the importance of explicitly incorporating several stylized facts of volatility, volatility clustering, the leverage effect and long memory, in the GARCH models as well as the potential value of IV as an information source for the purpose of forecasting.

Second, I examine the forecastability of IV itself using a range of autoregressive models that account for the leverage effect and the persistence of voaltility. The results show that IV follows a predictable pattern. An ARMA model that accounts for the contemporaneous asymmetric relationship between IV and stock index returns performs best. Moreover, IV contains incremental information about future volatility beyond that contained in GARCH models. The inclusion of the leverage effect and long memory in the GARCH model improves its performance. In particular, the GARCH specification that simultaneously accounts for the leverage effect and IV performs best. While IV is more informative than GARCH, the information content of both predictors are complementary. Results are consistent using both the ex post daily squared returns and RV as measure of true volatility, and for both US and European indices. Finally, this evidence is further supported by consideration of value-at-risk.

Chapter 4 investigates the performance of the under-utilized in the literature SV models. I examine whether the use of the leverage effect and IV improve both the in-sample and out-of-sample performance of the SV models, as in Chapters 2 and 3 significantly improve the accuracy of the GARCH models. I further compare the SV models with two popular GARCH specifications, the GARCH and EGARCH. The results indicate that incorporating implied volatility in the stochastic volatility model significantly enhances the performance of volatility forecasts. In contrast, the presence of the asymmetric effect seems not to significantly improve the performance of the SV models. Overall, the EGARCH-IV model produces the most accurate volatility forecast at one day horizon. For longer horizons, the GARCH-IV model performs best.

Chapter 5 explores the forecasting performance of ARFIMA and HAR models for realised volatility. For the purpose of forecasting I investigate the importance of explicitly incorporating several stylized facts of volatility in these models, the long memory, leverage effects, volatility of RV and IV. The results suggest that the HAR class of models performs better than the ARFIMA. Taking simultaneously into account IV and leverage effect significantly improve the forecasting performance of the models. In contrast, modelling the volatility of RV does not substantially improve the performance of the HAR models. Results are consistent under both the rolling and recursive scheme.

2 Forecasting stock return volatility: a comparison of GARCH models and implied volatility

The accurate estimation and forecasting of volatility in financial market is an issue of crucial importance and has been a popular subject of research with no general conclusion as to which model provides the most accurate forecasts. There is an extensive literature that addresses the question of whether implied volatility (IV) contains any additional information useful to predict future volatility beyond that embedded in GARCH models. Recent studies suggest that IV can be forecasted. This chapter builds on these two strands of literature by investigating whether the IV forecast is a better predictor of stock return volatility by analyzing the forecasting performance of GARCH and IV models for the S&P500, DJIA and Nasdaq100 stock indices. The results indicate that IV *per se* can be forecasted. Using both *ex post* daily squared returns and realized variance the results show that when IV forecast incorporates the contemporaneous positive and negative returns is a good predictor of future stock return volatility. In most cases, IV is more informative than GARCH. Nevertheless, a model which combines the information contained in an asymmetric GARCH with the information from option markets through an ARMAX model is the most appropriate for predicting future return volatility.

2.1 Introduction

Modelling and forecasting volatility is an important task in financial markets. Over the past few decades there is an extensive research agenda that has analyzed the importance of volatility in investment, option pricing and risk management. Thus, an accurate estimation and forecasting of asset returns volatility is crucial for assessing investment risk.

The topic of volatility forecasting has received extensive attention in the literature by both academics and practitioners. The main focus of the literature has been on the type of models used to produce accurate volatility forecasts. Broadly speaking, there are two approaches that the majority of researchers adopt to generate volatility forecasts. The first method is to extract information about the variance of future returns from historical data using simple models, GARCH-type models or stochastic volatility models. The second method is to extract market expectations about future volatility from observed option prices, using the implied volatility (henceforth IV) indices. The focus of this study lies on the GARCH-type models and implied volatility.

The observation of clustering in stock market volatility (Mandelbrot, 1963b; Fama, 1965) has been long ago recognized. However, it is only since the introduction of ARCH model by Engle (1982) and its generalization (GARCH) by Bollerslev (1986) that these temporal dependencies have been modelled using formal econometric techniques. The GARCH class of models describes the conditional variance of the returns. The empirical success of the GARCH models triggered the development of other more sophisticated models. Models that take into account the leverage effect, such as the exponential GARCH (EGARCH) model by Nelson (1991), the GJR-GARCH model of Glosten et al. (1993), the asymmetric power ARCH model of Ding et al. (1993b) and several others have been developed over the years. Moreover, GARCH models that accommodate the long memory feature of volatility have been proposed. Examples of such models are the integrated GARCH (IGARCH) by Engle & Bollerslev (1986), the component GARCH (CGARCH) of Engle & Lee (1993), the fractionally IGARCH (FIGARCH) of Baillie et al. (1996) and the FIEGARCH of Bollerslev & Mikkelsen (1996).

In contrast to GARCH models, implied volatility is a forward-looking measure of volatility. In the framework of an option pricing model, such as the Black-Scholes model (Black & Scholes, 1973; Merton, 1973), implied volatility is the volatility that equates the market price of the option with the model price. Implied volatility as a concept has gained a growing interest since 1993 when CBOE launched a volatility index (VIX) based on the S&P100 index options as a measure to assess the market expectations of the future volatility. IV is frequently considered as a measure of the market risk and hence as an input to many asset pricing models. Thus, the issue of the predictability of IV is very important. Over the last years, IV index has become a leading indicator for measuring and predicting the performance of stock markets.

The aim of this chapter is to make an empirical comparison between a wide range of GARCH-type models and IV indices models, so as to choose the model that produces the most accurate volatility forecasts. To this end, symmetric, asymmetric and long memory GARCH models have been used as well as ARMA type models for modelling and forecasting IV indices.

The remainder of the chapter is organized as follows: In the next Section, I review the literature. Section 2.3 introduces the data and the methodology employed. Section 2.4 presents the empirical results and finally, Section 2.5 summarizes and concludes.

2.2 Background and related work

There are several studies that investigate the forecasting ability of GARCH models against naive technical analysis with mixed results. For example, Akgiray (1989) is one of the first studies that investigates the performance of GARCH models. Using data from the US stock market the author reports that GARCH(1,1) consistently outperforms exponential weighted moving average (EWMA) and historical volatility. Cumby et al. (1993) conclude that EGARCH is better than historical volatility. On the other hand, Tse (1991), Tse & Tung (1992), Boudoukh et al. (1997) and Walsh & Tsou (1998), using different stock markets, provide evidence that some EWMA-type specifications are superior to the GARCH model for forecasting volatility of a wide range of assets. Finally, other studies find ambiguous results. For example, Brailsford & Faff (1996) examine the performance of different statistical methods and GARCH type models for the Australian stock market and are unable to identify a clearly superior model.

However, the usefulness of GARCH models in providing accurate volatility forecasts has been strengthened by the research of Andersen & Bollerslev (1998) and Andersen et al. (1999). They provide evidence that the use of ex post daily squared returns as the proxy for the 'true' volatility is defective and suggest the so-called realized volatility which is based upon the sum of squared intraday returns. Using the realized volatility as the measure of true volatility, McMillan & Speight (2004), among others, in a dataset of 17 daily exchange rate series, have provided evidence in favor of GARCH models.

An alternative to GARCH volatility forecasts have been proposed through the use of implied volatilities from options. A number of empirical studies (Latane & Rendleman, 1976; Chiras & Manaster, 1978) support the idea of using implied volatility as a predictor for future volatility and hence it is of interest to compare its forecasting accuracy with that of GARCH volatility forecasts. Early studies conclude that IV is biased and inefficient and performs very poorly when compared with volatility forecasts based on historical returns. For example, Day & Lewis (1992) compare the information content of IV for the S&P100 index options to GARCH type conditional volatility and find that IV contains predictive information about future volatility beyond that contained in GARCH models. A similar conclusion has been reached by Lamoureux & Lastrapes (1993) who study several individual stocks. But the findings in these studies are subject to a few measurement errors. Overcoming these problems, more recent papers favour the conclusion that IV is informationally efficient in forecasting future volatility. For example, Christensen & Prabhala (1998) utilize the non-overlapping samples to study S&P100 index options and document that IV outperforms historical volatility.

The original VIX has been launched by Chicago Board Options Exchange (CBOE) in 1993 and was based on the calculation of the S&P100 stock options. Since then the VIX has become a natural choice to study the dynamics of market IV and forecast the performance of stock markets. In 2003, the construction of VIX changed and since then it is based on a broader index, the S&P500. The VIX uses the current prices of the S&P500 index options to represent the expected future market volatility over the next 30 calendar days (Whaley, 2009). It essentially offers a forward-looking measure of one-month ahead stock market volatility. It is also referred to as the investor's 'fear gauge', because it reflects investors' expectations about near term volatility. A higher VIX indicates that market participants are expecting a higher volatility in the stock market, while a lower VIX proposes moderate fluctuations in the stock index (Simons, 2003). Over the last 15 years implied volatility indices have increased quickly in European and U.S. markets.

The accuracy of volatility forecasting has been the subject of extensive research. Literature that compares volatility forecasts embedded in option prices with those from time series models is voluminous. Nonetheless, no conclusion has been reached yet and hence, there is still an ongoing debate between GARCH-type models and IV indices models of finding the best model in estimating and forecasting future volatility.

Using daily index returns and/or intraday returns Blair et al. (2001) for the S&P100 index and the VIX find that VIX provides more accurate forecasts than GARCH-type models in particular as the forecast horizon increases. A combination of VIX and GJR forecast is more informative than VIX and GJR alone when forecasting one-day ahead. For the German economy Claessen & Mittnik (2002) find that, although the null hypothesis that the German IV index (VDAX) is an unbiased estimate for realized volatility is rejected, the GARCH volatility do not contain useful information beyond the volatility expectations already reflected in option prices. Giot (2005a) and Corrado & Miller (2005) conclude that the volatility forecast based on the VIX and VXN indices, i.e. the IV index based on NASDAQ100 index, have the highest information content both for volatility forecasting and for market risk assessment framework. However, Giot (2005a) concludes that combining GARCH and implied volatility often improves on the results from either one alone. Carr & Wu (2006) for the S&P500 stock index, Yu et al. (2010) using stock index options traded over-the-counter and on exchanges in Hong Kong and Japan and Yang & Liu (2012) for the Taiwan stock index reach similar conclusions. Frijns et al. (2010), for the Australian index, find that at short horizons combining GJR-GARCH and IV improve future volatility forecast, but overall IV outperforms the RiskMetrics and GJR-GARCH. In a similar vein, Cheng & Fung (2012) show that while IV is more informative than GARCH, the GARCH forecast improves the predictive ability of Iv for the Hong Kong market. On the other hand, and among others, the results of Becker et al. (2007) contradict the previous studies, because they show that VIX is not an efficient volatility predictor and does not provide any additional information relevant to future volatility. Bentes & Menezes (2012) using data of both emerging and developed economies conclude that GARCH volatility is a better predictor of future realized volatility than IV. Finally, Bentes (2015) using four stock markets show that GARCH is a better predictor of realized volatility than IV.

By contrast, relatively little work has been done on whether the dynamics of implied volatility *per se* can be forecasted. Ahoniemi (2006) uses linear and probit models to model the VIX index. The author finds that an ARIMA(1,1,1) model enhanced with exogenous regressors outperforms. The use of GARCH terms in the ARIMA(1,1,1) model are statistically significant, but do not improve the forecast accuracy of the model. Konstantinidi et al. (2008) examine five alternative model specifications to form both point and interval forecasts using a number of US and European

IV indices. They find that the ARIMA(1,1,1) and ARFIMA(1,d,1) specifications provide the best point forecast for the US indices. In a similar spirit, Dunis et al. (2013) investigate the forecastability of intraday IV on an underlying EUR-USD exchange rate for a number of maturities by combining a variety of forecasting models. They find that the GJR model and the principal component model perform better for one-month and three-months maturity, while ARFIMA and VAR models outperform for longer periods. Finally, Fernandes et al. (2014) perform a thorough statistical examination of the time series properties of the VIX. The out-of-sample analysis shows that ARMA models perform very well in the short run and very poorly in the long run, while the semiparametric heterogeneous autoregressive (HAR) process perform relatively well across all forecasting horizons.

Hence, several issues arise from the existing literature. First, there is no clear-cut conclusion regarding the superior volatility forecasting approach. Second, while IV is often considered as a measure of market risk and, therefore, an input to many asset pricing models, the question whether IV *per se* can be forecasted has received little attention.

The aim of this study is to provide a comparative evaluation of the ability of a wide range of GARCH models and IV models to forecast stock returns volatility. I provide evidence from the S&P500, DJIA and Nasdaq100 indices as well as their IV indices. Specifically, I attempt to answer the question whether implied volatility contains additional information about the future volatility beyond that contained in GARCH forecasts. I examine whether the dynamics of IV *per se* can be forecasted by parsimonious ARMA-type models. I address the question whether the IV forecasts are good forecasts of stock returns volatility, which to the best of my knowledge has not previously been considered in the literature. In my analysis, I also investigate the contemporaneous asymmetric relationship between stock index returns and implied volatility. In total, ten GARCH models are considered, GARCH, GJR, EGARCH, APGARCH and ACGARCH and their 'hybrid' specifications adding the lagged value of the implied volatility. For forecasting IV indices ARMA, ARIMA and ARFIMA models and their unrestricted specifications for capturing the asymmetric relationship between stock index returns and implied volatility are considered.

2.3 Data and empirical methodology

2.3.1 Data

The dataset used for the purposes of this study consists of the daily closing price data for the S&P Composite 500 (S&P500), Dow Jones Industrial Average (DJIA) and Nasdaq100 indices and their implied volatility indices, VIX, VXD and VXN, respectively. Since the various implied volatility indices have been listed on different dates, I consider the period from February 2, 2001 to February 28, 2013 in order to study the indices over the same time period. The in-sample period is from February 2, 2001 to February 23, 2010 consisting of 2,363 daily observations, and the remaining 787 observations (February 24, 2010 to February 28, 2013) will be used for the out-of-sample evaluation. Both the *ex post* squared daily returns and the realized variance are used as proxies for the true volatility. The data of the realized variance are taken from Oxford-Man Institute's Realized Library version 0.2 Heber et al. (2009).

I compute the stock index returns, r_t, by calculating the prices log differences, $r_t = ln\left(P_t/P_{t-1}\right)$. Figure 1 clearly shows that the mean of the returns is constant and around zero, but the variance changes over time showing evidence of volatility clustering.

A non-constant variance of returns indicates a non-normal distribution. Table 1 presents the descriptive statistics of the stock market returns plotted in Figure 1. The mean and the median are consistently close to zero. As far as the values of skewness and kurtosis are concerned, for a normal distribution, they should be zero and three, respectively. The negative skewness of all series indicates asymmetric distributions skewed to the left, while the kurtosis statistics show the leptokurtic characteristic of all returns distributions. The evidence of non-normality is further supported by the Jarque-Bera test statistic which rejects the null hypothesis of normal distribution at the 1% level.

Similarly, Table 2 shows the summary statistics of the IV indices along with Augmented Dickey-Fuller (ADF) test for unit roots. The p-values of the ADF tests show that implied volatility indices are stationary at conventional levels. The IV indices measure the market's expectation over the next 30 calendar days. Thus, the IV indices are expressed in annualized percentages. Therefore, following Blair et al. (2001), the daily implied index volatility is equal to $\frac{IV}{100*\sqrt{252}}$.

2.3.2 Empirical methodology

The aim of this chapter is to compare the volatility forecasting ability of GARCH models and implied volatility indices analyzing the information content of IV.

One way is to add implied volatility as an exogenous variable to GARCH models. By constructing a nested model I can assess whether implied volatility is an important determinant of conditional variance. As shown in the previous section, daily returns exhibit volatility clustering and fat tails. The family of GARCH models have been proven to be particularly suitable for capturing not only these characteristics, but also features like the leverage effect and long memory. In this section, I consider an array of symmetric, asymmetric and long memory GARCH specifications.

In order to establish the methods to be used, the return process is given by

$$r_t = \mu + \varepsilon_t \tag{17}$$

where μ is the constant mean and $\varepsilon_t = h_t z_t$ is the innovation term with $z_t \sim N(0, 1)$.[6]

To determine whether an ARCH process describes the innovation term sequence is equivalent to identify the presence of conditional heteroskedasticity. The squared residual series $\hat{\varepsilon}_t^2$ are employed to test the conditional heteroskedasticity which is known as ARCH effect. This is performed by testing the squared errors for serial correlation.

The two tests for conditional heteroskedasticity used in this exercise are the Ljung-Box test and the Lagrange Multiplier (LM) test. As referred to the Table 3, the Ljung-Box Q(p) statistics of all return series are significant with a p-value equal to zero, which indicates that the squared residuals are autocorrelated. In the same table, according to the LM test the null hypothesis of homoskedasticity is clearly rejected at 1% significant level, indicating the presence of ARCH effect in all return series. These results provide justification for the next stage in the analysis which

[6] Using the AIC and SBIC information criteria I found that an AR(0) model is appropriate for the mean equation.

involves estimating the conditional variance using an ARCH process.

GARCH

The generalized ARCH (GARCH) model, which has been developed by Engle (1982) and Bollerslev (1986), involves a joint estimation of the mean equation (17) and the conditional variance equation. On the assumption that $\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$, the GARCH(1,1) model provides a parsimonious parameterization for the conditional variance as follows

$$h_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \tag{18}$$

with $a_0 > 0, \quad and \quad a_1,\, \beta_1 \geqslant 0$. The model is covariance stationary if and only if $a_1 + \beta_1 < 1$.

The GARCH(1,1) specification augmented by implied volatility is given by

$$h_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + \theta I V_{t-1}^2 \tag{19}$$

Model (18) can be interpreted as the special case of model (19) when $\theta = 0$. The test of interest is given by $H_0 : \theta = 0$. If the null hypothesis is rejected, it means that IV contains incremental information useful for explaining the conditional variance.

The GARCH(1,1) model successfully captures some of the characteristics of asset returns, like volatility clustering and leptokurtosis and can be readily modified to capture features such as non-trading periods and forecastable events. However, its structure enforces important restrictions. For this reason, numerous extensions of the GARCH model have been developed.

GJR

One primary limitation of the GARCH model is its symmetric response to negative and positive shocks. However, negative shocks have been found to increase volatility by a greater amount than positive shocks of the same magnitude. In other words, returns are said to have an asymmetric impact on volatility. As noted by Black (1976) and Christie (1982), stock price fluctuations are negatively correlated with volatility, which entails more uncertainty and hence generates more volatility. This asymmetric behavior is also known as the leverage effect.

Since the first generation symmetric GARCH model is unable to account for the leverage effects observed in stock returns, I evaluate three widely known second generation asymmetric GARCH models.

The GJR model has been proposed by Glosten et al. (1993) and is specified as:

$$h_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1} \tag{20}$$

and its encompassing specification as

$$h_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1} + \theta IV_{t-1}^2 \tag{21}$$

where the leverage effect is captured by the dummy variable I_{t-1}, such that $I_{t-1} = 1 \quad if\ \varepsilon_{t-1} < 0$ and $I_{t-1} = 0 \quad if\ \varepsilon_{t-1} > 0$. $a_0 \geqslant 0,\ a_1 \geqslant 0,\ \beta_1 \geqslant 0\ and\ a_1 + \gamma \geqslant 0$ in order to ensure that conditional variance is positive. Hence, for the GJR-GARCH(1,1), positive news has an impact of a_1, negative news has an impact of $a_1 + \gamma$, with negative (positive) news having a greater effect on volatility if$\gamma > 0$ ($\gamma < 0$).

EGARCH

The exponential GARCH (EGARCH) model has been proposed by Nelson (1991) in order to capture the leverage effect. Nelson (1991) used the EGARCH model to model daily returns of the CRSP value-weighted stock market index in the period 1962-1987. Nelson confirmed that returns are significantly negatively correlated with volatility.

The EGARCH model and its embedded with IV specification are given byR^2

$$ln(h_t^2) = a_0 + a_1\frac{\mid \varepsilon_{t-1} \mid}{h_{t-1}} + \gamma\frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 ln(h_{t-1}^2) \tag{22}$$

and

$$ln(h_t^2) = a_0 + a_1\frac{\mid \varepsilon_{t-1} \mid}{h_{t-1}} + \gamma\frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 ln(h_{t-1}^2) + \theta IV_{t-1}^2 \tag{23}$$

where the coefficient γ captures the presence of the leverage effects if $\gamma < 0$. This model is successful, because, except that it captures the leverage effect, no inequality constraints need to be imposed on the model parameters. Since the $ln(h_t)$ is modeled, even if parameters are negative, h_t will always be positive.

Component GARCH

The component GARCH (CGARCH) model has been developed by Engle & Lee (1993) in order to investigate the log-run and short-run movement of volatility. While the GARCH model and its asymmetric extensions show mean reversion to the unconditional variance, which is constant for all time, the CGARCH model allows mean reversion to a time-varying long-run volatility level, q_t. The specification of the CGARCH model is:

$$h_t^2 = q_t + a_1\left(\varepsilon_{t-1}^2 - q_{t-1}\right) + \beta_1\left(h_{t-1}^2 - q_{t-1}\right) \tag{24}$$

and

$$h_t^2 = q_t + a_1\left(\varepsilon_{t-1}^2 - q_{t-1}\right) + \beta_1\left(h_{t-1}^2 - q_{t-1}\right)_1 + \theta IV_{t-1}^2 \tag{25}$$

the CGARCH model nested with IV. $q_t = a_o + \rho q_{t-1} + \phi\left(\varepsilon_{t-1}^2 - h_{t-1}^2\right)$ is the time-varying long-run volatility provided $\rho > (a_1 + \beta_1)$. The forecast error $(\varepsilon_{t-1}^2 - q_{t-1})$ drives the time-varying process of q_t and the difference between the conditional variance and its trend, $(h_t^2 - q_t)$, is the transitory or short-run component of the conditional variance. Stationarity is accomplished provided $(a_1 + \beta_1)(1 - \rho) + \rho < 1$, which in turn requires $\rho < 1$ and $a_1 + \beta_1 < 1$.

Asymmetric Component GARCH

The asymmetric specification, ACGARCH model, and its nested with IV specification are:

$$h_t^2 = q_t + (\alpha_1 - \gamma I_{t-1})\left(\varepsilon_{t-1}^2 - q_{t-1}\right) + \beta_1\left(h_{t-1}^2 - q_{t-1}\right) \tag{26}$$

and

$$h_t^2 = q_t + (\alpha_1 - \gamma I_{t-1})\left(\varepsilon_{t-1}^2 - q_{t-1}\right) + \beta_1\left(h_{t-1}^2 - q_{t-1}\right)_1 + \theta IV_{t-1}^2 \tag{27}$$

respectively. The asymmetric effect is captured by the dummy variable I_{t-1}, such that $I_{t-1} = 1 \quad if\ \varepsilon_{t-1} < 0$ and $I_{t-1} = 0 \quad if\ \varepsilon_{t-1} > 0$. Stationarity is accomplished provided $(a_1 + \beta_1 + {}^1\!/_2\gamma)(1-\rho) + \rho < 1$, which in turn requires $\rho < 1$ and $a_1 + \beta_1 + {}^1\!/_2\gamma < 1$.

Another way to compare IV with GARCH is to investigate the forecasting ability of IV indices. That is, whether implied volatility can *per se* be forecasted and whether the IV index model forecast will be more accurate than the GARCH type models. In line to previous research, for instance Konstantinidi et al. (2008) show that the ARIMA(1,1,1) and ARFIMA(1,d,1) specifications provide the best point forecast for the US indices, different autoregressive models are going to be used.

ARMA(1,1)

Univariate autoregressive moving average models are the most general class of models for forecasting stationary time series or time series that can be transformed to stationary by taking differences. Employing the augmented Dickey-Fuller test in the IV indices, the null hypothesis of a unit root is rejected for all series. For each IV index an ARMA(1,1) is employed of the form

$$IV_t = c_0 + \phi_1 IV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \tag{28}$$

One lag is used for both the autoregressive and moving average part since this is found to minimize the BIC criterion.

ARMAX(1,1)

For comparing the IV models to the asymmetric GARCH specifications, contemporaneous positive and negative returns of the underlying stock index are included in equation 28. The predictive regression has the form

$$IV_t = c_0 + \phi_1 IV_{t-1} + \theta_1 \varepsilon_{t-1} + c_1 r_t^+ + c_2 r_t^- + \varepsilon_t \tag{29}$$

where $r_t^+$and r_t^-denote the positive and negative stock index returns, respectively, so as to assess the contemporaneous asymmetric relationship between the index returns and the IV indices (see also Simons (2003) and Giot (2005b) for a similar approach).

ARIMA(1,1,1)

A generalization of the ARMA models is the autoregressive integrated moving average (ARIMA) model. It is usually denoted as ARIMA(p,d,q) and is employed to capture the possible presence of short memory features in the dynamics of implied volatility. The ARIMA(p,d,q) specification is defined by

$$\phi(L)\Delta^d IV_t = c_0 + \theta(L)\varepsilon_t$$

where d is a positive integer that imposes the order of integration needed to produce stationary and invertible process. The ARIMA(1,1,1) specification is going to be used here, it is given by

$$\Delta IV_t = c_0 + \phi_1 \Delta IV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \tag{30}$$

ARIMAX(1,1,1)

The ARIMAX(1,1,1) model is going to be used, takes into account the possible presence of the short memory and asymmetric effect of the index returns and is given by

$$\Delta IV_t = c_0 + \phi_1 \Delta IV_{t-1} + \theta_1 \varepsilon_{t-1} + c_1 r_t^+ + c_2 r_t^- + \varepsilon_t \tag{31}$$

ARFIMA(1,d,1)

Following Konstantinidi et al. (2008) and Dunis et al. (2013) I apply a franctionally integrated

ARMA model, which is defined by

$$\phi(L)(1-L)^d IV_t = c_0 + \theta(L)\varepsilon_t \tag{32}$$

where d dictates the order of fractional integration and takes non-integer values. If $\mid d \mid < 0.5$, the ARFIMA is both stationary and invertible. In particular, if $d \in (0, 0.5)$, the process is said to exhibit long memory, while if $d \in (-0.5, 0)$, the process exhibits antipersistence. The ARFIMA(1,d,1) model is employed based on the BIC criterion and estimated by maximum likelihood.

ARFIMAX(1,d,1)

The ARFIMAX(1,d,1) model takes into account the possible presence of the long memory and asymmetric effect of the index returns and is given by

$$(1-L)^d IV_t = c_0 + \phi_1 IV_{t-1} + \theta_1 \varepsilon_{t-1} + c_1 r_t^+ + c_2 r_t^- + \varepsilon_t \tag{33}$$

Random Walk

I assess the predictability of IV by comparing the above mentioned forecasting model against the random walk benchmark.

$$IV_t = IV_{t-1} + \varepsilon_t \tag{34}$$

2.3.3 Forecast evaluation

The next step in the analysis is to evaluate the forecasting performance of the various models described in Subsection 2.3.2. The forecasts are obtained recursively by increasing the sample length by one observation. In other words, the initial estimation date is fixed and, once I obtain a forecast I increase the sample size by one observation and re-estimate.

For examining the forecastability of IV itself, the Diebold-Mariano pairwise test (Diebold &

Mariano, 1995) is employed. This test evaluates the forecasting performance of two competing models. In short, let $L(y_t; \hat{y_t})$ denote the forecast loss where y_t is the 'true' value and $\hat{y_t}$ is the predicted value. The difference in loss of model i relative to a benchmark model o is defined as

$$d_{i,t} = L(y_t; \hat{y_{o,t}}) - L(y_t; \hat{y_{i,t}}) \tag{35}$$

The issue is whether the two models have equal predictive ability. That is, the null hypothesis that is tested is $H_0 : E(d_{i,t}) = 0$. The DM test statistic is then expressed as

$$DM = \frac{\overline{d}}{\sqrt{LRV_{\overline{d}}/T}} \sim N(0,1) \tag{36}$$

where $\overline{d} = \frac{1}{n}\sum_{t=1}^{n} dt$ and $LRV_{\overline{d}} = \gamma_0 + \sum_{j=1}^{\infty} \gamma_j$ - $\gamma_j = cov(d_t, d_{t-j})$ - is an estimator of the asymptotic variance of $\overline{d}\sqrt{T}$. In this application the DM test is used to assess whether any model under consideration outperforms the random walk model under the MSE and MAE metrics.

Given that volatility is latent, the ex post squared returns are used as a proxy for 'true' volatility against which the forecast performance of the volatility estimators is assessed. That is, 'true' volatility is developed by

$$\sigma_t^2 = \sum r_t^2 \tag{37}$$

where r_t is the daily return on day t.

However, as noted by (Andersen & Bollerslev, 1998) and (Andersen et al., 1999), although the use of squared returns as a measure of true volatility is a simple and unbiased measure it provides a very noisy one. Thus Andersen and Bollerslev suggest that the proxy of ex post volatility should be based on intraday squared returns. The so-called realized variance is defined by

$$\sigma_t^2 = \sum_{j=1}^{n} r_{t,j}^2 \tag{38}$$

where $r_{t,j}$ is the return in interval j on day t and n is the number of interval in a day.[7]

The ability of the models described in Subsection (2.3.2) to accurately forecast the 'true' volatility is assessed using two alternative types of measures for forecast comparisons. In the first one, two different forecast error statistic have been selected. The mean absolute error (MAE) and the root mean squared error (RMSE)

$$MAE = \frac{1}{\tau} \sum_{t=T+1}^{T+\tau} \mid h_t^2 - \sigma_t^2 \mid \tag{39}$$

$$RMSE = \sqrt{\frac{1}{\tau} \sum_{t=T+1}^{T+\tau} (h_t^2 - \sigma_t^2)} \tag{40}$$

where τ is the number of out-of-sample observations, h_t^2 is the GARCH or IV forecast and σ_t^2 is the 'true' volatility. The MAE measures the average absolute forecast error and by construction does not permit the offsetting effect of over- and underprediction. The RMSE is a conventional criterion which clearly weights greater forecast errors more heavily than smaller forecast errors in the forecast error penalty.

Following previous research, for all forecasting volatility models, the second type of measures for forecast comparisons is the testing procedure of Mincer-Zarnowitz (1969, hereafter MZ), which measures how much of the true volatility is explained by the forecasted series. The true volatility σ_t^2 is regressed on the forecasted series of the different GARCH models and IV models, denoted h_t^2, as shown below

$$\sigma_t^2 = a_0 + a_1 h_t^2 + \epsilon_t \tag{41}$$

The primary interest lies in the R^2, where the model with the highest R^2 is preferred.

In order to examine the relative forecasting performance of the GARCH and IV models, a forecast-encompassing exercise is also performed. To test for such forecast encompassing the fol-

[7] As mentioned in Subsection 2.3.1, the daily realized variance of all indices are obtained from the Oxford-Man Institute's Realized Library.

lowing extension of the regression model in equation (41) is considered:

$$\sigma_t^2 = a_0 + a_1 h_{1,t}^{2f} + a_2 h_{2,t}^{2f} + \epsilon_t \tag{42}$$

where $h_{1,t}^{2f}$ refers to the GARCH forecasts and $h_{2,t}^{2f}$ refers to the IV forecasts. If the IV forecast model carries no additional information then it is said that the GARCH forecast encompasses the IV forecast and the null hypothesis that $a_2 = 0$ is true. Similarly, If IV encompasses GARCH the null hypothesis that $a_1 = 0$ is true.

To the best of my knowledge previous studies that investigate whether IV contains incremental information regarding the future volatility have considered either that IV follows a random walk or the volatility of IV.[8] Since my aim is to examine whether the forecast of implied volatility is a good predictors for the stock market volatility, I run the equation 42 twice: first using the forecast of IV indices as $h_{2,t}^{2f}$ and second using IV following a random walk (see equation 34) as $h_{2,t}^{2f}$ in order to examine whether the forecasts of IV indices are better predictors for the future volatility than the random walk.

Finally, I assess the performance of the forecast encompassing regressions by calculating the value-at-risk (VaR). VaR is a popular approach to measure risk as it specifies the portfolio loss that occurs within a given time and with a given probability. More formally, VaR is calculated as $VaR = a(N)\sigma_{t+1}V$, where $a(N)$ is the appropriate left-hand cut-off of the normal distribution, σ_{t+1} is the one-step ahead volatility forecast and V is the portfolio's value. In this study, I want to assess the performance of the forecast encompassing regressions when both the squared daily retuns and the realized variance are used as the true volatility proxy. Thus, σ_{t+1} is the one-step ahead volatility forecast as it is estimated through the combinations of GARCH and IV model forecasts. In order to evaluate the performance of these forecasts for producing reasonable VaR estimates I examine the models failure rate that is the frequency that the actual loss exceeds the estimated VaR.

The Kupiec test (Kupiec, 1995) for the equality of the empirical failure rate to a specified

[8]The volatility of IV is obtained by adding the implied volatility to the variance equation of the various GARCH specifications under the constraint that the time series parameters a_1 and β_1 equal zero.

statistical level is computed. Moreover, I also compute the dynamic quantile (DQ) test proposed by Engle & Manganelli (2004) and argues that in addition to the failure rate, the conditional accuracy of the VaR estimates is important. Therefore, they test the joint null hypothesis that the violations should both occur at a specified rate and not be serially correlated. They define the hit sequence:

$$Hit_t = I(r_t < -VaR_t) - a$$

which assumes value $(1 - a)$ every time the actual return is less than the VaR quantile and $-a$ otherwise. The expected value of Hit_t is zero and the the hit sequence must be uncorrelated with any past information and have expected value equal to zero. If the hit sequence satisfies these conditions the hits will not be correlated an the fraction of exception will be correct. The DQ test statistic is computed as

$$DQ = \hat{\beta}' X' X \beta / a(\hat{1} - a)$$

where X is the vector of explanatory variables and $\hat{\beta}$ the OLS estimates. The test follows a χ^2 distribution with degree of freedom equal to the number of parameters.

2.4 Empirical results

2.4.1 In-sample results

Table 4 reports the parameter estimates of the alternative GARCH models defined above. The period used for the estimations is February 2, 2001 to February 23, 2010. For all stock index returns, the estimates of GARCH show that all the coefficients of the variance equation (a_0, a_1 and β_1) are statistically significant at 1% level and satisfy the non-negativity constraints. The sum $a_1 + \beta_1$ is less than one, but very close to the unity, which implies that shocks to volatility have a highly persistent effect on the conditional variance. Turning to the results for GARCH-IV model, which adds implied volatility as an exogenous variable in the conditional variance equation, I find that for all indices the IV parameter θ are significant at the 1% level. The likelihood ratio test[9] rejects the

[9] The likelihood ratio test is defined as $LR = -2(L_r - L_u)\ \chi^2(m)$ where L_u is the maximized value of the log

null hypothesis that implied volatility contains no incremental information useful for explaining the conditional variance.

In GJR and GJR-IV models, $a_1 + \gamma > 0$ holds for all indices indicating that bad news increase the conditional volatility more than good news a_1. Unlike the usual restriction of the GJR model, $a_1 < 0$ for all series in both models. Nonetheless, the restriction for positive unconditional volatility $(a_1 + \beta_1 + {}^1\!/\!{}_2\gamma)$ still holds. When the information of implied volatility is added the log-likelihood is significantly higher than in GJR.

The impact of implied volatility in the conditional volatility can also be found by comparing the EGARCH model and its encompassing specification EGARCH-IV. The coefficient of the lagged IV indices were found to be statistically significant at the 1% level. Similar to the results from the previous models, the likelihood ratio test show that the implied volatility indices information has incremental explanatory power for conditional volatility. To examine the asymmetric effect of news; the negative and significant γ in both EGARCH and EGARCH-IV specification show the existence of leverage effect in returns.

In CGARCH and CGARCH-IV models, the condition $a_1 + \beta_1 < \rho < 1$ holds for all indices implying that the long run index return conditional volatility will decay more slowly than the transitory component of volatility. This result further suggests that the permanent volatility controls the conditional volatility. The coefficient of the lagged IV indices were found to be statistically significant at 1% level.

ACGARCH and ACGARCH-IV models intend to capture the long memory characteristic of the returns. The conditional volatility shows the existence of both transitory and permanent components. The transitory asymmetric volatility is captured by γ and the leverage effect feature holds. Also, the condition $a_1 + \beta_1 + {}^1\!/\!{}_2\gamma < \rho < 1$ holds in both models for all indices implying that the long run index return conditional volatility will decay more slowly than the transitory component

likelihood for an unrestricted model (in our case GARCH-IV), $L_r$is the maximized value of the log likelihood for a model which has been estimated imposing the constraints(in our case GARCH(1,1) imposing the constraint that $\theta = 0$) and m is the number of the restrictions.

of volatility. Once more the likelihood ratio test shows the usefulness of implied volatility.

Diagnostic tests in the standardized residuals are performed for all the alternative GARCH specifications. The standardized residuals are expected to have skewness and kurtosis parameters close to those of a normal distribution as well as not to have remaining non-modelled ARCH effects. As it is referred to the Table 5, the results from diagnostics tests indicate that the standardized residuals are skewed to the left, while the values of kurtosis and the Jarque-Bera test have noticeably reduced in absolute values for all series compared to the statistics from the original return series in Table 1. Thus, it can be inferred that all the models are able to explain the asymmetric and fat tails characteristics of the return distributions to some extent. With the exception of GARCH-IV and ACGARCH-IV model in S&P500 index, GARCH-IV in the DJIA and EGARCH-IV in Nasdaq100, the Ljung-Box Q(m) statistics indicate that the autocorrelations of the residuals are all statistically insignificant at the 1% level for all GARCH family models. So, the null hypothesis of no autocorrelation is not rejected.

Finally, in order to test whether there are any remaining ARCH effects in the residuals the LM is carried out. If the conditional variance equations are correctly specified, there should be no ARCH effect in the standardized residuals. Indeed, as it can be seen in Table 5, the null hypothesis of no ARCH effect cannot be rejected at the 1% level with the exception of GARCH-IV and ACGARCH-IV model in S&P500 index, GARCH-IV in the DJIA index and EGARCH-IV in Nasdaq100 index.

Table 6 summarizes AR(FI)MA(X) models' coefficients and their p-values for all indices. The AR(1) and MA(1) terms are statistically significant at the 1% level for all models except the ARFIMAX specification for the VIX index. The coefficients of r_t^+ and $r_t^-$are also statistically significant at the 1% level. Moreover, the coefficient of r_t^- are greater in absolute values than the coefficients of r_t^+ for all models. It is apparent that there are contemporaneous asymmetric effects for all estimations. In other words, negative returns influence the implied volatility indices more than positive returns. The negative and positive stock index returns trigger the IV index to move asymmetrically in the opposite direction. That is, positive contemporaneous returns decrease the implied volatility, while negative contemporaneous returns raise implied volatility and thus the index level. The difference parameter d in the ARFIMA models is significant at 1% level and $d \in (0, 0.5)$ for all series indicating that the processes exhibit long memory. Based on the log-likelihood, the unrestricted ARMA models, these that allow for asymmetry, outperform their restricted counterparts. Overall, within the sample, the ARMAX specification performs best for VIX and VXD indices and the ARIMAX specification for the VXN index.

2.4.2 Out-of-sample results

Regarding the forecast of the implied volatility itself, Table 7 presents the Diebold-Mariano test in order to address the question whether the dynamics of implied volatility *per se* can be forecasted. The DM test using the MSE and MAE criteria of all models assesses the predictive ability of each forecasting model against the benchmark model. The null hypothesis of equal predictive ability of each model against the random walk is tested against the alternative hypothesis that random walk is outperformed. There are 24 cases (out of 36) in which I reject the null hypothesis of equal predictability. Therefore, in 66.67% of the different combinations of IV and predictability measures one of the models performs better than the random walk. This indicates that there is a predictable pattern in the dynamics of implied volatility indices.

In terms of how competing models perform, the ARMAX model performs best yielding the lowest loss versus the alternative models. According to the MSE metric, the ARFIMA and random walk perform poorly, while according to MAE, the ARMA and ARFIMA models are outperformed followed by the random walk. When the model under consideration is an ARMA model that takes into account the contemporaneous asymmetric effect - ARMAX, ARIMAX, ARFIMAX models - always outperforms the random walk. In those cases, the null hypothesis of equal predictive ability is always rejected at the 1% level.

Tables 8 and 9 report the mean absolute error and the root mean square error for the various models when the squared returns and the realized variance, respectively, are used as proxy for the true volatility. Of particular interest is the question whether the good performance of the IV in-sample carries over to out-of-sample comparisons.

According to the MAE, Table 8 shows that for both S&P500 and DJIA index the EGARCH specification seems to perform best closely followed by the EGARCH-IV, while for the Nasdaq100 the EGARCH-IV provides the best forecast. The majority of the models that are nested with implied volatility outperform their GARCH counterparts that exclude the implied volatility information. As for the IV forecasts, the results suggest that under MAE metrics, all the ARMA-type models perform poorly compare to the GARCH specifications. Nonetheless, focusing only on the performance of the various IV indices to provide accurate volatility forecasts I find that the ARIMAX model performs best for the S&P500 and Nasdaq100 index, while the ARMAX model provides the best forecast for the DJIA index.

On the other hand, using the RMSE, there are overwhelming evidence of the superiority of the GJR-IV specification. In all series, a GARCH specification combined with implied volatility outperforms its restricted version. Looking at the IV models, the ARMAX model is the best for the S&P500 and the ARFIMAX model for the DJIA and Nasdaq100. Furthermore, contrary to the MAE results, in many cases ARMA specifications yield lower RMSE than the restricted GARCH specifications.

Similar results are obtained in Table 9 where the realized variance is used as the proxy for the true volatility. According to the MAE, the EGARCH model yields the lowest loss for the S&P500 and Nasdaq100, while the EGARCH-IV performs best for the DJIA index. The IV forecasts perform poorly with the ARMAX and the ARIMAX specification to yield the lowest MAE for S&P500, and DJIA and Nasdaq100, respectively. When the RMSE is used, for both the S&P500 and DJIA the EGARCH-IV performs best, while for the Nasdaq100 the GJR-IV provides the best forecast. When the forecasting performance of IV models is assessed, the ARMAX model for the S&P500 index and the ARFIMAX model for the DJIA and Nasdaq100 indices provides the best forecast.

In sum, on both forecast error statistic, models that capture the leverage effect and/or long memory are superior to the simple GARCH model. In most cases, an asymmetric GARCH model nested with IV performs best, indicating that both the in-sample and out-of-sample IV contains incremental information useful for explaining the future volatility beyond that available from the GARCH models. As for the IV forecasts, the asymmetric ARMA models strictly outperform the random walk.

In order to examine how much of the 'true' volatility is explained by the GARCH forecasts and IV forecasts, the MZ procedure is employed. Tables 10 and 11 report the R^2 values from the forecasting regression in equation (41) using the squared returns and realized variance measure of true volatility, respectively. The model with the highest R^2 is preferred. Examining the results of the MZ test procedure I find that, for both measures of true volatility, the GJR specification embedded with implied volatility performs best followed by the EGARCH-IV model and the asymmetric ARMA specifications. For all series an unrestricted GARCH specification obtains strictly higher R^2 than its restricted version. In the case that squared returns is used as proxy, the R^2 value increases by about 3% to 6% when the IV is added in the conditional variance equations. When the realized variance is the proxy of the true volatility, the R^2 value rises by about 5% up to 13%. Looking at the IV forecasts, the ARMA-type models which take into account the contemporaneous asymmetric effect - ARMAX, ARIMAX and ARFIMAX - obtain higher R^2 values than the random walk for all cases. The random walk yields marginally higher R^2 than the symmetric ARMA specifications, implying that when the contemporaneous asymmetric effect is considered, the forecast of IV does a better job than the random walk in explaining the variability of the 'true' volatility. Among the IV forecast, the ARFIMAX specification reports the highest R^2 value across all indices and measures of true volatility. Finally, when realized volatility is the proxy of the true volatility, all models yields much higher R^2 values.

The next step is to investigate the relative forecasting performance of the GARCH and IV models so as to identify whether these forecasts contain independent information useful in predicting future volatility and whether the IV forecast through an ARMA-type specification is a better predictor than the random walk. Tables 12 to 14 present the results of the encompassing regressions described in equation (42) for all indices using the *ex post* daily squared returns measure of true volatility.

Estimation results from the encompassing regressions for the S&P500 are given in Table 12. In this comparison the significance of the a_2 coefficient is of primary interest as it would indicate that IV is not encompassed by the GARCH models. The a_2 coefficient is, in most cases, significantly different from zero. This implies that the IV forecast contains additional information over the GARCH forecast. In many cases, the a_1 coefficient is insignificant indicating that the GARCH information is subsumed by the VIX. There are also a few cases in which both coefficients are significant implying that both approaches complement each other. According to the R^2, a combination of the ACGARCH-IV forecast with the ARFIMAX forecast performs best. In this case, both forecasts contain independent information useful in forecasting future stock return volatility. This can be clearly observed by comparing the R^2 value of the encompassing regression with the one of the individual regressions presented in Table 10. Furthermore, when the GARCH forecast is combined with the forecast of an asymmetric IV model always yields higher R^2 values than when the GARCH is combined with the random walk process of IV.

Tables 13 and 14 report the encompassing regressions results for the DJIA and Nasdaq100 indices. The results show that, in most cases, the a_2 coefficient is significant which means that in these cases IV contains independent information than the one contained in GARCH. In many encompassing regressions, IV forecast subsumes GARCH forecast information, while there are few cases in which both forecasts are significant which means that they both contain information useful for predicting stock index return volatility. Looking at the R^2 values, for both indices the highest R^2 is reported when the GJR-IV is combined with the IV forecast through an ARMAX model. Although, IV is encompassed by the GJR forecast, the R^2 of the univariate regressions in Table 10, indicating that a combination of both predictors is preferred as it can further improve the forecasts. Similarly to the S&P500 index, when the GARCH forecast is combined with the unrestricted forecasts of IV yields higher R^2 values than when the GARCH is combined with the random walk process of IV.

Tables 15 and 17 report the results of the encompassing regressions for the forecast models using the realized variance as proxy for the true volatility. There is a remarkably consistency across all indices. First, the GJR-IV combined with the ARMAX model reports the highest R^2 values for all indices. This is also confirmed looking at the R^2 which is strictly higher than the R^2 for the univariate regressions in Table 11. Second, regarding the encompassing test, the null hypothesis that the GARCH forecasts encompasses the IV forecasts is rejected for all series and all indices, with the exception of the EGARCH-IV specification combined with the random walk for all indies and the GJR-IV combined with the random walk for the Nasdaq100 index. Nonetheless, in many cases in which one approach dominates the other, the adjusted R^2 is marginally higher than the R^2 of the univariate regressions indicating that combining GARCH and IV improves on the results from either one alone. Finally, similarly to the encompassing regressions results in which daily squared returns measure the true volatility, when the GARCH forecasts are combined with the asymmetric IV model forecasts yield higher R^2 values than when the GARCHs are combined with the random walk.

The VaR results for the encompassing regressions are reported in Tables 18 and 19 when the squared returns and the realized volatility are respectively used as the true volatility proxies. More specifically, in Table 18, at both 1% and 5% VaR levels, the combination providing the best VaR measures in terms of achieving the lowest average failure rate is the ACGARCH-IV combined with the ARIMAX. In most cases, when GARCH forecasts are combined with the IV forecasts through an ARMA-type models have lower average failure rate than when the GARCH models are combined with the IV following a random walk. In terms of the Kupiec and DQ tests, at the 1% level, both the ACGARCH-IV combined with the ARIMAX specification and the EGARCH combined with the ARMAX forecast perform best. In these cases only one market does not reject the null hypotheses of the equality of the number of violations at a specified rate, Kupiec test, and of the non autocorrelation in the sequence of exceptions, DQ test. Examining the 5% VaR results, I observe that the majority of the combinations perform well, with none or one index significant on both the Kupiec and the DQ test.

Similar results are reported in Table 19. At the 1% VaR probability level, combining the ACGARCH-IV with the ARFIMAX performs best having lowest average failure rate. In terms of the specification tests, the combination of the EGARCH-IV model with the ARMAX model performs best, with one index significant. When GARCH forecasts are combined with the asymmetric IV forecasts outperform the combinations of GARCH forecasts with the random walk. Examining the 5% VaR results, the GJR-IV combined with the ARMAX model performs best in terms of both the average failure rate and the DQ test. In terms of the Kupiec test the EGARCH-IV combined with the ARMAX performs well.

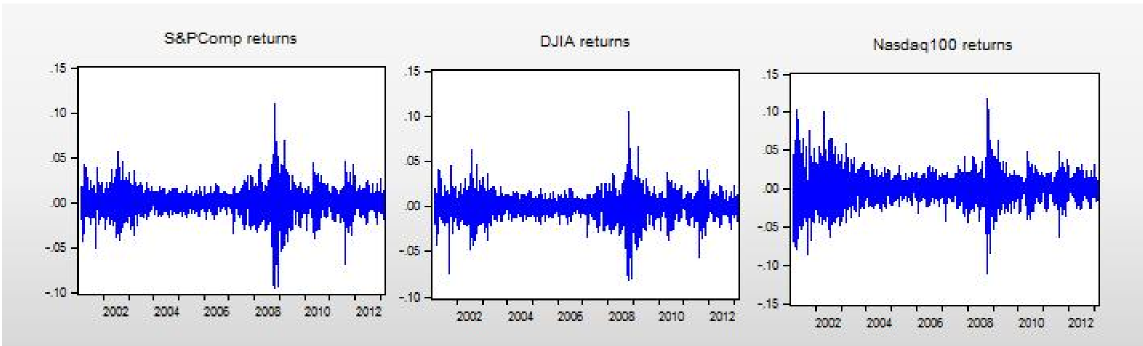
2.5 Conclusion

This chapter provides a comparative evaluation of the ability of a wide range of GARCH models and IV models to forecast stock index return volatility focused on the S&P500, DJIA and Nasdaq100 indices as well as their IV using the Mincer-Zarnowitz regression test of predictive power.There is a bulk of literature that investigates the information content of IV using IV as an exogenous variable in the conditional variance equation or considering that IV follows a random walk. More recent literature has shown that IV follows a predictable pattern. Therefore, this study analyzes whether the IV forecasts are good predictors for the stock market volatility. A total of ten GARCH models are considered, GARCH, GJR, EGARCH, CGARCH and ACGARCH model and the encompassing variants of these models including IV as a regressor in the variance equation. Additionally, six ARMA models have been taken into consideration for forecasting IV indices. Both the *ex post* daily squared returns and realized variance are used as measures of true volatility.

The results show that the IV forecast contains significant information regarding the future volatility. With regard to the forecastability of IV itself, I find that IV forecasts are statistically significant. When the IV model accounts for the contemporaneous asymmetric effect its forecast strictly outperforms the random walk. The ARMAX model perform best. As for the GARCH models, the inclusion of IV in the GARCH variance equations improves both the in-sample and out-of-sample performance of the GARCH models with and asymmetric GARCH to perform best. Encompassing regressions indicate that IV forecasts is generally more informative than GARCH forecasts, but combining both predictors can often improve the forecasts. Finally, with regard to VaR forecasts, a combination of an asymmetric GARCH model with an asymmetric ARMA model is preferred when both the *ex post* daily squared returns and realized variance are used as measures of true volatility.

To summarize, the results suggest the IV does contain additional information useful for the future stock market volatility beyond the information contained in the GARCH model based volatility forecasts. The presence of the asymmetric effect is really important as it significantly improves the performance of both the GARCH and IV indices models. Overall, a model that includes both an asymmetric GARCH and the option market information through an ARMAX model is the most appropriate for predicting future volatility.

Figure 1: Daily returns of the S&P500, DJIA and Nasdaq100 index



Notes: The figure shows daily returns for the S&P500, DJIA and Nasdaq100 index for the period February 2, 2001 to February 28, 2013.

Table 1: Summary statistics for the full sample and in-sample daily stock returns

a)	Full sample		
	S&P500	DJIA	Nasdaq100
Mean	3.80E-05	8.47E-05	3.37E-05
Median	0.0006	0.0004	0.0008
Maximum	0.1095	0.1051	0.1185
Minimum	-0.0947	-0.0820	-0.1111
Std. Dev.	0.0134	0.0125	0.0180
Skewness	-0.1704	0.0328	0.0592
Kurtosis	11.049	10.778	7.3248
Jarque-Bera	8223.4	7666.2	2371.7
p-value	0.0000*	0.0000*	0.0000*

b)	In-sample		
	S&P500	DJIA	Nasdaq100
Mean	-9.18E-05	-2.41E-05	-0.0001
Median	0.0006	0.0004	0.0007
Maximum	0.1096	0.1051	0.1185
Minimum	-0.0947	-0.0820	-0.1111
Std. Dev.	0.0139	0.0131	0.0194
Skewness	-0.1118	0.1148	0.1001
Kurtosis	11.420	10.981	6.7772
Jarque-Bera	6740.0	6056.5	1359.2
p-value	0.0000*	0.0000*	0.0000*

Notes: Entries report the summary statistics of the daily stock returns for a) the full sample period February 2, 2001 to February 28, 2013 and b) the in-sample period February 2, 2001 to February 23, 2010. In the last row, the p-values of the Jarque-Bera test for normality are reported. * denotes rejection of the null hypothesis at the 1% level, respectively.

Table 2: Summary statistics for implied volatility indices

	VIX	VXD	VXN
Mean	0.0137	0.0127	0.0179
Median	0.0123	0.0115	0.0150
Maximum	0.0509	0.0470	0.0508
Minimum	0.0062	0.0058	0.0079
Std. Dev.	0.0060	0.0056	0.0082
Skewness	1.9049	1.8183	1.2459
Kurtosis	8.4758	7.8965	3.7623
Jarque-Bera	5640.3	4715.2	860.67
p-value	0.0000*	0.0000*	0.0000*
ADF (p-value)	0.0051*	0.0091*	0.0113**

Notes: Entries report the summary statistics of the three implied volatility indices for the period February 2, 2001 to February 28, 2013. In the last two rows, the p-values of the Jarque-Bera test for normality and the Augmented Dickey-Fuller (ADF) test for unit root are reported. * and ** denote rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 3: Test for ARCH effects in returns

Index	$Q(p)$	LM
	$p = 7$	$p = 7$
S&P500	1509.0* (0.000)	634.901* (0.000)
DJIA	1386.8* (0.000)	595.446* (0.000)
Nasdaq100	987.01* (0.000)	414.152* (0.000)

Note: The Ljung-Box Q(7) test for squared residual autocorrelation and the Lagrange multiplier (LM) test for homoskedasticity are reported. p-values are in parentheses. * denotes rejection of the null hypothesis at the 1% level.

Table 4: Estimation models of the GARCH family

	$GARCH$	$GARCH-IV$	GJR	$GJR-IV$	$EGARCH$	$EGARCH-IV$	$CGARCH$	$CGARCH-IV$	$ACGARCH$	$ACGARCH-IV$
					S&P500					
a_0	$1.08*10^{-6}$** (0.020)	$-1.37*10^{-5}$** (0.012)	$9.06*10^{-7}$* (0.000)	$4.37*10^{-7}$ (0.269)	−0.18015* (0.000)	−0.00584 (0.919)	0.00016*** (0.068)	$-8.43*10^{-6}$ (0.418)	0.00015* (0.009)	$-1.26*10^{-5}$* (0.007)
a_1	0.07416* (0.000)	−0.04184*** (0.089)	−0.02695** (0.012)	−0.05644* (0.000)	0.08435* (0.000)	0.00884 (0.807)	0.00244 (0.355)	−0.08964* (0.000)	−0.14559* (0.000)	−0.01220 (0.993)
β_1	0.91892 * (0.000)	−0.06396 (0.799)	0.95343* (0.000)	0.90724 * (0.000)	0.98754* (0.000)	0.87468 * (0.000)	−0.99442* (0.000)	0.19152 (0.275)	0.66490* (0.000)	0.08828 (0.946)
γ			0.13144* (0.000)	0.18280* (0.000)	−0.11406* (0.000)	−0.19229* (0.000)			0.06728 (0.216)	0.05027 (0.361)
ρ							0.99349* (0.000)	0.97684* (0.000)	0.98790* (0.000)	0.03837 (0.977)
ϕ							0.07334* (0.000)	0.03265* (0.000)	0.10462* (0.000)	−0.04816 (0.971)
θ		0.86531* (0.000)		0.03889* (0.000)		0.13150* (0.000)		0.76574* (0.000)		0.70726** (0.020)
$Log-L$	7133.993	7177.89	7185.029	7199.599	7178.78	7202.876	7135.571	7198.065	7144.343	7179.536
χ^2	87.79	–	29.14	–	48.19	–	126.16	–	70.39	–
					DJIA					
a_0	$1.10*10^{-6}$* (0.009)	$-8.73*10^{-6}$*** (0.081)	$9.91*10^{-7}$* (0.000)	$-3.45*10^{-7}$ (0.483)	−0.19593* (0.000)	−0.06522 (0.233)	0.000142** (0.010)	$-3.54*10^{-6}$ (0.633)	0.00013* (0.009)	$-4.53*10^{-6}$* (0.000)
a_1	0.07787* (0.000)	−0.04274* (0.000)	−0.01647 (0.229)	−0.04959* (0.000)	0.09728* (0.000)	0.03378 (0.253)	−0.10033* (0.000)	−0.09671* (0.000)	−0.17666* (0.000)	−0.12228** (0.030)
β_1	0.91451 * (0.000)	−0.08851 (0.671)	0.94073* (0.000)	0.88927 * (0.000)	0.98704* (0.000)	0.89445 * (0.000)	0.54646* (0.004)	0.05205 (0.758)	0.59917* (0.000)	0.15721 (0.393)
γ			0.13532* (0.000)	0.18267* (0.000)	−0.11315* (0.000)	−0.17441* (0.000)			0.11191* (0.005)	0.06680** (0.042)
ρ							0.98804* (0.000)	0.96711* (0.000)	0.98896* (0.000)	0.96339* (0.000)
ϕ							0.1010* (0.000)	0.02895* (0.003)	0.09750* (0.000)	0.03257* (0.000)
θ		0.87797* (0.000)		0.04918* (0.000)		0.10574* (0.000)		0.81980* (0.000)		0.74265* (0.000)
$Log-L$	7233.753	7281.437	7278.800	7296.655	7279.334	7301.833	7242.652	7297.520	7248.263	7302.492
χ^2	95.37	–	35.71	–	45.00	–	109.74	–	108.46	–
					Nasdaq100					
a_0	$1.14*10^{-6}$** (0.029)	$-1.66*10^{-5}$*** (0.073)	$1.04*10^{-6}$** (0.013)	$-2.63*10^{-7}$ (0.843)	−0.12253* (0.000)	0.06063 (0.432)	−0.00219 (0.942)	$-1.26*10^{-5}$ (0.359)	0.000299 (0.109)	$-1.57*10^{-5}$ (0.282)
a_1	0.05582* (0.000)	−0.02184 (0.442)	−0.00811 (0.363)	−0.03667** (0.025)	0.07336* (0.000)	−0.01776 (0.624)	0.01046 (0.439)	−0.04893 (0.104)	−0.08887* (0.005)	−0.06980** (0.044)
β_1	0.94013 * (0.000)	−0.04489 (0.887)	0.95980* (0.000)	0.83725 * (0.000)	0.99234* (0.000)	0.76755 * (0.000)	0.95099* (0.000)	0.09177 (0.711)	0.35552 (0.322)	0.42994 ** (0.032)
γ			0.08657* (0.000)	0.14345* (0.000)	−0.07811* (0.000)	−0.76755* (0.000)			0.04185 (0.348)	0.06627*** (0.076)
ρ							0.99999* (0.000)	0.97961* (0.000)	0.99516* (0.000)	0.98051* (0.000)
ϕ							0.05269* (0.000)	0.01848** (0.032)	0.06868* (0.000)	0.02152** (0.021)
θ		0.89134* (0.000)		0.10058* (0.007)		0.24650* (0.001)		0.82964* (0.000)		0.54522* (0.005)
$Log-L$	6273.628	6315.092	6300.433	6317.492	6303.448	6328.670	6272.703	6323.778	6279.973	6325.611
χ^2	82.93	–	34.12	–	50.44	–	102.15	–	91.28	–

Note: Entries report results of the alternative GARCH models as described in equations (18) - (27). The p-values of the estimated coefficients are in parentheses, the log-likelihood (Log-L) and χ^2 for testing the restricted GARCH specifications, where $\theta = 0$, against the augmented GARCH specifications, where $\theta \neq 0$ are reported. *, ** and *** denote significance at the 1%, 5% and 10% level, respectively.

Table 5: Diagnostics tests in squared standardized residuals

	Skewness	Kurtosis	Jarque-Bera	Q(7)	LM(7)
			S&P500		
GARCH	−0.3185	4.1268	159.17 (0.000)	5.569 (0.591)	5.648 (0.581)
GARCH-IV	−0.3739	4.1484	178.42 (0.000)	16.799* (0.019)	16.039** (0.025)
GJR	−0.3374	3.9274	124.95 (0.000)	8.600 (0.283)	8.958 (0.256)
GJR-IV	−0.3711	4.1599	180.14 (0.000)	10.130 (0.181)	10.649 (0.155)
EGARCH	−0.4015	4.2664	213.62 (0.000)	8.279 (0.309)	8.629 (0.208)
EGARCH-IV	−0.4196	4.1792	199.01 (0.000)	11.007 (0.138)	11.934 (0.103)
CGARCH	−0.3101	4.0811	147.58 (0.000)	5.744 (0.570)	5.786 (0.565)
CGARCH-IV	−0.3784	4.0548	160.10 (0.000)	1.256 (0.990)	1.293 (0.989)
ACGARCH	−0.3436	4.1139	162.74 (0.000)	1.866 (0.967)	1.904 (0.965)
ACGARCH-IV	−0.3906	4.1927	193.11 (0.000)	17.273** (0.016)	16.911** (0.018)
			DJIA		
GARCH	−0.2706	4.1045	143.72 (0.000)	7.872 (0.344)	7.751 (0.355)
GARCH-IV	−0.3188	3.9265	120.17 (0.000)	15.995** (0.025)	15.523** (0.030)
GJR	−0.2835	3.9357	113.73 (0.000)	8.042 (0.329)	8.097 (0.324)
GJR-IV	−0.3079	4.0607	142.92 (0.000)	9.008 (0.252)	9.151 (0.242)
EGARCH	−0.3217	4.0930	152.84 (0.000)	7.282 (0.400)	7.361 (0.392)
EGARCH-IV	−0.3134	3.8984	115.20 (0.000)	10.436 (0.165)	10.861 (0.145)
CGARCH	−0.2809	4.0226	129.31 (0.000)	3.989 (0.781)	4.006 (0.779)
CGARCH-IV	−0.3333	3.9386	125.91 (0.000)	2.029 (0.958)	2.023 (0.959)
ACGARCH	−0.3138	4.1584	164.89 (0.000)	3.041 (0.881)	3.074 (0.878)
ACGARCH-IV	−0.3359	3.9431	127.35 (0.000)	2.162 (0.950)	2.176 (0.950)
			Nasdaq100		
GARCH	−0.1271	3.6512	46.42 (0.000)	7.137 (0.415)	7.171 (0.411)
GARCH-IV	−0.1776	3.4726	32.32 (0.000)	11.641 (0.113)	11.548 (0.116)
GJR	−0.1891	3.4943	36.80 (0.000)	10.934 (0.142)	11.024 (0.138)
GJR-IV	−0.1836	3.4803	34.73 (0.000)	11.437 (0.121)	11.514 (0.118)
EGARCH	−0.2263	3.5187	45.02 (0.000)	11.457 (0.120)	11.713 (0.110)
EGARCH-IV	−0.2282	3.5177	45.25 (0.000)	14.750** (0.039)	15.207** (0.033)
CGARCH	−0.1406	3.6793	51.34 (0.000)	7.544 (0.374)	7.528 (0.376)
CGARCH-IV	−0.1672	3.3971	25.61 (0.000)	5.237 (0.631)	5.0532 (0.654)
ACGARCH	−0.1284	3.5689	37.01 (0.000)	2.163 (0.950)	2.149 (0.951)
ACGARCH-IV	−0.1746	3.4103	27.57 (0.000)	5.933 (0.548)	5.849 (0.558)

Note: Entries report the diagnostic residual test results of the GARCH models. The Ljung-Box Q(7) test and the Lagrange multiplier (LM) test for the squared standardized residuals are reported. p-values are in parentheses. * and ** denote rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 6: Estimation output of time series models for implied volatility prediction

	ARMA	ARMAX	ARIMA	ARIMAX	ARFIMA	ARFIMAX
			VIX			
c_0	0.01383*	0.01374*	$-1.85 * 10^{-7}$	$-4.64 * 10^{-7}$	0.01457	0.01454
	(0.000)	(0.000)	(0.991)	(0.982)	(0.440)	(0.436)
$AR(1)$	0.98932*	0.98813*	0.56069*	−0.44888*	0.89105*	0.71781*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$MA(1)$	−0.14282*	0.15798*	−0.70967*	0.59274*	−0.59004*	−0.06291
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.124)
d					0.49314*	0.49651*
					(0.000)	(0.000)
r_t^+		−0.02337*		−0.02331*		−0.02323*
		(0.000)		(0.000)		(0.000)
r_t^-		−0.04256*		−0.04160*		−0.04284*
		(0.000)		(0.000)		(0.000)
$Log-L$	12332.11	12914.04	12338.69	12910.83	12317.91	12874.81
			VXD			
c_0	0.01291*	0.01288*	$-2.81 * 10^{-7}$	$-4.99 * 10^{-7}$	0.01350	0.01323
	(0.000)	(0.000)	(0.985)	(0.978)	(0.447)	(0.347)
$AR(1)$	0.99098*	0.99029*	0.43836*	−0.67812*	0.91920*	0.81770*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$MA(1)$	−0.18672*	0.05046*	−0.61659*	0.74711*	−0.65026*	−0.28437*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
d					0.49199*	0.49335*
					(0.000)	(0.000)
r_t^+		−0.02607*		−0.02602*		−0.02566*
		(0.000)		(0.000)		(0.000)
r_t^-		−0.03674*		−0.03557*		−0.03733*
		(0.000)		(0.000)		(0.000)
$Log-L$	12590.77	13097.63	12590.79	13095.62	12579.71	13064.72
			VXN			
c_0	0.01978*	0.01973*	$-9.27 * 10^{-6}$	$-9.57 * 10^{-6}$	0.02128	0.02127
	(0.000)	(0.000)	(0.60)	(0.655)	(0.320)	(0.324)
$AR(1)$	0.99405*	0.99373*	0.73707*	−0.63943*	0.86512*	0.79548*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$MA(1)$	−0.05258*	0.10309*	−0.80766*	0.74476*	−0.45675*	−0.21329*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
d					0.49423*	0.49583*
					(0.000)	(0.000)
r_t^+		−0.01351*		−0.01367*		−0.01352*
		(0.000)		(0.000)		(0.000)
r_t^-		−0.02111*		−0.02041*		−0.02118*
		(0.000)		(0.000)		(0.000)
$Log-L$	12418.49	12702.3	12421.5	12707.57	129397.01	12666.19

Note: Entries report results of the alternative implied volatility models as described in equations (28) - (33). The p-values of the estimated coefficients are in parentheses. * denotes significance at the 1% level.

Table 7: Diebold-Mariano test for the implied volatility models

	MSE			MAE		
	VIX	VXD	VXN	VIX	VXD	VXN
Random walk	0.00152	0.00127	0.00129	0.758	0.692	0.723
ARMA(1,1)	0.00146	0.00121	0.00128	0.756	0.694	0.728**
ARMAX(1,1)	0.00085*	0.00081*	0.00093*	0.583*	0.558*	0.622*
ARIMA(1,1,1)	0.00145***	0.0012***	0.00127**	0.753	0.692	0.719***
ARIMAX(1,1,1)	0.00087*	0.00083*	0.00096*	0.589*	0.561*	0.624*
ARFIMA(1,d,1)	0.00148	0.00122	0.00130	0.765	0.702	0.740*
ARFIMAX(1,d,1)	0.00086*	0.00082*	0.00095*	0.590*	0.565*	0.631*

Note: The Diebold-Mariano test results using the mean squared forecast error (MSE) and the mean absolute forecast error (MAE) of the IV models are reported. The null hypothesis that the random walk and the model under consideration perform equally well is tested against the alternative that the model under consideration performs better. All numbers are multiplied by 10^3. *, ** and *** denote rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 8: MAE and RMSE using *ex post* squared returns measure of true volatility

	MAE			RMSE		
	$S\&P500$	$DJIA$	$Nasdaq100$	$S\&PComp$	$DJIA$	$Nasdaq100$
EWMA	0.1450	0.1189	0.1675	0.3021	0.2395	0.3099
GARCH	0.1451	0.1195	0.1710	0.2993	0.2370	0.3092
GARCH-IV	0.1421	0.1170	0.1644	0.2934	0.2333	0.2989
GJR	0.1390	0.1160	0.1673	0.2931	0.2324	0.3036
GJR-IV	0.1385	0.1152	0.1637	0.2860*	0.2267*	0.2945*
EGARCH	0.1347*	0.1126*	0.1638	0.2936	0.2320	0.3057
EGARCH-IV	0.1376	0.1139	0.1615*	0.2876	0.2282	0.2957
CGARCH	0.1462	0.1195	0.1721	0.3002	0.2411	0.3091
CGARCH-IV	0.1412	0.1156	0.1630	0.2926	0.2321	0.2981
ACGARCH	0.1442	0.1184	0.1706	0.3034	0.2396	0.3099
ACGARCH-IV	0.1423	0.1156	0.1632	0.2930	0.2315	0.2987
ARMA	0.1723	0.1402	0.1978	0.2985	0.2372	0.3042
ARMAX	0.1704	0.1396†	0.1904	0.2935†	0.2343	0.3025
ARIMA	0.1725	0.1402	0.1902	0.2992	0.2377	0.3045
ARIMAX	0.1704†	0.1397	0.1899†	0.2937	0.2346	0.3026
ARFIMA	0.1727	0.1404	0.1911	0.2987	0.2373	0.3043
ARFIMAX	0.1709	0.1397	0.1907	0.2936	0.2340†	0.3024†
Random walk	0.1722	0.1402	0.1902	0.2981	0.2371	0.3038

Note: The mean absolute forecast error (MAE) and the root mean squared forecast error (RMSE) defined in equations (39) and (40), respectively, of both GARCH and IV models when *ex post* squared returns measure true volatility are reported. All numbers are multiplied by 10^3. * denotes the lowest forecast error. † denotes the lowest forecast error among the IV models.

Table 9: MAE and RMSE using realized variance measure of true volatility

	MAE			RMSE		
	$S\&P500$	$DJIA$	$Nasdaq100$	$S\&P500$	$DJIA$	$Nasdaq100$
EWMA	0.0809	0.0714	0.0945	0.1495	0.1394	0.1542
GARCH	0.0796	0.0710	0.1016	0.1461	0.1365	0.1551
GARCH-IV	0.0777	0.0667	0.0974	0.1367	0.1288	0.1414
GJR	0.0763	0.0689	0.1004	0.1429	0.1337	0.1532
GJR-IV	0.0740	0.0661	0.0962	0.1332	0.1254	0.1409*
EGARCH	0.0679*	0.0637	0.0933*	0.1341	0.1289	0.1459
EGARCH-IV	0.0725	0.0634*	0.0950	0.1313*	0.1241*	0.1412
CGARCH	0.0808	0.0729	0.1036	0.1473	0.1458	0.1582
CGARCH-IV	0.0782	0.0664	0.0976	0.1449	0.1307	0.1500
ACGARCH	0.0807	0.0719	0.1035	0.1593	0.1439	0.1632
ACGARCH-IV	0.0778	0.0659	0.0979	0.1357	0.1288	0.1517
ARMA	0.1180	0.0945	0.1453	0.1591	0.1396	0.1807
ARMAX	0.1167†	0.0940	0.1449	0.1562†	0.1380	0.1793
ARIMA	0.1183	0.0946	0.1442	0.1600	0.1401	0.1798
ARIMAX	0.1169	0.0939†	0.1440†	0.1569	0.1384	0.1788
ARFIMA	0.1184	0.0947	0.1455	0.1593	0.1398	0.1801
ARFIMAX	0.1171	0.0942	0.1451	0.1558	0.1377†	0.1786†
Random walk	0.1178	0.0942	0.1445	0.1588	0.1388	0.1802

Note: The mean absolute forecast error (MAE) and the root mean squared forecast error (RMSE) defined in equations (39) and (40), respectively, of both GARCH and IV models when realized variance measure true volatility are reported. All numbers are multiplied by 10^3. * denotes the lowest forecast error. † denotes the lowest forecast error among the IV models.

Table 10: Out-of-sample predictive power for alternative forecasts using *ex post* squared returns measure of true volatility

Models	$adj-R^2$		
	$S\&P500$	$DJIA$	$Nasdaq100$
GARCH	0.1348	0.1370	0.0983
GARCH-IV	0.1771	0.1689	0.1600
GJR	0.1720	0.1744	0.1296
GJR-IV	0.2090*	0.2090*	0.1846*
EGARCH	0.1699	0.1722	0.1158
EGARCH-IV	0.2037	0.2014	0.1768
CGARCH	0.1305	0.1152	0.1007
CGARCH-IV	0.1730	0.1709	0.1603
ACGARCH	0.1202	0.1249	0.0974
ACGARCH-IV	0.1790	0.1750	0.1570
ARMA	0.1743	0.1693	0.1573
ARMAX	0.2047	0.1909	0.1673
ARIMA	0.1697	0.1660	0.1544
ARIMAX	0.2030	0.1880	0.1657
ARFIMA	0.1734	0.1690	0.1572
ARFIMAX	0.2051	0.1935	0.1675
RW: IV_{t-1}	0.1767	0.1707	0.1588

Note: Entries are the adjusted R^2 values from the Mincer-Zarnowitz regression described in equation (41) when the *ex post* squared daily returns measure the true volatility. * denotes the highest adjusted R^2 value.

Table 11: Out-of-sample predictive power of daily volatility forecasts using realized variance measure of true volatility

Models	$adj - R^2$		
	$S\&P500$	$DJIA$	$Nasdaq100$
GARCH	0.3207	0.2771	0.1946
GARCH-IV	0.3786	0.3447	0.2970
GJR	0.3849	0.3368	0.2416
GJR-IV	0.4387*	0.3888*	0.3406*
EGARCH	0.3797	0.3355	0.2018
EGARCH-IV	0.4303	0.3877	0.3197
CGARCH	0.3135	0.2163	0.1985
CGARCH-IV	0.3613	0.3293	0.2852
ACGARCH	0.2567	0.2317	0.1617
ACGARCH-IV	0.3900	0.3593	0.2943
ARMA	0.3960	0.3541	0.3072
ARMAX	0.4197	0.3684	0.3179
ARIMA	0.3884	0.3488	0.3017
ARIMAX	0.4161	0.3657	0.3163
ARFIMA	0.3921	0.3518	0.3074
ARFIMAX	0.4206	0.3709	0.3196
RW: IV_{t-1}	0.4060	0.3656	0.3106

Note: Entries are the adjusted R^2 values from the Mincer-Zarnowitz regression described in equation (41) when the realized variance measures the true volatility. * denotes the highest adjusted R^2 value.

Table 12: Forecast encompassing regression results for the S&P500 index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-6.39*10^{-5}$* (0.001)	0.0211 (0.900)	1.0008* (0.000)	0.1732
GARCH-IV & ARMA	$-4.64*10^{-5}$** (0.035)	1.4168 (0.104)	−0.0974 (0.888)	0.1762
GJR & ARMAX	$-8.09*10^{-5}$* (0.000)	0.0222 (0.895)	1.0883* (0.000)	0.2036
GJR-IV & ARMAX	$-4.97*10^{-5}$** (0.018)	0.6081* (0.001)	0.5092** (0.011)	0.2147
EGARCH & ARMAX	$-9.17*10^{-5}$* (0.000)	−0.2307 (0.356)	1.3046* (0.000)	0.2045
EGARCH-IV & ARMAX	$-5.99*10^{-5}$* (0.005)	0.5653** (0.039)	0.5971** (0.022)	0.2081
CGARCH & ARIMA	$-6.18*10^{-5}$* (0.002)	−0.0053 (0.975)	1.0080* (0.000)	0.1686
CGARCH & ARFIMA	$-6.81*10^{-5}$* (0.001)	−0.0272 (0.870)	1.0546* (0.000)	0.1723
CGARCH-IV & ARIMA	$-2.81*10^{-5}$ (0.294)	0.5948** (0.027)	0.3925 (0.171)	0.1740
CGARCH-IV & ARFIMA	$-3.89*10^{-5}$ (0.118)	0.4718* (0.078)	0.5380*** (0.065)	0.1757
ACGARCH & ARIMAX	$-8.35*10^{-5}$* (0.000)	−0.1058 (0.386)	1.1902* (0.000)	0.2027
ACGARCH & ARFIMAX	$-9.12*10^{-5}$* (0.000)	−0.0868 (0.467)	1.2167* (0.000)	0.2046
ACGARCH-IV & ARIMAX	−0.00011* (0.000)	−2.3353* (0.000)	2.9451* (0.000)	0.2183
ACGARCH-IV & ARFIMAX	−0.00013* (0.000)	−2.2897* (0.000)	3.0072* (0.000)	0.2213†
GARCH & IV_{t-1}	$-6.04*10^{-5}$* (0.002)	0.0306 (0.850)	0.9737* (0.000)	0.1757
GARCH-IV & IV_{t-1}	$-5.41*10^{-5}$* (0.008)	0.7710 (0.410)	0.4072 (0.574)	0.1764
GJR & IV_{t-1}	$-3.70*10^{-5}$*** (0.082)	0.3973** (0.021)	0.5958* (0.002)	0.1814
GJR-IV & IV_{t-1}	$-7.47*10^{-6}$ (0.716)	1.0893* (0.000)	−0.0523 (0.798)	0.2081
EGARCH & IV_{t-1}	$-4.66*10^{-5}$** (0.25)	0.4059 (0.122)	0.6697* (0.003)	0.1782
EGARCH-IV & IV_{t-1}	$-4.20*10^{-6}$ (0.847)	1.7699* (0.000)	−0.5817 *** (0.054)	0.2065
CGARCH & IV_{t-1}	$-6.19*10^{-5}$* (0.001)	−0.0236 (0.883)	1.0193 * (0.000)	0.1757
CGARCH-IV & IV_{t-1}	$-4.21*10^{-5}$*** (0.062)	0.3742 (0.140)	0.6261** (0.018)	0.1780
ACGARCH & IV_{t-1}	$-5.89*10^{-5}$* (0.002)	0.0772 (0.522)	0.9337* (0.000)	0.1761
ACGARCH-IV & IV_{t-1}	$-4.48*10^{-5}$** (0.039)	1.5215 (0.142)	−0.1871 (0.818)	0.1780

Note: Entries are the estimated coefficients, their p-values in parentheses and the adjusted R^2 values from the encompassing regression described in equation (42) when the *ex post* squared daily returns measure the true volatility. *, ** and *** denote significance at the 1% level. A significant p-value indicates that the forecast under consideration is not encompassed by the alternative model, † denotes the highest adjusted R^2 value.

Table 13: Forecast encompassing regression results for the DJIA index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-4.38*10^{-5}$** (0.004)	0.1043 (0.531)	0.8966* (0.000)	0.1687
GARCH-IV & ARMA	$-4.17*10^{-5}$** (0.011)	0.5299 (0.489)	0.5712 (0.346)	0.1688
GJR & ARMAX	$-4.26*10^{-5}$** (0.012)	0.2611*** (0.085)	0.7740* (0.000)	0.1930
GJR-IV & ARMAX	$-2.54*10^{-5}$ (0.118)	0.7744* (0.000)	0.2844 (0.135)	0.2103†
EGARCH & ARMAX	$-5.06*10^{-5}$* (0.001)	0.1874 (0.404)	0.8911* (0.000)	0.1906
EGARCH-IV & ARMAX	$-3.27*10^{-5}$** (0.046)	0.8637* (0.001)	0.2637 (0.239)	0.2018
CGARCH & ARIMA	$-4.25*10^{-5}$* (0.005)	0.0561 (0.663)	0.9230* (0.000)	0.1647
CGARCH & ARFIMA	$-4.66*10^{-5}$* (0.003)	0.0380 (0.766)	0.9612* (0.000)	0.1680
CGARCH-IV & ARIMA	$-1.97*10^{-5}$ (0.274)	0.7436** (0.017)	0.2742 (0.443)	0.1707
CGARCH-IV & ARFIMA	$-2.70*10^{-5}$ (0.145)	0.5978*** (0.055)	0.4256 (0.163)	0.1720
ACGARCH & ARIMAX	$-5.33*10^{-5}$* (0.000)	0.0068 (0.957)	1.0277* (0.000)	0.1869
ACGARCH & ARFIMAX	$-6.04*10^{-5}$* (0.000)	-0.0091 (0.941)	1.0834* (0.000)	0.1925
ACGARCH-IV & ARIMAX	$-5.05*10^{-5}$* (0.004)	0.0880 (0.748)	0.9499* (0.000)	0.1870
ACGARCH-IV & ARFIMAX	$-6.20*10^{-5}$* (0.001)	-0.0483 (0.663)	1.1224* (0.000)	0.1925
GARCH & IV_{t-1}	$-4.06*10^{-5}$* (0.007)	0.1256 (0.433)	0.8592* (0.000)	0.1702
GARCH-IV & IV_{t-1}	$-4.23*10^{-5}$* (0.007)	0.1001 (0.912)	0.8869 (0.204)	0.1696
GJR & IV_{t-1}	$-1.93*10^{-5}$ (0.239)	0.4905* (0.002)	0.4566* (0.009)	0.1806
GJR-IV & IV_{t-1}	$-1.90*10^{-6}$ (0.905)	1.1159* (0.000)	-0.1171 (0.543)	0.2083
EGARCH & IV_{t-1}	$-2.66*10^{-5}$*** (0.097)	0.5900** (0.011)	0.4692** (0.025)	0.1766
EGARCH-IV & IV_{t-1}	$3.95*10^{-7}$ (0.981)	1.7068* (0.000)	-0.5287 ** (0.050)	0.2044
CGARCH & IV_{t-1}	$-4.17*10^{-5}$* (0.005)	0.0670 (0.586)	0.9082 * (0.000)	0.1699
CGARCH-IV & IV_{t-1}	$-2.77*10^{-5}$ (0.104)	0.5279*** (0.076)	0.4815*** (0.088)	0.1730
ACGARCH & IV_{t-1}	$-4.01*10^{-5}$* (0.007)	0.1367 (0.270)	0.8488* (0.000)	0.1709
ACGARCH-IV & IV_{t-1}	$-2.09*10^{-5}$ (0.234)	0.6858** (0.023)	0.3211 (0.3211)	0.1752

Note: As Table 12.

Table 14: Forecast encompassing regression results for the Nasdaq100 index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-6.03*10^{-5}$* (0.004)	-0.2049 (0.239)	1.1629* (0.000)	0.1577
GARCH-IV & ARMA	$-9.04*10^{-5}$ (0.788)	3.0665*** (0.051)	-1.5663 (0.236)	0.1604
GJR & ARMAX	$-6.91*10^{-5}$* (0.000)	-0.0354 (0.851)	1.0760* (0.000)	0.1662
GJR-IV & ARMAX	$-4.15*10^{-5}$*** (0.056)	1.1037* (0.000)	0.0745 (0.771)	0.1858†
EGARCH & ARMAX	$-6.93*10^{-5}$* (0.000)	-0.2314 (0.266)	1.2165* (0.000)	0.1675
EGARCH-IV & ARMAX	$-3.92*10^{-5}$*** (0.087)	1.0543* (0.003)	0.1114 (0.731)	0.1759
CGARCH & ARIMA	$-5.96*10^{-5}$* (0.005)	-0.1536 (0.363)	1.1294* (0.000)	0.1542
CGARCH & ARFIMA	$-6.46*10^{-5}$* (0.002)	-0.1408 (0.391)	1.1352* (0.000)	0.1569
CGARCH-IV & ARIMA	$-1.23*10^{-5}$ (0.672)	0.9800** (0.022)	0.0318 (0.941)	0.1592
CGARCH-IV & ARFIMA	$-2.66*10^{-5}$ (0.367)	0.7401*** (0.072)	0.2821 (0.499)	0.1597
ACGARCH & ARIMAX	$-6.63*10^{-5}$* (0.001)	-0.1341 (0.361)	1.1446* (0.000)	0.1656
ACGARCH & ARFIMAX	$-7.39*10^{-5}$* (0.000)	-0.1170 (0.416)	1.1604* (0.000)	0.1672
ACGARCH-IV & ARIMAX	$-6.67*10^{-5}$* (0.018)	-0.0191 (0.957)	1.0571* (0.001)	0.1646
ACGARCH-IV & ARFIMAX	$-7.54*10^{-5}$* (0.007)	-0.0416 (0.901)	1.1093* (0.002)	0.1665
GARCH & IV_{t-1}	$-5.79*10^{-5}$* (0.005)	-0.2031 (0.238)	1.1547* (0.000)	0.1592
GARCH-IV & IV_{t-1}	$-3.15*10^{-5}$ (0.325)	1.8640 (0.271)	-0.5481 (0.698)	0.1590
GJR & IV_{t-1}	$-5.71*10^{-5}$* (0.006)	0.0858 (0.648)	0.9285* (0.000)	0.1579
GJR-IV & IV_{t-1}	$-2.85*10^{-5}$ (0.179)	1.4066* (0.000)	-0.2193 (0.397)	0.1843
EGARCH & IV_{t-1}	$-5.87*10^{-5}$* (0.005)	-0.1112 (0.595)	1.0822* (0.000)	0.1580
EGARCH-IV & IV_{t-1}	$-1.54*10^{-6}$ (0.499)	1.7028* (0.000)	-0.4921 (0.168)	0.1778
CGARCH & IV_{t-1}	$-5.90*10^{-5}$* (0.004)	-0.1570 (0.338)	1.1258 * (0.000)	0.1587
CGARCH-IV & IV_{t-1}	$-3.11*10^{-5}$ (0.260)	0.6225 (0.133)	0.3946 (0.338)	0.1602
ACGARCH & IV_{t-1}	$-5.86*10^{-5}$* (0.005)	-0.0572 (0.693)	1.0461* (0.000)	0.1578
ACGARCH-IV & IV_{t-1}	$-3.91*10^{-5}$ (0.158)	0.3790 (0.293)	0.6111*** (0.099)	0.1589

Note: As Table 12.

Table 15: Forecast encompassing regression results for the S&P500 index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-4.56*10^{-5}$* (0.000)	0.0854 (0.254)	0.7286* (0.000)	0.3962
GARCH-IV & ARMA	$-6.32*10^{-5}$* (0.000)	−1.1479* (0.003)	1.7076* (0.000)	0.4020
GJR & ARMAX	$-4.10*10^{-5}$* (0.000)	0.1939** (0.011)	0.6288* (0.000)	0.4239
GJR-IV & ARMAX	$-2.63*10^{-5}$* (0.004)	0.5151* (0.000)	0.3222* (0.000)	0.4477†
EGARCH & ARMAX	$-4.91*10^{-5}$* (0.000)	0.1215 (0.278)	0.7308* (0.000)	0.4198
EGARCH-IV & ARMAX	$-3.15*10^{-5}$* (0.000)	0.5651* (0.000)	0.3181* (0.006)	0.4352
CGARCH & ARIMA	$-4.42*10^{-5}$* (0.000)	0.0692 (0.363)	0.7328* (0.000)	0.3882
CGARCH & ARFIMA	$-4.75*10^{-5}$* (0.000)	0.0669 (0.370)	0.7508* (0.000)	0.3920
CGARCH-IV & ARIMA	$-4.41*10^{-5}$* (0.000)	0.0432 (0.721)	0.3893* (0.000)	0.3877
CGARCH-IV & ARFIMA	$-5.00*10^{-5}$* (0.000)	−0.00063 (0.996)	0.8108* (0.000)	0.3913
ACGARCH & ARIMAX	$-5.37*10^{-5}$* (0.000)	−0.0470 (0.390)	0.8628* (0.000)	0.4159
ACGARCH & ARFIMAX	$-5.94*10^{-5}$* (0.000)	−0.0345 (0.517)	0.8837* (0.000)	0.4202
ACGARCH-IV & ARIMAX	$-6.05*10^{-5}$* (0.000)	−0.6735** (0.011)	1.3543* (0.000)	0.4203
ACGARCH-IV & ARFIMAX	$-7.11*10^{-5}$* (0.000)	−0.7448 (0.003)	1.4604* (0.000)	0.4266
GARCH & IV_{t-1}	$-4.46*10^{-5}$* (0.000)	0.0730 (0.310)	0.7304* (0.000)	0.4060
GARCH-IV & IV_{t-1}	$-6.99*10^{-5}$* (0.000)	−2.4939* (0.000)	2.7074* (0.000)	0.4333
GJR & IV_{t-1}	$-3.08*10^{-5}$* (0.001)	0.2590* (0.001)	0.2587* (0.000)	0.4141
GJR-IV & IV_{t-1}	$-1.66*10^{-5}$*** (0.066)	0.6071* (0.000)	0.2057* (0.022)	0.4419
EGARCH & IV_{t-1}	$-3.87*10^{-5}$* (0.000)	0.2186*** (0.061)	0.6143* (0.000)	0.4079
EGARCH-IV & IV_{t-1}	$-1.61*10^{-5}$* (0.000)	0.8253* (0.000)	0.0545 (0.683)	0.4297
CGARCH & IV_{t-1}	$-4.54*10^{-5}$* (0.000)	0.0424 (0.553)	0.7560 * (0.000)	0.4055
CGARCH-IV & IV_{t-1}	$-5.20*10^{-5}$* (0.000)	−0.1051 (0.352)	0.8966* (0.000)	0.4059
ACGARCH & IV_{t-1}	$-4.66*10^{-5}$* (0.000)	$6.89*10^{-5}$ (0.999)	0.7917* (0.000)	0.4052
ACGARCH-IV & IV_{t-1}	$-6.46*10^{-5}$* (0.000)	−1.6572* (0.000)	2.0841* (0.000)	0.4153

Note: Entries are the estimated coefficients, their p-values in parentheses and the adjusted R^2 values from the encompassing regression described in equation (42) when the realized variance measures the true volatility. *, ** and *** denote significance at the 1% level. A significant p-value indicates that the forecast under consideration is not encompassed by the alternative model, † denotes the highest adjusted R^2 value.

Table 16: Forecast encompassing regression results for the DJIA index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-3.76*10^{-5}$* (0.000)	0.0345 (0.706)	0.8519* (0.000)	0.3534
GARCH-IV & ARMA	$-4.11*10^{-5}$* (0.000)	−0.3138 (0.454)	1.1277* (0.001)	0.3537
GJR & ARMAX	$-2.99*10^{-5}$* (0.001)	0.2249* (0.007)	0.6641* (0.000)	0.3736
GJR-IV & ARMAX	$-1.89*10^{-5}$** (0.032)	0.5713* (0.000)	0.3372* (0.001)	0.3966†
EGARCH & ARMAX	$-3.59*10^{-5}$* (0.000)	0.1917 (0.118)	0.7388* (0.000)	0.3696
EGARCH-IV & ARMAX	$-2.17*10^{-5}$** (0.014)	0.7307* (0.000)	0.2534*** (0.052)	0.3899
CGARCH & ARIMA	$-3.82*10^{-5}$* (0.000)	−0.0656 (0.351)	0.9278* (0.000)	0.3487
CGARCH & ARFIMA	$-4.10*10^{-5}$* (0.000)	−0.0690 (0.324)	0.9468* (0.000)	0.3518
CGARCH-IV & ARIMA	$-3.41*10^{-5}$* (0.001)	0.0801 (0.640)	0.7961* (0.000)	0.3482
CGARCH-IV & ARFIMA	$-3.83*10^{-5}$* (0.000)	0.0292 (0.864)	0.8588* (0.000)	0.3510
ACGARCH & ARIMAX	$-4.09*10^{-5}$* (0.000)	−0.0431 (0.530)	0.9287* (0.000)	0.3652
ACGARCH & ARFIMAX	$-4.56*10^{-5}$* (0.000)	−0.0405 (0.548)	0.9561* (0.000)	0.3703
ACGARCH-IV & ARIMAX	$-3.27*10^{-5}$* (0.001)	0.2103 (0.161)	0.6913* (0.000)	0.3665
ACGARCH-IV & ARFIMAX	$-3.85*10^{-5}$* (0.001)	0.1590 (0.271)	0.7658* (0.000)	0.3710
GARCH & IV_{t-1}	$-3.67*10^{-5}$* (0.000)	0.0202 (0.815)	0.8545* (0.000)	0.3648
GARCH-IV & IV_{t-1}	$-4.91*10^{-5}$* (0.000)	−2.1207* (0.000)	2.4926* (0.000)	0.3804
GJR & IV_{t-1}	$-2.56*10^{-5}$* (0.004)	0.2374* (0.005)	0.6261* (0.000)	0.3714
GJR-IV & IV_{t-1}	$-1.49*10^{-5}$*** (0.082)	0.6017* (0.000)	0.2887* (0.006)	0.3941
EGARCH & IV_{t-1}	$-3.15*10^{-5}$* (0.000)	0.1990 (0.115)	0.7046* (0.000)	0.3668
EGARCH-IV & IV_{t-1}	$-1.54*10^{-5}$*** (0.082)	0.8539* (0.000)	0.1249 (0.395)	0.3874
CGARCH & IV_{t-1}	$-3.81*10^{-5}$* (0.000)	−0.0619 (0.353)	0.9224 * (0.000)	0.3655
CGARCH-IV & IV_{t-1}	$-4.16*10^{-5}$* (0.000)	−0.1601 (0.352)	1.0175* (0.000)	0.3656
ACGARCH & IV_{t-1}	$-3.74*10^{-5}$* (0.000)	−0.0175 (0.794)	0.3665* (0.000)	0.3648
ACGARCH-IV & IV_{t-1}	$-3.18*10^{-5}$* (0.001)	0.1636 (0.318)	0.7181* (0.000)	0.3656

Note: As Table 15.

Table 17: Forecast encompassing regression results for the Nasdaq100 index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-4.00*10^{-5}*$ (0.000)	-0.1043*** (0.096)	0.6378* (0.000)	0.3088
GARCH-IV & ARMA	$-8.02*10^{-5}*$ (0.000)	-2.4014* (0.000)	2.5724* (0.000)	0.3227
GJR & ARMAX	$-4.37*10^{-5}*$ (0.000)	-0.0436 (0.521)	0.6098* (0.000)	0.3174
GJR-IV & ARMAX	$-3.03*10^{-5}*$ (0.000)	0.5167* (0.000)	0.1176* (0.000)	0.3412†
EGARCH & ARMAX	$-4.38*10^{-5}*$ (0.000)	-0.2324* (0.002)	0.7439* (0.000)	0.3257
EGARCH-IV & ARMAX	$-3.34*10^{-5}*$ (0.000)	0.3424* (0.007)	0.2688** (0.022)	0.3235
CGARCH & ARIMA	$-3.96*10^{-5}*$ (0.000)	-0.0805 (0.187)	0.6228* (0.000)	0.3024
CGARCH & ARFIMA	$-4.25*10^{-5}*$ (0.000)	-0.0741 (0.210)	0.6268* (0.000)	0.3079
CGARCH-IV & ARIMA	$-4.44*10^{-5}*$ (0.000)	-0.1119 (0.467)	0.6683* (0.000)	0.3013
CGARCH-IV & ARFIMA	$-5.21*10^{-5}*$ (0.000)	-0.2047 (0.166)	0.7710* (0.000)	0.3082
ACGARCH & ARIMAX	$-4.24*10^{-5}*$ (0.000)	-0.1628 (0.001)	0.6989* (0.000)	0.3240
ACGARCH & ARFIMAX	$-4.68*10^{-5}*$ (0.000)	-0.1497* (0.004)	0.7053* (0.000)	0.3263
ACGARCH-IV & ARIMAX	$-4.71*10^{-5}*$ (0.000)	-0.1034 (0.420)	0.6750* (0.000)	0.3160
ACGARCH-IV & ARFIMAX	$-5.16*10^{-5}*$ (0.000)	-0.1022 (0.392)	0.6911* (0.000)	0.3194
GARCH & IV_{t-1}	$-3.88*10^{-5}*$ (0.000)	-0.1049*** (0.091)	0.6350* (0.000)	0.3123
GARCH-IV & IV_{t-1}	$-9.51*10^{-5}*$ (0.000)	-3.8683* (0.000)	3.7715* (0.000)	0.3408
GJR & IV_{t-1}	$-3.93*10^{-5}*$ (0.001)	-0.0108 (0.874)	0.5650* (0.000)	0.3097
GJR-IV & IV_{t-1}	$-2.64*10^{-5}*$ (0.001)	0.5962* (0.000)	0.0385 (0.678)	0.3399
EGARCH & IV_{t-1}	$-3.93*10^{-5}*$ (0.000)	-0.2023* (0.007)	0.7029* (0.000)	0.3163
EGARCH-IV & IV_{t-1}	$-2.70*10^{-5}*$ (0.001)	0.4791* (0.001)	0.1356 (0.293)	0.3197
CGARCH & IV_{t-1}	$-3.93*10^{-5}*$ (0.000)	-0.0832 (0.158)	0.6218 * (0.000)	0.3115
CGARCH-IV & IV_{t-1}	$-5.17*10^{-5}*$ (0.000)	-0.2855*** (0.056)	0.8341* (0.000)	0.3130
ACGARCH & IV_{t-1}	$-3.93*10^{-5}*$ (0.000)	-0.1335** (0.010)	0.6600* (0.000)	0.3156
ACGARCH-IV & IV_{t-1}	$-3.98*10^{-5}*$ (0.000)	-0.0135 (0.917)	0.5695* (0.000)	0.3097

Note: As Table 15.

Table 18: Summary of 1% and 5% VaR failure rates of forecast encompassing regressions when squared returns is the measure of true volatility

	1%			5%		
	Ave. failure rate	Sig. Kupiec test	Sig. DQ test	Ave. failure rate	Sig. Kupiec test	Sig. DQ test
GARCH & ARMA	0.0241	All	All	0.0622	None	None
GARCH-IV & ARMA	0.0249	All	All	0.0612	None	None
GJR & ARMAX	0.0232	S&P, Nasdaq	S&P, Nasdaq	0.0631	S&P	S&P
GJR-IV & ARMAX	0.0215	S&P, Nasdaq	S&P, Nasdaq	0.0600	None	None
EGARCH & ARMAX	0.0232	S&P	S&P	0.0613	S&P	S&P
EGARCH-IV & ARMAX	0.0259	All	All	0.0617	None	None
CGARCH & ARIMA	0.0241	All	All	0.0604	None	None
CGARCH & ARFIMA	0.0245	All	All	0.0626	S&P	S&P
CGARCH-IV & ARIMA	0.0219	S&P, Nasdaq	S&P, Nasdaq	0.0604	None	None
CGARCH-IV & ARFIMA	0.0219	S&P, Nasdaq	S&P, Nasdaq	0.0613	None	None
ACGARCH & ARIMAX	0.0249	S&P, Nasdaq	S&P, Nasdaq	0.0609	S&P	S&P
ACGARCH & ARFIMAX	0.0259	S&P, Nasdaq	S&P, Nasdaq	0.0618	S&P	S&P
ACGARCH-IV & ARIMAX	0.0188	Nasdaq	Nasdaq	0.0530	None	None
ACGARCH-IV & ARFIMAX	0.0215	S&P, Nasdaq	S&P, Nasdaq	0.0552	None	None
GARCH & IV_{t-1}	0.0245	All	All	0.0609	None	S&P
GARCH-IV & IV_{t-1}	0.0245	All	All	0.0600	None	None
GJR & IV_{t-1}	0.0228	S&P, Nasdaq	S&P, Nasdaq	0.0600	None	None
GJR-IV & IV_{t-1}	0.0241	All	All	0.0569	None	None
EGARCH & IV_{t-1}	0.0246	All	All	0.0600	None	None
EGARCH-IV & IV_{t-1}	0.0298	All	All	0.0618	None	None
CGARCH & IV_{t-1}	0.0232	All	All	0.0604	None	None
CGARCH-IV & IV_{t-1}	0.0232	All	All	0.0622	None	None
ACGARCH & IV_{t-1}	0.0241	All	All	0.0614	S&P	S&P
ACGARCH-IV & IV_{t-1}	0.0249	All	All	0.0622	S&P	S&P

Note: Entries are the average failure rate of the forecasts encompassing regressions. The series for which the Kupiec test for the equality of the empirical failure rate at a specified statistical level and the DQ test for the autocorrelation in VaR violations are significant are listed.

Table 19: Summary of 1% and 5% VaR failure rates of forecast encompassing regressions when realized variance is the measure of true volatility

	1%			5%		
	Ave. failure rate	Sig. Kupiec test	Sig. DQ test	Ave. failure rate	Sig. Kupiec test	Sig. DQ test
GARCH & ARMA	0.0386	All	All	0.0823	S&P, Nasdaq	S&P, Nasdaq
GARCH-IV & ARMA	0.0425	All	All	0.0824	S&P, Nasdaq	S&P, Nasdaq
GJR & ARMAX	0.0359	S&P, Nasdaq	S&P, Nasdaq	0.0784	S&P, Nasdaq	S&P, Nasdaq
GJR-IV & ARMAX	0.0355	S&P, Nasdaq	S&P, Nasdaq	0.0758	S&P, Nasdaq	None
EGARCH & ARMAX	0.0364	S&P500	S&P500	0.0832	S&P, Nasdaq	S&P, Nasdaq
EGARCH-IV & ARMAX	0.0363	All	All	0.0766	Nasdaq	Nasdaq
CGARCH & ARIMA	0.0394	All	All	0.0835	S&P, Nasdaq	S&P, Nasdaq
CGARCH & ARFIMA	0.0399	All	All	0.0837	S&P, Nasdaq	S&P, Nasdaq
CGARCH-IV & ARIMA	0.0421	All	All	0.0828	S&P, Nasdaq	S&P, Nasdaq
CGARCH-IV & ARFIMA	0.0421	All	All	0.0854	S&P, Nasdaq	S&P, Nasdaq
ACGARCH & ARIMAX	0.0359	S&P, Nasdaq	S&P, Nasdaq	0.0797	S&P, Nasdaq	S&P, Nasdaq
ACGARCH & ARFIMAX	0.0351	S&P, Nasdaq	S&P, Nasdaq	0.0810	S&P, Nasdaq	S&P, Nasdaq
ACGARCH-IV & ARIMAX	0.0332	S&P, Nasdaq	S&P, Nasdaq	0.0775	Nasdaq	Nasdaq
ACGARCH-IV & ARFIMAX	0.0328	S&P, Nasdaq	S&P, Nasdaq	0.0788	Nasdaq	Nasdaq
GARCH & IV_{t-1}	0.0386	All	All	0.0832	S&P, Nasdaq	S&P, Nasdaq
GARCH-IV & IV_{t-1}	0.0390	All	All	0.0810	S&P, Nasdaq	S&P, Nasdaq
GJR & IV_{t-1}	0.0390	All	All	0.0815	S&P, Nasdaq	S&P, Nasdaq
GJR-IV & IV_{t-1}	0.0399	All	All	0.0775	S&P, Nasdaq	Nasdaq100
EGARCH & IV_{t-1}	0.0386	All	All	0.0845	S&P, Nasdaq	S&P, Nasdaq
EGARCH-IV & IV_{t-1}	0.0381	All	All	0.0789	S&P, Nasdaq	S&P, Nasdaq
CGARCH & IV_{t-1}	0.0386	All	All	0.0837	S&P, Nasdaq	S&P, Nasdaq
CGARCH-IV & IV_{t-1}	0.0416	All	All	0.0854	S&P, Nasdaq	S&P, Nasdaq
ACGARCH & IV_{t-1}	0.0390	All	All	0.0832	S&P, Nasdaq	S&P, Nasdaq
ACGARCH-IV & IV_{t-1}	0.0399	All	All	0.0817	S&P, Nasdaq	S&P, Nasdaq

Note: Entries are the average failure rate of the forecasts encompassing regressions. The series for which the Kupiec test for the equality of the empirical failure rate at a specified statistical level and the DQ test for the autocorrelation in VaR violations are significant are listed.

3 Forecasting stock return volatility: Further international evidence

In this study, I repeat the analysis of Chapter 2 in order to investigate whether the implied volatility forecast is a good predictor of stock market volatility when European data are examined. for this purpose, six European indices - EURO STOXX, CAC40, DAX30, AEX, SMI, FTSE100 - and their IV indices are used. The results are consistent with those obtained for the US data suggesting that an ARMAX model is the best model for modelling and forecasting IV. Moreover, implied volatility forecast is a good predictor of future volatility and a model which includes the information contained in an asymmetric GARCH and the information contained in IV through an asymmetric ARMA model is the best for predicting future stock market volatility.

3.1 Introduction

Volatility forecasting has received extensive attention in literature. Since the construction of the VIX index by CBOE in 1993, the IV indices have mushroomed. Te IV indices have been used in the continuing debate of finding the model that produces the most accurate volatility forecast. The question whether IV contains incremental information relevant to future volatility beyond that captured in GARCH model forecasts has been extensively analyzed. While there is an extensive literature addressing this issue using US data, there are few evidence using data from other international stock markets.[10]

For example, Claessen & Mittnik (2002), for the stock market of Germany an the DAX index, find that IV derived from time series models contains all the information useful in predicting future volatility. Frijns et al. (2010) and Yang & Liu (2012) examine the stock markets of Australia and Taiwan respectively, and they find that IV contains additional information about future volatility. On the other hand, the predictability of IV itself have received little attention.[11] To the best of my knowledge, Konstantinidi et al. (2008) is the only study that provide international evidence on this

[10]For a more complete literature review, see Chapter 3, Section 2.
[11]For a literature review, see Chapter 3, Section 2.

issue. They examine four American implied volatility indices (VIX, VXO, VXN, VXD) and three European (VDAX, VCAC, VSTOXX) performing a horse race among alternative models and find that there is a predictable pattern in the dynamics of IV indices.

The aim of this study is to repeat the analysis of Chapter 3 using six European indices - EURO STOXX, CAC40, DAX30, AEX, SMI, FTSE100 - and their IV indices in order to check the robustness of the obtained results. I address the question whether IV can be forecasted. In an MZ regression framework, I attempt to answer whether the IV forecast is a good predictor of stock index return volatility.

The remainder of the chapter is structured as follows. In the next Section, the dataset is described. Section 3.3 presents both the in-sample and out-of-sample performance of the models. The last Section concludes.

3.2 Data and Empirical Methodology

The dataset used in estimating and forecasting exercise consists of the daily closing price data for six European indices and their IV indices over the period February 2, 2001 to February 28, 2013, as in Chapter 3. The in-sample period is again from February 2, 2001 to February 23, 2010 and the remaining period, from February 24, 2010 to February 28, 2013, is reserved for the out-of-sample evaluation. More specifically, the stock indices are the EURO STOXX, CAC40, DAX30, AEX, SMI, FTSE100 and their IV indices are VSTOXX, VCAC, VDAX, VAEX, VSMI, VFTSE100, respectively. Both the *ex post* daily squared returns and the realized variance[12] are used as proxies for the true volatility.

The price indices are converted to returns by calculating the prices log difference. Figure 2 shows clearly that returns are centered around zero with their amplitude to vary over time showing evidence of volatility clustering. The summary statistics of the returns are presented in Table 20 for the full sample and Table 21 for the in-sample period. The mean and median are consistent and close to zero. As the skewness is concerned, for the STOXX and CAC the skewness value is positive

[12] I obtain the daily realized variances from Realized Library version 0.2 of the Oxford-Man Institute of Quantitative Finance Heber et al. (2009). These realized variances are based on the sum of 5-minute intra-day squared returns.

in both tables indicating asymmetric distributions skewed to the right, while AEX and FTSE100 returns are skewed to the left. DAX and SMI returns' skewness is negative for the full-sample and positive in-sample. Looking at the kurtosis value both tables show the leptokurtic characteristic of all returns distributions. Finally, the Jarque-Bera test statistic for normality rejects the null hypothesis that returns follow a normal distribution.

The IV indices have been constructed to measure the market's expectations of the underlying index's volatility for the next 22 trading days. European markets encouraged by the success of CBOE introducing VIX have developed several indices using the same methodology. Thus, EUREX and NYSE Euronext exchange have introduced VSTOXX, VDAX, VSMI and VCAC,VAEX, VFTSE100, respectively. Table 22 show the summary statistics of the IV indices as well as the Augmented Dickey-Fuller (ADF) test for unit roots. The p-values of the ADF test show that implied volatility indices are stationary at the 1% level.

In this chapter, I follow the same empirical methodology of Chapter 3[13], because the aim of this Chapter is to investigate whether the strong results obtained for the US indices hold for the EU indices.

3.3 Empirical results

3.3.1 In-sample results

To consider whether an ARCH process appears in the innovation term sequence in return equation $(r_t = \mu + \varepsilon_t)$[14] is the same as to identify the presence of conditional heteroskedasticity. The squared residual series $\hat{\varepsilon}_t^2$ are conducted to test the conditional heteroskedasticity which is known as ARCH effect. This is performed by testing for serial correlation in squared errors. The two tests for conditional heteroskedasticity are carried out in this exercise are the Ljung-Box test and the Lagrange Multiplier test. As referred to the Table 23, the Ljung-Box Q(m) statistics of all return series are significant with p-value equal to zero, which indicates that the squared residuals are autocorrelated. In the same table, according to the Lagrange Multiplier the null hypothesis of

[13]More details for the methodology has been used can be found in Chapter 3, Subsection 3.2.
[14]For more details see Chapter 3, Subsection 3.2.

homoskedasticity is clearly rejected at 1% significant level, indicating the presence of ARCH effect in all return series. These results provide justification for the next stage in the analysis which involves estimating the conditional variance using an ARCH process.

Table 24 show the in-sample performance of the alternative GARCH models. For all index returns, the estimates of GARCH show that the coefficients satisfy the non-negativity constraint. The models are stationary, because the sum $a_1 + \beta_1$ is less than one, although close to unity. This implies that shocks to volatility have a highly persistent effect on the conditional variance. In GARCH-IV models, the IV is added as an exogenous variable in the variance equation. Both the fact that the IV estimated coefficient is significantly different from zero for all indices and the value of the likelihood ratio test reject the null hypothesis that IV does not contain incremental information other than the information contained in GARCH useful for explaining the conditional variance.

In both GJR and GJR-IV models, the impact of the bad news $a_1+\gamma$ on the conditional variance is much greater than the one of the good news, a_1, indicating a substantial negative asymmetric effect. When the information of the IV is added in the GJR-IV model, the log-likelihood is significantly higher than in the GJR model. Similar information can be extracted from the EGARCH and EGARCH-IV models. The presence of the leverage effect in returns is captured by the coefficient γ which is significantly less than zero. Once more, the presence of IV improves the model's fit indicating that IV has incremental explanatory power for the conditional variance.

The usefulness of IV is also captured by comparing the CGARCH-IV and its asymmetric specification ACGARCH-IV with their restricted version. The stationarity constraints are satisfied implying that the long run index return conditional volatility will decay more slowly than the transitory component of volatility. Moreover, the asymmetric effect in models ACGARCH and ACGARCH-IV is captured by γ with negative news having greater effect on volatility because $\gamma > 0$.

Diagnostic tests in the squared normalized residuals for all alternative GARCH specifications are reported in Table 25. The value of skewness indicates asymmetric distributions skewed to the left for all series. The kurtosis and the Jarque-Bera test have noticeably reduced in absolute values for all series compared to the statistics from the original return series in Table 21. According to the Ljung-Box Q(7) statistic, the alternative GARCH specifications have considerably reduced the intertemporal dependence of the squared standardized residuals. Nevertheless, the null hypothesis of no autocorrelation is not rejected at the 1% level for all models and indices. Similar results are obtained looking at the LM(7) test, where the null hypothesis of homoskedasticity cannot be rejected at the 1% level in all cases.

Table 26 presents parameter estimates and the log-likelihood of the six ARMA models defined in Chapter 3, Subsection 3.2. The $AR(1)$ and $MA(1)$ coefficients are statistically significant for all models apart from the ARIMA model in VDAX indices. Focusing on the coefficients of r_t^+ and r_t^-, it is apparent that there is an asymmetric effect for all indices. In all cases, the coefficient of r_t^- is greater in absolute value than the coefficient of r_t^+, which indicates that negative returns yield much higher implied volatility than positive ones. In the table, negative coefficients are reported for both contemporaneous positive and negative returns. Hence, contemporaneous positive returns reduce IV, while negative contemporaneous returns raise the IV. Focusing on the ARFIMA(X) models, the fractional integration parameter d is significant at 1% level throughout and lies between $0 < d < 0.5$ implying that IV exhibits long memory. Finally, According to the log-likelihood, the inclusion of both positive and negative returns improves the model's fit. An ARMAX and an ARIMAX model performs best for the VSTOXX, VDAX, VAEX and VCAC, VSMI,VFTSE100, respectively.

3.3.2 Out-of-sample results

Regarding the forecast of the implied volatility models, Table 27 reports the Diebold-Mariano (DM) test in order to address the question whether the dynamics of implied volatility *per se* can be forecasted. The DM test uses the MSE and MAE criteria in order to assess the predictive ability of each ARMA forecasting model against the benchmark random walk process. The null hypothesis of equal predictive ability is tested against the alternative hypothesis that random walk is outperformed by the ARMA models. There are 42 cases(out of 72) in which we reject the null hypothesis of equal predictability. Therefore, in 58.33% of the different possible combinations of IV and predictability measures an ARMA type models performs better than the random walk. This indicates that there is a predictable pattern in the dynamics of implied volatility indices.

In terms of how which model performs best, using both the MSE and MAE, the ARMAX model yields the lowest loss versus the alternative models for all indices with the exception of the VCAC in which the ARIMAX model performs best closely followed by the ARMAX when the MSE forecast error is used. The ARFIMA and random walk yield the highest MSE and MAE. Only when is the model under consideration an ARMA model that takes into account the contemporaneous asymmetric effect - ARMAX, ARIMAX, ARFIMAX models - outperforms the random walk. In these cases, the null hypothesis of equal predictive ability is always rejected at the 1% level.

In Tables 28 and 29, the ability of both GARCH and IV models to adequately predict volatility is assessed using the MAE and RMSE. Table 28 present the mean absolute error (MAE) and root mean square error (RMSE) forecast statistics for each model when *ex post* daily squared returns is used as measure of true volatility. On the basis of the MAE the results suggest that the EGARCH-IV model provides the most accurate forecast for all indices apart from the SMI index in which the EGARCH model yields the lowest error. With the exception of the GARCH-IV, GJR-IV and EGARCH-IV for the SMI index, the GARCH models augmented with IV provide better forecasts than their counterpart restricted GARCH models. The forecast of IV through an ARMA-type specification or a random walk perform poorly yielding the highest MAE. Focusing just on the forecasting performance of the IV, the results show that the ARMAX model yields the lowest MAE for all indices except the DAX index in which the ARIMAX model performs best. When the IV forecast takes into consideration the asymmetric relationship between returns and IV it always perform better than the random walk.

On the other hand, in Table 28 and under the RMSE, the IV forecasts perform better than using the MAE. Specifically, the EGARCH-IV model yields the lowest loss for the CAC, AEX, SMI and FTSE100 indices, while the ARMAX model performs best for the STOXX and DAX indices. In all series, a GARCH specification nested with implied volatility outperforms when it is compared with the respective restricted specification. As for the performance of the various IV indices to provide accurate forecasts for the true volatility, the results show the importance of the asymmetry, as only the ARMA models that captures the asymmetry perform better than the random walk. Among the IV models, the ARMAX model yields the lowest RMSE.

Table 29 reports the MAE and RMSE using the realized variance measure of true volatility. According to the MAE, the EGARCH-IV provides the best forecast for the CAC, AEX and FTSE100 indices, while the EGARCH performs best for the STOXX and SMI indices. As for the DAX index, the GJR-IV yields the lowest forecast error. Apart from the SMI index, for all indices a GARCH model that embeds IV outperforms its restricted version. As the IV models are concerned, they generally perform poorly. Similarly to the results of Table 28, among the IV models, the ARMAX and ARIMAX specifications yield the lowest MAE. The ARMA models that capture the contemporaneous asymmetric relationship between IV and index returns provide more accurate forecast for the true volatility that their restricted versions and the random walk.

According to the RMSE, the EGARCH-IV provides the best forecast for the STOXX, CAC and AEX index, while the GJR-IV and GARCH-IV performs best for the DAX and FTSE100 index, respectively. As for the SMI index, similarly to the MAE results, the EGARCH preforms best. Among the IV forecasts, the ARFIMAX specification perform best for all indices apart from the SMI in which the ARFIMA specification yields the lowest RMSE.

In order to evaluate the predictive power of the models, i.e. how much of the 'true' volatility is explained by the GARCH forecasts and IV forecasts, the MZ procedure is employed. Tables 30 and 31 report the results of the MZ procedure over the forecast period when the squared returns and realized variance are used as proxies for the true volatility, respectively. The primary interest lies in the R^2 values, where the model with the highest R^2 is preferred.

Examining Table 30, it is first seen the good performance of the forecasts produced by an asymmetric IV models. The ARFIMAX model is the most informative model for the STOXX, DAX and AEX indices and the ARMAX model yields the strictly highest R^2 values for the FTSE100 index. As for the CAC index, the EGARCH-IV model performs best followed by the ARMAX model. Only in the case of SMI index, where the EGARCH-IV has the highest predictive power, the IV forecasts perform noticeably worse. Second, the inclusion of IV in the various GARCH specifications improves the predictive power of the models indicating that IV contains incremental information beyond the GARCH models. Third, the asymmetric IV model forecasts obtain higher

R^2 values than the random walk.

Table 31 reports the results of the univariate MZ regressions for both GARCH and IV forecasts when the realized variance measures true volatility. At first sight, I observe that, using the realized variance, I obtain much higher R^2 values than using the *ex post* squared returns. Second, the results are remarkably consistent across all indices. The GJR-IV obtains the highest R^2 value for the DAX and FTSE100 indices and the EGARCH-IV performs best for all the other indices. Third, and similar to the results in Table 30, when the GARCH models are augmented with the IV performs better than their restricted counterparts. Finally, the asymmetric IV model forecasts increases the R^2 values than the random walk with the exception of the CAC index in which the random walk outperforms the ARMA forecasts.

The next step is to investigate the relative forecasting performance of the models so as to identify whether IV forecasts contain different information from GARCH forecasts. Tables 32 to 37 present the results of the encompassing exercise considered in this study for all indices when the *ex post* daily squared returns is used as measure of true volatility.

Encompassing regression results for the STOXX index are given in Table 32. The a_2 coefficient is significantly different from zero in almost all cases implying that IV carries information useful for predicting future volatility. As for the GARCH models, in many cases, it is dominated by the IV forecasts, as the a_1 coefficient is insignificant. There also are some cases in which both coefficients are significant which means that both approaches complement each other. On the basis of the adjusted R^2 values reported, a combination of the ACGARCH model with the ARFIMAX model performs best. Both approaches are highly significant implying that they both contain independent information useful in forecasting future volatility. This can also be noticed from the R^2 value which is higher than the R^2 for the univariate regressions presented in Table 30. Moreover, in some cases, even if one forecast dominates the other a combination of both predictors marginally increases the R^2 value. Finally, similarly to the univariate regressions results, when the GARCH models are combined with an asymmetric IV model yield higher R^2 values than when they are combined with IV following a random walk process.

The results for the DAX index, Table 34, are very similar, in that IV is almost everywhere highly significant and in most cases more informative than GARCH. A combination of the ACGARCH forecast with the ARFIMAX forecast to obtain the highest R^2 value. When the GARCH forecasts are combined with the ARMA forecasts have stronger predictive power than when they are combined with the random walk apart from the case in which the RW is combined with the GARCH-IV.

Similar results are reported in Tables 33, 35 and 37 for the CAC, AEX and FTSE100 indices, respectively. For all these indices, the highest R^2 value is reported when EGARCH-IV is combined with the IV forecast through an ARMAX model. In the case of the CAC index, the EGARCH-IV is more informative than the ARMAX and the combination of these two forecasts marginally improves the forecast of future volatility, reporting slightly higher R^2. With regard to the encompassing tests, the results indicate the strong predictive power of the IV forecasts as the a_2 coefficient is highly significant.

For the CAC and FTSE100 indices, Tables 33 and 37 respectively, show that when the IV forecast takes into account the contemporaneous asymmetric relationship between returns and IV, the encompassing regression yields higher R^2 than when IV follows a random walk. Table 35 for the AEX index indicates that when the different ARMA models are combined with the GARCH models always report higher R^2 than when IV following a random walk is combined with the various GARCH.

Finally, Table 36 reports the encompassing regression results of the SMI index. Looking at the encompassing tests, there are few cases in which both forecasts complement each other. Nonetheless, in many cases a combination of these forecasts slightly improves the forecast of future volatility. Although, in most cases, the IV forecast through an ARMA model combined with a GARCH forecast yields higher R^2 values, the highest R^2 value is reported when the EGARCH-IV is combined with the random walk.

Tables 38 to 43 report the encompassing regressions results when the realized variance is used as measure of true volatility. The results are very similar to those obtained when the *ex post* daily squared returns is used. Using the realized variance, the coefficient a_2 continues to be highly significant in most combinations and indices, while the GARCH forecast is significant in many cases indicating more its importance.

Tables 38, 39 and 42 report the encompassing regression results for the STOXX, CAC and SMI index, respectively. The EGARCH-IV model combined with the ARMAX model yields the highest R^2 value for the STOXX index, while a combination of EGARCH-IV with the random walk performs best for the CAC and SMI indices. For all indices, in most cases, a combination of both GARCH and IV, even when one of the two is not statistically significant, yields higher R^2 values than the R^2 of the univariate regressions reported in Table 31.

Table 40 and 43 report the results for the DAX and FTSE100 indices. The highest R^2 is obtained when the GJR-IV forecast is combined with the ARMAX forecast. In Table 40, when a GARCH approach is combined with an IV forecast always performs better in terms of R^2 than when it is combined with the random walk with the exception of the EGARCH-IV model combined with the random walk. Similar to the univariate regressions results in Table 31 for the FTSE100 index, only when is IV forecasted through an asymmetric ARMA model performs better than when IV follows a random walk process. Finally, Table 41 show that for the AEX index a combination of the ACGARCH-IV forecast with the ARFIMAX forecast is statistically significant and superior in terms of the R^2 values.

The VaR results for the encompassing regressions are reported in Tables 44 and 45 when the squared returns and the realized volatility are respectively used as the true volatility proxies. More specifically, in Table 44, at the 1% VaR probability level, the combination providing the best VaR measures in terms of achieving the lowest average failure rate is the GJR combined with the ARMAX, while at the 5% level the CGARCH-IV combined with the ARIMA performs best. In most cases, when GARCH forecasts are combined with the IV forecasts through an ARMA-type models have lower average failure rate than when the GARCH models are combined with the IV following a random walk. In terms of the specification tests, at the 1% level, several combinations perform well with only two market significant on both the Kupiec test and the DQ test. At the 5% level, the CGARCH-IV combined with the ARIMA performs best with no index significant on the Kupiec test and only one index significant on the DQ test.

In Table 45 the VaR results of the forecast encompassing regressions when realized variance is the measure of true volatility are reported. At the 1% VaR probability level, combining the

ACGARCH-IV with the ARIMAX performs best having lowest average failure rate, while at the 5% level, the GJR combined with the ARMAX model performs best. When GARCH forecasts are combined with the asymmetric IV forecasts outperform the combinations of GARCH forecasts with the random walk. Contrary to the previous table in which the squared returns is the proxy of the true volatility, in this table, in terms of the specification tests, no model rejects the null hypotheses of the equality of the number of violations at a specified rate, Kupiec test, and of the non autocorrelation in the sequence of exceptions, DQ test.

3.4 Conclusion

This study repeat the analysis of Chapter 3 in order to investigate whether the implied volatility forecast is a good predictor of stock market volatility when European data are examined.

The results are robust to those obtained for the US data. First, IV can be forecasted and its forecast contains incremental information regarding the future stock return volatility. Second, asymmetry proves to be important for both GARCH and IV models both in-sample and out-of-sample. When IV is modelled and forecasted through an ARMA model that captures the contemporaneous asymmetric relationship between returns and IV performs better than the random walk. Second, IV indices can be forecasted and the ARMAX model performs best. Third, IV contains additional information useful for the future stock market volatility beyond the information contained in the GARCH model based volatility forecasts. Actually, in many cases, it proves to be more informative than GARCH. Nonetheless, a combination of both approaches is the most appropriate for predicting future stock index return volatility.

Overall, and consistently with the results obtained for the US data, an asymmetric GARCH model augmented with IV combined with the ARMA model that captures the asymmetric relationship between returns and IV performs best both when the squared returns and the realized variance are used as proxies of the true volatility.

Table 20: Summary statistics for the full sample daily stock returns

	Full sample					
	STOXX	CAC	DAX	AEX	SMI	FTSE100
Mean	-0.0002	-0.0001	5.04E-05	-0.0002	-1.76E-05	-5.43E-06
Median	7.91E-05	0.0002	0.0009	0.0003	0.0005	0.0003
Maximum	0.1044	0.1059	0.1079	0.1003	0.1078	0.0938
Minimum	-0.0821	-0.0947	-0.0887	-0.0959	-0.0810	-0.0926
Std. Dev.	0.0161	0.0157	0.0163	0.0159	0.0128	0.0129
Skewness	0.0314	0.0526	-0.0004	-0.0505	0.0133	-0.1116
Kurtosis	7.2469	7.8069	7.2920	8.8121	9.0066	9.2049
Jarque-Bera	2293.3	2966.8	2341.1	4343.6	4538.6	4892.7
p-value	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*

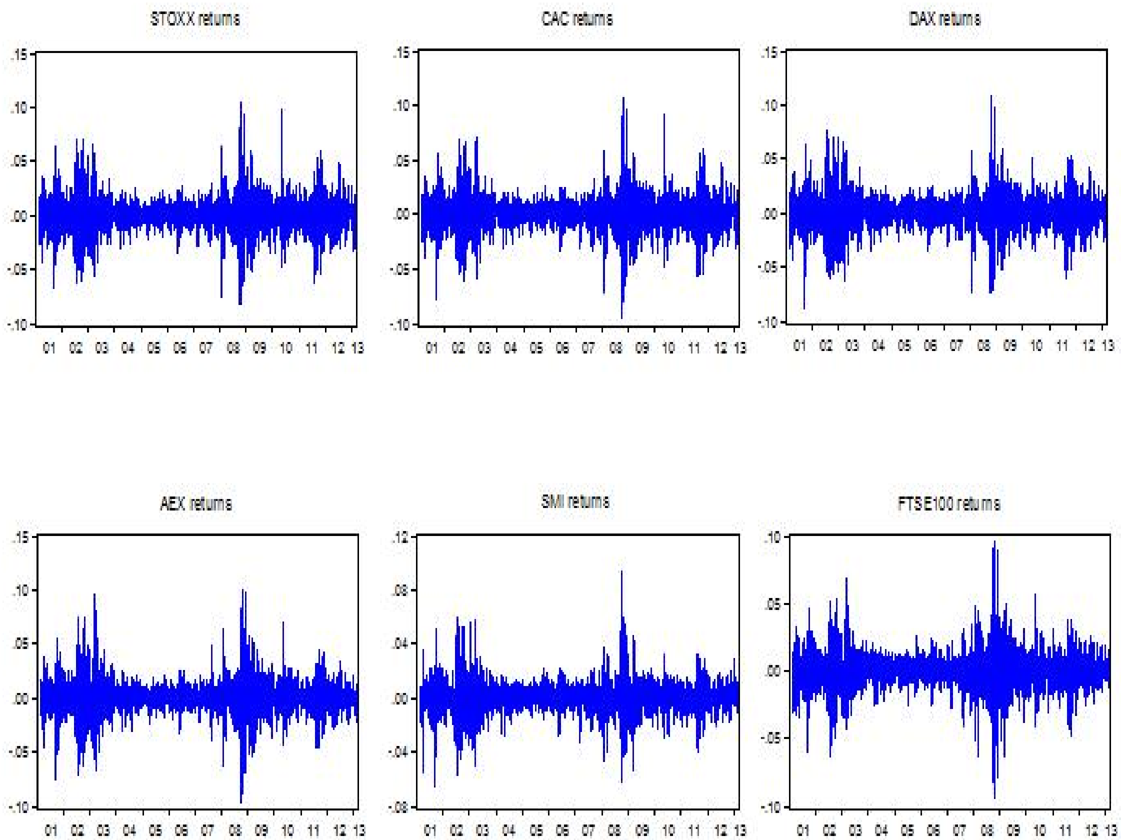
Notes: Entries report the summary statistics of the daily stock returns for the full sample period February 2, 2001 to February 28, 2013. In the last row, the p-values of the Jarque-Bera test for normality are reported. * denotes rejection of the null hypothesis at the 1% level, respectively.

Table 21: Summary statistics for the in-sample daily stock returns

	In-sample					
	STOXX	CAC	DAX	AEX	SMI	FTSE100
Mean	-0.0002	-0.0001	-7.42E-05	-0.0003	-8.22E-05	-7.12E-05
Median	0.0001	0.0002	0.0007	0.0004	0.0005	0.0004
Maximum	0.1043	0.1059	0.1079	0.1003	0.1078	0.0938
Minimum	-0.0820	-0.0947	-0.0887	-0.0959	-0.0810	-0.0926
Std. Dev.	0.0162	0.0158	0.0170	0.0168	0.0135	0.0134
Skewness	0.0027	0.0472	0.0445	-0.0448	0.0537	-0.1010
Kurtosis	7.4630	8.3199	7.4332	8.6479	8.8149	9.6058
Jarque-Bera	1894.7	2720.2	1870.2	3071.0	3188.1	4169.3
Probability	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*

Notes: Entries report the summary statistics of the daily stock returns for the in-sample period February 2, 2001 to February 23, 2010. In the last row, the p-values of the Jarque-Bera test for normality are reported. * denotes rejection of the null hypothesis at the 1% level, respectively.

Figure 2: Daily returns of the European indices



Notes: The figure shows daily returns of the European indices for the period February 2, 2001 to February 28, 2013.

Table 22: Summary statistics for implied volatility indices

	STOXX	CAC	DAX	AEX	SMI	FTSE100
Mean	0.0165	0.0154	0.0162	0.0157	0.0124	0.0137
Median	0.0150	0.0140	0.0142	0.0137	0.0108	0.0122
Maximum	0.0551	0.0491	0.0524	0.0512	0.0534	0.0475
Minimum	0.0073	0.0058	0.0073	0.0036	0.0054	0.0057
Std. Dev.	0.0068	0.0062	0.0068	0.0073	0.0055	0.0064
Skewness	1.3638	1.4541	1.4997	1.5231	2.0087	1.5452
Kurtosis	5.0875	5.6244	5.3390	5.2907	8.6726	6.0283
Jarque-Bera	1500.3	1970.1	1839.2	1867.9	6080.2	1787.9
p-value	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*
ADF (p-value)	0.0027*	0.0124**	0.0484**	0.0151**	0.0009*	0.0186**

Notes: Entries report the summary statistics of the three implied volatility indices for the period February 2, 2001 to February 28, 2013. In the last two rows, the p-values of the Jarque-Bera test for normality and the Augmented Dickey-Fuller (ADF) test for unit root are reported. * and ** denote rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 23: Test for ARCH effects in returns.

Index	$Q(p)$	LM
	$p = 7$	$p = 7$
STOXX	1080.4* (0.000)	455.897* (0.000)
CAC	976.98* (0.000)	427.998* (0.000)
DAX	882.21* (0.000)	391.023* (0.000)
AEX	1372.7* (0.000)	579.891* (0.000)
SMI	1408.9* (0.000)	522.133* (0.000)
FTSE100	1293.2* (0.000)	514.570* (0.000)

Note: The Ljung-Box Q(7) test for squared residual autocorrelation and the Lagrange multiplier (LM) test for homoskedasticity are reported. p-values are in parentheses. * denotes rejection of the null hypothesis at the 1% level.

Table 24: Estimation models of the GARCH family

	$GARCH$	$GARCH-IV$	GJR	$GJR-IV$	$EGARCH$	$EGARCH-IV$	$CGARCH$	$CGARCH_{-IV}$	$ACGARCH$	$ACGARCH_{-IV}$
					STOXX					
a_0	$1.96*10^{-6}$* (0.006)	$-2.35*10^{-5}$* (0.004)	$2.08*10^{-6}$* (0.000)	$-4.01*10^{-7}$ (0.642)	−0.22782* (0.000)	−0.03001 (0.656)	0.00030 (0.140)	$-1.61*10^{-5}$*** (0.077)	0.00027 (0.101)	$-9.22*10^{-6}$ (0.334)
a_1	0.09924* (0.000)	−0.03374*** (0.076)	−0.02963** (0.028)	−0.05658* (0.002)	0.09986* (0.000)	0.01493 (0.620)	−0.09934* (0.000)	−0.07147* (0.003)	−0.15578* (0.000)	−0.10666** (0.038)
β_1	0.89497 * (0.000)	−0.19213 (0.413)	0.92327* (0.000)	0.84992 * (0.000)	0.98306* (0.000)	0.85311 * (0.000)	0.22116 (0.380)	−0.03697 (0.870)	−0.29396 (0.129)	0.85317 * (0.000)
γ			0.19281* (0.000)	0.23541* (0.000)	−0.14268* (0.000)	−0.21891* (0.000)			0.07897*** (0.075)	0.22791 * (0.000)
ρ							0.99087* (0.000)	0.94925* (0.000)	0.99022* (0.000)	0.95510* (0.000)
ϕ							0.12021* (0.000)	0.03368* (0.008)	0.11979* (0.000)	0.04138 (0.232)
θ		1.0083** (0.013)		0.06574* (0.000)		0.15162* (0.000)		0.94036* (0.000)		0.12167* (0.000)
$Log-L$	6686.527	6742.20	6749.725	6767.10	6755.204	6778.36	6694.007	6751.69	6696.348	6770.05
χ^2	111.35	–	34.75	–	46.31	–	115.37	–	147.40	–
					CAC					
a_0	$1.68*10^{-6}$* (0.007	$-1.98*10^{-5}$* (0.013)	$2.11*10^{-6}$* (0.000)	$-6.41*10^{-7}$ (0.473)	−0.22727* (0.000)	−0.08526 (0.125)	0.00031 (0.221)	$-1.24*10^{-5}$ (0.291)	0.000301 (0.206)	$-1.88*10^{-6}$ (0.884)
a_1	0.09545* (0.000)	−0.00601 (0.827)	−0.02152** (0.016)	−0.04391** (0.029)	0.10031* (0.000)	0.03507 (0.240)	−0.10302* (0.000)	−0.06261** (0.049)	−0.13245* (0.001)	−0.07648* (0.007)
β_1	0.89978 * (0.000)	−0.08983 (0.735)	0.92074* (0.000)	0.84986 * (0.000)	0.98330* (0.000)	0.89022 * (0.000)	0.22439 (0.353)	0.10647 (0.648)	0.27937 (0.181)	0.85353 * (0.000)
γ			0.18058* (0.000)	0.22424* 0.000)	−0.13812* (0.000)	−0.20843* (0.000)			0.03964 (0.412)	0.23350* (0.000)
ρ							0.99239* (0.000)	0.94904* (0.000)	0.99210* (0.000)	0.98063* (0.000)
ϕ							0.11479* (0.000)	0.04448* (0.001)	0.11544* (0.000)	0.02664 (0.129)
θ		0.98099* (0.000)		0.06475* (0.000)		0.10705* (0.000)		0.87733* (0.000)		0.10471* (0.000)
$Log-L$	6797.022	6839.06	6853.044	6870.034	6861.553	6882.579	6806.076	6854.197	6806.631	6873.540
χ^2	84.08	–	33.98	–	42.05	–	96.24	–	133.82	–
					DAX					
a_0	$2.13*10^{-6}$** (0.011)	$-2.41*10^{-5}$** (0.031)	$2.33*10^{-6}$* (0.000)	$-4.54*10^{-7}$ (0.754)	−0.24979* (0.000)	0.01262 (0.885)	0.00031*** (0.093)	$-1.21*10^{-8}$ (0.399)	0.00026** (0.027)	$-9.12*10^{-6}$ (0.395)
a_1	0.09716* (0.000)	−0.03470 (0.159)	−0.01624 (0.209)	−0.06741* (0.000)	0.11825* (0.000)	0.02040 (0.540)	−0.09788* (0.001)	−0.08008* (0.007)	−0.17904* (0.000)	−0.13274* (0.000)
β_1	0.89649 * (0.000)	−0.35508 (0.135)	0.92195* (0.000)	0.80342 * (0.000)	0.98194* (0.000)	0.81710 * (0.000)	0.13691 (0.617)	−0.20550 (0.402)	0.13031 (0.458))	0.84850 * (0.000)
γ			0.16423* (0.000)	0.23016* (0.000)	−0.12319* (0.000)	−0.20101* (0.000)			0.09802** (0.016)	0.22452* (0.000)
ρ							0.99042* (0.000)	0.95321* (0.000)	0.98836* (0.000)	0.93829* (0.000)
ϕ							0.11497* (0.000)	0.03844* (0.002)	0.11032* (0.000)	0.05019 (0.102)
θ		1.20901* (0.000)		0.11909* (0.000)		0.19314* (0.000)		1.15515* (0.000)		0.15472* (0.002)
$Log-L$	6565.562	6611.021	6614.939	6637.981	6617.632	6646.187	6573.857	6626.591	6576.207	6645.623
χ^2	90.92	–	46.08	–	57.11	–	105.47	–	138.83	–

Note: Entries report results of the alternative GARCH models. The p-values of the estimated coefficients are in parentheses, the log-likelihood (Log-L) and χ^2 for testing the restricted GARCH specifications, where $\theta = 0$, against the augmented GARCH specifications, where $\theta \neq 0$ are reported. *, ** and *** denote significance at the 1%, 5% and 10% level, respectively.

cont. Table 24

	$GARCH$	$GARCH-IV$	GJR	$GJR-IV$	$EGARCH$	$EGARCH-IV$	$CGARCH$	$CGARCH_{-IV}$	$ACGARCH$	$ACGARCH_{-IV}$
AEX										
a_0	$1.73*10^{-6}$* (0.005)	$-3.35*10^{-6}$ (0.108)	$1.57*10^{-6}$* (0.000)	$-5.23*10^{-7}$ (0.183)	−0.21163* (0.000)	−0.0832*** (0.096)	−0.00033 (0.248)	$-9.43*10^{-5}$ (0.402)	0.00025*** (0.089)	$-1.37*10^{-6}$ (0.892)
a_1	0.11211* (0.000)	0.09806* (0.000)	−0.02490** (0.031)	−0.04870* (0.004)	0.10870* (0.000)	0.06286** (0.024)	−0.10231* (0.001)	−0.07056** (0.019)	−0.17208* (0.000)	−0.06634* (0.000)
β_1	0.88412 * (0.000)	0.68052 * (0.000)	0.93031* (0.000)	0.88095 * (0.000)	0.98584* (0.000)	0.90864 * (0.000)	0.31255 (0.267)	0.01300 (0.957)	0.52526* (0.001)	0.86242 * (0.000)
γ			0.17049* (0.000)	0.21329* (0.000)	−0.12954* (0.000)	−0.18372* (0.000)			0.10432** (0.027)	0.23328 * (0.000)
ρ							0.99242* (0.000)	0.94827* (0.000)	0.98946* (0.000)	0.98539* (0.000)
ϕ							0.13093* (0.000)	0.06028* (0.001)	0.13058* (0.000)	0.01023 (0.178)
θ		0.18239* (0.000)		0.04206* (0.000)		0.09219* (0.000)		0.89043* (0.000)		0.06686* (0.000)
$Log-L$	6824.646	6844.823	6888.584	6904.411	6889.335	6907.572	6833.153	6878.072	6836.427	6905.589
χ^2	40.35	–	31.65	–	36.47	–	89.84	–	138.32	–
SMI										
a_0	$1.97*10^{-6}$* (0.002)	$-6.19*10^{-7}$ (0.629)	$1.85*10^{-6}$* (0.000)	$7.54*10^{-7}$ (0.192)	−0.30126* (0.000)	−0.20398* (0.000)	0.000235 (0.155)	$-1.03*10^{-5}$ (0.168)	0.00021 (0.112)	$-2.40*10^{-6}$ (0.630)
a_1	0.11998* (0.000)	0.10946* (0.000)	−0.01492 (0.256)	−0.04180*** (0.094)	0.12606* (0.000)	0.09029* (0.001)	−0.07073* (0.007)	−0.04494*** (0.057)	−0.00056 (0.949)	−0.19317* (0.004)
β_1	0.87120 * (0.000)	0.69996 * (0.000)	0.90905* (0.000)	0.85658 * (0.000)	0.97798* (0.000)	0.92812 * (0.000)	0.54977** (0.049)	−0.16836 (0.455)	−0.96361* (0.000)	0.90885 * (0.000)
γ			0.18418* (0.000)	0.22158* (0.000)	−0.14406* (0.000)	−0.17209* (0.000)			0.00435 (0.817)	0.23192* (0.000)
ρ							0.98998* (0.000)	0.94798* (0.000)	0.99030* (0.000)	0.90346* (0.000)
ϕ							0.13799* (0.000)	0.06267* (0.000)	0.11911* (0.000)	0.11692*** (0.084)
θ		0.17122* (0.001)		0.05809* (0.000)		0.05905* (0.000)		1.26382* (0.000)		0.15127* (0.000)
$Log-L$	7046.055	7065.777	7099.047	7115.003	7104.553	7119.914	7049.345	7096.709	7046.239	7117.283
χ^2	39.44	–	31.91	–	30.72	–	94.73	–	142.09	–
FTSE100										
a_0	$1.00*10^{-6}$* (0.000)	$-1.73*10^{-5}$* (0.000)	$1.30*10^{-6}$* (0.000)	$-1.61*10^{-7}$ (0.830)	−0.22184* (0.000)	−0.07930 (0.319)	0.00044 (0.668)	$-1.24*10^{-5}$** (0.019)	0.00021 (0.314)	−0.00223 (0.985)
a_1	0.10708* (0.000)	0.00197 (0.930)	−0.00865 (0.548)	−0.05746* (0.000)	0.11588* (0.000)	0.07424** (0.044)	−0.06068** (0.047)	−0.04298*** (0.099)	−0.16546* (0.000)	−0.07981* (0.000)
β_1	0.88981 * (0.000)	−0.21332 (0.312)	0.91611* (0.000)	0.78603 * (0.000)	0.98577* (0.000)	0.84541 * (0.000)	0.08033 (0.866)	−0.13848 (0.453)	0.47503* (0.003)	0.77548 * (0.000)
γ			0.16455* (0.000)	0.25583* (0.000)	−0.12618* (0.000)	−0.19106* (0.000)			0.17395* (0.000)	0.27410* (0.000)
ρ							0.99723* (0.000)	0.96376* (0.000)	0.99469* (0.000)	0.99999* (0.000)
ϕ							0.12178* (0.000)	0.03517* (0.003)	0.11188* (0.000)	0.01993* (0.001)
θ		0.96386* (0.000)		0.10110* (0.000)		0.16016* (0.000)		0.98847* (0.000)		0.11835* (0.000)
$Log-L$	7226.517	7264.410	7270.586	7302.434	7278.475	7299.520	7229.383	7281.293	7237.988	7298.488
χ^2	75.79	–	63.70	–	24.09	–	103.82	–	121.00	–

Table 25: Diagnostics tests in squared standardized residuals

	Skewness	Kurtosis	Jarque-Bera	Q(7)	LM(7)
			STOXX		
GARCH	−0.3232	4.0096	136.71 (0.000)	14.89** (0.037)	15.402** (0.031)
GARCH-IV	−0.3034	3.6246	72.14 (0.000)	23.77* (0.001)	21.585* (0.003)
GJR	−0.3011	3.4058	50.15 (0.000)	19.27* (0.007)	19.622* (0.006)
GJR-IV	−0.2885	3.3532	43.54 (0.000)	16.91* (0.018)	17.634* (0.014)
EGARCH	−0.2754	3.3331	39.42 (0.000)	14.34** (0.045)	14.473** (0.043)
EGARCH-IV	−0.2901	3.2915	40.11 (0.000)	10.96 (0.140)	11.568 (0.116)
CGARCH	−0.3289	4.0894	154.05 (0.000)	6.10 (0.528)	6.229 (0.513)
CGARCH-IV	−0.3158	3.6567	78.98 (0.000)	6.87 (0.442)	6.859 (0.444)
ACGARCH	−0.3202	4.0854	151.08 (0.000)	6.31 (0.504)	6.431 (0.490)
ACGARCH-IV	−0.2932	3.4194	49.45 (0.000)	11.05 (0.137)	11.115 (0.134)
			CAC		
GARCH	−0.3023	3.9437	120.69 (0.000)	12.66*** (0.081)	12.765*** (0.078)
GARCH-IV	−0.2731	3.6370	67.66 (0.000)	23.55* (0.001)	20.295* (0.005)
GJR	−0.2889	3.4728	53.56 (0.000)	16.67** (0.020)	6.838** (0.019)
GJR-IV	−0.2792	3.4286	47.61 (0.000)	16.20** (0.024)	16.663** (0.020)
EGARCH	−0.2607	3.3792	39.95 (0.000)	12.34 (0.100)	12.177 (0.100)
EGARCH-IV	−0.2829	3.3643	43.50 (0.000)	10.21 (0.177)	10655 (0.154)
CGARCH	−0.2839	3.9021	109.17 (0.000)	3.38 (0.848)	3.441 (0.841)
CGARCH-IV	−0.2797	3.6082	65.60 (0.000)	3.34 (0.852)	3.320 (0.854)
ACGARCH	−0.2799	3.9289	113.02 (0.000)	3.64 (0.820)	3.696 (0.814)
ACGARCH-IV	−0.2794	3.4280	47.60 (0.000)	13.63*** (0.058)	13.790*** (0.055)
			DAX		
GARCH	−0.3464	3.9854	138.02 (0.000)	11.55 (0.116)	11.822 (0.107)
GARCH-IV	−0.3250	3.6551	81.01 (0.000)	39.77* (0.001)	34.038* (0.000)
GJR	−0.3293	3.4821	63.46 (0.000)	16.22** (0.023)	16.357** (0.022)
GJR-IV	−0.2859	3.3679	43.98 (0.000)	12.56*** (0.084)	13.339*** (0.064)
EGARCH	−0.3158	3.4880	60.60 (0.000)	11.18 (0.131)	11.049 (0.137)
EGARCH-IV	−0.2953	3.3484	44.73 (0.000)	10.29 (0.173)	10.783 (0.148)
CGARCH	−0.3376	3.9701	132.89 (0.000)	3.35 (0.851)	3.364 (0.849)
CGARCH-IV	−0.3378	3.6677	85.83 (0.000)	7.06 (0.423)	7.051 (0.424)
ACGARCH	−0.3285	3.9748	131.45 (0.000)	3.67 (0.817)	3.694 (0.814)
ACGARCH-IV	−0.2894	3.4722	53.08 (0.000)	6.35 (0.500)	6.260 (0.510)

Note: Entries report the diagnostic residual test results of the GARCH models. The Ljung-Box Q(7) test and the Lagrange multiplier (LM) test for the squared standardized residuals are reported. p-values are in parentheses. *, ** and *** denote rejection of the null hypothesis at the 1%, 5% and 10% level, respectively.

cont. Table 25

	Skewness	Kurtosis	Jarque-Bera	Q(7)	LM(7)
			AEX		
GARCH	−0.2689	3.5751	59.69 (0.000)	20.99 * (0.004)	20.753* (0.005)
GARCH-IV	−0.2700	3.5491	57.10 (0.000)	27.37 * (0.000)	27.207* (0.000)
GJR	−0.2478	3.2295	28.72 (0.000)	16.05 ** (0.025)	15.883** (0.026)
GJR-IV	−0.2271	3.1803	22.99 (0.000)	14.56 ** (0.042)	14.494** (0.043)
EGARCH	−0.2394	3.2557	28.36 (0.000)	12.67 *** (0.081)	12.316*** (0.091)
EGARCH-IV	−0.2329	3.2018	24.81 (0.000)	11.97 (0.101)	12.204 (0.100)
CGARCH	−0.2629	3.5599	56.79 (0.000)	6.64 (0.467)	6.584 (0.473)
CGARCH-IV	−0.2756	3.4797	51.38 (0.000)	6.29 (0.506)	6.318 (0.503)
ACGARCH	−0.2291	3.5766	52.22 (0.000)	8.09 (0.324)	8.001 (0.333)
ACGARCH-IV	−0.2138	3.1774	20.63 (0.000)	12.05 (0.100)	11.955 (0.102)
			SMI		
GARCH	−0.3625	3.8642	119.92 (0.000)	7.57 (0.373)	7.296 (0.399)
GARCH-IV	−0.3560	3.7120	96.62 (0.000)	14.67 ** (0.040)	14.737** (0.039)
GJR	−0.3221	3.5297	65.56 (0.000)	11.03 (0.137)	11.018 (0.138)
GJR-IV	−0.3156	3.4493	56.58 (0.000)	10.92 (0.142)	10.839 (0.146)
EGARCH	−0.3005	3.5160	59.14 (0.000)	10.99 (0.139)	11.052 (0.136)
EGARCH-IV	−0.3003	3.4139	50.16 (0.000)	11.67 (0.112)	11.671 (0.112)
CGARCH	−0.3464	3.8196	108.55 (0.000)	5.16 (0.641)	5.200 (0.636)
CGARCH-IV	−0.3507	3.5969	79.94 (0.000)	4.01 (0.779)	3.997 (0.781)
ACGARCH	−0.3638	3.8815	123.14 (0.000)	7.69 (0.360)	7.436 (0.385)
ACGARCH-IV	−0.3040	3.4074	50.48 (0.000)	6.25 (0.511)	6.120 (0.526)
			FTSE100		
GARCH	−0.3208	3.5887	72.38 (0.000)	7.33 (0.396)	7.170 (0.411)
GARCH-IV	−0.3354	3.5852	75.64 (0.000)	57.05 (0.147)	45.106 (0.000)
GJR	−0.3416	3.4823	66.76 (0.000)	6.86 (0.444)	6.529 (0.479)
GJR-IV	−0.3347	3.3562	54.89 (0.000)	8.46 (0.294)	9.014 (0.252)
EGARCH	−0.3215	3.4167	56.06 (0.000)	5.81 (0.563)	5.452 (0.605)
EGARCH-IV	−0.3526	3.4130	63.75 (0.000)	5.97 (0.543)	6.012 (0.538)
CGARCH	−0.3126	3.5449	65.65 (0.000)	3.14 (0.872)	3.047 (0.881)
CGARCH-IV	−0.3192	3.4249	56.15 (0.000)	5.80 (0.563)	5.537 (0.595)
ACGARCH	−0.3099	3.5704	67.72 (0.000)	4.67 (0.701)	4.604 (0.708)
ACGARCH-IV	−0.2992	3.3300	44.58 (0.000)	6.72 (0.458)	7.076 (0.421)

Table 26: Estimation output of time series models for implied volatilitity prediction

	ARMA	ARMAX	ARIMA	ARIMAX	ARFIMA	ARFIMAX
			VSTOXX			
c_0	0.01631* (0.000)	0.01615* (0.000)	$1.21 * 10^{-6}$ (0.955)	$7.33 * 10^{-7}$ (0.978)	0.01681 (0.394)	0.01666 (0.483)
$AR(1)$	0.98719* (0.000)	0.9857* (0.0000)	0.72841* (0.000)	−0.20120* (0.000)	0.81172* (0.000)	0.6288* (0.000)
$MA(1)$	−0.03327* (0.000)	0.26393* (0.000)	−0.81077* (0.000)	0.43779* (0.000)	−0.38881* (0.000)	0.13773* (0.000)
d					0.49416* (0.000)	0.49755* (0.000)
r_t^+		−0.01334* (0.000)		−0.01336* (0.000)		−0.01327* (0.000)
r_t^-		−0.04193* (0.000)		−0.04201* (0.000)		−0.04203* (0.000)
$Log-L$	12112.88	12613.67	12116.53	12607.85	12091.433	12577.895
			VCAC			
c_0	0.01526* (0.000)	0.01516* (0.000)	$8.84 * 10^{-7}$ (0.961)	$7.01 * 10^{-7}$ (0.97)	0.01564 (0.391)	0.01554 (0.393)
$AR(1)$	0.98705* (0.000)	0.98664* (0.000)	0.63586* (0.000)	0.70893* (0.000)	0.83666* (0.000)	0.79220* (0.000)
$MA(1)$	−0.11204* (0.000)	−0.0624* (0.000)	−0.76407* (0.000)	−0.80871* (0.000)	−0.48726* (0.000)	−0.37946* (0.000)
d					0.49377* (0.000)	0.49476* (0.000)
r_t^+		−0.00584* (0.000)		−0.00563* (0.000)		−0.00567* (0.000)
r_t^-		−0.02523* (0.000)		−0.02546* (0.000)		−0.02542* (0.000)
$Log-L$	12245.82	12353.27	12252.14	12359.72	12231.433	12335.508
			VDAX			
c_0	0.01637* (0.000)	0.01627* (0.000)	$1.17 * 10^{-7}$ (0.963)	$8.17 * 10^{-7}$ (0.974)	0.01671 (0.406)	0.01659 (0.453)
$AR(1)$	0.98799* (0.000)	0.98733* (0.000)	−0.16364 (0.295)	−0.09649* (0.000)	0.76888* (0.000)	0.68912* (0.000)
$MA(1)$	0.04778* (0.000)	0.25313* (0.000)	0.20695 (0.177)	0.33505* (0.000)	−0.22889* (0.000)	0.07608** (0.021)
d					0.49517* (0.000)	0.49681* (0.000)
r_t^+		−0.00966* (0.000)		−0.00972* (0.000)		−0.00947* (0.000)
r_t^-		−0.03164* (0.000)		−0.03167* (0.000)		−0.03171* (0.000)
$Log-L$	12368.85	12737.62	12358.74	12727.85	12342.294	12704.853

Note: Entries report results of the alternative implied volatility models. The p-values of the estimated coefficients are in parentheses. * denotes significance at the 1% level.

cont. Table 26

	ARMA	ARMAX	ARIMA	ARIMAX	ARFIMA	ARFIMAX
			VAEX			
c_0	0.01664* (0.000)	0.016044* (0.000)	$-1.29*10^{-7}$ (0.976)	$1.60*10^{-6}$ (0.947)	0.01656 (0.339)	0.01648 (0.390)
$AR(1)$	0.99069* (0.000)	0.98974* (0.000)	0.99137* (0.000)	−0.15503* (0.001)	0.86060* (0.000)	0.76315* (0.000)
$MA(1)$	0.00164 (0.889)	0.22246* (0.000)	−0.998518* (0.0000)	0.36406* (0.0000)	−0.38700* (0.000)	−0.02962* (0.000)
d					0.48959* (0.000)	0.49434* (0.000)
r_t^+		−0.01234* (0.000)		−0.01229* (0.000)		−0.01239* (0.000)
r_t^-		−0.03163* (0.000)		−0.03166* (0.000)		−0.03153* (0.000)
$Log-L$	12528.8	12915.25	12867.81	12907.46	12504.22	12871.15
			VSMI			
c_0	0.01256* (0.000)	0.01249* (0.000)	$1.32*10^{-6}$ (0.947)	$1.31*10^{-6}$ (0.951)	0.01274 (0.395)	0.01267 (0.464)
$AR(1)$	0.98799* (0.000)	0.98664* (0.000)	−0.64609* (0.000)	−0.40999* (0.000)	0.79753* (0.000)	0.71904* (0.000)
$MA(1)$	0.08761* (0.0000)	0.20666* (0.000)	0.73828* (0.000)	0.59853* (0.000)	−0.23567* (0.000)	−0.03138* (0.000)
d					0.49314* (0.000)	0.49600* (0.000)
r_t^+		−0.0381* (0.0000)		−0.00356* (0.000)		−0.00350* (0.000)
r_t^-		−0.02222* (0.000)		−0.02227* (0.000)		−0.02202* (0.000)
$Log-L$	12786.11	12911.64	12785.96	12915.69	12749.72	12863.42
			VFTSE100			
c_0	0.01356* (0.000)	0.01340* (0.000)	$7.42*10^{-7}$ (0.971)	$6.58*10^{-7}$* (0.000)	0.01403 (0.285)	0.01386 (0.377)
$AR(1)$	0.98882* (0.000)	0.98596* (0.000)	0.22729* (0.000)	−0.72407* (0.000)	0.89094* (0.000)	0.79806* (0.000)
$MA(1)$	−0.15092* (0.000)	0.07735* (0.000)	−0.37955* (0.000)	−0.84062* (0.000)	−0.55649* (0.000)	−0.30453* (0.000)
d					0.48459* (0.000)	0.49260* (0.000)
r_t^+		−0.00652* (0.000)		−0.00904* (0.000)		−0.00551* (0.007)
r_t^-		−0.04693* (0.000)		−0.04634* (0.000)		−0.04629* (0.000)
$Log-L$	12328.58	12641.44	12319.91	12656.95	12321.56	12607.63

Table 27: Diebold-Mariano test

	MSE						
	Random walk	ARMA	ARMAX	ARIMA	ARIMAX	ARFIMA	ARFIMAX
$VSTOXX$	0.00154	0.00152***	0.00095*	0.00151	0.00096*	0.00154	0.00095*
$VCAC$	0.00141	0.00141	0.00109*	0.00139	0.00108*	0.00142	0.00109*
$VDAX$	0.00105	0.00102**	0.00070*	0.00103***	0.00071*	0.00104	0.00071*
$VAEX$	0.00129	0.00128*	0.00099**	0.00128*	0.00099**	0.00131	0.00100**
$VSMI$	0.00059	0.00057	0.00046*	0.00058	0.00047*	0.00058	0.00046*
$VFTSE$	0.00102	0.00105	0.00072*	0.00106	0.00073*	0.00108	0.00076*
	MAE						
	Random walk	ARMA	ARMAX	ARIMA	ARIMAX	ARFIMA	ARFIMAX
$VSTOXX$	0.830	0.827	0.652*	0.826	0.656*	0.835	0.657*
$VCAC$	0.806	0.808	0.709*	0.807	0.711*	0.816	0.714*
$VDAX$	0.689	0.683	0.560*	0.684**	0.562*	0.687	0.564*
$VAEX$	0.732	0.731	0.604*	0.731	0.606*	0.741	0.612*
$VSMI$	0.510	0.503	0.445*	0.505	0.449*	0.510	0.449*
$VFTSE$	0.706	0.713	0.582*	0.714	0.583*	0.722	0.594*

Note: The Diebold-Mariano test results using the mean squared forecast error (MSE) and the mean absolute forecast error (MAE) of the IV models are reported. The null hypothesis that the random walk and the model under consideration perform equally well is tested against the alternative that the model under consideration performs better. All numbers are multiplied by 10^3. *, ** and *** denote rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 28: MAE and RMSE using *ex post* squared returns measure of true volatility

	MAE						RMSE					
	STOXX	CAC	DAX	AEX	SMI	FTSE100	STOXX	CAC	DAX	AEX	SMI	FTSE100
EWMA	0.2569	0.2532	0.2082	0.1590	0.1103	0.1322	0.5380	0.5103	0.3790	0.3101	0.2219	0.2309
GARCH	0.2587	0.2534	0.2101	0.1631	0.1119	0.1343	0.5376	0.5108	0.3775	0.3101	0.2165	0.2303
GARCH-IV	0.2420	0.2340	0.1978	0.1573	0.1129	0.1248	0.5153	0.4955	0.3639	0.3047	0.2129	0.2216
GJR	0.2440	0.2424	0.2009	0.1563	0.1084	0.1259	0.5228	0.4965	0.3719	0.3014	0.2048	0.2234
GJR-IV	0.2417	0.2371	0.1980	0.1537	0.1089	0.1226	0.5184	0.4920	0.3675	0.2988	0.2021	0.2209
EGARCH	0.2405	0.2367	0.1994	0.1571	0.1059*	0.1251	0.5198	0.4925	0.3682	0.3005	0.2028	0.2215
EGARCH-IV	0.2404*	0.2329*	0.1968*	0.1526*	0.1070	0.1216*	0.5142	0.4880*	0.3642	0.2977*	0.2015*	0.2187*
CGARCH	0.2581	0.2534	0.2081	0.1616	0.1128	0.1338	0.5389	0.5108	0.3763	0.3108	0.2197	0.2302
CGARCH-IV	0.2449	0.2390	0.1980	0.1538	0.1099	0.1261	0.5160	0.4949	0.3633	0.3007	0.2064	0.2222
ACGARCH	0.2567	0.2529	0.2064	0.1597	0.1118	0.1316	0.5372	0.5099	0.3751	0.3090	0.2168	0.2285
ACGARCH-IV	0.2474	0.2370	0.2001	0.1547	0.1093	0.1250	0.5190	0.4939	0.3678	0.2989	0.2042	0.2221
ARMA	0.2763	0.2538	0.2215	0.1781	0.1250	0.1495	0.5193	0.4959	0.3678	0.3044	0.2118	0.2285
ARMAX	0.2719†	0.2514†	0.2189	0.1760†	0.1238†	0.1468†	0.5141*†	0.4935†	0.3632*†	0.3016†	0.2100†	0.2240†
ARIMA	0.2769	0.2544	0.2213	0.1780	0.1249	0.1495	0.5203	0.4962	0.3678	0.3044	0.2118	0.2286
ARIMAX	0.2721	0.2518	0.2188†	0.1758	0.1237	0.1470	0.5144	0.4938	0.3634	0.3017	0.2110	0.2243
ARFIMA	0.2767	0.2541	0.2217	0.1781	0.1251	0.1496	0.5198	0.4962	0.3679	0.3043	0.2118	0.2285
ARFIMAX	0.2725	0.2516	0.2193	0.1762	0.1240	0.1470	0.5144	0.4938	0.3633	0.3016	0.2103	0.2241
Random walk	0.2766	0.2539	0.2213	0.1778	0.1250	0.1494	0.5189	0.4950	0.3680	0.3044	0.2122	0.2285

Note: The mean absolute forecast error (MAE) and the root mean squared forecast error (RMSE) of both GARCH and IV models when *ex post* squared returns measure true volatility are reported. All numbers are multiplied by 10^3. * denotes the lowest forecast error. † denotes the lowest forecast error among the IV models.

Table 29: MAE and RMSE using realized variance measure of true volatility

	MAE						RMSE					
	STOXX	CAC	DAX	AEX	SMI	FTSE100	STOXX	CAC	DAX	AEX	SMI	FTSE100
EWMA	0.1222	0.1218	0.0990	0.0802	0.0557	0.0676	0.2013	0.1910	0.1764	0.1345	0.1149	0.0956
GARCH	0.1206	0.1236	0.0994	0.0837	0.0567	0.0704	0.2019	0.2050	0.1685	0.1365	0.1086	0.1013
GARCH-IV	0.1044	0.0978	0.0882	0.0736	0.0606	0.0654	0.1603	0.1463	0.1497	0.1204	0.1058	0.0858*
GJR	0.1085	0.1120	0.0911	0.0787	0.0541	0.0651	0.1819	0.1769	0.1590	0.1275	0.0969	0.0974
GJR-IV	0.1003	0.995	0.0838*	0.0725	0.0557	0.0614	0.1644	0.1536	0.1401*	0.1186	0.0926	0.0894
EGARCH	0.0976*	0.1002	0.0862	0.0793	0.0507*	0.0634	0.1564	0.1498	0.1451	0.1220	0.0863*	0.0897
EGARCH-IV	0.0989	0.0956*	0.0842	0.0710*	0.0546	0.0610*	0.1556*	0.1437*	0.1416	0.1153*	0.0880	0.0873
CGARCH	0.1229	0.1236	0.1008	0.0842	0.0582	0.0714	0.2147	0.2050	0.1753	0.1430	0.1140	0.1044
CGARCH-IV	0.1083	0.1061	0.0913	0.0742	0.0590	0.0670	0.1717	0.1633	0.1563	0.1216	0.1053	0.0949
ACGARCH	0.1220	0.1227	0.0994	0.0815	0.0563	0.0681	0.2142	0.2045	0.1749	0.1391	0.1080	0.0996
ACGARCH-IV	0.1090	0.1007	0.0907	0.0742	0.0582	0.0652	0.1733	0.1568	0.1511	0.1205	0.0989	0.0955
ARMA	0.1598	0.1350	0.1272	0.1181	0.0847	0.1119	0.2015	0.1736	0.1750	0.1512	0.1183	0.1334
ARMAX	0.1585†	0.1342†	0.1259†	0.1172	0.0846	0.1114	0.1996	0.1735	0.1732	0.1499	0.1187	0.1322
ARIMA	0.1607	0.1357	0.1271	0.1181	0.0846	0.1119	0.2035	0.1752	0.1755	0.1512	0.1186	0.1341
ARIMAX	0.1589	0.1350	0.1259	0.1170†	0.0845†	0.1113†	0.2007	0.1751	0.1738	0.1502	0.1192	0.1329
ARFIMA	0.1604	0.1355	0.1273	0.1183	0.0849	0.1122	0.2013	0.1736	0.1743	0.1507	0.1181†	0.1332
ARFIMAX	0.1590	0.1347	0.1261	0.1175	0.0849	0.1117	0.1991†	0.1733†	0.1726†	0.1491†	0.1185	0.1317†
Random walk	0.1601	0.1351	0.1272	0.1178	0.0847	0.1116	0.2023	0.1736	0.1758	0.1514	0.1191	0.1342

Note: The mean absolute forecast error (MAE) and the root mean squared forecast error (RMSE) of both GARCH and IV models when realized variance measures true volatility are reported. All numbers are multiplied by 10^3. * denotes the lowest forecast error. † denotes the lowest forecast error among the IV models.

Table 30: Out-of-sample predictive power for alternative forecasts using *ex post* squared returns measure of true volatility

Models	$adj - R^2$					
	STOXX	CAC	DAX	AEX	SMI	FTSE100
GARCH	0.0700	0.0752	0.1539	0.0961	0.1885	0.1082
GARCH-IV	0.1470	0.1255	0.2176	0.1163	0.2139	0.1645
TGARCH	0.1151	0.1182	0.1834	0.1399	0.2721	0.1562
TGARCH-IV	0.1254	0.1295	0.1943	0.1510	0.2924	0.1706
EGARCH	0.1209	0.1275	0.1916	0.1417	0.2897	0.1650
EGARCH-IV	0.1402	0.1446*	0.2084	0.1564	0.3029*	0.1856
CGARCH	0.0700	0.0752	0.1594	0.0952	0.1710	0.1113
CGARCH-IV	0.1343	0.1185	0.2126	0.1410	0.2634	0.1619
ACGARCH	0.0744	0.0777	0.1636	0.1019	0.1865	0.1192
ACGARCH-IV	0.1240	0.1226	0.1953	0.1522	0.2777	0.1654
ARMA	0.1377	0.1205	0.2073	0.1454	0.2474	0.1617
ARMAX	0.1580	0.1313	0.2288	0.1596	0.2623	0.1927*
ARIMA	0.1341	0.1190	0.2072	0.1454	0.2469	0.1616
ARIMAX	0.1567	0.1296	0.2280	0.1593	0.2616	0.1909
ARFIMA	0.1366	0.1197	0.2074	0.1456	0.2482	0.1616
ARFIMAX	0.1585*	0.1312	0.2301*	0.1601*	0.2613	0.1918
RW: IV_{t-1}	0.1391	0.1232	0.2064	0.1454	0.2438	0.1633

Note: Entries are the adjusted R^2 values from the univariate Mincer-Zarnowitz regression when the *ex post* squared daily returns measure the true volatility. * denotes the highest adjusted R^2 value.

Table 31: Out-of-sample predictive power of daily volatility forecasts using realized variance measure of true volatility

Models	$adj-R^2$					
	STOXX	CAC	DAX	AEX	SMI	FTSE100
GARCH	0.3406	0.3188	0.4524	0.3548	0.4545	0.4308
GARCH-IV	0.4902	0.4958	0.5122	0.4205	0.4793	0.5530
TGARCH	0.4843	0.5143	0.5541	0.4548	0.5999	0.5313
TGARCH-IV	0.5274	0.5711	0.6068*	0.5015	0.6370	0.6034*
EGARCH	0.5374	0.5603	0.5769	0.4831	0.6472	0.5390
EGARCH-IV	0.5626*	0.5890*	0.5951	0.5157*	0.6524*	0.6007
CGARCH	0.2963	0.3188	0.4192	0.3118	0.4193	0.4136
CGARCH-IV	0.4712	0.4784	0.5209	0.4757	0.5478	0.5536
ACGARCH	0.2998	0.3226	0.4127	0.3314	0.4564	0.4362
ACGARCH-IV	0.5196	0.5309	0.5812	0.5147	0.5921	0.6013
ARMA	0.5007	0.4857	0.5201	0.4711	0.4840	0.5276
ARMAX	0.5133	0.4835	0.5306	0.4804	0.4786	0.5468
ARIMA	0.4894	0.4753	0.5197	0.4709	0.4830	0.5249
ARIMAX	0.5107	0.4724	0.5293	0.4784	0.4752	0.5463
ARFIMA	0.4977	0.4827	0.5196	0.4729	0.4844	0.5272
ARFIMAX	0.5098	0.4816	0.5296	0.4838	0.4766	0.5444
RW: IV_{t-1}	0.5037	0.4954	0.5171	0.4712	0.4764	0.5413

Note: Entries are the adjusted R^2 values from the Mincer-Zarnowitz regression described in equation (41) when the realized variance measures the true volatility. * denotes the highest adjusted R^2 value.

Table 32: Forecast encompassing regression results for the STOXX index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	-0.00013^{*} (0.000)	-0.5024^{*} (0.004)	1.6024^{*} (0.000)	0.1461
GARCH-IV & ARMA	$-3.56*10^{-5}$ (0.381)	3.0617^{*} (0.001)	-1.4285^{***} (0.055)	0.1500
TGARCH & ARMAX	-0.00016^{*} (0.000)	-0.2444 (0.191)	1.4821^{*} (0.000)	0.1588
TGARCH-IV & ARMAX	-0.00013^{*} (0.000)	-0.0414 (0.840)	1.2498^{*} (0.000)	0.1570
EGARCH & ARMAX	-0.00015^{*} (0.000)	-0.3136 (0.201)	1.5057^{*} (0.000)	0.1587
EGARCH-IV & ARMAX	-0.00012^{*} (0.002)	0.1872 (0.433)	1.0282^{*} (0.000)	0.1576
CGARCH & ARIMA	-0.00011^{*} (0.003)	-0.2530^{***} (0.089)	1.3466^{*} (0.000)	0.1363
CGARCH & ARFIMA	-0.00012^{*} (0.001)	-0.2474^{***} (0.091)	1.3812^{*} (0.000)	0.1387
CGARCH-IV & ARIMA	$-6.51*10^{-5}$ (0.125)	0.5671 (0.146)	0.5503 (0.163)	0.1354
CGARCH-IV & ARFIMA	$-8.10*10^{-5***}$ (0.066)	0.4272 (0.267)	0.7139^{***} (0.073)	0.1368
ACGARCH & ARIMAX	-0.00014^{*} (0.000)	-0.3367^{**} (0.018)	1.5107^{*} (0.000)	0.1617
ACGARCH & ARFIMAX	-0.00016^{*} (0.000)	-0.3213^{**} (0.022)	1.5601^{*} (0.000)	0.1631†
ACGARCH-IV & ARIMAX	-0.00014^{*} (0.000)	-0.1612 (0.457)	1.3582^{*} (0.000)	0.1562
ACGARCH-IV & ARFIMAX	-0.00015^{*} (0.000)	-0.1652 (0.436)	1.4231^{*} (0.000)	0.1580
GARCH & IV_{t-1}	-0.00012^{*} (0.001)	-0.5082^{*} (0.003)	1.5845^{*} (0.000)	0.1478
GARCH-IV & IV_{t-1}	$-4.73*10^{-5}$ (0.226)	2.8366^{*} (0.002)	-1.2150 (0.107)	0.1488
TGARCH & IV_{t-1}	-0.00010^{**} (0.015)	-0.0148 (0.941)	1.1256^{*} (0.000)	0.1380
TGARCH-IV & IV_{t-1}	$-8.39*10^{-5**}$ (0.032)	0.2137 (0.345)	0.8951^{*} (0.000)	0.1400
EGARCH & IV_{t-1}	$-9.88*10^{-5**}$ (0.012)	0.0113 (0.967)	1.0985^{*} (0.000)	0.1380
EGARCH-IV & IV_{t-1}	$-6.50*10^{-6***}$ (0.095)	0.5901^{**} (0.032)	0.5419^{***} (0.055)	0.1432
CGARCH & IV_{t-1}	-0.00011^{*} (0.002)	-0.2726^{***} (0.063)	1.3646^{*} (0.000)	0.1419
CGARCH-IV & IV_{t-1}	$-8.51*10^{-5**}$ (0.040)	0.2659 (0.500)	0.8526^{**} (0.030)	0.1385
ACGARCH & IV_{t-1}	-0.00011^{*} (0.003)	-0.1855 (0.195)	1.2829^{*} (0.000)	0.1399
ACGARCH-IV & IV_{t-1}	$-9.24*10^{-5**}$ (0.021)	0.0945 (0.697)	1.0100^{*} (0.000)	0.1382

Note: Entries are the estimated coefficients, their p-values in parentheses and the adjusted R^2 values from the encompassing regression when the *ex post* squared daily returns measure the true volatility. *, ** and *** denote significance at the 1% level. A significant p-value indicates that the forecast under consideration is not encompassed by the alternative model, † denotes the highest adjusted R^2 value.

Table 33: Forecast encompassing regression results for the CAC index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-7.59*10^{-6}$** (0.033)	-0.0090 (0.949)	1.2031* (0.000)	0.1194
GARCH-IV & ARMA	$8.71*10^{-5}$ (0.125)	7.0397* (0.000)	-5.2605* (0.003)	0.1342
TGARCH & ARMAX	$-6.85*10^{-5}$*** (0.089)	0.2165 (0.246)	0.9733* (0.000)	0.1317
TGARCH-IV & ARMAX	$-5.35*10^{-5}$ (0.166)	0.4723** (0.021)	0.7022* (0.001)	0.1362
EGARCH & ARMAX	$-6.16*10^{-5}$ (0.107)	0.4492*** (0.054)	0.7563* (0.008)	0.1344
EGARCH-IV & ARMAX	$-3.50*10^{-5}$ (0.358)	0.8624* (0.000)	0.3065 (0.285)	0.1447†
CGARCH & ARIMA	$-7.32*10^{-5}$** (0.039)	-0.0069 (0.961)	1.1890* (0.000)	0.1178
CGARCH & ARFIMA	$-8.06*10^{-5}$** (0.026)	0.0013 (0.993)	1.2112* (0.000)	0.1186
CGARCH-IV & ARIMA	$-4.38*10^{-5}$ (0.274)	0.4982 (0.138)	0.6305 (0.105)	0.1203
CGARCH-IV & ARFIMA	$-5.07*10^{-5}$ (0.229)	0.4620 (0.169)	0.6898*** (0.083)	0.1208
ACGARCH & ARIMAX	$-9.19*10^{-5}$** (0.010)	-0.0606 (0.668)	1.3110* (0.000)	0.1286
ACGARCH & ARFIMAX	-0.00010* (0.005)	-0.0534 (0.701)	1.3437* (0.000)	0.1302
ACGARCH-IV & ARIMAX	$-6.32*10^{-5}$*** (0.087)	0.4140** (0.031)	0.7899* (0.001)	0.1337
ACGARCH-IV & ARFIMAX	$-7.18*10^{-5}$*** (0.058)	0.3909** (0.041)	0.8438* (0.001)	0.1347
GARCH & IV_{t-1}	$-7.31*10^{-5}$** (0.036)	-0.0152 (0.913)	1.1941* (0.000)	0.1221
GARCH-IV & IV_{t-1}	-0.00014*** (0.075)	11.271* (0.002)	-8.9263* (0.007)	0.1325
TGARCH & IV_{t-1}	$-3.86*10^{-5}$ (0.326)	0.3415*** (0.077)	0.7431* (0.006)	0.1257
TGARCH-IV & IV_{t-1}	$-2.46*10^{-5}$ (0.518)	0.6288* (0.005)	0.4540*** (0.099)	0.1315
EGARCH & IV_{t-1}	$-3.38*10^{-5}$ (0.365)	0.6330* (0.009)	0.4902*** (0.089)	0.1297
EGARCH-IV & IV_{t-1}	$-3.66*10^{-5}$ (0.992)	1.1576* (0.000)	-0.0770 (0.802)	0.1435
CGARCH & IV_{t-1}	$-7.31*10^{-5}$** (0.036)	-0.0152 (0.913)	1.1941 * (0.000)	0.1221
CGARCH-IV & IV_{t-1}	$-5.66*10^{-5}$ (0.145)	0.2300 (0.374)	0.0853* (0.026)	0.1230
ACGARCH & IV_{t-1}	$-7.18*10^{-5}$** (0.039)	0.0220 (0.873)	1.1551* (0.000)	0.1221
ACGARCH-IV & IV_{t-1}	$-4.31*10^{-5}$ (0.235)	0.5015* (0.013)	0.6358* (0.009)	0.1291

Note: As Table 32.

Table 34: Forecast encompassing regression results for the DAX index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-8.28*10^{-5}$* (0.000)	-0.5104* (0.008)	-1.5664* (0.000)	0.2135
GARCH-IV & ARMA	$-1.25*10^{-5}$ (0.963)	3.1942* (0.000)	-1.7447** (0.013)	0.2229
TGARCH & ARMAX	$-8.87*10^{-5}$* (0.001)	-0.1426 (0.357)	1.2809* (0.000)	0.2286
TGARCH-IV & ARMAX	$-7.95*10^{-5}$* (0.001)	-0.0334 (0.856)	1.1539* (0.000)	0.2278
EGARCH & ARMAX	$-8.44*10^{-5}$* (0.006)	-0.1269 (0.503)	1.2484* (0.000)	0.2282
EGARCH-IV & ARMAX	$-7.08*10^{-5}$* (0.004)	0.1537 (0.443)	0.9682* (0.000)	0.2284
CGARCH & ARIMA	$-6.68*10^{-5}$* (0.004)	-0.1861 (0.266)	1.2300* (0.000)	0.2074
CGARCH & ARFIMA	$-7.55*10^{-5}$* (0.002)	-0.1556 (0.341)	1.2411* (0.000)	0.2073
CGARCH-IV & ARIMA	$-1.86*10^{-5}$ (0.518)	0.8404** (0.020)	0.1853 (0.623)	0.2118
CGARCH-IV & ARFIMA	$-2.26*10^{-5}$ (0.456)	0.8044** (0.019)	0.2319 (0.535)	0.2119
ACGARCH & ARIMAX	$-8.49*10^{-5}$* (0.000)	-0.3044*** (0.057)	1.4003* (0.000)	0.2307
ACGARCH & ARFIMAX	$-9.65*10^{-5}$* (0.000)	-0.2888*** (0.064)	1.4361* (0.000)	0.2325†
ACGARCH-IV & ARIMAX	$-8.18*10^{-5}$* (0.001)	-0.1268 (0.506)	1.2423* (0.000)	0.2274
ACGARCH-IV & ARFIMAX	$-9.37*10^{-5}$* (0.000)	-0.1375 (0.463)	1.3011* (0.000)	0.2296
GARCH & IV_{t-1}	$-7.80*10^{-5}$* (0.000)	-0.5122* (0.009)	1.5484* (0.000)	0.2125
GARCH-IV & IV_{t-1}	$-6.67*10^{-5}$ (0.795)	3.1561* (0.000)	-1.6923* (0.009)	0.2234
TGARCH & IV_{t-1}	$-5.23*10^{-5}$** (0.041)	0.1073 (0.510)	0.9284* (0.000)	0.2058
TGARCH-IV & IV_{t-1}	$-4.75*10^{-5}$*** (0.051)	0.2779 (0.162)	0.7732* (0.000)	0.2073
EGARCH & IV_{t-1}	$-4.99*10^{-5}$** (0.043)	0.2086 (0.303)	0.8396* (0.000)	0.2064
EGARCH-IV & IV_{t-1}	$-3.47*10^{-5}$ (0.159)	0.5635** (0.011)	0.4915 ** (0.033)	0.2120
CGARCH & IV_{t-1}	$-6.67*10^{-5}$* (0.000)	-0.1864 (0.269)	1.2300 * (0.000)	0.2066
CGARCH-IV & IV_{t-1}	$-1.57*10^{-5}$ (0.585)	0.8890** (0.013)	0.1336 (0.722)	0.2117
ACGARCH & IV_{t-1}	$-6.18*10^{-5}$* (0.008)	-0.0497 (0.757)	1.0954* (0.000)	0.2054
ACGARCH-IV & IV_{t-1}	$-4.72*10^{-5}$*** (0.065)	0.2237 (0.284)	0.8085* (0.001)	0.2065

Note: As Table 32.

Table 35: Forecast encompassing regression results for the AEX index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-3.33*10^{-5}$*** (0.085)	-0.2292 (0.160)	1.0877* (0.000)	0.1465
GARCH-IV & ARMA	$-3.73*10^{-5}$*** (0.058)	-0.4098 (0.115)	1.2320* (0.000)	0.1470
TGARCH & ARMAX	$-3.62*10^{-5}$*** (0.079)	0.0915 (0.627)	0.8507* (0.000)	0.1587
TGARCH-IV & ARMAX	$-2.73*10^{-5}$ (0.188)	0.3053 (0.120)	0.6491* (0.001)	0.1611
EGARCH & ARMAX	$-3.81*10^{-5}$*** (0.057)	0.0711 (0.746)	0.8744* (0.000)	0.1586
EGARCH-IV & ARMAX	$-2.44*10^{-5}$ (0.242)	0.4110*** (0.061)	0.5556** (0.011)	0.1623†
CGARCH & ARIMA	$-3.12*10^{-5}$ (0.105)	-0.0958 (0.503)	0.9718* (0.000)	0.1448
CGARCH & ARFIMA	$-3.34*10^{-5}$*** (0.086)	-0.0872 (0.538)	0.9750* (0.000)	0.1449
CGARCH-IV & ARIMA	$-1.89*10^{-5}$ (0.394)	0.2829 (0.310)	0.6198** (0.026)	0.1454
CGARCH-IV & ARFIMA	$-2.08*10^{-5}$ (0.356)	0.2732 (0.327)	0.6363** (0.024)	0.1456
ACGARCH & ARIMAX	$-3.98*10^{-5}$** (0.037)	-0.1269 (0.373)	1.0387* (0.000)	0.1591
ACGARCH & ARFIMAX	$-4.48*10^{-5}$** (0.021)	-0.1063 (0.447)	1.0455* (0.000)	0.1596
ACGARCH-IV & ARIMAX	$-2.43*10^{-5}$ (0.240)	0.3238*** (0.089)	0.6165* (0.002)	0.1614
ACGARCH-IV & ARFIMAX	$-2.83*10^{-5}$ (0.182)	0.3166*** (0.090)	0.6405* (0.001)	0.1621
GARCH & IV_{t-1}	$-3.08*10^{-5}$ (0.107)	-0.2291 (0.160)	1.0771* (0.000)	0.1464
GARCH-IV & IV_{t-1}	$-3.44*10^{-5}$* (0.076)	-0.4092 (0.116)	1.2196* (0.000)	0.1470
TGARCH & IV_{t-1}	$-1.55*10^{-5}$* (0.449)	0.3277*** (0.096)	0.5677* (0.005)	0.1473
TGARCH-IV & IV_{t-1}	$-5.25*10^{-5}$ (0.800)	0.5763* (0.006)	0.3387 (0.108)	0.1527
EGARCH & IV_{t-1}	$-1.90*10^{-5}$ (0.338)	0.3727 (0.108)	0.5441** (0.016)	0.1471
EGARCH-IV & IV_{t-1}	$-2.80*10^{-5}$ (0.989)	0.7752* (0.001)	0.1628 (0.487)	0.1558
CGARCH & IV_{t-1}	$-2.93*10^{-5}$ (0.126)	-0.0958 (0.502)	0.9637 * (0.000)	0.1448
CGARCH-IV & IV_{t-1}	$-1.76*10^{-5}$* (0.418)	0.2827 (0.311)	0.6147** (0.026)	0.1454
ACGARCH & IV_{t-1}	$-2.83*10^{-5}$ (0.138)	-0.0012 (0.993)	0.8841* (0.000)	0.1443
ACGARCH-IV & IV_{t-1}	$-2.91*10^{-5}$ (0.889)	0.5864* (0.004)	0.3110 (0.142)	0.1535

Note: As Table 32.

Table 36: Forecast encompassing regression results for the SMI index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-6.01*10^{-5}$* (0.000)	-0.1945 (0.199)	1.3039* (0.000)	0.2480
GARCH-IV & ARMA	$-5.72*10^{-5}$* (0.000)	-0.1658 (0.441)	1.2673* (0.000)	0.2470
TGARCH & ARMAX	$-3.05*10^{-5}$** (0.032)	0.6232* (0.000)	0.4443** (0.020)	0.2764
TGARCH-IV & ARMAX	$-2.26*10^{-5}$ (0.101)	0.9968* (0.000)	0.0679 (0.737)	0.2915
EGARCH & ARMAX	$-3.80*10^{-5}$* (0.003)	0.8804* (0.000)	0.3188** (0.045)	0.2925
EGARCH-IV & ARMAX	$-3.44*10^{-5}$* (0.006)	1.1440* (0.000)	0.0513 (0.774)	0.3020
CGARCH & ARIMA	$-6.01*10^{-5}$* (0.000)	-0.2604** (0.052)	1.3570* (0.000)	0.2497
CGARCH & ARFIMA	$-6.46*10^{-5}$* (0.000)	-0.2183*** (0.092)	1.3548* (0.000)	0.2500
CGARCH-IV & ARIMA	$-1.14*10^{-5}$ (0.470)	0.8802* (0.000)	0.0724 (0.778)	0.2625
CGARCH-IV & ARFIMA	$-1.51*10^{-5}$ (0.355)	0.8273* (0.000)	0.1431** (0.578)	0.2628
ACGARCH & ARIMAX	$-6.71*10^{-5}$* (0.000)	-0.3443** (0.021)	1.4711* (0.000)	0.2659
ACGARCH & ARFIMAX	$-7.19*10^{-5}$* (0.000)	-0.2733 (0.057)	1.4498* (0.000)	0.2639
ACGARCH-IV & ARIMAX	$-2.89*10^{-5}$** (0.036)	0.7667* (0.000)	0.2919 (0.155)	0.2787
ACGARCH-IV & ARFIMAX	$-3.09*10^{-5}$** (0.032)	0.7606* (0.000)	0.1352 (0.902)	0.2789
GARCH & IV_{t-1}	$-5.65*10^{-5}$* (0.000)	-0.1754 (0.255)	1.2643* (0.000)	0.2441
GARCH-IV & IV_{t-1}	$-5.35*10^{-5}$* (0.000)	-0.1248** (0.571)	1.2080* (0.000)	0.2431
TGARCH & IV_{t-1}	$-1.17*10^{-5}$ (0.409)	0.8879* (0.000)	0.0995 (0.609)	0.2714
TGARCH-IV & IV_{t-1}	$-3.36*10^{-5}$ (0.805)	1.3536* (0.000)	-0.3611*** (0.081)	0.2943
EGARCH & IV_{t-1}	$-2.62*10^{-5}$** (0.037)	1.0949* (0.000)	0.0704 (0.661)	0.2889
EGARCH-IV & IV_{t-1}	$-2.16*10^{-5}$*** (0.084)	1.4516* (0.000)	-0.2873 (0.114)	0.3042†
CGARCH & IV_{t-1}	$-5.90*10^{-5}$* (0.000)	-0.2459*** (0.069)	1.3382 * (0.000)	0.2461
CGARCH-IV & IV_{t-1}	$-7.13*10^{-6}$ (0.650)	0.9520* (0.000)	-0.0174 (0.946)	0.2625
ACGARCH & IV_{t-1}	$-5.71*10^{-5}$* (0.000)	-0.1985 (0.199)	1.2858* (0.000)	0.2444
ACGARCH-IV & IV_{t-1}	$-9.96*10^{-6}$ (0.476)	1.1074* (0.000)	-0.1265 (0.555)	0.2770

Note: As Table 32.

Table 37: Forecast encompassing regression results for the FTSE100 index using *ex post* squared returns measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-3.27*10^{-5}$** (0.030)	-0.3044*** (0.070)	1.1139* (0.000)	0.1643
GARCH-IV & ARMA	$-4.78*10^{-5}$ (0.818)	1.3001 (0.107)	-0.2001* (0.000)	0.1635
TGARCH & ARMAX	$-4.46*10^{-5}$* (0.005)	-0.0474 (0.776)	0.9913* (0.000)	0.1917
TGARCH-IV & ARMAX	$-3.70*10^{-5}$** (0.019)	0.1781 (0.306)	0.7893* (0.000)	0.1928
EGARCH & ARMAX	$-4.14*10^{-5}$* (0.007)	0.0586 (0.761)	0.8982* (0.000)	0.1917
EGARCH-IV & ARMAX	$-3.10*10^{-5}$** (0.053)	0.3775*** (0.068)	0.6155* (0.002)	0.1952†
CGARCH & ARIMA	$-2.96*10^{-5}$** (0.048)	-0.1972 (0.211)	1.0174* (0.000)	0.1622
CGARCH & ARFIMA	$-3.26*10^{-5}$** (0.032)	-0.1734 (0.264)	1.0157* (0.000)	0.1619
CGARCH-IV & ARIMA	$-1.01*10^{-5}$ (0.606)	0.4815 (0.198)	0.4074 (0.249)	0.1623
CGARCH-IV & ARFIMA	$-1.16*10^{-5}$ (0.565)	0.4796 (0.190)	0.4173 (0.236)	0.1624
ACGARCH & ARIMAX	$-4.37*10^{-5}$* (0.003)	-0.3001** (0.047)	1.1698* (0.000)	0.1940
ACGARCH & ARFIMAX	$-4.97*10^{-5}$* (0.001)	-0.2772*** (0.062)	1.1867* (0.000)	0.1945
ACGARCH-IV & ARIMAX	$-3.58*10^{-5}$** (0.022)	0.1224 (0.456)	0.8178* (0.000)	0.1904
ACGARCH-IV & ARFIMAX	$-4.06*10^{-5}$** (0.012)	0.1195 (0.461)	0.8474* (0.000)	0.1913
GARCH & IV_{t-1}	$-2.85*10^{-5}$*** (0.053)	-0.2800*** (0.086)	1.0704* (0.000)	0.1654
GARCH-IV & IV_{t-1}	$-1.80*10^{-6}$ (0.944)	1.5438 (0.264)	-0.3908 (0.725)	0.1635
TGARCH & IV_{t-1}	$-1.40*10^{-5}$ (0.377)	0.3279*** (0.065)	0.5492* (0.002)	0.1659
TGARCH-IV & IV_{t-1}	$-6.68*10^{-6}$ (0.672)	0.6063* (0.002)	0.3144*** (0.091)	0.1727
EGARCH & IV_{t-1}	$-1.43*10^{-5}$ (0.349)	0.5298** (0.011)	0.4111 ** (0.026)	0.1693
EGARCH-IV & IV_{t-1}	$-7.59*10^{-6}$ (0.639)	1.1041* (0.000)	-0.1132 (0.609)	0.1848
CGARCH & IV_{t-1}	$-2.76*10^{-5}$*** (0.062)	-0.1650 (0.278)	0.9810 * (0.000)	0.1635
CGARCH-IV & IV_{t-1}	$-1.41*10^{-5}$ (0.476)	0.3492 (0.389)	0.5266 (0.165)	0.1630
ACGARCH & IV_{t-1}	$-2.60*10^{-5}$*** (0.078)	-0.0390 (0.800)	0.8777* (0.000)	0.1622
ACGARCH-IV & IV_{t-1}	$-8.90*10^{-6}$ (0.577)	0.4810 (0.001)	0.4029** (0.030)	0.1696

Note: As Table 32.

Table 38: Forecast encompassing regression results for the STOXX index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-7.54*10^{-5}$* (0.000)	-0.0717 (0.146)	0.8671* (0.000)	0.5014
GARCH-IV & ARMA	$-7.35*10^{-5}$* (0.000)	-0.0670 (0.791)	0.8546* (0.000)	0.5001
TGARCH & ARMAX	$-5.02*10^{-5}$* (0.000)	0.2437* (0.000)	0.5397* (0.000)	0.5261
TGARCH-IV & ARMAX	$-4.44*10^{-5}$* (0.000)	0.4343* (0.000)	0.3763* (0.000)	0.5479
EGARCH & ARMAX	$-4.40*10^{-5}$* (0.000)	0.5256* (0.000)	0.3140* (0.000)	0.5489
EGARCH-IV & ARMAX	$-3.98*10^{-5}$* (0.000)	0.6294* (0.000)	0.2072* (0.002)	0.5674†
CGARCH & ARIMA	$-7.24*10^{-5}$* (0.000)	-0.0737*** (0.085)	0.8574* (0.000)	0.4907
CGARCH & ARFIMA	$-8.01*10^{-5}$* (0.000)	-0.0722*** (0.084)	0.8825* (0.000)	0.4990
CGARCH-IV & ARIMA	$-5.88*10^{-5}$* (0.000)	0.1741 (0.120)	0.6167* (0.000)	0.4903
CGARCH-IV & ARFIMA	$-7.07*10^{-5}$* (0.000)	0.0863 (0.431)	0.7261* (0.000)	0.4975
ACGARCH & ARIMAX	$-7.73*10^{-5}$* (0.000)	-0.0807** (0.048)	0.8785* (0.000)	0.5125
ACGARCH & ARFIMAX	$-8.59*10^{-5}$* (0.000)	-0.0647 (0.108)	0.8966* (0.000)	0.5109
ACGARCH-IV & ARIMAX	$-4.39*10^{-5}$* (0.000)	0.4045* (0.000)	0.3791* (0.000)	0.5375
ACGARCH-IV & ARFIMAX	$-4.83*10^{-5}$* (0.000)	0.4068* (0.000)	0.3927* (0.000)	0.5381
GARCH & IV_{t-1}	$-7.23*10^{-5}$* (0.000)	-0.0740 (0.130)	0.8564* (0.000)	0.5045
GARCH-IV & IV_{t-1}	$-7.41*10^{-5}$* (0.000)	-0.2696 (0.301)	1.0081* (0.000)	0.5037
TGARCH & IV_{t-1}	$-4.18*10^{-5}$* (0.000)	0.2630 (0.000)	0.4954* (0.000)	0.5169
TGARCH-IV & IV_{t-1}	$-3.42*10^{-5}$ (0.002)	0.4797* (0.000)	0.3069* (0.000)	0.5391
EGARCH & IV_{t-1}	$-3.35*10^{-5}$* (0.002)	0.5967 (0.000)	0.2255* (0.003)	0.5423
EGARCH-IV & IV_{t-1}	$-2.53*10^{-5}$** (0.015)	0.7487* (0.000)	0.0675 (0.371)	0.5625
CGARCH & IV_{t-1}	$-7.28*10^{-5}$* (0.000)	-0.0845** (0.042)	0.8664 * (0.000)	0.5057
CGARCH-IV & IV_{t-1}	$-6.93*10^{-5}$* (0.000)	-0.0028 (0.980)	0.7900* (0.000)	0.5031
ACGARCH & IV_{t-1}	$-7.19*10^{-5}$* (0.002)	-0.0595 (0.143)	0.8430* (0.000)	0.5044
ACGARCH-IV & IV_{t-1}	$-3.58*10^{-5}$* (0.001)	0.4440* (0.000)	0.3204* (0.000)	0.5300

Note: Entries are the estimated coefficients, their p-values in parentheses and the adjusted R^2 values from the encompassing regression when the realized variance measures the true volatility. *, ** and *** denote significance at the 1% level. A significant p-value indicates that the forecast under consideration is not encompassed by the alternative model, † denotes the highest adjusted R^2 value.

Table 39: Forecast encompassing regression results for the CAC index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-6.51*10^{-5}$* (0.000)	0.0241 (0.520)	0.8049* (0.000)	0.4853
GARCH-IV & ARMA	$-3.87*10^{-5}$ (0.796)	2.6952* (0.000)	−1.6400* (0.000)	0.5029
TGARCH & ARMAX	$-2.45*10^{-5}$** (0.018)	0.4016* (0.000)	0.3113* (0.000)	0.5260
TGARCH-IV & ARMAX	$-1.63*10^{-5}$ (0.086)	0.6327* (0.000)	0.0940 (0.150)	0.5717
EGARCH & ARMAX	$-2.22*10^{-5}$** (0.019)	0.6748* (0.000)	0.0852 (0.559)	0.5606
EGARCH-IV & ARMAX	$-1.41*10^{-5}$ (0.126)	0.8114* (0.000)	−0.0572 (0.409)	0.5888
CGARCH & ARIMA	$-6.20*10^{-5}$* (0.000)	0.0320 (0.402)	0.7845* (0.000)	0.4751
CGARCH & ARFIMA	$-6.83*10^{-5}$* (0.000)	0.0309 (0.409)	0.8105* (0.000)	0.4825
CGARCH-IV & ARIMA	$-4.07*10^{-5}$* (0.000)	0.3879* (0.000)	0.3901* (0.000)	0.4872
CGARCH-IV & ARFIMA	$-4.84*10^{-5}$* (0.000)	0.3284* (0.000)	0.4728* (0.000)	0.4910
ACGARCH & ARIMAX	$-6.25*10^{-5}$* (0.000)	0.0434 (0.256)	0.7769* (0.000)	0.4726
ACGARCH & ARFIMAX	$-6.98*10^{-5}$* (0.000)	0.0410 (0.272)	0.8082* (0.000)	0.4818
ACGARCH-IV & ARIMAX	$-3.21*10^{-5}$* (0.000)	0.5214* (0.000)	0.2521* (0.000)	0.5405
ACGARCH-IV & ARFIMAX	$-3.74*10^{-5}$* (0.000)	0.4972* (0.000)	0.2938* (0.000)	0.5435
GARCH & IV_{t-1}	$-6.31*10^{-5}$* (0.000)	0.0205 (0.576)	0.7979* (0.000)	0.4949
GARCH-IV & IV_{t-1}	$-4.95*10^{-5}$** (0.016)	0.7704** (0.016)	0.1287 (0.884)	0.4951
TGARCH & IV_{t-1}	$-2.71*10^{-5}$* (0.007)	0.3698* (0.000)	0.3483* (0.000)	0.5292
TGARCH-IV & IV_{t-1}	$-1.59*10^{-5}$*** (0.088)	0.6279* (0.000)	0.0962* (0.153)	0.5717
EGARCH & IV_{t-1}	$-2.39*10^{-5}$* (0.009)	0.6528* (0.000)	0.1100 (0.123)	0.5611
EGARCH-IV & IV_{t-1}	$-1.09*10^{-5}$ (0.230)	0.8476* (0.000)	−0.0997 (0.178)	0.5894†
CGARCH & IV_{t-1}	$-6.31*10^{-5}$* (0.000)	0.0205 (0.576)	0.7979 * (0.000)	0.4949
CGARCH-IV & IV_{t-1}	$-5.18*10^{-5}$* (0.000)	0.2255** (0.011)	0.5748* (0.000)	0.4989
ACGARCH & IV_{t-1}	$-6.27*10^{-5}$* (0.000)	0.0330 (0.364)	0.7849* (0.000)	0.4953
ACGARCH-IV & IV_{t-1}	$-3.61*10^{-5}$* (0.000)	0.4703* (0.000)	0.3107* (0.000)	0.5452

Note: As Table 38.

Table 40: Forecast encompassing regression results for the DAX index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-5.73*10^{-5}*$ (0.000)	0.0123 (0.866)	0.8078* (0.000)	0.5195
GARCH-IV & ARMA	$-6.08*10^{-5}*$ (0.000)	−0.1549 (0.609)	0.9561* (0.000)	0.5196
TGARCH & ARMAX	$-2.63*10^{-5}*$ (0.005)	0.4511* (0.000)	0.3248* (0.000)	0.5660
TGARCH-IV & ARMAX	$-2.38*10^{-5}*$ (0.005)	0.7814* (0.000)	0.0548 (0.426)	0.6077†
EGARCH & ARMAX	$-2.75*10^{-5}*$ (0.002)	0.6477* (0.000)	0.1799** (0.015)	0.5796
EGARCH-IV & ARMAX	$-2.43*10^{-5}*$ (0.005)	0.7835* (0.000)	0.0547 (0.466)	0.5949
CGARCH & ARIMA	$-5.70*10^{-5}*$ (0.000)	−0.0533 (0.402)	0.8612* (0.000)	0.5195
CGARCH & ARFIMA	$-6.30*10^{-5}*$ (0.000)	−0.0320 (0.607)	0.8690* (0.000)	0.5192
CGARCH-IV & ARIMA	$-3.45*10^{-5}*$ (0.002)	0.4136* (0.003)	0.3843* (0.007)	0.5248
CGARCH-IV & ARFIMA	$-3.79*10^{-5}*$ (0.001)	0.4129* (0.002)	0.3990* (0.005)	0.5252
ACGARCH & ARIMAX	$-6.08*10^{-5}*$ (0.000)	−0.0797 (0.191)	0.8989* (0.000)	0.5298
ACGARCH & ARFIMAX	$-6.72*10^{-5}*$ (0.000)	−0.0590 (0.322)	0.9088* (0.000)	0.5296
ACGARCH-IV & ARIMAX	$-2.19*10^{-5}**$ (0.019)	0.6633 (0.000)	0.1208 (0.127)	0.5816
ACGARCH-IV & ARFIMAX	$-1.98*10^{-5}**$ (0.027)	0.6755* (0.000)	0.1019 (0.191)	0.5820
GARCH & IV_{t-1}	$-5.41*10^{-5}*$ (0.000)	0.0208 (0.780)	0.7880* (0.000)	0.5166
GARCH-IV & IV_{t-1}	$-5.29*10^{-5}**$ (0.000)	0.1145 (0.691)	0.7089* (0.005)	0.5166
TGARCH & IV_{t-1}	$-1.73*10^{-5}***$ (0.063)	0.5130* (0.000)	0.2373* (0.001)	0.5599
TGARCH-IV & IV_{t-1}	$-1.35*10^{-5}$ (0.105)	0.9056* (0.000)	−0.0868 (0.229)	0.6071
EGARCH & IV_{t-1}	$-1.79*10^{-5}**$ (0.041)	0.7516* (0.000)	0.0581 (0.449)	0.5767
EGARCH-IV & IV_{t-1}	$-1.20*10^{-5}$ (0.163)	0.9475* (0.000)	−0.1272 (0.115)	0.5959
CGARCH & IV_{t-1}	$-5.66*10^{-5}*$ (0.000)	−0.0503 (0.433)	0.8575 * (0.000)	0.5169
CGARCH-IV & IV_{t-1}	$-3.18*10^{-5}*$ (0.004)	0.4613* (0.001)	0.3339** (0.019)	0.5237
ACGARCH & IV_{t-1}	$-5.58*10^{-5}*$ (0.000)	−0.0285 (0.641)	0.8356* (0.000)	0.5167
ACGARCH-IV & IV_{t-1}	$-8.43*10^{-5}$ (0.353)	0.8019* (0.000)	−0.0497 (0.552)	0.5809

Note: As Table 38.

Table 41: Forecast encompassing regression results for the AEX index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-3.57*10^{-5}$* (0.000)	-0.0168 (0.750)	0.6705* (0.000)	0.4705
GARCH-IV & ARMA	$-3.51*10^{-5}$* (0.000)	0.0234 (0.780)	0.6367* (0.000)	0.4705
TGARCH & ARMAX	$-2.87*10^{-5}$* (0.000)	0.2164* (0.000)	0.4563* (0.000)	0.4883
TGARCH-IV & ARMAX	$-1.96*10^{-5}$* (0.003)	0.4323* (0.000)	0.2553* (0.000)	0.5112
EGARCH & ARMAX	$-2.78*10^{-5}$* (0.000)	0.3655* (0.000)	0.3314* (0.000)	0.4978
EGARCH-IV & ARMAX	$-1.71*10^{-5}$* (0.008)	0.5382* (0.000)	0.1639** (0.016)	0.5186
CGARCH & ARIMA	$-3.61*10^{-5}$* (0.000)	0.0639 (0.166)	0.7095* (0.000)	0.4716
CGARCH & ARFIMA	$-3.78*10^{-5}$* (0.000)	-0.0599 (0.189)	0.7145* (0.000)	0.4734
CGARCH-IV & ARIMA	$-2.03*10^{-5}$* (0.004)	0.3774* (0.000)	0.2940* (0.000)	0.4824
CGARCH-IV & ARFIMA	$-2.20*10^{-5}$* (0.001)	0.3608* (0.000)	0.3144* (0.000)	0.4832
ACGARCH & ARIMAX	$-3.64*10^{-5}$* (0.000)	-0.0195 (0.672)	0.6770* (0.000)	0.4778
ACGARCH & ARFIMAX	$-4.04*10^{-5}$* (0.000)	-0.0136 (0.762)	0.6906* (0.000)	0.4831
ACGARCH-IV & ARIMAX	$-1.53*10^{-5}$** (0.017)	0.4846* (0.000)	0.1880* (0.003)	0.5197
ACGARCH-IV & ARFIMAX	$-1.85*10^{-5}$* (0.005)	0.4582* (0.000)	0.2237* (0.000)	0.5220†
GARCH & IV_{t-1}	$-3.43*10^{-5}$* (0.000)	-0.0174 (0.742)	0.6647* (0.000)	0.4706
GARCH-IV & IV_{t-1}	$-3.37*10^{-5}$* (0.000)	0.0223 (0.790)	0.6316* (0.000)	0.4706
TGARCH & IV_{t-1}	$-2.44*10^{-5}$* (0.000)	0.2472* (0.000)	0.4120* (0.000)	0.4810
TGARCH-IV & IV_{t-1}	$-1.46*10^{-5}$** (0.024)	0.4853* (0.000)	0.1916* (0.004)	0.5062
EGARCH & IV_{t-1}	$-2.36*10^{-5}$* (0.000)	0.4183* (0.000)	0.2696* (0.000)	0.4919
EGARCH-IV & IV_{t-1}	$-1.09*10^{-5}$*** (0.092)	0.6272* (0.000)	0.0672 (0.356)	0.5156
CGARCH & IV_{t-1}	$-3.47*10^{-5}$* (0.000)	-0.0645 (0.161)	0.7042 * (0.000)	0.4719
CGARCH-IV & IV_{t-1}	$-1.99*10^{-5}$* (0.004)	0.3750* (0.000)	0.2939* (0.001)	0.4825
ACGARCH & IV_{t-1}	$-3.41*10^{-5}$* (0.000)	-0.0009* (0.985)	0.6507* (0.000)	0.4705
ACGARCH-IV & IV_{t-1}	$-1.09*10^{-5}$*** (0.094)	0.5358* (0.000)	0.1272*** (0.053)	0.5164

Note: As Table 38.

Table 42: Forecast encompassing regression results for the SMI index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-3.99*10^{-5}*$ (0.000)	0.2650* (0.000)	0.5731* (0.000)	0.4937
GARCH-IV & ARMA	$-4.07*10^{-5}*$ (0.000)	0.3888* (0.000)	0.4675* (0.000)	0.4945
TGARCH & ARMAX	$-2.42*10^{-6}$ (0.670)	0.9864* (0.000)	−0.2730* (0.000)	0.6061
TGARCH-IV & ARMAX	$1.83*10^{-6}$ (0.719)	1.3507* (0.000)	−0.6234* (0.000)	0.6670
EGARCH & ARMAX	$-2.06*10^{-5}*$ (0.000)	1.1301* (0.000)	−0.2243* (0.0000)	0.6532
EGARCH-IV & ARMAX	$-1.84*10^{-5}*$ (0.000)	1.3571* (0.000)	−0.4612* (0.000)	0.6730
CGARCH & ARIMA	$-4.20*10^{-5}*$ (0.000)	0.1489* (0.013)	0.6781* (0.000)	0.4865
CGARCH & ARFIMA	$-4.48*10^{-5}*$ (0.000)	0.1617* (0.005)	0.6879* (0.000)	0.4890
CGARCH-IV & ARIMA	$-1.48*10^{-6}$ (0.824)	0.9748* (0.000)	−0.3042* (0.005)	0.5519
CGARCH-IV & ARFIMA	$-3.21*10^{-6}$ (0.642)	0.9262* (0.000)	−0.2514** (0.021)	0.5504
ACGARCH & ARIMAX	$-3.70*10^{-5}*$ (0.000)	0.3139* (0.000)	0.5155* (0.000)	0.4893
ACGARCH & ARFIMAX	$-4.01*10^{-5}*$ (0.000)	0.3165* (0.000)	0.5360* (0.000)	0.4922
ACGARCH-IV & ARIMAX	$-5.13*10^{-5}$ (0.354)	1.1104* (0.000)	−0.3913* (0.000)	0.6034
ACGARCH-IV & ARFIMAX	$-5.44*10^{-6}$ (0.349)	1.0706* (0.000)	−0.3559* (0.000)	0.6010
GARCH & IV_{t-1}	$-3.72*10^{-5}*$ (0.000)	0.2928* (0.000)	0.5323* (0.000)	0.4881
GARCH-IV & IV_{t-1}	$-3.79*10^{-5}*$ (0.000)	0.4487* (0.000)	0.3985* (0.000)	0.4899
TGARCH & IV_{t-1}	$3.42*10^{-7}***$ (0.079)	1.0381* (0.000)	−0.3339* (0.000)	0.6091
TGARCH-IV & IV_{t-1}	$5.20*10^{-6}$ (0.299)	1.4436* (0.000)	−0.7235* (0.000)	0.6756
EGARCH & IV_{t-1}	$-1.91*10^{-5}*$ (0.000)	1.1702* (0.000)	−0.2656* (0.000)	0.6556
EGARCH-IV & IV_{t-1}	$-1.66*10^{-5}*$ (0.000)	1.4279* (0.000)	−0.5324 * (0.000)	0.6791†
CGARCH & IV_{t-1}	$-4.07*10^{-5}*$ (0.000)	0.1667* (0.006)	0.6553 * (0.000)	0.4809
CGARCH-IV & IV_{t-1}	$1.86*10^{-5}$ (0.779)	1.0300* (0.000)	−0.3740* (0.001)	0.5543
ACGARCH & IV_{t-1}	$-3.70*10^{-5}*$ (0.000)	0.3042* (0.000)	0.5232* (0.000)	0.4890
ACGARCH-IV & IV_{t-1}	$-2.15*10^{-5}$ (0.700)	1.1727* (0.000)	−0.4642* (0.000)	0.6068

Note: As Table 38.

Table 43: Forecast encompassing regression results for the FTSE100 index using realized variance measure of true volatility

	a_0	a_1	a_2	$adj-R^2$
GARCH & ARMA	$-1.97*10^{-5}$* (0.000)	0.0396 (0.325)	0.4645* (0.000)	0.5276
GARCH-IV & ARMA	$-7.60*10^{-6}$ (0.113)	1.5447* (0.000)	−0.7665* (0.000)	0.5669
TGARCH & ARMAX	$-1.39*10^{-5}$* (0.000)	0.2203* (0.000)	0.3048* (0.000)	0.5646
TGARCH-IV & ARMAX	$-8.53*10^{-6}$** (0.015)	0.4258* (0.000)	0.1313* (0.000)	0.6092†
EGARCH & ARMAX	$-1.60*10^{-5}$* (0.000)	0.2655* (0.000)	0.2864* (0.000)	0.5662
EGARCH-IV & ARMAX	$-7.29*10^{-5}$* (0.043)	0.4786* (0.000)	0.0883** (0.044)	0.6024
CGARCH & ARIMA	$-1.89*10^{-5}$* (0.000)	0.0172 (0.650)	0.4761* (0.000)	0.5244
CGARCH & ARFIMA	$-2.06*10^{-5}$* (0.000)	0.0224 (0.549)	0.4819* (0.000)	0.5268
CGARCH-IV & ARIMA	$-1.79*10^{-5}$ (0.695)	0.6114* (0.000)	−0.0761 (0.356)	0.5535
CGARCH-IV & ARFIMA	$-2.28*10^{-5}$ (0.962)	0.5701* (0.000)	−0.0369 (0.653)	0.5531
ACGARCH & ARIMAX	$-2.03*10^{-5}$* (0.000)	0.0602*** (0.096)	0.4539* (0.000)	0.5474
ACGARCH & ARFIMAX	$-2.24*10^{-5}$* (0.000)	0.0724** (0.042)	0.4569* (0.000)	0.5463
ACGARCH-IV & ARIMAX	$-7.68*10^{-6}$** (0.027)	0.3986* (0.000)	0.1355* (0.000)	0.6078
ACGARCH-IV & ARFIMAX	$-8.33*10^{-6}$** (0.020)	0.4002* (0.000)	0.1380* (0.000)	0.6078
GARCH & IV_{t-1}	$-1.92*10^{-6}$* (0.000)	0.0254* (0.511)	0.4714* (0.000)	0.5410
GARCH-IV & IV_{t-1}	$1.09*10^{-5}$*** (0.069)	1.9681* (0.000)	−1.0864* (0.000)	0.5628
TGARCH & IV_{t-1}	$-1.14*10^{-5}$* (0.002)	0.2296* (0.000)	0.2831* (0.000)	0.5589
TGARCH-IV & IV_{t-1}	$-4.56*10^{-5}$ (0.191)	0.4776* (0.000)	0.0721*** (0.080)	0.6045
EGARCH & IV_{t-1}	$-1.35*10^{-5}$ (0.000)	0.2790* (0.000)	0.2621 * (0.000)	0.5602
EGARCH-IV & IV_{t-1}	$-2.27*10^{-5}$ (0.531)	0.5719* (0.000)	−0.0057 (0.908)	0.6002
CGARCH & IV_{t-1}	$-1.93*10^{-5}$* (0.000)	0.0075 (0.834)	0.4856 * (0.000)	0.5407
CGARCH-IV & IV_{t-1}	$-4.97*10^{-5}$ (0.282)	0.4407* (0.000)	0.0872* (0.000)	0.5536
ACGARCH & IV_{t-1}	$-1.87*10^{-5}$* (0.000)	0.0687*** (0.059)	0.4378* (0.000)	0.5429
ACGARCH-IV & IV_{t-1}	$-4.06*10^{-6}$ (0.255)	0.4452* (0.000)	0.0805** (0.050)	0.6029

Note: As Table 38.

Table 45: Summary of 1% and 5% VaR failure rates of forecast encompassing regressions when realized variance is the measure of true volatility

	1%			5%		
	Ave. failure rate	Sig. Kupiec test	Sig. DQ test	Ave. failure rate	Sig. Kupiec test	Sig. DQ test
GARCH & ARMA	0.0531	All	All	0.1036	All	All
GARCH-IV & ARMA	0.0500	All	All	0.1046	All	All
GJR & ARMAX	0.0453	All	All	0.0978	All	All
GJR-IV & ARMAX	0.0497	All	All	0.1046	All	All
EGARCH & ARMAX	0.0548	All	All	0.1089	All	All
EGARCH-IV & ARMAX	0.0568	All	All	0.1097	All	All
CGARCH & ARIMA	0.0499	All	All	0.1039	All	All
CGARCH & ARFIMA	0.0516	All	All	0.1052	All	All
CGARCH-IV & ARIMA	0.0472	All	All	0.1008	All	All
CGARCH-IV & ARFIMA	0.0466	All	All	0.1012	All	All
ACGARCH & ARIMAX	0.0431	All	All	0.0984	All	All
ACGARCH & ARFIMAX	0.0431	All	All	0.0999	All	All
ACGARCH-IV & ARIMAX	0.0479	All	All	0.1036	All	All
ACGARCH-IV & ARFIMAX	0.0481	All	All	0.1036	All	All
GARCH & IV_{t-1}	0.0494	All	All	0.1028	All	All
GARCH-IV & IV_{t-1}	0.0492	All	All	0.1045	All	All
GJR & IV_{t-1}	0.0501	All	All	0.0991	All	All
GJR-IV & IV_{t-1}	0.0497	All	All	0.1061	All	All
EGARCH & IV_{t-1}	0.0571	All	All	0.1098	All	All
EGARCH-IV & IV_{t-1}	0.0566	All	All	0.1111	All	All
CGARCH & IV_{t-1}	0.0494	All	All	0.1034	All	All
CGARCH-IV & IV_{t-1}	0.0455	All	All	0.1019	All	All
ACGARCH & IV_{t-1}	0.0499	All	All	0.1032	All	All
ACGARCH-IV & IV_{t-1}	0.0499	All	All	0.1043	All	All

Note: Entries are the average failure rate of the forecasts encompassing regressions. The series for which the Kupiec test for the equality of the empirical failure rate at a specified statistical level and the DQ test for the autocorrelation in VaR violations are significant are listed.

Table 44: Summary of 1% and 5% VaR failure rates of forecast encompassing regressions when squared returns is the measure of true volatility

	1%			5%		
	Ave. failure rate	Sig. Kupiec test	Sig. DQ test	Ave. failure rate	Sig. Kupiec test	Sig. DQ test
GARCH & ARMA	0.0270	All	All	0.0699	STOXX, DAX, SMI	STOXX, DAX, SMI, FTSE100
GARCH-IV & ARMA	0.0246	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0670	STOXX, SMI	STOXX, SMI
GJR & ARMAX	0.0183	STOXX, DAX	STOXX, DAX	0.0618	STOXX	STOXX
GJR-IV & ARMAX	0.0189	STOXX, SMI	STOXX, SMI	0.0631	STOXX, SMI	STOXX, SMI
EGARCH & ARMAX	0.0200	STOXX, DAX, SMI	STOXX, DAX, SMI	0.0627	STOXX, SMI	STOXX, SMI
EGARCH-IV & ARMAX	0.0218	STOXX, SMI	STOXX, SMI	0.0649	STOXX, SMI	STOXX, SMI
CGARCH & ARIMA	0.0262	All	All	0.0679	STOXX, SMI	STOXX, SMI
CGARCH & ARFIMA	0.0270	All	All	0.0692	STOXX, SMI	STOXX, SMI
CGARCH-IV & ARIMA	0.0198	AEX, SMI	AEX, SMI	0.0607	None	FTSE100
CGARCH-IV & ARFIMA	0.0205	STOXX, AEX, SMI	STOXX, AEX, SMI	0.0627	STOXX	STOXX, FTSE100
ACGARCH & ARIMAX	0.0240	STOXX, DAX, SMI	STOXX, DAX, SMI	0.0653	STOXX, SMI	STOXX, SMI, FTSE100
ACGARCH & ARFIMAX	0.0245	STOXX, DAX, SMI	STOXX, DAX, SMI	0.0655	STOXX, SMI	STOXX, SMI, FTSE100
ACGARCH-IV & ARIMAX	0.0185	STOXX, SMI	STOXX, SMI	0.0620	STOXX, SMI	STOXX, SMI
ACGARCH-IV & ARFIMAX	0.0189	STOXX, DAX, SMI	STOXX, DAX, SMI	0.0626	STOXX, SMI	STOXX
GARCH & IV_{t-1}	0.0259	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0692	STOXX, DAX, SMI	STOXX, DAX, SMI, FTSE100
GARCH-IV & IV_{t-1}	0.0235	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0664	STOXX, CAC, SMI	STOXX
GJR & IV_{t-1}	0.0209	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0629	STOXX	STOXX
GJR-IV & IV_{t-1}	0.0209	STOXX, DAX, SMI	STOXX, DAX, SMI	0.0618	SMI	SMI
EGARCH & IV_{t-1}	0.0222	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0623	STOXX, SMI	STOXX, SMI
EGARCH-IV & IV_{t-1}	0.0220	STOXX, SMI	STOXX, SMI	0.0656	SMI	SMI
CGARCH & IV_{t-1}	0.0253	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0681	STOXX, SMI	STOXX, SMI
CGARCH-IV & IV_{t-1}	0.0206	STOXX, AEX, SMI	STOXX, AEX, SMI	0.0618	STOXX	STOXX
ACGARCH & IV_{t-1}	0.0248	STOXX, DAX, AEX, SMI	STOXX, DAX, AEX, SMI	0.0670	STOXX, SMI	STOXX, SMI
ACGARCH-IV & IV_{t-1}	0.0201	STOXX, DAX, SMI	STOXX, DAX, SMI	0.0627	STOXX, SMI	STOXX, SMI

Note: Entries are the average failure rate of the forecasts encompassing regressions. The series for which the Kupiec test for the equality of the empirical failure rate at a specified statistical level and the DQ test for the autocorrelation in VaR violations are significant are listed.

4 Forecasting stock index return volatility with Stochastic Volatility models

In this chapter the performance of the stochastic volatility model for forecasting the daily volatility of a number of European and US stock indices is assessed and compared with two popular GARCH specifications, the GARCH(1,1) and EGARCH(1,1). The chapter examines whether the leverage effect and implied volatility have any significant effect on the performance of the SV model as, in the previous chapter, they played an important role. The one-, five- and twenty two-day out-of-sample volatility forecasts of the GARCH and SV models are evaluated. The findings indicate that incorporating implied volatility in the stochastic volatility model significantly enhances the performance of volatility forecasts. In contrast, the presence of the asymmetric effect seems not to significantly improve the performance of the SV models. Overall, the EGARCH-IV model produces the most accurate volatility forecast at one day horizon. For longer horizons, the GARCH-IV model performs best.

4.1 Introduction

The accurate estimation and forecasting of volatility in financial markets plays a crucial role in decision making in a number of areas, such as option and derivatives pricing, hedging strategies, portfolio allocation and Value-at-Risk calculations. Whilst one of the most well known phenomenon exhibited by the volatility of many financial return series is the volatility clustering, it is only since the introduction of the benchmark GARCH model (Engle, 1982; Bollerslev, 1986) that financial economists have developed alternative model specifications to capture this empirical characteristic. GARCH models have been extensively used be both academics and practitioners. In GARCH models, the conditional variance is expressed as a deterministic function of the past squared residuals and the past conditional variance.[15]

A rival class of time-varying volatility models is known as the stochastic volatility (SV) models

[15] See Bera & Higgins (1993) and Bollerslev et al. (1994) for a review of the models.

(Taylor, 1986). In theses models, volatility is modeled as an unobserved component that follows some latent stochastic process. Similarly to GARCH models, various SV specifications have been developed in order to capture volatility's empirical stylized facts. The leptokurtosis that many financial series exhibit, can be incorporated in SV models and, amongst others, has been studied by Harvey et al. (1994b) and Chib et al. (2002). Regarding the leverage effect, Harvey & Shephard (1996) and Jacquier et al. (2004) suggest alternative approaches to allow for the correlation between the two error terms. Finally, Breidt et al. (1998), amongst others, have investigated the feature of long memory.[16]

The aim of this chapter is to investigate both the in-sample and out-of-sample performance of the alternative SV specifications for forecasting stock returns volatility. One of the main advantages of the SV models is that since it contains an additional innovative term in the dynamics of the conditional variance, it is more flexible than the GARCH models in describing the stylized facts. Furthermore, SV models the unobserved variance process as a logarithmic first order autoregressive process which can be viewed as a discrete-time approximation of the continuous-time models used in the option pricing literature.[17]

While the stochastic volatility approach is both theoretically and economically more attractive than the GARCH it has been under-utilized in empirical research. This is due to the fact that SV models are more difficult to estimate, because an exact likelihood function cannot be derived when the volatility itself is stochastic. However, in recent years several methods have been developed for estimating the SV models. Such methods include the quasi-maximum likelihood (QML) of Harvey et al. (1994a), the generalized methods of moments (GMM) of Melino & Turnbull (1990), the efficient method of moments (EMM) of Gallant et al. (1997) and the Markov chain Monte Carlo (MCMC) of Kim et al. (1998). In this chapter, the QML method is used to estimate the SV models parameters and obtain one step ahead volatility forecasts.

The goal of this study is to explore the in-sample and out-of-sample performance of SV models. I investigate whether, in the context of stochastic volatility, the presence of the leverage effect and the information content of IV play such an important role as in the GARCH context. The

[16]See Taylor (1994), Shephard (1996) and Ghysels et al. (1996) for a review of the stochastic volatility models.
[17]See, for example, Hull & White (1987b) and Hull & White (1987a).

GARCH and EGARCH models as well as their augmented versions with IV are forecasted in order to provide a comparative evaluation of the volatility forecasting ability of SV and GARCH models. The one-, five- and twenty two-day out-of-sample volatility forecasts of the GARCH and SV models are evaluated.

The remainder of the chapter is organized as follows: In the next Section, I review the literature. Sections 4.3 and 4.4 introduce the data and the methodology employed. Section 4.5 presents the empirical results and, finally, Section 4.5.3 summarizes and concludes.

4.2 Literature review

The SV models are considered as a successful alternative to GARCH models. This class of model has a long history that goes back to the work of Clark (1973) where asset returns are modeled as a function of a random information arrival process. Tauchen & Pitts (1983) refined this work suggesting that if information flows are positively autocorrelated, the return process reveals volatility clustering and gives rise to the idea that returns volatility follows its own stochastic process. Later, Taylor (1986) formulated a discrete-time SV model as an alternative to GARCH models in which a logarithmic first order autoregressive process is modeled. Although the SV models were developed in parallel with the GARCH models, they have received much less attention in the volatility forecasting literature, because of their estimation complexity.

Heynen & Kat (1994) forecast both stock index and exchange rate volatility and find that SV models provide the most accurate forecast for the indices, but performs poorly when the exchange rates volatility is forecasted, where a GARCH(1,1) model performs best. So et al. (1999) compare the usefulness of the SV model with GARCH models in forecasting exchange rates volatility and find that although the two approaches perform similarly, the SV model does not, in general, outperforms the GARCH approach. Yu (2002) forecasts New Zealand stock market volatility and find that the SV model outperforms GARCH models. Pederzoli (2006) forecasts volatility for the UK stock market and ranks EGARCH top, while there is no difference between GARCH(1,1) and SV. Chortareas et al. (2011) using intraday data find evidence that the SV model performs poorly compare to other time series models for forecasting daily volatility of the euro bilateral exchange rates. Other studies, such as Bluhm & Yu (2000), Dunis et al. (2000) and Hol & Koopman (2000) compare SV and other time series models with implied volatility without a clear-cut result. Dunis et al. (2000) conclude that combined forecast is the best for currencies. Both Bluhm & Yu (2000) and Hol & Koopman (2000) find that implied volatility is better than SV when stock index volatility is forecasted. The mixed results in the existing literature suggest that further research needs to be done on the merits of SV models with the aim of producing accurate volatility forecasts. Sadorsky (2005) using different assets compares a discrete-time range-based SV model with simple models and find that simple models outperform the SV.

4.3 Data

The dataset used for the purposes of this chapter consists of the daily closing price data of three major US indices (S&P500, DJIA, Nasdaq100) and six European (STOXX, CAC, DAX, AEX, SMI and FTSE100) as well as their implied volatility indices (VIX, VXD,VXN, VSTOXX, VCAC, VDAX, VAEX, VSMI and VFTSE100). The data have been collected from Datastream for the period 2 February 2001 to 28 February 2013. I have also obtained daily realized variances from Realized Library of the Oxford-Man Institute of Quantitative Finance. These realized variances are based on the sum of 5-minute intra-day squared returns.

4.4 Methodology

4.4.1 Stochastic volatility model

The aim is to investigate both the in-sample and out-of-sample performance of the SV models. Like GARCH models, the SV models are defined by their first and second moments. That is, the conditional mean and conditional variance, respectively.

The standard SV model proposed by Taylor (1986) is a log-normal AR(1) which is formulated as

$$r_t = \sigma_t \varepsilon_t = exp({}^{h_t}\!/_2)\varepsilon_t, \qquad \varepsilon_t \overset{i.i.d}{\sim} N(0,1) \tag{43}$$

$$h_{t+1} = \phi_0 + \phi_1 h_t + \eta_{t+1}, \qquad \eta_t \overset{i.i.d.}{\sim} N(0, \sigma_\eta^2) \tag{44}$$

where

$$corr(\varepsilon_t, \eta_{t+1}) = 0$$

The parameter ϕ_1 measures the volatility persistence and it is restricted to be positive and lower than one in order the volatility process to be stationary. Under this assumption, the unconditional variance is $Var(h) = \sigma_\eta^2/(1-\phi_1^2)$. It is assumed that ε_t and η_{t+1} are independent.[18] The SV model, as an alternative to the GARCH models, is supposed to describe the returns features better than the GARCH-type models, since the additional innovation in the variance equation makes this model class more flexible than GARCH.

The SV model can be augmented with IV as explanatory variable in the variance equation. Thus, equation (44) can be written as

$$h_{t+1} = \phi_0 + \phi_1 h_t + \theta IV_t + \eta_{t+1}, \qquad \eta_t \overset{i.i.d.}{\sim} N(0, \sigma_\eta^2) \tag{45}$$

where IV is the daily implied volatility index computed from the annualized percentages as $IV/(100*\sqrt{252})$. IV_t is in a logarithmic form so that $IV_t = lnIV_t^2$. Whilst the parameter ϕ is constrained to be positive in equation (44), in this specification the stationarity is ensured when $\mid \phi \mid < 1$.

The SV models in equations (44) and (45) respond symmetrically to positive and negative shocks. However, a crucial and well documented stylized fact of the financial returns is the leverage effect, which, in the previous chapters, has been proved to significantly improve the performance of GARCH and IV models.

Harvey & Shephard (1996) and later Jacquier et al. (2004) generalized the basic SV model proposing two alternative specifications that take the leverage effect into account. Yu (2005) compares the two specifications and show that the model suggested by Jacquier et al. (2004) is inferior to the Harvey & Shephard (1996) one. Thus, based on his results, I follow Harvey & Shephard

[18] For review of the SV models see Taylor (1994), Ghysels et al. (1996) and Shephard (1996)

(1996) specification and, in both equations (44) and (45), I relax the assumption that ε_t and η_{t+1} are uncorrelated. A contemporaneous correlation between the innovations

$$corr(\varepsilon_t, \eta_{t+1}) = \rho$$

is allowed. When ρ is negative, negative shocks in the return series are linked with contemporaneous volatility shock, while a positive shock in the return series followed by a decrease in volatility.

The parameters are estimated using the quasi-maximum likelihood, which is consistent and easy to implement numerically method.

4.4.2 Forecast evaluation

I obtain one-, five- and twenty two-day out-of-sample volatility forecasts which in terms of trading days corresponds to one-day, one-week and one-month forecasting horizons. The forecasts do not overlap, because they are generated by a rolling window estimation process. That is, the initial period is rolled forward by adding one, five and twenty two observations and removing the most distant, thus keeping the sample size fixed. I compare the predictive abilities of the SV models with those of the GARCH models. More specifically, the GARCH(1,1) and EGARCH(1,1) models which are both extended to include the IV and have been evaluated in the previous chapter are used in this chapter.

The ability of the models described in Subsection 4.4.1 to accurately forecast the true volatility is evaluated using three of the most popular measures for forecast comparison. The mean absolute error (MAE), the root mean squared error (RMSE) and the goodness-of-fit R^2 statistic of the Mincer-Zarnowitz (MZ) (Mincer & Zarnowitz, 1969) regression, which measures how much of the true volatility is explained by the forecast series.

$$MSE = \frac{1}{\tau}\sum_{t=1}^{\tau}(h_t^2 - \sigma_t^2)^2 \tag{46}$$

$$RMSE = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau}(h_t^2 - \sigma_t^2)} \tag{47}$$

$$\sigma_t^2 = a_0 + a_1 h_t^f + \epsilon_t \tag{48}$$

where τ is the number of out-of-sample observations and σ_t^2 is the 'true' volatility. These measures are useful for forecast comparisons, but they do not provide any statistical test of the difference among the models. Thus, there is the possibility that the model with the lower forecast error may not be inherently better than the competing model, as their difference may be statistically insignificant. Consequently, the Giacomini-White test is employed.

Giacomini and White test

Giacomini-White (GW) pairwise test is a test of conditional predictive ability proposed by Giacomini & White (2006). The test evaluates the forecasting performance of two competing models, accounting for parameter uncertainty. In short, let $L(y_t;\hat{y}_t)$ denote the forecast loss where y_t is the 'true' value and $\hat{y}_t$ is the predicted value. The difference in loss of model i relative to a benchmark model o is defined as

$$d_{i,t} = L(y_t;\hat{y_{o,t}}) - L(y_t;\hat{y_{i,t}}) \tag{49}$$

The issue is whether the two models have equal predictive ability. That is, the null hypothesis tested is $H_0 : E(d_{i,t+\tau} \mid h_t) = 0$, where h_t is some information set. The CPA test statistic is then computed as a Wald statistic

$$CPA_t = T(T^{-1}\sum_{t=1}^{T-\tau} h_t d_{i,t+\tau})'\hat{\Omega}_T^{-1}(T^{-1}\sum_{t=1}^{T-\tau} h_t d_{i,t+\tau}) \sim \chi_1^2 \tag{50}$$

where $\hat{\Omega_T}$ is the Newey an West (1987) HAC estimator of the asymptotic variance of the $h_t d_{i,t+\tau}$. In this application the CPA test is used to assess whether any model under consideration outperforms the random walk model under the squared error metric.

SPA test

The GW test is a pairwise test that evaluates the forecasts of any two competing models. In order to investigate the relative performance of various volatility models Hansen (2005) introduced the Superior Predictive Ability (SPA) test. That is, it evaluates the performance of several alternative models simultaneously against a benchmark model The test uses a bootstrap procedure to assess whether the same outcome can be obtained from more than one sample. Forecasts are evaluated by a pre-specified loss function and the model that produces the smallest expected loss is the best model.

In short, let the difference in loss of model i relative to a benchmark model o is defined as in equation (49). The issue is whether any of the competing models $i = 1, ..., K$ significantly outperforms the benchmark model testing the null hypothesis that $\mu_i = E(d_{i,t}) \leq 0$. It is tested with the statistic

$$T_n^{SPA} = max_i n^{0.5} \frac{\bar{d}_i}{\sigma_i} \tag{51}$$

where $\bar{d}_i = \frac{1}{n} \sum d_{i,t}$ and $\sigma_i = \lim\limits_{n \to \infty} var(n^{0.5} \bar{d}_i)$ which is estimated via a bootstrap procedure.

4.5 Empirical results

4.5.1 In-sample results

In this subsection, the estimation results obtained with the different SV models are reported. The results are based on the period 2 February 2001 to 23 February 2010.

Table 46 reports the parameters estimates for the daily returns on the US indices. For the SV model the volatility persistence estimate, ϕ_1, is statistically significant and very close to unity for all series which is a typical finding for daily stock index return series and consistent with the observed volatility clustering.

The incorporation of the implied volatility in the variance equation of the SV model, i.e. SV-IV model, always shows significant estimates for the coefficient θ. This confirms earlier findings in the literature that IV contains incremental information useful for explaining the conditional variance. For example, Hol & Koopman (2000) and Koopman et al. (2005) found that the incorporation of the implied volatility in the SV model has a significant effect on the fit of the model for the S&P100 index. The estimates of the persistence coefficient ϕ_1 are negative and statistically significant and in absolute value less than one.The inclusion of IV has considerably increased the estimates for σ_η^2. Finally, also the likelihood ratio test statistic indicates that IV has incremental explanatory power for conditional volatility.

Turning to the ASV model the negative value of the parameter ρ denotes the presence of leverage effect, which is significant across all indices. Despite this, likelihood ratio test show that ASV model does not fit the data significantly better than the SV. The volatility process is highly persistent for all series as shown by the close to the unity coefficient ϕ_1. When the IV is added as an exogenous variable in the variance equation, I find that parameter ρ is negative for all series, but significant only for the DJIA index. However, asymmetry has an insignificant effect on the fit of the model as shown by the likelihood ratio test. On the other hand, the effect of IV is strong since the log-likelihood increases in relation to the ASV model. Finally, I find that while the σ_η^2 has highly significant values the coefficient $\phi_1$is no longer significant.

Similar results I find for the European indices that are presented in Table 47. For the SV models I indicate that the coefficient for persistence ϕ_1 is statistically significant and very close to one. When the IV is added in the volatility equation the coefficient ϕ_1 is negative and less than one in absolute values, but in most indices it is not significant. At the same time, the value of the variance of η_t has significantly increased. As in the US data, SV-IV fit the data significantly better than the SV.

Regarding the ASV model, the coefficient ρ is not negative and significant for most indices. However, when IV is incorporated in the variance equation, ρ is negative and significant for STOXX, CAC, DAX and AEX indices, while it is positive and insignificant for FTSE100. Looking at the likelihood ratio test, I conclude that while the inclusion of IV has a strong effect on the fit of the models, the same does not apply when asymmetry is taken into consideration.

4.5.2 Out-of-sample results

The one-, five- and twenty two-day out-of-sample volatility forecasts of the different GARCH and SV models are constructed using the rolling forecasting methodology discussed in subsection 4.4.2 for the period 24 February 2010 to 28 February 2013.

Table 48 presents the mean square forecast error results. For the one-day ahead forecast the results are mixed. For 5 of the 9 indices the EGARCH-IV model yields the lowest forecast error. As for the rest 4 indices, the SV-IV model provides the best forecast for the Nasdaq100 index, the GARCH-IV and SV outperform for the CAC and AEX and FTSE100 index, respectively. Focusing in these indices, the EGARCH-IV model follows closely, apart from the case of Nasdaq100 where it performs poorly. The performance of GARCH and ASV is generally poor. GARCH provides the worst forecast for four indices, while ASV yields the highest loss for five indices. Finally, within the group of SV models, the SV-IV specification is superior for all indices with the exception of the FTSE100, where the basic SV model perform best. In the majority of the series, both SV and ASV models that are nested with IV provide more accurate forecasts that their SV and ASV counterparts where IV is precluded. The results are mixed when I compare the forecasting performance of the symmetric and asymmetric models.

When the predictive ability of the various models is examined for longer forecasting horizons there is overwhelming evidence of the superiority of the GARCH-IV model. More specifically, for the forecasting horizon of the five days, the GARCH-IV provides the best forecast for all indices with the exception of the Nasdaq100 and STOXX indices in which the SV-IV and EGARCH-IV, respectively, perform best. For all indices the GARCH model performs poorly yielding the highest loss apart from the DAX index, where the ASV specification performs worst. Among the SV-class models, and similar to the one-day ahead results, the SV-IV and ASV-IV specifications provide the best forecasts. As for the twenty two days ahead forecasts, the GARCH-IV performs best for five indices. As for the rest, the EGARCH-IV is superior for the STOXX and AEX indices and the SV-IV yields the lowest loss for the Nasdaq100 and FTSE100 indices. When the information of the IV is taken into account both the GARCH and SV models perform better than their restricted counterparts.

Table 49 reports the root mean squared forecast error results. First, the results are consistent with those obtained in Table 48. For the forecasting horizon of one day there is evidence for the superiority of the EGARCH-IV model in forecasting volatility. More specifically, the EGARCH-IV provides the best forecast for six of the nine indices. Whereas, for longer forecasting horizons, the GARCH-IV specification performs best for most indices. Second, both GARCH and SV models that account for IV perform better than those that preclude IV. Third, the GARCH and ASV specifications perform generally poor for most indices. Finally, and withing the group of SV models, the SV-IV and ASV-IV provide the best forecasts for all indices with the two specifications following very close to one another.

Table 50 presents the goodness-of-fit R^2 statistic, with now overwhelming evidence of the superiority of the GARCH genre of models in forecasting volatility. That is, the EGARCH-IV has the best forecasting performance obtaining the highest R^2 values for the forecasting horizon of the one day for all indices and the GARCH-IV performs best for the longer horizons. More specifically, for the five days ahead with the exception of the STOXX index where the EGARCH-IV perform best, the GARCH-IV yields the highest R^2 value for all indices. As for the twenty two steps ahead, the GARCH-IV is superior for all indices apart from the STOXX and AEX indices where the EGARCH-IV obtains the highest R^2 value. Furthermore, the models that are nested with IV outperform their counterparts that discount IV. Among the different SV specifications, the SV-IV seems to perform best for almost all indices.

Tables 51 to 56 present the GW pairwise test for the squared forecast errors for all the US and European indices and forecasting horizons. In the tables, I report the p-values for testing the null hypothesis of equal forecasting performance between the row and column models in terms of squared forecast error. The signs in bracket indicate which model performs best. A positive sign shows that the row model forecast yields larger loss than the column model forecast, which implies that the column model is significantly superior. Similarly, a negative sign denotes that the row model forecast performs significantly better than the column model forecast, since the latter produces larger loss.

Tables 51 and 52 report the GW test results for the one-day ahead forecast for the US and European indices, respectively. As for the US indices, the GARCH genre of models performs significantly better than the SV class of models. More specifically, for the S&P500 index only the GARCH-IV model outperforms the SV specifications. In all the other cases the null hypothesis of equal predictive ability between a GARCH model and a SV model cannot be rejected. For the DJIA index, the GARCH-IV performs significantly better than the SV models, while the ASV specification performs poorly as it is outperformed by all the other models with the exception of the GARCH models, where they perform equally well. In the case of the Nasdaq100 index, the GARCH-IV model is superior to all the other GARCH models as well as to the SV and ASV models. The majority of the rest GARCH models are outperformed by the SV models. Furthermore, within the group of the SV models, the models that account for the IV perform better than their restricted counterparts apart from the S&P500 index where they perform equally well. The basic SV model perform better than the ASV for both the S&P500 and DJIA indices, while for all indices the SV-IV and ASV-IV perform equally well.

As for one-day ahead of the European indices, Table 52, the results are similar. First, for all indices the GARCH-IV and the EGARCH-IV perform significantly better than the other models, with the exception of the FTSE100 index in which the SV specification perform best, supporting the results of the MSE and RMSE. Second, in the majority of the indices the GARCH models performs worst. Furthermore, when the SV models class is examined, the results are mixed. In half cases, the basic SV performs significantly better than the ASV and in the other half the SV models is outperformed by the ASV. Finally, in three of the six indices the SV-IV models outperforms the ASV-IV. In the other indices I cannot reject that both models forecast equally well.

Tables 53 and 55 displays the GW test results for the US indices for the five- and twenty two-day horizons. The results supports the findings of the MSE and RMSE. The GARCH-IV performs significantly better than the other models for all indices. The GARCH model performs poor across all indices. Within the group of the SV models, the results of the predictive ability of the models are same with those obtained at the one day horizon. Finally, Tables 54 and 56 show the results for the European indices for the forecasting horizons of five and twenty two days. Once more the results are in accordance with those obtained using the MSE and RMSE. The GARCH genre of models in most cases are superior to the SV models. More specifically, the GARCH-IV specification outperforms the other models for all indices apart from the FTSE100 index. In this case, the SV class of models is significantly better than the GARCH models and the SV model performs best.

I complement the above results by running the SPA test. Table 57 reports the p-values for testing the null hypothesis that none of the alternative models is better than the benchmark model. The p-values are based on 1,000 bootstrap samples under the mean squared forecast error loss function. The first column lists the names of the benchmark models and hence the remaining seven models are treated as competitive ones. Small p-values indicate that at least one of the competing models performs better than the base model. Thus, the higher the p-value is, the better the forecasting performance of the benchmark model is.

The results show that the GARCH class of models performs better than the SV models. More specifically, at the one-day horizon, the EGARCH-IV performs best for five of the nine indices. The EGARCH and GARCH-IV models outperform the competing ones for the SMI and AEX indices and just for two indices the SV models perform best. That is, the SV-IV and SV specifications provide the best forecasts for the Nasdaq100 and FTSE100, respectively. Overall, the SV models that take into account IV are found to be superior to SV and ASV. For longer horizons, five and twenty two days ahead, results are in accordance with those obtained before. The GARCH class of models performs better than the SV models. The GARCH-IV provides the best forecast for most indices. The EGARCH-IV perform best for the STOXX index, while the SV class of models outperforms for the Nasdaq100 and FTSE100 indices. The ASV-IV and SV-IV specifications are superior at the five and twenty two days ahead.

4.5.3 Conclusion

In this chapter I consider daily volatility forecasts of various US and European stock indices and examine the predictive ability of the SV models. In the previous chapter, it has been found that the leverage effect and implied volatility proved to have a significant effect on the relative performance of alternative GARCH models. Thus, I extend the basic SV model to a volatility model that allows for the presence of leverage effect and the inclusion of IV.

In both in-sample and out-of-sample results there is a consensus about the usefulness of incorporating IV in the variance equation. IV contains incremental information regarding the future volatility beyond that captured by SV. In contrast, the presence of the asymmetric effect seems not to significantly improve the performance of the SV models. Within the group of the SV models, the SV-IV model provides the most accurate forecast for both US and European indices.

This study provides a comparative evaluation of the volatility forecasting ability of SV and GARCH models, GARCH and EGARCH models. The one-, five- and twenty two-day out-of-sample volatility forecasts of the GARCH and SV models are evaluated. The results show that, overall, the GARCH genre of models perform better than the SV models. More specifically, the EGARCH-IV performs best at the one-day horizon for all indices and the GARCH-IV provides the most accurate forecast for the longer horizons (five and twenty two days ahead), with the exception of the Nasdaq100 and FTSE100, in which a SV model performs best.

Table 46: Estimation results of the SV models for the US indices

	SV	$SV-IV$	ASV	$ASV-IV$
		$S\&P$		
ϕ_0	-0.0565* (0.034)	1.9459* (0.007)	-0.0527** (0.046)	1.6813* (0.010)
ϕ_1	0.9939* (0.000)	-0.2724 (0.162)	0.9943* (0.000)	-0.0750 (0.769)
ρ			-0.2371** (0.044)	-0.2052 (0.101)
θ		1.5708* (0.000)		1.3314* (0.000)
σ_η^2	0.0128* (0.002)	0.3071* (0.008)	0.0145* (0.003)	0.2733** (0.017)
$log-L$	-5156.60	-5119.87	-5155.38	-5118.75
$LR(\theta=0)$	73.46		73.26	
$LR(\rho=0)$	2.44	2.24		
		$DJIA$		
ϕ_0	-0.0326* (0.000)	1.8483** (0.013)	-0.0448*** (0.059)	1.3405** (0.017)
ϕ_1	0.9965* (0.000)	-0.3318*** (0.059)	0.9952* (0.000)	0.0884 (0.696)
ρ			-0.2476*** (0.063)	-0.3078* (0.007)
θ		1.6152* (0.000)		1.1140* (0.000)
σ_η^2	0.0084* (0.001)	0.3361** (0.014)	0.0119* (0.005)	0.3073** (0.018)
$log-L$	-5154.67	-5127.09	-5153.06	-5125.18
$LR(\theta=0)$	55.16		55.76	
$LR(\rho=0)$	3.22	3.82		
		$Nasdaq100$		
ϕ_0	-0.0227 (0.128)	0.9427 (0.130)	-0.0209 (0.156)	0.8373 (0.115)
ϕ_1	0.9973* (0.000)	-0.4017** (0.013)	0.9975* (0.000)	-0.2156 (0.299)
ρ			-0.3470* (0.004)	-0.1970 (0.132)
θ		1.5800* (0.000)		1.3726* (0.000)
σ_η^2	0.0065* (0.003)	0.3782* (0.005)	0.0078* (0.009)	0.3665* (0.004)
$log-L$	-5169.45	-5142.20	-5166.84	-5141.37
$LR(\theta=0)$	54.50		50.94	
$LR(\rho=0)$	5.22	1.66		

Note: Entries report results of the alternative SV models as described in equations (!!!!). The p-values of the estimated coefficients are in parentheses. $LR(\theta=0)$ and $LR(\rho=0)$ are the likelihood ratio statistics for the hypotheses $\theta=0$ and $\rho=0$, respectively. *, ** and *** denote significance at the 1%, 5% and 10% level, respectively.

Table 47: Estimation results of the SV models for the US indices

	SV	$SV-IV$	ASV	$ASV-IV$		SV	$SV-IV$	ASV	$ASV-IV$
		$STOXX$						CAC	
ϕ_0	−0.0577** (0.037)	2.0602** (0.012)	−0.0565** (0.042)	1.8793* (0.007)	ϕ_0	−0.0650** (0.018)	1.3258*** (0.055)	−0.0599** (0.027)	1.2317** (0.033)
ϕ_1	0.9934* (0.000)	−0.5990* (0.000)	0.9936* (0.000)	−0.4201* (0.003)	ϕ_1	0.9926* (0.000)	−0.3604** (0.022)	0.9932* (0.000)	−0.2063 (0.257)
ρ			−0.0654 (0.569)	−0.3687* (0.001)	ρ			−0.2625** (0.034)	−0.3431** (0.010)
θ		1.9224** (0.000)		1.7132* (0.000)	θ		1.5706* (0.000)		1.3993* (0.000)
σ_η^2	0.0126* (0.002)	0.2100** (0.029)	0.0131* (0.003)	0.0735 (0.459)	σ_η^2	0.0132* (0.002)	0.4309* (0.001)	0.0155* (0.003)	0.3253* (0.008)
$log-L$	−5127.20	−5084.03	−5127.11	−5080.08	$log-L$	−5218.19	−5187.98	−5216.70	−5184.06
$LR(\theta=0)$	86.34		94.06		$LR(\theta=0)$	60.42			65.28
$LR(\rho=0)$	0.18	7.9			$LR(\rho=0)$	2.98	7.84		
		DAX						AEX	
ϕ_0	−0.0604** (0.024)	1.8233** (0.015)	−0.0628** (0.024)	1.1888** (0.031)	ϕ_0	−0.0708** (0.015)	1.2945** (0.027)	−0.0720** (0.016)	1.1311** (0.023)
ϕ_1	0.9930* (0.000)	−0.3045 (0.275)	0.9928* (0.000)	−0.0290 (0.906)	ϕ_1	0.9920* (0.000)	−0.3101 (0.252)	0.9919* (0.000)	−0.1345 (0.600)
ρ			0.1469 (0.250)	−0.2712* (0.009)	ρ			0.0676 (0.575)	−0.2180** (0.035)
θ		1.5751* (0.000)		1.2129* (0.000)	θ		1.5321* (0.000)		1.3280* (0.000)
σ_η^2	0.0139* (0.002)	0.1025*** (0.061)	0.0125* (0.003)	0.0001 (0.996)	σ_η^2	0.0173* (0.000)	0.1148** (0.020)	0.0166* (0.008)	0.0423 (0.249)
$log-L$	−5083.35	−5060.19	−5082.86	−5057.19	$log-L$	−5158.25	−5128.32	−5158.15	−5126.71
$LR(\theta=0)$	46.32		51.34		$LR(\theta=0)$	59.86		62.88	
$LR(\rho=0)$	0.98	6.00			$LR(\rho=0)$	0.2	3.22		
		SMI						$FTSE100$	
ϕ_0	−0.0928** (0.018)	0.2845 (0.388)	−0.1049* (0.009)	−0.0711 (0.228)	ϕ_0	−0.0973* (0.009)	0.8495 (0.178)	−0.1013* (0.007)	0.8475 (0.181)
ϕ_1	0.9898* (0.000)	0.4275 (0.170)	0.9885* (0.000)	0.9517* (0.000)	ϕ_1	0.9894* (0.000)	−0.1300 (0.690)	0.9890* (0.000)	−0.1272 (0.698)
ρ			0.4748* (0.000)	0.4259* (0.000)	ρ			0.1162 (0.325)	0.0085 (0.933)
θ		0.6185*** (0.065)		0.0415*** (0.095)	θ		1.2862* (0.001)		1.2830* (0.001)
σ_η^2	0.0187* (0.002)	0.0003 (0.997)	0.0109** (0.030)	0.0002 (0.989)	σ_η^2	0.0214* (0.000)	0.0163 (0.881)	0.0199* (0.000)	0.0185 (0.866)
$log-L$	−4970.49	−4952.86	−4965.20	−4963.76	$log-L$	−5119.56	−5090.12	−5119.26	−5090.12
$LR(\theta=0)$	35.26		2.88		$LR(\theta=0)$	58.88		58.28	
$LR(\rho=0)$	10.58	−21.8			$LR(\rho=0)$	0.6	0.0		

Note: Entries report results of the alternative SV models as described in equations (!!!!). The p-values of the estimated coefficients are in parentheses. $LR(\theta = 0)$ and $LR(\rho = 0)$ are the likelihood ratio statistics for the hypotheses $\theta = 0$ and $\rho = 0$, respectively. *, ** and *** denote significance at the 1%, 5% and 10% level, respectively.

Table 48: Mean square forecast error results

	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
one day ahead									
GARCH	0.0213	0.0185	0.0235	0.0412	0.0367	0.0273	0.0183	0.0117	0.1005
GARCH-IV	0.0188	0.0167	0.0191	0.0248	**0.0211**	0.0216	**0.0120**	0.0100	0.0708
EGARCH	0.0182	0.0165	0.0222	0.0249	0.0230	0.0210	0.0149	0.0076	0.0837
EGARCH-IV	**0.0171**	**0.0152**	0.0200	**0.0237**	0.0213	**0.0198**	0.0129	**0.0075**	0.0769
SV	0.0230	0.0212	0.0194	0.0364	0.0280	0.0334	0.0191	0.0122	**0.0648**
SV-IV	0.0200	0.0178	**0.0181**	0.0328	0.0255	0.0237	0.0132	0.0120	0.0661
ASV	0.0232	0.0213	0.0192	0.0364	0.0283	0.0331	0.0192	0.0121	0.0649
ASV-IV	0.0204	0.0182	0.0182	0.0361	0.0279	0.0247	0.0134	0.0128	0.0662
five days ahead									
GARCH	0.0261	0.0222	0.0259	0.0406	0.0389	0.0316	0.0204	0.0143	0.1147
GARCH-IV	**0.0177**	**0.0160**	0.0186	0.0249	**0.0208**	**0.0205**	**0.0123**	**0.0104**	**0.0703**
EGARCH	0.0236	0.0208	0.0253	0.0248	0.0275	0.0272	0.0188	0.0112	0.0959
EGARCH-IV	0.0219	0.0191	0.0212	**0.0237**	0.0234	0.0246	0.0162	0.0111	0.0795
SV	0.0239	0.0220	0.0199	0.0378	0.0297	0.0344	0.0195	0.0133	0.0711
SV-IV	0.0198	0.0175	**0.0176**	0.0306	0.0238	0.0231	0.0133	0.0118	0.0664
ASV	0.0240	0.0219	0.0198	0.0378	0.0298	0.0344	0.0195	0.0133	0.0710
ASV-IV	0.0198	0.0178	0.0176	0.0309	0.0240	0.0230	0.0131	0.0140	0.0664
twenty-two days head									
GARCH	0.0304	0.0254	0.0292	0.0400	0.0458	0.0369	0.0215	0.0212	0.1309
GARCH-IV	**0.0177**	**0.0161**	0.0186	0.0247	**0.0208**	**0.0204**	0.0126	**0.0103**	0.0702
EGARCH	0.0263	0.0230	0.0269	0.0247	0.0267	0.0331	0.0166	0.0175	0.1131
EGARCH-IV	0.0221	0.0201	0.0201	**0.0236**	0.0271	0.0256	**0.0113**	0.0159	0.0751
SV	0.0251	0.0229	0.0208	0.0410	0.0338	0.0371	0.0218	0.0158	0.0701
SV-IV	0.0197	0.0176	**0.0176**	0.0296	0.0232	0.0233	0.0132	0.0122	**0.0664**
ASV	0.0253	0.0230	0.0207	0.0412	0.0340	0.0376	0.0218	0.0159	0.0701
ASV-IV	0.0197	0.0177	0.0177	0.0291	0.0231	0.0235	0.0130	0.0147	0.0665

Note: The mean squared forecast error (MSE) defined in equation (46) of GARCH and SV models for the one, five and twenty-two days ahead are reported. All numbers are multiplied by 10^6. * denotes the lowest forecast error.

Table 49: Root mean square forecast error results

	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
one day ahead									
GARCH	0.1458	0.1361	0.1532	0.2029	0.1916	0.1651	0.1355	0.1083	0.1003
GARCH-IV	0.1371	0.1291	0.1380	0.1574	0.1453	0.1468	**0.1094**	0.1000	0.0841
EGARCH	0.1347	0.1283	0.1491	0.1577	0.1516	0.1448	0.1221	0.0873	0.0915
EGARCH-IV	**0.1312**	**0.1232**	0.1414	**0.1540**	**0.1459**	**0.1407**	0.1136	**0.0868**	0.0877
SV	0.1518	0.1457	0.1393	0.1909	0.1673	0.1827	0.1382	0.1107	**0.0805**
SV-IV	0.1414	0.1333	**0.1344**	0.1811	0.1598	0.1540	0.1151	0.1094	0.0813
ASV	0.1523	0.1459	0.1387	0.1908	0.1681	0.1820	0.1385	0.1101	0.0806
ASV-IV	0.1428	0.1349	0.1350	0.1899	0.1671	0.1572	0.1156	0.1132	0.0814
five days ahead									
GARCH	0.1616	0.1491	0.1608	0.2015	0.1972	0.1778	0.1429	0.1196	0.1071
GARCH-IV	**0.1331**	**0.1267**	0.1365	0.1576	**0.1443**	**0.1431**	**0.1110**	**0.1021**	0.0839
EGARCH	0.1536	0.1441	0.1590	0.1576	0.1657	0.1650	0.1371	0.1052	0.0979
EGARCH-IV	0.1480	0.1382	0.1457	**0.1540**	0.1530	0.1567	0.1273	0.1057	0.0892
SV	0.1547	0.1482	0.1412	0.1945	0.1722	0.1854	0.1396	0.1155	0.0843
SV-IV	0.1406	0.1324	**0.1327**	0.1748	0.1542	0.1519	0.1154	0.1088	**0.0815**
ASV	0.1550	0.1481	0.1406	0.1944	0.1728	0.1854	0.1397	0.1153	0.0843
ASV-IV	0.1406	0.1336	0.1327	0.1757	0.1549	0.1515	0.1145	0.1183	0.0816
twenty-two days head									
GARCH	0.1743	0.1592	0.1710	0.1999	0.2141	0.1922	0.1467	0.1457	0.1144
GARCH-IV	**0.1331**	**0.1269**	0.1363	0.1572	**0.1442**	**0.1430**	0.1122	**0.1017**	0.0838
EGARCH	0.1622	0.1518	0.1640	0.1570	0.1635	0.1819	0.1287	0.1321	0.1064
EGARCH-IV	0.1488	0.1417	0.1416	**0.1535**	0.1646	0.1601	**0.1065**	0.1262	0.0866
SV	0.1586	0.1512	0.1442	0.2025	0.1840	0.1925	0.1476	0.1259	0.0837
SV-IV	0.1405	0.1326	**0.1328**	0.1720	0.1523	0.1528	0.1151	0.1105	**0.0815**
ASV	0.1591	0.1517	0.1439	0.2029	0.1845	0.1939	0.1478	0.1261	0.0837
ASV-IV	0.1402	0.1332	0.1329	0.1705	0.1520	0.1533	0.1140	0.1212	0.0816

Note: The root mean squared forecast error (RMSE) defined in equation (47) of GARCH and SV models for the one, five and twenty-two days ahead are reported. All numbers are multiplied by 10^3. * denotes the lowest forecast error.

Table 50: Out-of-sample predictive power of daily volatility forecasts

	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
one day ahead									
GARCH	0.3243	0.2809	0.2047	0.3360	0.3723	0.4512	0.3436	0.4391	0.4180
GARCH-IV	0.3695	0.3405	0.2950	0.4888	0.4870	0.5051	0.4807	0.4983	0.5440
EGARCH	0.3878	0.3436	0.2282	0.5429	0.5726	0.5826	0.4824	0.6374	0.5348
EGARCH-IV	**0.4425**	**0.3989**	**0.3334**	**0.5625**	**0.5871**	**0.5856**	**0.5260**	**0.6407**	**0.5955**
SV	0.2049	0.1706	0.1106	0.2101	0.2920	0.2561	0.2312	0.2887	0.3216
SV-IV	0.3177	0.2884	0.2168	0.3256	0.3370	0.4295	0.3588	0.3783	0.4373
ASV	0.1990	0.1681	0.1034	0.2058	0.2738	0.2514	0.2332	0.2967	0.3193
ASV-IV	0.3035	0.2706	0.2123	0.2868	0.2996	0.4081	0.3494	0.3123	0.4353
five days ahead									
GARCH	0.2017	0.1701	0.1230	0.3386	0.3197	0.3599	0.2484	0.3158	0.3107
GARCH-IV	**0.4080**	**0.3678**	**0.3163**	0.4873	**0.4954**	**0.5338**	**0.4577**	**0.4742**	**0.5472**
EGARCH	0.2264	0.1942	0.1191	0.5425	0.4518	0.4320	0.3049	0.4234	0.3671
EGARCH-IV	0.2820	0.2477	0.2117	**0.5620**	0.4788	0.4507	0.3319	0.4207	0.4333
SV	0.1794	0.1474	0.0859	0.1887	0.2568	0.2296	0.1988	0.2265	0.2637
SV-IV	0.3247	0.2984	0.2341	0.3566	0.3694	0.4455	0.3564	0.3851	0.4365
ASV	0.1750	0.1460	0.0810	0.1851	0.2416	0.2227	0.2010	0.2318	0.2634
ASV-IV	0.3222	0.2851	0.2326	0.3509	0.3634	0.4475	0.3577	0.2539	0.4363
twenty-two days head									
GARCH	0.1255	0.0971	0.0656	0.3407	0.1769	0.2531	0.2456	0.1009	0.1638
GARCH-IV	**0.4077**	**0.3647**	**0.3162**	0.4912	**0.4970**	**0.5338**	0.4476	**0.4846**	**0.5518**
EGARCH	0.1296	0.1027	0.0542	0.5435	0.3429	0.2522	0.3453	0.1285	0.1712
EGARCH-IV	0.2505	0.1950	0.1760	**0.5655**	0.3424	0.3887	**0.4646**	0.1706	0.3292
SV	0.1399	0.1163	0.0647	0.1422	0.1684	0.1727	0.1422	0.1188	0.1888
SV-IV	0.3260	0.2954	0.2349	0.3745	0.3834	0.4411	0.3590	0.3690	0.4397
ASV	0.1349	0.1105	0.0588	0.1369	0.1588	0.1615	0.1432	0.1209	0.1878
ASV-IV	0.3261	0.2888	0.2331	0.3805	0.3844	0.4368	0.3642	0.2199	0.4390

Note: Entries are the adjusted R^2 values from the Mincer-Zarnowitz regression described in equation (48).
* denotes the highest R^2 value.

Table 51: Conditional Giacomini-White test results for the one day ahead volatility forecasts of the US indices

	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
S&P							
GARCH	$0.094^{(+)}$	$0.000^{(+)}$	$0.031^{(+)}$	0.277	0.349	0.284	0.711
GARCH-IV	-	0.379	0.401	$0.054^{(-)}$	$0.040^{(-)}$	$0.051^{(-)}$	$0.030^{(-)}$
EGARCH		-	0.358	0.168	0.150	0.161	0.135
EGARCH-IV			-	0.155	0.160	0.150	0.163
SV				-	0.468	$0.032^{(-)}$	0.461
SV-IV					-	0.444	0.199
ASV						-	0.434
DJIA							
GARCH	0.114	$0.000^{(+)}$	$0.002^{(+)}$	0.231	0.685	0.237	0.391
GARCH-IV	-	0.384	0.451	$0.004^{(-)}$	$0.068^{(-)}$	$0.006^{(-)}$	$0.058^{(-)}$
EGARCH		-	0.108	$0.029^{(-)}$	0.246	$0.034^{(-)}$	0.191
EGARCH-IV			-	0.112	$0.091^{(-)}$	$0.018^{(-)}$	0.130
SV				-	$0.025^{(+)}$	$0.090^{(-)}$	$0.022^{(+)}$
SV-IV					-	$0.032^{(-)}$	0.612
ASV						-	$0.028^{(+)}$
Nasdaq100							
GARCH	$0.001^{(+)}$	$0.000^{(+)}$	$0.001^{(+)}$	$0.000^{(+)}$	$0.003^{(+)}$	$0.000^{(+)}$	$0.003^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.005^{(-)}$	$0.050^{(-)}$	0.232	$0.033^{(-)}$	0.325
EGARCH		-	$0.001^{(+)}$	$0.000^{(+)}$	$0.024^{(+)}$	$0.000^{(+)}$	$0.021^{(+)}$
EGARCH-IV			-	$0.002^{(+)}$	0.534	$0.002^{(+)}$	0.514
SV				-	$0.070^{(+)}$	$0.000^{(+)}$	$0.038^{(+)}$
SV-IV					-	$0.069^{(-)}$	0.550
ASV						-	$0.038^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts +and −indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 52: Conditional Giacomini-White test results for the one day ahead volatility forecasts of the European indices

	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
STOXX							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.022^{(+)}$	$0.037^{(+)}$	$0.020^{(+)}$	0.399
GARCH-IV	-	$0.000^{(-)}$	$0.001^{(+)}$	$0.001^{(-)}$	$0.004^{(-)}$	$0.001^{(-)}$	$0.002^{(-)}$
EGARCH		-	$0.001^{(+)}$	$0.054^{(-)}$	0.130	$0.057^{(-)}$	$0.056^{(-)}$
EGARCH-IV			-	$0.051^{(-)}$	$0.095^{(-)}$	$0.055^{(-)}$	$0.045^{(-)}$
SV				-	0.363	$0.000^{(+)}$	0.628
SV-IV					-	0.369	$0.001^{(-)}$
ASV						-	0.613
CAC							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.007^{(+)}$	$0.002^{(+)}$	$0.06^{(+)}$	$0.016^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.000^{(-)}$	$0.001^{(-)}$	$0.012^{(-)}$	$0.002^{(-)}$	$0.003^{(-)}$
EGARCH		-	$0.000^{(+)}$	$0.024^{(-)}$	$0.029^{(-)}$	$0.032^{(-)}$	$0.060^{(-)}$
EGARCH-IV			-	$0.035^{(-)}$	$0.072^{(-)}$	$0.044^{(-)}$	$0.099^{(-)}$
SV				-	0.169	$0.010^{(-)}$	0.251
SV-IV					-	0.205	$0.012^{(-)}$
ASV						-	0.273
DAX							
GARCH	$0.000^{(+)}$	$0.039^{(+)}$	$0.019^{(+)}$	$0.084^{(-)}$	$0.093^{(+)}$	0.106	0.275
GARCH-IV	-	$0.017^{(+)}$	$0.060^{(+)}$	$0.001^{(-)}$	0.167	$0.001^{(-)}$	$0.089^{(-)}$
EGARCH		-	$0.036^{(+)}$	$0.094^{(-)}$	$0.079^{(-)}$	0.107	0.169
EGARCH-IV			-	$0.064^{(-)}$	0.136	$0.075^{(-)}$	0.176
SV				-	$0.005^{(+)}$	$0.007^{(+)}$	$0.023^{(+)}$
SV-IV					-	$0.007^{(-)}$	$0.033^{(-)}$
ASV						-	$0.029^{(+)}$
AEX							
GARCH	$0.000^{(+)}$	$0.012^{(+)}$	$0.005^{(+)}$	$0.002^{(-)}$	$0.001^{(+)}$	$0.002^{(-)}$	$0.002^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.001^{(-)}$	$0.003^{(-)}$	0.484	$0.0013^{(-)}$	0.485
EGARCH		-	$0.000^{(+)}$	$0.047^{(-)}$	$0.000^{(+)}$	$0.043^{(-)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.051^{(-)}$	$0.016^{(-)}$	$0.046^{(-)}$	$0.029^{(-)}$
SV				-	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$
SV-IV					-	$0.000^{(-)}$	0.404
ASV						-	$0.000^{(+)}$
SMI							
GARCH	$0.094^{(+)}$	$0.012^{(+)}$	$0.013^{(+)}$	$0.068^{(-)}$	0.243	$0.066^{(-)}$	$0.059^{(-)}$
GARCH-IV	-	0.188	0.135	0.140	$0.066^{(-)}$	0.141	0.115
EGARCH		-	0.540	0.139	$0.046^{(-)}$	0.140	0.134
EGARCH-IV			-	0.132	$0.040^{(-)}$	0.133	0.130
SV				-	$0.062^{(+)}$	$0.066^{(+)}$	$0.074^{(-)}$
SV-IV					-	$0.054^{(-)}$	0.118
ASV						-	$0.046^{(-)}$
FTSE100							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.000^{(-)}$	$0.001^{(+)}$	0.417	$0.002^{(+)}$	0.409
EGARCH		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.000^{(+)}$	$0.004^{(+)}$	$0.000^{(+)}$	$0.004^{(+)}$
SV				-	$0.016^{(-)}$	$0.001^{(-)}$	$0.014^{(-)}$
SV-IV					-	$0.017^{(+)}$	0.286
ASV						-	$0.015^{(-)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts +and −indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 53: Conditional Giacomini-White test results for the five day ahead volatility forecasts of the US indices

	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
S&P							
GARCH	$0.003^{(+)}$	$0.003^{(+)}$	$0.019^{(+)}$	0.345	$0.008^{(+)}$	0.396	$0.004^{(+)}$
GARCH-IV	-	$0.025^{(-)}$	$0.024^{(-)}$	$0.039^{(-)}$	$0.044^{(-)}$	$0.037^{(-)}$	$0.046^{(-)}$
EGARCH		-	0.159	0.461	0.101	0.441	$0.077^{(+)}$
EGARCH-IV			-	0.115	0.246	0.112	0.250
SV				-	0.315	$0.001^{(-)}$	0.274
SV-IV					-	0.304	0.163
ASV						-	0.262
DJIA							
GARCH	$0.002^{(+)}$	$0.017^{(+)}$	$0.006^{(+)}$	0.707	$0.002^{(+)}$	0.763	$0.001^{(+)}$
GARCH-IV	-	$0.012^{(-)}$	$0.024^{(-)}$	$0.006^{(-)}$	$0.054^{(-)}$	$0.006^{(-)}$	$0.046^{(-)}$
EGARCH		-	$0.023^{(+)}$	$0.057^{(-)}$	$0.024^{(+)}$	$0.072^{(-)}$	$0.014^{(+)}$
EGARCH-IV			-	$0.020^{(-)}$	0.173	$0.023^{(-)}$	0.182
SV				-	$0.015^{(+)}$	$0.014^{(-)}$	$0.013^{(+)}$
SV-IV					-	$0.018^{(-)}$	0.595
ASV						-	$0.015^{(+)}$
Nasdaq100							
GARCH	$0.001^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.001^{(-)}$	$0.048^{(-)}$	$0.070^{(-)}$	0.216	$0.062^{(-)}$	0.276
EGARCH		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.001^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
SV				-	$0.096^{(+)}$	$0.000^{(+)}$	$0.086^{(+)}$
SV-IV					-	0.106	0.288
ASV						-	$0.095^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts $+$and $-$indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 54: Conditional Giacomini-White test results for the five day ahead volatility forecasts of the European indices

	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
STOXX							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.010^{(+)}$	$0.002^{(+)}$	$0.009^{(+)}$	$0.003^{(+)}$
GARCH-IV	-	$0.000^{(+)}$	$0.001^{(+)}$	$0.000^{(-)}$	$0.003^{(-)}$	$0.000^{(-)}$	$0.001^{(-)}$
EGARCH		-	$0.001^{(+)}$	$0.036^{(-)}$	0.164	$0.039^{(-)}$	0.168
EGARCH-IV			-	$0.036^{(-)}$	0.104	$0.038^{(-)}$	0.111
SV				-	$0.060^{(+)}$	$0.000^{(+)}$	$0.068^{(+)}$
SV-IV					-	$0.064^{(-)}$	0.721
ASV						-	$0.072^{(+)}$
CAC							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.001^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.004^{(-)}$	0.284	$0.000^{(-)}$	$0.068^{(-)}$	$0.000^{(-)}$	$0.067^{(-)}$
EGARCH		-	$0.000^{(+)}$	$0.022^{(-)}$	$0.004^{(+)}$	$0.029^{(-)}$	$0.005^{(+)}$
EGARCH-IV			-	$0.070^{(-)}$	0.127	$0.083^{(-)}$	0.129
SV				-	$0.007^{(+)}$	$0.009^{(-)}$	$0.008^{(+)}$
SV-IV					-	$0.008^{(-)}$	0.388
ASV						-	$0.008^{(+)}$
DAX							
GARCH	$0.000^{(+)}$	$0.053^{(+)}$	$0.003^{(+)}$	0.169	$0.000^{(+)}$	0.160	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.015^{(-)}$	$0.002^{(-)}$	0.164	$0.002^{(-)}$	0.232
EGARCH		-	$0.000^{(+)}$	0.175	$0.004^{(+)}$	0.181	$0.000^{(+)}$
EGARCH-IV			-	$0.054^{(-)}$	0.146	$0.060^{(-)}$	$0.029^{(+)}$
SV				-	$0.001^{(+)}$	$0.000^{(+)}$	$0.001^{(+)}$
SV-IV					-	$0.001^{(-)}$	0.566
ASV						-	$0.001^{(+)}$
AEX							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.001^{(+)}$	$0.000^{(+)}$	$0.001^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.007^{(-)}$	$0.006^{(-)}$	0.484	$0.005^{(-)}$	0.634
EGARCH		-	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.016^{(-)}$	$0.021^{(+)}$	$0.014^{(-)}$	$0.009^{(+)}$
SV				-	$0.002^{(+)}$	$0.000^{(-)}$	$0.001^{(+)}$
SV-IV					-	$0.002^{(-)}$	0.164
ASV						-	$0.001^{(+)}$
SMI							
GARCH	$0.002^{(+)}$	$0.002^{(+)}$	$0.004^{(+)}$	$0.001^{(+)}$	$0.053^{(+)}$	$0.001^{(+)}$	$0.006^{(+)}$
GARCH-IV	-	0.498	0.587	$0.001^{(+)}$	$0.077^{(-)}$	$0.007^{(-)}$	$0.010^{(-)}$
EGARCH		-	$0.004^{(-)}$	0.102	0.149	0.103	$0.055^{(-)}$
EGARCH-IV			-	0.121	0.314	0.122	$0.052^{(-)}$
SV				-	0.188	$0.013^{(+)}$	$0.008^{(-)}$
SV-IV					-	0.192	0.164
ASV						-	$0.006^{(-)}$
FTSE100							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.024^{(-)}$	$0.009^{(-)}$	0.549	$0.009^{(-)}$	0.533
EGARCH		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.000^{(+)}$	$0.018^{(+)}$	$0.000^{(+)}$	$0.017^{(+)}$
SV				-	$0.030^{(+)}$	$0.007^{(+)}$	$0.029^{(+)}$
SV-IV					-	$0.032^{(-)}$	0.386
ASV						-	$0.032^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts $+$and $-$indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 55: Conditional Giacomini-White test results for the twenty-two days ahead volatility forecasts of the US indices

	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
S&P							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.002^{(+)}$	$0.000^{(+)}$	$0.003^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.003^{(-)}$	$0.069^{(-)}$	$0.017^{(-)}$	$0.044^{(-)}$	$0.015^{(-)}$	$0.047^{(-)}$
EGARCH		-	$0.000^{(+)}$	0.192	$0.035^{(+)}$	0.298	$0.024^{(+)}$
EGARCH-IV			-	$0.023^{(-)}$	0.594	$0.019^{(-)}$	0.535
SV				-	0.175	$0.003^{(-)}$	0.145
SV-IV					-	0.156	0.469
ASV						-	0.127
DJIA							
GARCH	$0.000^{(+)}$	$0.002^{(+)}$	$0.000^{(+)}$	$0.043^{(+)}$	$0.000^{(+)}$	$0.070^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.002^{(-)}$	$0.020^{(-)}$	$0.003^{(-)}$	$0.051^{(-)}$	$0.002^{(-)}$	$0.044^{(-)}$
EGARCH		-	$0.000^{(+)}$	0.882	$0.007^{(+)}$	0.942	$0.004^{(+)}$
EGARCH-IV			-	$0.001^{(-)}$	0.133	$0.001^{(-)}$	0.126
SV				-	$0.018^{(+)}$	$0.013^{(-)}$	$0.011^{(+)}$
SV-IV					-	$0.013^{(-)}$	0.676
ASV						-	$0.007^{(+)}$
Nasdaq100							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	0.169	$0.099^{(-)}$	0.214	$0.094^{(-)}$	0.240
EGARCH		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.000^{(-)}$	$0.017^{(+)}$	$0.000^{(-)}$	$0.010^{(+)}$
SV				-	$0.045^{(+)}$	$0.000^{(+)}$	$0.037^{(+)}$
SV-IV					-	$0.050^{(-)}$	0.205
ASV						-	$0.042^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts $+$and $-$indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 56: Conditional Giacomini-White test results for the twenty-two days ahead volatility forecasts of the European indices

	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
STOXX							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.008^{(-)}$	$0.000^{(+)}$	$0.008^{(-)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(+)}$	$0.001^{(+)}$	$0.000^{(-)}$	$0.007^{(-)}$	$0.000^{(-)}$	$0.010^{(-)}$
EGARCH		-	$0.001^{(+)}$	$0.025^{(-)}$	$0.072^{(-)}$	$0.024^{(-)}$	$0.055^{(-)}$
EGARCH-IV			-	$0.025^{(-)}$	$0.049^{(-)}$	$0.024^{(-)}$	$0.043^{(-)}$
SV				-	$0.007^{(+)}$	$0.000^{(-)}$	$0.003^{(+)}$
SV-IV					-	$0.006^{(-)}$	$0.082^{(+)}$
ASV						-	$0.003^{(+)}$
CAC							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	$0.000^{(-)}$	$0.000^{(-)}$	$0.084^{(-)}$	$0.000^{(-)}$	$0.089^{(-)}$
EGARCH		-	$0.000^{(-)}$	$0.005^{(-)}$	$0.005^{(+)}$	$0.012^{(-)}$	$0.003^{(+)}$
EGARCH-IV			-	$0.007^{(-)}$	$0.003^{(+)}$	$0.016^{(-)}$	$0.002^{(+)}$
SV				-	$0.000^{(+)}$	$0.001^{(-)}$	$0.000^{(+)}$
SV-IV					-	$0.000^{(-)}$	0.827
ASV						-	$0.000^{(+)}$
DAX							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	0.100	$0.000^{(+)}$	0.157	$0.000^{(+)}$
GARCH-IV	-	$0.002^{(-)}$	0.125	$0.000^{(-)}$	0.113	$0.000^{(-)}$	0.115
EGARCH		-	$0.000^{(+)}$	$0.047^{(-)}$	$0.014^{(+)}$	$0.031^{(-)}$	$0.010^{(+)}$
EGARCH-IV			-	$0.000^{(-)}$	0.213	$0.000^{(-)}$	0.220
SV				-	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$
SV-IV					-	$0.000^{(-)}$	0.458
ASV						-	$0.000^{(+)}$
AEX							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	0.212	$0.000^{(-)}$	0.766	$0.000^{(-)}$	0.884
EGARCH		-	$0.000^{(+)}$	$0.025^{(-)}$	$0.085^{(+)}$	$0.023^{(-)}$	$0.065^{(+)}$
EGARCH-IV			-	$0.002^{(-)}$	0.228	$0.002^{(-)}$	0.305
SV				-	$0.000^{(+)}$	$0.000^{(-)}$	$0.000^{(+)}$
SV-IV					-	$0.000^{(-)}$	$0.072^{(+)}$
ASV						-	$0.000^{(+)}$
SMI							
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.008^{(-)}$	$0.034^{(-)}$	$0.083^{(-)}$	$0.048^{(-)}$	$0.079^{(-)}$	$0.054^{(-)}$
EGARCH		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.023^{(+)}$	$0.000^{(+)}$	$0.004^{(+)}$
EGARCH-IV			-	$0.000^{(+)}$	0.117	$0.000^{(+)}$	$0.042^{(+)}$
SV				-	0.196	$0.004^{(-)}$	$0.051^{(+)}$
SV-IV					-	0.194	0.191
ASV						-	$0.058^{(+)}$
FTSE100	GARCH-IV	EGARCH	EGARCH-IV	SV	SV-IV	ASV	ASV-IV
GARCH	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
GARCH-IV	-	$0.000^{(-)}$	0.124	$0.016^{(+)}$	0.563	$0.017^{(+)}$	0.564
EGARCH		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
EGARCH-IV			-	$0.000^{(+)}$	0.237	$0.000^{(+)}$	0.235
SV				-	$0.087^{(+)}$	$0.000^{(-)}$	$0.085^{(+)}$
SV-IV					-	$0.087^{(-)}$	0.184
ASV						-	$0.085^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts +and −indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 57: SPA test (MSE)

	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE
one day ahead									
GARCH	0.009	0.006	0.015	0.004	0.001	0.001	0.002	0.051	0.001
GARCH-IV	0.602	0.373	0.457	0.427	0.570	0.658	**0.994**	0.152	0.361
EGARCH	0.680	0.120	0.028	0.523	0.221	0.337	0.005	**0.934**	0.042
EGARCH-IV	**0.886**	**0.968**	0.325	**0.853**	**0.807**	**0.916**	0.495	0.814	0.243
SV	0.094	0.035	0.329	0.031	0.026	0.056	0.022	0.204	**0.930**
SV-IV	0.352	0.167	**0.979**	0.013	0.039	0.161	0.341	0.070	0.914
ASV	0.094	0.036	0.474	0.025	0.038	0.049	0.025	0.294	0.618
ASV-IV	0.311	0.156	0.392	0.008	0.034	0.076	0.307	0.139	0.263
five day ahead									
GARCH	0.020	0.022	0.001	0.004	0.000	0.002	0.002	0.018	0.000
GARCH-IV	**0.876**	**0.905**	0.304	0.420	**0.870**	**0.912**	**0.787**	**0.968**	0.361
EGARCH	0.061	0.036	0.001	0.542	0.002	0.002	0.000	0.620	0.001
EGARCH-IV	0.039	0.046	0.001	**0.852**	0.263	0.025	0.014	0.542	0.045
SV	0.099	0.034	0.195	0.021	0.017	0.064	0.024	0.264	0.592
SV-IV	0.335	0.281	0.726	0.020	0.075	0.191	0.076	0.081	0.828
ASV	0.099	0.028	0.238	0.018	0.022	0.048	0.026	0.367	0.603
ASV-IV	0.388	0.188	**0.975**	0.012	0.090	0.213	0.405	0.113	**0.945**
twenty-two day ahead									
GARCH	0.000	0.000	0.000	0.003	0.000	0.001	0.000	0.001	0.000
GARCH-IV	**0.953**	**0.921**	0.377	0.427	**0.942**	**0.952**	0.242	**0.948**	0.336
EGARCH	0.010	0.007	0.000	0.550	0.000	0.013	0.000	0.015	0.000
EGARCH-IV	0.076	0.052	0.034	**0.841**	0.001	0.120	**0.863**	0.099	0.244
SV	0.064	0.026	0.115	0.017	0.009	0.009	0.002	0.151	0.696
SV-IV	0.355	0.213	**0.911**	0.040	0.113	0.141	0.111	0.027	**0.988**
ASV	0.034	0.026	0.117	0.015	0.009	0.008	0.006	0.173	0.505
ASV-IV	0.418	0.186	0.790	0.031	0.116	0.155	0.361	0.101	0.197

5 Forecasting realised volatility: the role of implied volatility, leverage effects and the volatility of realised volatility

I assess the forecasting performance of time series models for realised volatility, which take into consideration implied volatility, leverage effects, as well as the volatility of realised volatility. Realised volatility is modeled and forecasted with ARFIMA and HAR models for a number of US and European indices. I find that accounting for these stylized facts of volatility leads to a significant improvement of the models' predictive performance. The results suggest that a HAR model which accommodates implied volatility and leverage effects produces the most accurate volatility forecast

.

5.1 Introduction

Volatility, and in particular volatility forecasting, is of crucial importance for derivative pricing, asset allocation and risk management. Over the last fifteen years the availability of high-frequency data has shed more light on modelling and forecasting daily volatility. Andersen & Bollerslev (1998) first used the high-frequency data to construct a new volatility measure. They showed that the so-called realised variance (RV), computed by the sum of squared intraday returns, is a more precise measure of volatility than the ex-post daily squared returns. There is now a range of volatility estimators that are constructed using high-frequency data. (see, for example, Andersen et al. (2006) and Barndorff-Nielsen & Shephard (2007))

Since then realised volatility has been used not only as a measure of the true volatility but also in modelling and forecasting future volatility. The potential value of realised volatility as an information source to improve existing volatility models has been extensively examined. Among others, Blair et al. (2001), Martens (2001; 2002) and Engle (2002) were some of the first to incorporate realised volatility as an exogenous variable in the GARCH equation. They found that realised volatility is highly informative about future volatility.

One of the most salient features of volatility is the long memory, that is the autocorrelation function decays hyperbolically.[19] Reduced-form time series models that directly model and forecast realised volatility have been developed in order to capture its persistence. Andersen et al. (2003) suggests the use of autoregressive fractionally integrated moving average (ARFIMA) models for this purpose. They show that long memory models outperform the traditional GARCH and SV models which use low frequency returns for future volatility forecasting. Since then several studies employ the ARFIMA models. Among others, Martens & Zein (2004), Pong et al. (2004) and Koopman et al. (2005) find that ARFIMA models produce more accurate forecast than GARCH and SV models for different asset classes.

A related-type of reduced form volatility forecasts is the Heterogeneous Autoregressive (HAR) model of Corsi (2009). Inspired by the Heterogeneous Market Hypothesis and the HARCH model of Muller et al. (1997), Corsi (2009) proposed a regression based approach - an additive cascade model of volatility components over different time horizons. Its ability to reproduce the volatility persistence combined with the fact that it is easy to implement has encouraged its use in several studies.

In this chapter, I assess the forecasting performance of the long memory models. I also examine the importance of embedding in these models other stylized facts of realised volatility for the purpose of forecasting. First, I take into account the leverage effect, that is volatility tend to increase more after a negative shock than a positive shock of the same magnitude as first noted by Black (1976). Bollerslev et al. (2009) show a prolonged leverage effect at the intradaily level of S&P500 futures returns. Martens et al. (2009) provide evidence that accounting for the leverage effect improves the performance of the different models. Corsi & Renò (2012) extends the HAR model to capture the heterogeneous leverage effect. They find that not only the daily but also the weekly and monthly negative returns have a significant effect on future volatility. Similar results are provided in Wang et al. (2015) for the Chinese stock market.

Second, I account for the potential value of implied volatility as an information source for volatility forecasting. It has been widely perceived as a natural forecast of future volatility. In the context of forecasting the realised volatility, Busch et al. (2011) find that IV contains incremental

[19]See Andersen et al. (2001a,b) for more details on the dynamic properties of realised volatility.

information about future realised volatility in foreign, stock and bond market.

Third, following the empirical evidence of Corsi et al. (2008), I allow for time-varying volatility of the realised volatility. It is common in the HAR literature to assume that the residuals of the HAR model are i.i.d. However, volatility clustering in the residuals of the realised volatility models are often observed. Corsi et al. (2008) show that for the S&P500 index futures the volatility of realised volatility is important and a GARCH model should be taken into account. However, Bubak & Zikes (2009) and Todorova (2015), who compare the forecasting performance of the HAR and HAR-GARCH models for the exchange rate and metal market, respectively, find that while the HAR-GARCH model performs better in-sample, it cannot significantly improve the out-of-sample performance of the simple HAR model.

The main contribution of this chapter is that it considers models that simultaneously capture long memory, leverage effects, IV and volatility of realised volatility. The main focus is on the importance of these features for the purpose of forecasting. I assess the predictive ability of several reduced form time series models for forecasting realised volatility for a number of US and European indices. I use both a rolling and recursive sample to estimate the parameters of forecasting models. I employ three loss functions to analyze the accuracy of competing forecasts and evaluate the statistical significance by implementing the Giacomini-White and the SPA test.

Two are the main results of this study. First, accounting for the leverage effects and the information content of implied volatility improves the predictive power of the models. While taking into account implied volatility is more important than the leverage effect, accounting for both features significantly improves the forecast performance of the models. Second, it seems not to be beneficial to model the volatility of realised volatility as it does not lead to a substantial improvement of the forecast performance of the HAR models. These results generally holds for all loss functions and indices under both the rolling and recursive scheme.

The remainder of the chapter is organized as follows. Section 5.2 discusses the empirical framework, including the realized measure and the volatility models, and describing the data.. Section 5.3 presents the empirical results and Section 5.4 concludes.

5.2 Methodology and Data

5.2.1 Realised measures

The fact that volatility is latent makes it hard to assess the performance of volatility models. Thus, a proxy for the true volatility is used. For several years, the daily squared returns has been used, but it is now widely accepted that they provide a poor proxy of the true volatility. As noted by Andersen & Bollerslev (1998), although the use squared returns is justified because it is an unbiased estimate of volatility, it provides a noisy measure. However, Andersen & Bollerslev (1998) advocate that estimator of volatility based on cumulative intraday squared returns are more accurate. Building upon this line of research Andersen et al. (2003) defined the so-called realised volatility (RV) on day t as

$$RV_t = \sum_{i=1}^{m} r_{t,i}^2 \tag{52}$$

where m is the number of intraday returns during day t. Letting $m \to \infty$, that is in case of continuous time sampling, RV_t converges to the true integrated volatility. Since the introduction of the standard RV measure several RV estimates have been developed using a variety of sampling frequency and capturing different characteristics of RV. Here, following the results of Liu et al. (2015), who compare over 400 different realized voaltility measures and find that it is difficult to significantly beat the simple realised variance estimator, I use the simple RV which is constructed based on five-minute returns data.

5.2.2 Modeling volatility

ARFIMA model

By treating volatility as an observed variable rather than latent, it allows the direct estimation and forecasting using reduced-form time series approaches. My empirical approach adopts the ARFIMA model and HAR model.

I employ the ARFIMAX(p,d,q) developed by Granger & Joyeux (1980) expressed as

$$\phi(L)(1-L)^d(y_t-\alpha^{'}\mathbf{X})=\theta(L)\varepsilon_t \tag{53}$$

where $\phi(L)$ is the lag operator that defines the autoregressive components, $\theta(L)$ is the moving average polynomial and ε_t is an approximately Gaussian white noise. d is the degree of fractional integration and $0 < d < 0.5$ in order to capture the long memory characteristic. The model allows for $k \times 1$ vector $\mathbf{X}$ of explanatory variables.

Andersen et al. (2003) suggests the use of a ARFIMA model for the log(RV) in order to deal with the long memory behavior of the volatility series. As Andersen et al. (2001a; 2001b) pointed out, while RV is heavily skewed and exhibits fat tails the log(RV) is approximately bell shaped. In this study, I forecast the logarithmic realised variance and equation (54) displays the nature of this log transformation. This is motivated by the fact that Andersen et al. (2007) find similar results when the realised variance, realised volatility or the logarithmic realised volatility is used for estimating HAR models parameters. In this study,

$$y_t = log(RV_t^2) \tag{54}$$

By placing various restrictions on parameters of equation (53) four different models are obtained that assess the relative importance of different stylized facts of RV. More specifically, I consider i) the linear ARFIMA model setting $\alpha^{'}\mathbf{X}=\alpha_{\mathbf{0}}$, which is a well-known realized volatility model first proposed by Andersen et al. (2003) and then examined by Koopman et al. (2005), Pong et al. (2004), Martens et al. (2009) among others. ii) the ARFIMAX model that captures the leverage effect replacing $\alpha^{'}\mathbf{X}=\alpha_0+\alpha_1\mathrm{r}_{\mathrm{t-1}}^{-}$ (hereafter ARFIMA-L), iii) a ARFIMAX model that uses the information provided by the IV with $\alpha^{'}\mathbf{X}=\alpha_0+\beta_1\log(\mathrm{IV}_{\mathrm{t-1}}^2)$ (hereafter ARFIMA-IV), and iv) a ARFIMAX model that simultaneously accounts for the leverage effect and IV with $\alpha^{'}\mathbf{X}=\alpha_0+\alpha_1\mathrm{r}_{\mathrm{t-1}}^{-}+\beta_1\log(\mathrm{IV}_{\mathrm{t-1}}^2)$, (hereafter ARFIMA-L-IV).

HAR model

An alternative to the ARFIMA model that successfully reproduces the volatility persistence, though formally not a long memory model, is the HAR model developed by Corsi (2009). Corsi (2009) proposes a simple autoregressive-type model for realised volatility considering RVs over different time horizons. The standard HAR model in the realised volatility literature includes daily, weekly and monthly realised volatility components. In this study, I use a slightly different lag structure from the one in Corsi (2009) following the HAR model recently implemented in Patton & Sheppard (2015) in order to avoid overlapping horizons. This reparameterization allows for the direct interpretation of the effect of each component.

Hence, the HAR model for the logarithmic realised variance I use is as follows

$$y_t = \alpha_0 + \alpha_d y_{t-1} + \alpha_w y_{t-2,t-5} + \alpha_m y_{t-6,t-22} + u_t \tag{55}$$

where

$$y_{t-2,t-5} = \frac{1}{4}\sum_{i=2}^{5} y_{t-i}$$

$$y_{t-6,t-22} = \frac{1}{17}\sum_{i=6}^{22} y_{t-i}$$

with $y_{t-2,t-5}$ and $y_{t-6,t-22}$ be the average weekly and monthly RV components.

In order to assess whether the explicit incorporation of various realised volatility features in HAR model improve its forecasting performance I consider a more general HAR model of the form

$$y_t = \alpha_0 + \alpha_d y_{t-1} + \alpha_w y_{t-2,t-5} + \alpha_m y_{t-6,t-22} + \beta^{'}\mathbf{X} + u_t \tag{56}$$

so that the model allows for $k \times 1$ vector $\mathbf{X}$ of exogenous variables. In this study I employ the following models.

First, Corsi & Renò (2012) extends the HAR model of Corsi (2009) by taking into account the leverage effect. More specifically, they extend the Heterogeneous Market Hypothesis and consider that realised volatility reacts asymmetrically not only to previous daily returns, but also to weekly and monthly returns. I take into account the leverage effect by replacing $\beta^{'}\mathbf{X} = \beta_d r^{-}_{t-1} + \beta_w r^{-}_{t-2,t-5} + \beta_m r^{-}_{t-6,t-22}$, where $r_{t-2,t-5}$ and $r^{-}_{t-6,t-22}$ are the weekly and monthly negative returns (hereafter HAR-L). Second, I use a HAR model that investigates the information content of IV (hereafter, HAR-IV) setting $\beta^{'}\mathbf{X} = \gamma IV_{t-1}^2$. Third, I consider a HAR model that simultaneously includes the asymmetry and IV (HAR-L-IV) replacing $\beta^{'}\mathbf{X} = \beta_d r_{t-1}^{-} + \beta_w r_{t-2,t-5}^{-} + \beta_m r_{t-6,t-22}^{-} + \gamma IV_{t-1}^2$.

Moreover, an important empirical issue is the conditional heteroskedasticity of the innovations of realized volatility. Corsi et al. (2008) observed that the innovations of realised volatility are not i.i.d., but exhibit volatility clustering. To account for the volatility of realised volatility they extend the HAR model in by incorporating a GARCH component (hereafter, HAR-G). So, the innovation term is not anymore a Gaussian white noise, but its variance is time-varying $u_t = h_t z_t$, where $z_t \sim N(0,1)$ and $h_t^2 = a_0 + a_1 u_{t-1}^2 + b_1 h_{t-1}^2$. I also extend the HAR-G specification in order to account for the leverage effect (HAR-L-G), the IV (HAR-IV-G) and simultaneously the asymmetry and IV (HAR-L-IV-G). Since the ARFIMA models are inferior to HAR in terms of forecasting and the estimation of the extended ARFIMA model to include the volatility of the realised volatility will be even more challenging, I focus on extending only the HAR model by the GARCH component.

5.2.3 Forecast evaluation

Forecasting models are estimated using both the rolling and recursive methods.[20] While most of the realised volatility literature uses rolling samples it is not clear whether the rolling or recursive scheme should be used. For example, Corsi et al. (2008) uses recursive sample for forecasting the realised volatility for the S&P500 index. Vortelinos (2015) uses both recursive and rolling samples for forecasting the realised volatility in several US financial markets finding no differences of forecast accuracy between recursive and rolling samples.

The ability of the models described in Subsection 5.2.2 to accurately forecast the true volatility is evaluated using four popular measures for forecast comparison. The mean absolute error (MAE), the mean squared error (MSE) , the quasi-Gaussian log-likelihood (QLIKE) and the goodness-of-fit R^2 statistic of the Mincer-Zarnowitz (MZ) (Mincer & Zarnowitz, 1969) regression, which measures how much of the true volatility is explained by the forecast series.

[20] According to the rolling scheme forecasts are generated by a moving average window of size N, while according to the recursive scheme the initial window increases adding new observations.

$$MAE = \frac{1}{\tau} \sum_{t=T+1}^{T+\tau} \mid \hat{RV_t^2} - RV_t^2 \mid \tag{57}$$

$$MSE = \frac{1}{\tau} \sum_{t=T+1}^{T+\tau} (\hat{RV_t^2} - RV_t^2)^2 \tag{58}$$

$$QLIKE = \frac{1}{\tau} \sum_{t=T+1}^{T+\tau} \left[log(\hat{RV_t^2}) + \frac{\hat{RV_t^2}}{RV_t^2} \right] \tag{59}$$

$$RV_{t+1}^2 = a_0 + a_1 \hat{RV_{t+1}^2} + \epsilon_{t+1} \tag{60}$$

where τ is the number of out-of-sample observations, $\hat{RV_t^2}$ is the volatility point forecast and RV_t^2 is the proxy for the 'true' volatility. As a proxy is needed to measure the true volatility, Patton (2011) provided the necessary and sufficient conditions to ensure that the ranking of the various forecasts is preserved when noisy volatility proxies are used. Moreover, Patton & Sheppard (2009) showed that the MSE and QLIKE loss functions are the most robust to noise in the volatility proxy, here the realised variance.

However, these measures are useful for forecast comparisons, but they do not provide any statistical test of the difference among the models. Thus, there is the possibility that the model with the lower forecast error may not be inherently better than the competing model, as their difference may be statistically insignificant. the significance of any difference in the MAE, MSE and QLIKE loss functions is tested via the Giacomini-White test and SPA test.

Giacomini and White test

Giacomini-White (GW) pairwise test is a test of conditional predictive ability proposed by Giacomini & White (2006). The test evaluates the forecasting performance of two competing models, accounting for parameter uncertainty. In short, let $L(y_t; \hat{y}_t)$ denote the forecast loss where y_t is the 'true' value and $\hat{y}_t$ is the predicted value. The difference in loss of model i relative to a benchmark model o is defined as

$$d_{i,t} = L(y_t; \hat{y_{o,t}}) - L(y_t; \hat{y_{i,t}}) \tag{61}$$

The issue is whether the two models have equal predictive ability. That is, the null hypothesis tested is $H_0 : E(d_{i,t+\tau} \mid h_t) = 0$, where h_t is some information set. The CPA test statistic is then computed as a Wald statistic

$$CPA_t = T(T^{-1} \sum_{t=1}^{T-\tau} h_t d_{i,t+\tau})' \hat{\Omega}_T^{-1} (T^{-1} \sum_{t=1}^{T-\tau} h_t d_{i,t+\tau}) \sim \chi_1^2 \tag{62}$$

where $\hat{\Omega_T}$ is the Newey and West (1987) HAC estimator of the asymptotic variance of the $h_t d_{i,t+\tau}$. In this application the CPA test is used to assess whether any model under consideration outperforms the random walk model under the squared error metric.

SPA test

The GW test is a pairwise test that evaluates the forecasts of any two competing models. In order to investigate the relative performance of various volatility models Hansen (2005) introduced the Superior Predictive Ability (SPA) test. That is, it evaluates the performance of several alternative models simultaneously against a benchmark model The test uses a bootstrap procedure to assess whether the same outcome can be obtained from more than one sample. Forecasts are evaluated by a pre-specified loss function and the model that produces the smallest expected loss is the best model.

In short, let the difference in loss of model i relative to a benchmark model o is defined as in equation (61). The issue is whether any of the competing models $i = 1, ..., K$ significantly outperforms the benchmark model testing the null hypothesis that $\mu_i = E(d_{i,t}) \leq 0$. It is tested with the statistic

$$T_n^{SPA} = max_i n^{0.5} \frac{\bar{d_i}}{\sigma_i} \tag{63}$$

where $\bar{d}_i = \frac{1}{n}\sum d_{i,t}$ and $\sigma_i = \lim_{n\to\infty} var(n^{0.5}\bar{d}_i)$ which is estimated via a bootstrap procedure.

5.2.4 Data

The same dataset from Chapter (2) and (3) is used. The daily closing price data and the volatility index data have been collected from Datastream. The daily realized variances based on five-minute returns are sourced from the Realized Library of the Oxford-Man Institute of Quantitative Finance.

Table 58 shows the descriptive statistics for the RV, the log(RV) as well as the daily returns. The returns distribution is skewed and leptokurtic for all series. The RV distribution is severely skewed and exhibits fat tails while it is highly peaked around the mean relative to the normal distribution. In contrast, the skewness and kurtosis for the log(RV) appears approximately Gaussian as previously documented by Andersen et al. (2001a; 2001b). This is also illustrated by Figure 3, which shows the distribution of RV and log(RV) along with the normal distribution. For this reason, in this chapter, I model the log(RV). Moreover, the sample autocorrelation function of RV and log(RV) appears to decay hyperbolically evidence of the presence of long memory.

5.3 Results

Tables 59 to 66 report the MAE, MSE, QLIKE and R^2 for the various models using both the rolling and recursive methods. The results show that all evaluation statistical criteria yield more or less the same performance ranking for the models.

Several interesting conclusions arise. First, the best forecast per index and forecasting method is consistent across different criteria. More specifically, the HAR-L-IV-G model performs best for both S&P500 and DJIA indices, the HAR-L for the SMI, while the HAR-L-IV provides the best forecast for all other indices. The results are consistent between the rolling and recursive forecasting methods.

Second, comparing the relative performance of the basic ARFIMA model versus the simple HAR model and their specifications augmented with the IV component and the leverage effect component, I find that the HAR class of models consistently performs better than their ARFIMA counterparts across all indices and loss functions. The remarkably higher R^2 values of the HAR specifications than those of the ARFIMA confirm this conclusion. For example, the R^2 for the simple HAR model for the S&P500 index is 37.9% compared to 33.1% for the simple ARFIMA. When the leverage effect and IV are included in the models their R^2 values increase, but still the HAR-L-IV performs better than the ARFIMA-L-IV. More specifically, the R^2 value is 37.2% for the ARFIMA-L-IV model and 46.9% for the HAR-L-IV specification. Moreover, explicitly accounting for the leverage effect and IV in both ARFIMA and HAR models enhances the forecast accuracy. In particular, models that simultaneously incorporate the asymmetry and IV yield lower loss than their restricted counterparts. This finding is also confirmed by the R^2 values. This is consistent with the findings of previous studies. For example, Martens et al. (2009) and Wang et al. (2015) show that taking into account the leverage effect significantly improves the forecast performance of the models for the S&P500 index and the Chinese stock market, respectively, and Busch et al. (2011) find that the IV is important in forecasting future realised volatility.

Third, the conditional heteroskedasticity of the innovations of realized volatility is taken into account in order to evaluate whether it significantly improves the forecasting performance of the HAR models. According to Corsi et al. (2008) allowing for time-varying volatility of the realised volatility improves the predictive ability of the S&P500 index. I find that only for the S&P500 and DJIA indices accounting for the GARCH effect yields lower loss than the restricted HAR versions. In particular, only when is the HAR-L-IV-G is compared to the HAR-L-IV leads to relatively modest increase of the R^2 value of about 0.2% for the S&P500 index and 0.1% for the DJIA. For all other indices accounting for the volatility clustering of realised volatility does not noticeably improve the predictive power of the model. Similar results are reported in Todorova (2015) for the LME non-ferrous metal market.

As in many cases there is little difference in the forecasts errors of the competing models I use the GW pairwise test and the SPA test in order to investigate whether these differences are statistically significant.

Tables 67 and 68 present the GW pairwise test for the US and European indices. The p-values reported on the tables are based on the mean differences between the row model and the column model. The null hypothesis of equal forecasting performance between the row and column models in terms of squared forecast error. The signs in bracket indicate which model performs best. A positive sign shows that the row model forecast yields larger loss than the column model forecast, which implies that the column model is significantly superior. Similarly, a negative sign denotes that the row model forecast performs significantly better than the column model forecast, since the latter produces larger loss.

The results are consistent for all indices. First, the ARFIMA model is significantly outperformed by the other models. In general, all ARFIMA specifications are inferior to the HAR counterparts. Second, the simple HAR model is significantly worse than its more sophisticated rivals except when the HAR models is competing the HAR-G model. In this case, the high p-value indicates that the null hypothesis cannot be rejected, which means that the two models forecast equally well. Third, the HAR-G is significantly inferior to its more sophisticated specifications. Moreover, allowing for time-varying volatility of realised volatility does not lead to a substantial improvement of the model's predictive ability. This result is in line with the study of Bubak & Zikes (2009) and Todorova (2015) who compare the forecasting performance of the HAR and HAR-G models for the exchange rate and metal market, respectively. They find that while the HAR-G model performs better in-sample, it cannot significantly improve the out-of-sample performance of the simple HAR model. Finally, simultaneously accounting for the leverage effect and IV significantly improves the accuracy of the volatility forecasts.

The SPA test of Hansen (2005) using the MAE, MSE and QLIKE loss functions is implemented to assess the significance of the relative forecasting performance of the models. The null hypothesis is that the forecast under consideration, i.e. the benchmark model, is not inferior to any alternative model. Tables 69 to 71 and Tables 72 to 74 present the SPA test results for the rolling and recursive scheme, respectively, using consecutively all models as benchmark models. The p-values reported are based on 1,000 bootstrap samples under the MAE, MSE and QLIKE loss functions. Small p-values indicate that at least one of the competing models perform better than the model under consideration. Thus, the higher the p-value, the better is the predictive ability of the model under consideration.

In Tables 69 to 71, for all the loss functions employed under the rolling scheme, the p-values of the SPA test show that the HAR-type models perform better than the ARFIMA specifications. Comparing the simple HAR and ARFIMA to their more sophisticated rivals I find evidence to suggest that explicitly incorporating the leverage effect and IV in these models significantly improves the accuracy of the volatility forecasts. Conversely, accounting for the volatility of realised volatility does not significantly enhances the HAR models. Moreover, the small p-value of the models that include the leverage effect but not the IV indicate that the null hypothesis is rejected, which means that these models are outperformed by the competing models. However, models that simultaneously account for the asymmetry and IV produces higher p-values. The results are consistent between the rolling and recursive techniques.

In particular, the HAR-L-IV-G model produces the highest p-values for both the S&P500 and DJIA indices indicating that the null hypothesis cannot be rejected which means that HAR-L-IV-G model is not outperformed by the competing models. This result is consistent across loss functions and for both the rolling and recursive methods. The only exception is under the QLIKE loss function for the DJIA where the HAR-IV-G model performs best. For the Nasdaq, STOXX, CAC, DAX, AEX and FTSE100 indices the HAR-L-IV model produces the highest p-values. For these indices the inclusion of the conditional heteroskedasticity in the innovations of realized volatility does not significantly improve the forecasting performance only of the HAR models. The HAR-L specification proved the best forecast for the SMI index.

In sum, the results clearly show that accounting for the volatility clustering significantly improves the forecasting performance of the S&P500, similarly to the Corsi et al. (2008) findings, and DJIA indices. Modelling the IV is more important than the leverage effect, but accounting for both features significantly improves the predictive ability of the models.

5.4 Conclusion

In this chapter I evaluate the forecasting performance of several reduced-form time series models for realised volatility. I mainly explore the effect of explicitly accounting for important stylized facts of realised volatility. More specifically, I examine the role of leverage effects, implied volatilities and volatility clustering of realised volatility. The analysis is based on daily realised variances of a number of US and European indices. I focus on the class of ARFIMA and HAR models as well as to extensions in order to capture the well-known features of volatility.

The empirical results lead to two main conclusions. First, the out-of-sample results show that accounting for the leverage effects and the information content of implied volatility improves the predictive power of the models. While taking into account implied volatility is more important than the leverage effect, accounting for both features significantly improves the forecast performance of the models. Second, it seems not to be beneficial to model the volatility of realised volatility as it does not lead to a substantial improvement of the forecast performance of the HAR models.

Overall, the HAR models perform better than the ARFIMA models. The HAR-L-IV seems to be the most appropriate for predicting realised volatility.

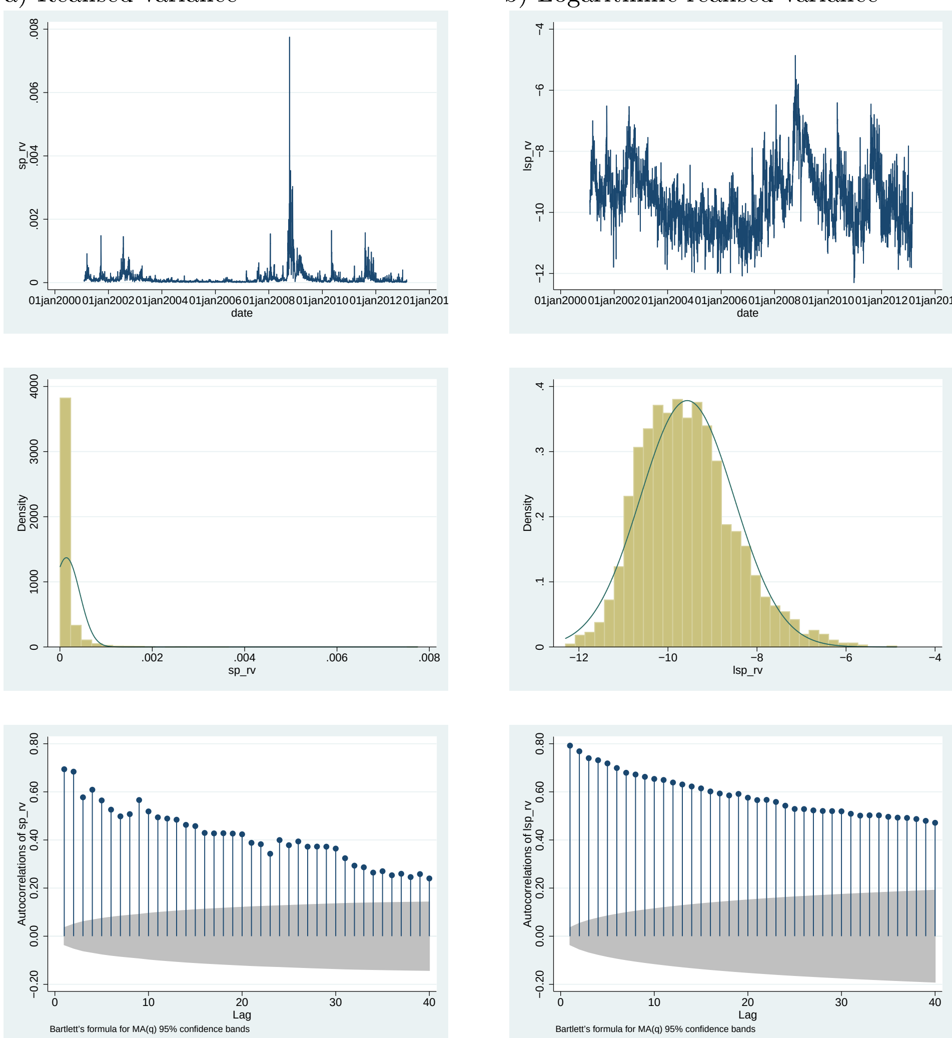
Table 58: Descriptive statistics

	Mean	Std. Dev.	Skew	Kurt
S&P500				
r_t	$3.80 * 10^{-5}$	0.0134	-0.1704	11.0489
rv_t	0.00014	0.0003	10.175	188.585
$log(rv_t)$	-9.5664	1.0547	0.5769	3.4244
DJIA				
r_t	$8.47 * 10^{-5}$	0.0125	0.0328	10.778
rv_t	0.00014	0.0003	11.734	244.481
$log(rv_t)$	-9.5970	1.0531	0.6152	3.5312
Nasdaq100				
r_t	$3.37 * 10^{-5}$	0.0179	0.0592	7.3248
rv_t	0.00013	0.0002	6.5738	74.906
$log(rv_t)$	-9.4848	0.9744	0.4455	3.0527
STOXX				
r_t	-0.00019	0.0161	0.0314	7.2470
rv_t	0.00020	0.0004	11.5445	246.611
$log(rv_t)$	-9.1458	1.0377	0.3570	3.2080
CAC				
r_t	-0.00014	0.0158	0.0526	7.8070
rv_t	0.00016	0.0003	7.8074	101.719
$log(rv_t)$	-9.2888	1.0134	0.3189	3.0304
DAX				
r_t	$5.04 * 10^{-5}$	0.0163	-0.0004	7.2921
rv_t	0.00021	0.0004	6.6967	74.834
$log(rv_t)$	-9.1197	1.0647	0.3866	2.9911
AEX				
r_t	-0.00020	0.0159	-0.0506	8.8121
rv_t	0.00014	0.0002	5.5914	50.501
$log(rv_t)$	-9.4690	1.0492	0.4504	2.9598
SMI				
r_t	$-1.76 * 10^{-5}$	0.0128	0.0133	9.0066
rv_t	0.00010	0.00018	6.4170	68.204
$log(rv_t)$	-9.7506	0.9665	0.7485	3.3368
FTSE100				
r_t	$5.43 * 10^{-6}$	0.0129	-0.0112	9.2049
rv_t	$9.99 * 10^{-5}$	0.00019	9.7089	164.582
$log(rv_t)$	-9.8500	1.0586	0.4177	2.9650

Figure 3: Daily realised variance of the S&P500 index

a) Realised variance

b) Logarithmic realised variance



Note: The graph shows the time series (first row), histogram (second row) and correlogram (third row) of the S&P500 realised variance in level and logarithms over the period February 2, 2001 to February 28, 2013.

Table 59: MAE under the rolling scheme

	MAE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	0.05733	0.05648	0.04124	0.08048	0.06917	0.07194	0.04446	0.03262	0.03251
ARFIMA-IV	0.05712	0.05670	0.03475	0.06590	0.05797	0.05370	0.03856	0.03009	0.02426
ARFIMA-L	0.05959	0.05834	0.03818	0.08055	0.06260	0.06554	0.03988	0.02907	0.03019
ARFIMA-L-IV	0.05654	0.05611	0.03475	0.06504	0.05818	0.05351	0.03870	0.03035	0.02486
HAR	0.05688	0.05650	0.03372	0.06564	0.05637	0.05588	0.03607	0.02755	0.02503
HAR-IV	0.05387	0.05349	0.03284	0.06167	0.05252	0.05256	0.03459	0.02676	0.02359
HAR-L	0.05442	0.05424	0.03219	0.06101	0.05169	0.05255	0.03408	**0.02381**	0.02373
HAR-L-IV	0.05293	0.05278	**0.03178**	**0.05900**	**0.04995**	**0.05070**	**0.03342**	0.02400	**0.02299**
HAR-G	0.05689	0.05647	0.03377	0.06571	0.05642	0.05594	0.03608	0.02750	0.02495
HAR-IV-G	0.05357	0.05337	0.03293	0.06198	0.05298	0.05270	0.03468	0.02670	0.02350
HAR-L-G	0.05434	0.05428	0.03233	0.06107	0.05208	0.05236	0.03426	0.02398	0.02380
HAR-L-IV-G	**0.05280**	**0.05266**	0.03190	0.05952	0.05012	0.05101	0.03362	0.02415	0.02305

Note: This table presents out-of-sample mean absolute forecast errors (MAE) defined in equation (57) for the twelve volatility models considered. Out-of-sample forecasts are obtained using a rolling window method. In bold is the lowest forecast error. In order to facilitate the presentation of my results, all numbers are multiplied by 10^3.

Table 60: MSE under the rolling scheme

	MSE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	0.02145	0.01917	0.01464	0.03509	0.02778	0.03242	0.01390	0.01338	0.00512
ARFIMA-IV	0.01877	0.01774	0.01289	0.02315	0.02062	0.01808	0.01120	0.01032	0.00292
ARFIMA-L	0.01861	0.01767	0.01367	0.03440	0.02319	0.02843	0.01193	0.01129	0.00476
ARFIMA-L-IV	0.01852	0.01750	0.01269	0.02248	0.02045	0.01690	0.01110	0.01024	0.00294
HAR	0.01833	0.01750	0.01118	0.02082	0.01596	0.01708	0.00886	0.00816	0.00291
HAR-IV	0.01641	0.01562	0.01076	0.01884	0.01467	0.01566	0.00824	0.00774	0.00250
HAR-L	0.01604	0.01583	0.01006	0.01501	**0.01084**	**0.01279**	0.00706	**0.00481**	0.00226
HAR-L-IV	0.01528	**0.01487**	**0.00993**	**0.01481**	0.01121	0.01303	**0.00696**	0.00512	**0.00220**
HAR-G	0.01866	0.01766	0.01136	0.02118	0.01635	0.01750	0.00905	0.00835	0.00294
HAR-IV-G	0.01662	0.01577	0.01097	0.01958	0.01527	0.01618	0.00856	0.00800	0.00254
HAR-L-G	0.01603	0.01578	0.01026	0.01534	0.01124	0.01317	0.00729	0.00497	0.00229
HAR-L-IV-G	**0.01520**	**0.01487**	0.01015	0.01553	0.01137	0.01369	0.00730	0.00532	0.00226

Note: This table presents out-of-sample mean squared forecast errors (MSE) defined in equation (58) for the twelve volatility models considered. Out-of-sample forecasts are obtained using a rolling window method. In bold is the lowest forecast error. In order to facilitate the presentation of my results, all numbers are multiplied by 10^6.

Table 61: QLIKE under the rolling scheme

	QLIKE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	-8.40648	-8.46261	-8.63000	-7.83379	-7.95368	-8.03037	-8.42849	-8.84181	-8.73819
ARFIMA-IV	-8.43070	-8.48491	-8.69841	-7.93078	-8.04093	-8.16551	-8.45511	-8.89825	-8.86314
ARFIMA-L	-8.39537	-8.45059	-8.67500	-7.82019	-8.01431	-8.09504	-8.47729	-8.89464	-8.77605
ARFIMA-L-IV	-8.43221	-8.48665	-8.70430	-7.93192	-8.03971	-8.16647	-8.46004	-8.89460	-8.85560
HAR	-8.42339	-8.47580	-8.72077	-7.92481	-8.04941	-8.14607	-8.50394	-8.91659	-8.85587
HAR-IV	-8.46692	-8.52513	-8.73476	-7.94610	-8.06631	-8.16713	-8.50672	-8.92787	-8.87205
HAR-L	-8.43721	-8.48815	-8.73068	-7.93527	-8.06220	-8.15861	-8.51327	-8.93099	-8.86391
HAR-L-IV	-8.46852	-8.52435	**-8.73879**	**-7.94933**	-8.07143	**-8.17149**	-8.51357	**-8.93674**	**-8.87396**
HAR-G	-8.42274	-8.47442	-8.71918	-7.92304	-8.04796	-8.14510	-8.50236	-8.91505	-8.85455
HAR-IV-G	-8.46856	**-8.52578**	-8.73412	-7.94342	-8.06438	-8.16662	-8.50431	-8.92692	-8.87146
HAR-L-G	-8.43738	-8.48748	-8.72919	-7.93534	-8.06184	-8.15791	-8.51223	-8.93009	-8.86311
HAR-L-IV-G	**-8.46985**	-8.52537	-8.73770	-7.94825	-8.07087	-8.17093	-8.51198	-8.93614	-8.87347

Note: This table presents the quasi–likelihood loss (QLIKE) defined in equation (59) for the twelve volatility models considered. Out-of-sample forecasts are obtained using a rolling window method. In bold is the lowest forecast error.

Table 62: R^2 values under the rolling scheme

	R^2								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	0.33119	0.29631	0.19417	0.36778	0.34296	0.29290	0.32457	0.30079	0.21907
ARFIMA-IV	0.35591	0.30403	0.30628	0.51099	0.49036	0.58717	0.46672	0.48228	0.55554
ARFIMA-L	0.37533	0.32181	0.27459	0.38042	0.47326	0.41400	0.43327	0.44310	0.28945
ARFIMA-L-IV	0.37197	0.31908	0.33138	0.53650	0.51168	0.61862	0.48858	0.51905	0.56137
HAR	0.37910	0.32596	0.35210	0.54369	0.54838	0.59641	0.51596	0.53974	0.53276
HAR-IV	0.44326	0.39979	0.38555	0.59946	0.60231	0.63426	0.56617	0.57282	0.59305
HAR-L	0.44789	0.38223	0.41891	0.66016	0.69496	0.68660	0.63268	**0.75072**	0.63796
HAR-L-IV	0.46985	0.41540	**0.43122**	**0.67294**	**0.69741**	**0.68786**	**0.64669**	0.73765	**0.64148**
HAR-G	0.36984	0.32291	0.34420	0.54086	0.54274	0.59018	0.51063	0.53801	0.53238
HAR-IV-G	0.44277	0.39963	0.37922	0.59114	0.59416	0.62662	0.56100	0.56818	0.58955
HAR-L-G	0.44944	0.38605	0.40961	0.65465	0.68497	0.68050	0.62092	0.74109	0.63373
HAR-L-IV-G	**0.47413**	**0.41863**	0.42331	0.66370	0.69712	0.67752	0.63324	0.72712	0.63370

Note: Entries are the R^2 values from the Mincer-Zarnowitz regression described in equation (60). Out-of-sample forecasts are obtained using a rolling window method. In bold is the highest R^2 value.

Table 63: MAE under the recursive scheme

	MAE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	0.05758	0.05663	0.04301	0.07842	0.06776	0.06970	0.04488	0.03145	0.03049
ARFIMA-IV	0.05791	0.05679	0.03512	0.06626	0.05860	0.05649	0.03846	0.03142	0.02482
ARFIMA-L	0.05682	0.05842	0.03986	0.07104	0.06085	0.06304	0.03914	0.02830	0.02794
ARFIMA-L-IV	0.05724	0.05606	0.03504	0.06516	0.05905	0.05592	0.03848	0.03154	0.02562
HAR	0.05695	0.05659	0.03373	0.06582	0.05652	0.05599	0.03613	0.02759	0.02505
HAR-IV	0.05397	0.05362	0.03293	0.06190	0.05276	0.05326	0.03454	0.02689	0.02377
HAR-L	0.05433	0.05426	0.03235	0.06086	0.05155	0.05245	0.03400	**0.02353**	0.02358
HAR-L-IV	0.05287	0.05277	**0.03206**	**0.05881**	**0.04974**	**0.05100**	**0.03327**	0.02365	**0.02295**
HAR-G	0.05679	0.05645	0.03377	0.06582	0.05659	0.05593	0.03611	0.02743	0.02495
HAR-IV-G	0.05373	0.05347	0.03299	0.06200	0.05311	0.05308	0.03461	0.02670	0.02364
HAR-L-G	0.05424	0.05424	0.03250	0.06088	0.05195	0.05228	0.03418	0.02369	0.02364
HAR-L-IV-G	**0.05275**	**0.05275**	0.03218	0.05924	0.05028	0.05107	0.03346	0.02358	0.02299

Note: This table presents out-of-sample mean absolute forecast errors (MAE) defined in equation (57) for the twelve volatility models considered. Out-of-sample forecasts are obtained using a recursive window method. In bold is the lowest forecast error. In order to facilitate the presentation of my results, all numbers are multiplied by 10^3.

Table 64: MSE under the recursive scheme

	MSE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	0.02155	0.01936	0.01468	0.03378	0.02630	0.03015	0.01348	0.01257	0.00461
ARFIMA-IV	0.01906	0.01783	0.01315	0.02289	0.02092	0.02047	0.01131	0.01059	0.00293
ARFIMA-L	0.01927	0.01881	0.01385	0.02744	0.02125	0.02612	0.01118	0.01010	0.00402
ARFIMA-L-IV	0.01877	0.01752	0.01292	0.02210	0.02083	0.01963	0.01115	0.01032	0.00313
HAR	0.01829	0.01740	0.01118	0.02085	0.01587	0.01697	0.00880	0.00802	0.00290
HAR-IV	0.01639	0.01558	0.01088	0.01863	0.01442	0.01540	0.00814	0.00751	0.00249
HAR-L	0.01600	0.01583	0.01030	0.01503	0.01073	0.01283	0.00696	**0.00456**	0.00221
HAR-L-IV	**0.01526**	**0.01491**	**0.01023**	**0.01472**	**0.01062**	**0.01272**	**0.00683**	0.00471	**0.00214**
HAR-G	0.01853	0.01758	0.01136	0.02117	0.01630	0.01740	0.00899	0.00821	0.00293
HAR-IV-G	0.01658	0.01575	0.01107	0.01930	0.01510	0.01598	0.00844	0.00778	0.00253
HAR-L-G	0.01599	0.01585	0.01049	0.01528	0.01112	0.01314	0.00714	0.00472	0.00224
HAR-L-IV-G	0.01524	0.01493	0.01044	0.01529	0.01118	0.01328	0.00711	0.00462	0.00218

Note: This table presents out-of-sample mean squared forecast errors (MSE) defined in equation (58) for the twelve volatility models considered. Out-of-sample forecasts are obtained using a recursive window method. In bold is the lowest forecast error. In order to facilitate the presentation of my results, all numbers are multiplied by 10^6.

Table 65: QLIKE under the recursive scheme

	QLIKE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	-8.40834	-8.46313	-8.61109	-7.84800	-7.97785	-8.06969	-8.43280	-8.86509	-8.78077
ARFIMA-IV	-8.42239	-8.47899	-8.69052	-7.93160	-8.03958	-8.15582	-8.45707	-8.88821	-8.85909
ARFIMA-L	-8.40241	-8.45949	-8.65788	-7.89767	-8.02787	-8.11558	-8.48518	-8.90651	-8.82158
ARFIMA-L-IV	-8.42586	-8.48195	-8.69851	-7.93335	-8.03675	-8.15963	-8.46317	-8.88513	-8.85084
HAR	-8.42423	-8.47674	-8.72104	-7.92535	-8.04987	-8.14688	-8.50443	-8.91718	-8.85643
HAR-IV	-8.46699	-8.52315	-8.73299	-7.94717	-8.06625	-8.16748	-8.50786	-8.92833	-8.87255
HAR-L	-8.43873	-8.48940	-8.72981	-7.93620	-8.06285	-8.15930	-8.51389	-8.93162	-8.86489
HAR-L-IV	-8.46880	-8.52294	**-8.73674**	**-7.95019**	-8.07158	**-8.17172**	-8.51485	-8.93738	**-8.87470**
HAR-G	-8.42354	-8.47521	-8.71934	-7.92325	-8.04832	-8.14572	-8.50284	-8.91560	-8.85517
HAR-IV-G	-8.46854	**-8.52407**	-8.73177	-7.94437	-8.06420	-8.16690	-8.50553	-8.92744	-8.87200
HAR-L-G	-8.43908	-8.48857	-8.72836	-7.93571	-8.06231	-8.15837	-8.51289	-8.93074	-8.86412
HAR-L-IV-G	**-8.47042**	-8.52379	-8.73560	-7.94871	-8.07056	-8.17114	-8.51337	**-8.93763**	-8.87438

Note: This table presents the quasi–likelihood loss (QLIKE) defined in equation (59) for the twelve volatility models considered. Out-of-sample forecasts are obtained using a recursive window method. In bold is the lowest forecast error.

Table 66: R^2under the recursive scheme

	R^2								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA	0.32708	0.29331	0.16486	0.37653	0.37969	0.37191	0.33800	0.37321	0.33372
ARFIMA-IV	0.34037	0.29812	0.29847	0.50595	0.46789	0.50771	0.43866	0.44596	0.54623
ARFIMA-L	0.36823	0.27251	0.23936	0.49957	0.50639	0.46296	0.46272	0.51752	0.43371
ARFIMA-L-IV	0.35737	0.31635	0.32072	0.53409	0.49062	0.53672	0.46491	0.49999	0.52471
HAR	0.37924	0.32752	0.35059	0.54019	0.54827	0.59533	0.51672	0.54298	0.53278
HAR-IV	0.44244	0.39963	0.38257	0.59756	0.60046	0.62863	0.56639	0.57499	0.59025
HAR-L	0.44917	0.38134	0.40961	0.65962	0.69723	0.68583	0.63820	**0.76282**	0.64573
HAR-L-IV	0.47029	0.41355	**0.42226**	**0.67206**	**0.70692**	**0.68821**	**0.65119**	0.75223	**0.65084**
HAR-G	0.37462	0.32432	0.34322	0.53819	0.54149	0.58979	0.51167	0.54060	0.53263
HAR-IV-G	0.44153	0.39839	0.37672	0.59039	0.59118	0.62123	0.56146	0.57063	0.58723
HAR-L-G	0.45013	0.38217	0.40033	0.65576	0.68813	0.68111	0.63000	0.75466	0.64177
HAR-L-IV-G	**0.47240**	**0.41417**	0.41474	0.66479	0.69699	0.67961	0.64133	0.75616	0.64486

Note: Entries are the R^2 values from the Mincer-Zarnowitz regression described in equation (60). Out-of-sample forecasts are obtained using a recursive window method. In bold is the highest R^2 value.

Table 69: SPA test (MAE) under the rolling scheme

		SPA-MAE								
		S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
1	ARFIMA-log(RV)	0.050	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	ARFIMA-log(RV)-IV	0.008	0.002	0.038	0.037	0.026	0.171	0.020	0.015	0.117
3	ARFIMAX-log(RV)-alt2	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.056	0.000
4	ARFIMAX-log(RV)-IV-alt2	0.068	0.071	0.035	0.052	0.020	0.174	0.016	0.015	0.000
5	HAR-log(RV)	0.000	0.000	0.003	0.002	0.000	0.000	0.003	0.012	0.002
6	HARX-log(RV)	0.200	0.389	0.031	0.140	0.048	0.092	0.068	0.020	0.091
7	AHAR-log(RV)-alt	0.005	0.001	0.021	0.017	0.009	0.018	0.015	**0.872**	0.002
8	AHARX-log(RV)-alt	0.505	0.452	**0.805**	**0.941**	**0.868**	**0.864**	**0.931**	0.568	**0.891**
9	HAR-log(RV)-GARCH	0.002	0.000	0.005	0.001	0.000	0.000	0.004	0.016	0.002
10	HARX-log(RV)-GARCH	0.379	0.376	0.049	0.129	0.053	0.168	0.077	0.035	0.187
11	AHAR-log(RV)-GARCH-alt	0.017	0.002	0.031	0.014	0.008	0.011	0.015	0.035	0.005
12	AHARX-log(RV)-GARCH-alt	**1.000**	**0.991**	0.285	0.267	0.160	0.719	0.376	0.134	0.534

Note: The p-values of the SPA test are reported. The null hypothesis is that the forecast under consideration is not inferior to any alternative forecast.

Table 70: SPA test (MSE) under the rolling scheme

	SPA-MSE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA-log(RV)	0.028	0.061	0.021	0.008	0.018	0.013	0.018	0.029	0.001
ARFIMA-log(RV)-IV	0.161	0.098	0.093	0.104	0.101	0.132	0.120	0.127	0.193
ARFIMAX-log(RV)-alt2	0.023	0.036	0.030	0.001	0.053	0.037	0.097	0.095	0.025
ARFIMAX-log(RV)-IV-alt2	0.317	0.286	0.083	0.133	0.088	0.221	0.117	0.138	0.178
HAR-log(RV)	0.040	0.036	0.061	0.011	0.033	0.052	0.116	0.066	0.062
HARX-log(RV)	0.393	0.496	0.215	0.109	0.070	0.107	0.110	0.071	0.077
AHAR-log(RV)-alt	0.064	0.078	0.034	0.618	**0.801**	0.803	0.403	**0.926**	0.417
AHARX-log(RV)-alt	0.652	0.797	**0.972**	**0.976**	0.737	**0.808**	**0.924**	0.308	**0.982**
HAR-log(RV)-GARCH	0.051	0.040	0.054	0.006	0.033	0.073	0.101	0.041	0.072
HARX-log(RV)-GARCH	0.378	0.285	0.195	0.064	0.090	0.129	0.140	0.102	0.102
AHAR-log(RV)-GARCH-alt	0.047	0.085	0.082	0.313	0.089	0.540	0.146	0.068	0.099
AHARX-log(RV)-GARCH-alt	**0.987**	**0.969**	0.318	0.143	0.155	0.335	0.244	0.085	0.177

Note: The p-values of the SPA test are reported. The null hypothesis is that the forecast under consideration is not inferior to any alternative forecast.

Table 67: Conditional Giacomini-White test results for the US indices

	Giacomini-White test										
	ARFIMA-IV	ARFIMA-L	ARFIMA-L-IV	HAR	HAR-IV	HAR-L	HAR-L-IV	HAR-G	HAR-IV-G	HAR-L-G	HAR-L-IV-G
S&P500											
ARFIMA	0.878	0.387	0.567	0.717	$0.025^{(+)}$	0.162	$0.052^{(+)}$	0.708	$0.010^{(+)}$	0.122	$0.031^{(+)}$
ARFIMA-IV	-	0.344	$0.003^{(+)}$	0.881	$0.021^{(+)}$	0.169	$0.036^{(+)}$	0.882	$0.008^{(+)}$	0.127	$0.018^{(+)}$
ARFIMA-L		-	0.236	0.254	$0.010^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	0.265	$0.009^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
ARFIMA-L-IV			-	0.830	$0.052^{(+)}$	0.265	$0.062^{(+)}$	0.823	$0.023^{(+)}$	0.212	$0.033^{(+)}$
HAR				-	$0.000^{(+)}$	0.139	$0.032^{(+)}$	0.966	$0.000^{(+)}$	$0.087^{(+)}$	$0.013^{(+)}$
HAR-IV					-	0.682	0.498	$0.000^{(-)}$	0.108	0.692	0.359
HAR-L						-	$0.007^{(+)}$	0.151	0.560	0.680	$0.004^{(+)}$
HAR-L-IV							-	$0.036^{(-)}$	0.666	$0.026^{(-)}$	0.598
HAR-G								-	$0.000^{(+)}$	$0.097^{(+)}$	$0.015^{(+)}$
HAR-IV-G									-	0.552	0.539
HAR-L-G										-	$0.005^{(+)}$
DJIA											
ARFIMA	0.852	0.396	0.752	0.988	$0.011^{(+)}$	0.159	$0.033^{(+)}$	0.987	$0.005^{(+)}$	0.126	$0.017^{(+)}$
ARFIMA-IV	-	0.478	$0.001^{(+)}$	0.884	$0.011^{(+)}$	0.137	$0.016^{(+)}$	0.864	$0.006^{(+)}$	0.118	$0.007^{(+)}$
ARFIMA-L		-	0.330	0.388	$0.016^{(+)}$	$0.001^{(+)}$	$0.000^{(+)}$	0.380	$0.016^{(+)}$	$0.004^{(+)}$	$0.000^{(+)}$
ARFIMA-L-IV			-	0.787	$0.032^{(+)}$	0.245	$0.035^{(+)}$	0.794	$0.020^{(+)}$	0.223	$0.017^{(+)}$
HAR				-	$0.000^{(+)}$	0.105	$0.014^{(+)}$	0.820	$0.000^{(+)}$	$0.066^{(+)}$	$0.005^{(+)}$
HAR-IV					-	0.513	0.518	$0.000^{(-)}$	0.343	0.427	0.370
HAR-L						-	$0.002^{(+)}$	0.109	0.468	0.860	$0.002^{(+)}$
HAR-L-IV							-	$0.015^{(-)}$	0.616	$0.006^{(-)}$	0.579
HAR-G								-	$0.000^{(+)}$	$0.070^{(+)}$	$0.005^{(+)}$
HAR-IV-G									-	0.382	0.475
HAR-L-G										-	$0.001^{(+)}$
Nasdaq100											
ARFIMA	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
ARFIMA-IV	-	$0.000^{(-)}$	0.969	0.321	$0.051^{(+)}$	$0.031^{(+)}$	$0.010^{(+)}$	0.312	$0.041^{(+)}$	$0.027^{(+)}$	$0.006^{(+)}$
ARFIMA-L		-	$0.000^{(+)}$	$0.000^{(+)}$	$0.010^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$	$0.000^{(+)}$
ARFIMA-L-IV			-	0.326	$0.051^{(+)}$	$0.026^{(+)}$	$0.008^{(+)}$	0.320	$0.042^{(+)}$	$0.023^{(+)}$	$0.005^{(+)}$
HAR				-	$0.002^{(+)}$	$0.025^{(+)}$	$0.008^{(+)}$	0.646	$0.004^{(+)}$	$0.015^{(+)}$	$0.003^{(+)}$
HAR-IV					-	0.253	$0.060^{(+)}$	$0.003^{(-)}$	0.496	0.276	$0.035^{(+)}$
HAR-L						-	$0.026^{(+)}$	$0.031^{(-)}$	0.246	0.305	0.194
HAR-L-IV							-	$0.010^{(-)}$	$0.069^{(-)}$	$0.023^{(-)}$	0.432
HAR-G								-	$0.001^{(+)}$	$0.018^{(+)}$	$0.004^{(+)}$
HAR-IV-G									-	0.254	$0.040^{(+)}$
HAR-L-G										-	$0.017^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis is that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts +and −indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 68: Conditional Giacomini-White test results for the European indices

	Giacomini-White test										
	ARFIMA-IV	ARFIMA-L	ARFIMA-L-IV	HAR	HAR-IV	HAR-L	HAR-L-IV	HAR-G	HAR-IV-G	HAR-L-G	HAR-L-IV-G
STOXX											
ARFIMA	0.000$^{(+)}$	0.952	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-IV	-	0.000$^{(-)}$	0.030$^{(+)}$	0.876	0.004$^{(+)}$	0.070$^{(+)}$	0.004$^{(+)}$	0.907	0.003$^{(+)}$	0.078$^{(+)}$	0.003$^{(+)}$
ARFIMA-L		-	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-L-IV			-	0.741	0.028$^{(+)}$	0.118	0.008$^{(+)}$	0.706	0.029$^{(+)}$	0.132	0.006$^{(+)}$
HAR				-	0.000$^{(+)}$	0.020$^{(+)}$	0.000$^{(+)}$	0.644	0.000$^{(+)}$	0.021$^{(+)}$	0.000$^{(+)}$
HAR-IV					-	0.725	0.080$^{(+)}$	0.000$^{(-)}$	0.196	0.754	0.072$^{(+)}$
HAR-L						-	0.004$^{(+)}$	0.022$^{(-)}$	0.625	0.842	0.077$^{(+)}$
HAR-L-IV							-	0.000$^{(-)}$	0.072$^{(-)}$	0.009$^{(-)}$	0.156
HAR-G								-	0.000$^{(+)}$	0.023$^{(+)}$	0.000$^{(+)}$
HAR-IV-G									-	0.651	0.060$^{(+)}$
HAR-L-G										-	0.090$^{(+)}$
CAC											
ARFIMA	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-IV	-	0.000$^{(-)}$	0.531	0.405	0.002$^{(+)}$	0.018$^{(+)}$	0.000$^{(+)}$	0.406	0.002$^{(+)}$	0.020$^{(+)}$	0.000$^{(+)}$
ARFIMA-L		-	0.000$^{(+)}$	0.003$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.003$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-L-IV			-	0.376	0.003$^{(+)}$	0.015$^{(+)}$	0.000$^{(+)}$	0.377	0.002$^{(+)}$	0.017$^{(+)}$	0.000$^{(+)}$
HAR				-	0.000$^{(+)}$	0.003$^{(+)}$	0.000$^{(+)}$	0.778	0.000$^{(+)}$	0.003$^{(+)}$	0.000$^{(+)}$
HAR-IV					-	0.582	0.019$^{(+)}$	0.000$^{(-)}$	0.049$^{(-)}$	0.750	0.022$^{(+)}$
HAR-L						-	0.007$^{(+)}$	0.004$^{(-)}$	0.418	0.024$^{(-)}$	0.015$^{(+)}$
HAR-L-IV							-	0.000$^{(-)}$	0.011$^{(-)}$	0.000$^{(-)}$	0.279
HAR-G								-	0.000$^{(+)}$	0.004$^{(+)}$	0.000$^{(+)}$
HAR-IV-G									-	0.537	0.012$^{(+)}$
HAR-L-G										-	0.001$^{(+)}$
DAX											
ARFIMA	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-IV	-	0.000$^{(-)}$	0.635	0.104	0.281	0.558	0.092$^{(+)}$	0.074$^{(-)}$	0.263	0.493	0.072$^{(+)}$
ARFIMA-L		-	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-L-IV			-	0.080$^{(-)}$	0.352	0.580	0.064$^{(+)}$	0.060$^{(-)}$	0.366	0.504	0.047$^{(+)}$
HAR				-	0.000$^{(+)}$	0.026$^{(+)}$	0.001$^{(+)}$	0.711	0.000$^{(+)}$	0.014$^{(+)}$	0.000$^{(+)}$
HAR-IV					-	0.995	0.108	0.000$^{(-)}$	0.521	0.883	0.066$^{(+)}$
HAR-L						-	0.014$^{(+)}$	0.030$^{(-)}$	0.919	0.576	0.058$^{(+)}$
HAR-L-IV							-	0.001$^{(-)}$	0.115	0.011$^{(-)}$	0.420
HAR-G								-	0.000$^{(+)}$	0.018$^{(+)}$	0.000$^{(+)}$
HAR-IV-G									-	0.812	0.072$^{(+)}$
HAR-L-G										-	0.065$^{(+)}$

continue

	Giacomini-White test										
	ARFIMA-IV	ARFIMA-L	ARFIMA-L-IV	HAR	HAR-IV	HAR-L	HAR-L-IV	HAR-G	HAR-IV-G	HAR-L-G	HAR-L-IV-G
AEX											
ARFIMA	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-IV	-	0.038$^{(-)}$	0.496	0.055$^{(+)}$	0.001$^{(+)}$	0.002$^{(+)}$	0.000$^{(+)}$	0.043$^{(+)}$	0.000$^{(+)}$	0.002$^{(+)}$	0.000$^{(+)}$
ARFIMA-L		-	0.063$^{(+)}$	0.006$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.003$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-L-IV			-	0.056$^{(+)}$	0.001$^{(+)}$	0.002$^{(+)}$	0.000$^{(+)}$	0.045$^{(+)}$	0.001$^{(+)}$	0.002$^{(+)}$	0.000$^{(+)}$
HAR				-	0.000$^{(+)}$	0.005$^{(+)}$	0.001$^{(+)}$	0.946	0.001$^{(+)}$	0.004$^{(+)}$	0.001$^{(+)}$
HAR-IV					-	0.378	0.033$^{(+)}$	0.001$^{(-)}$	0.673	0.538	0.041$^{(+)}$
HAR-L						-	0.015$^{(+)}$	0.005$^{(-)}$	0.336	0.053$^{(-)}$	0.103
HAR-L-IV							-	0.001$^{(-)}$	0.036$^{(-)}$	0.006$^{(-)}$	0.193
HAR-G								-	0.000$^{(+)}$	0.005$^{(+)}$	0.000$^{(+)}$
HAR-IV-G									-	0.457	0.029$^{(+)}$
HAR-L-G										-	0.017$^{(+)}$
SMI											
ARFIMA	0.001$^{(+)}$	0.000$^{(+)}$	0.004$^{(+)}$	0.003$^{(+)}$	0.001$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.002$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-IV	-	0.000$^{(-)}$	0.000$^{(-)}$	0.208	0.214	0.454	0.054$^{(+)}$	0.246	0.130	0.506	0.048$^{(+)}$
ARFIMA-L		-	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-L-IV			-	0.766	0.040$^{(+)}$	0.138	0.009$^{(+)}$	0.850	0.019$^{(+)}$	0.156	0.007$^{(+)}$
HAR				-	0.000$^{(+)}$	0.010$^{(+)}$	0.000$^{(+)}$	0.298	0.000$^{(+)}$	0.010$^{(+)}$	0.000$^{(+)}$
HAR-IV					-	0.713	0.038$^{(+)}$	0.001$^{(-)}$	0.295	0.569	0.021$^{(+)}$
HAR-L						-	0.002$^{(+)}$	0.021$^{(-)}$	0.575	0.103	0.006$^{(+)}$
HAR-L-IV							-	0.001$^{(-)}$	0.114	0.001$^{(-)}$	0.399
HAR-G								-	0.000$^{(+)}$	0.021$^{(+)}$	0.000$^{(+)}$
HAR-IV-G									-	0.442	0.084$^{(+)}$
HAR-L-G										-	0.002$^{(+)}$
FTSE											
ARFIMA	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-IV	-	0.000$^{(-)}$	0.969	0.321	0.051$^{(+)}$	0.031$^{(+)}$	0.010$^{(+)}$	0.312	0.041$^{(+)}$	0.027$^{(+)}$	0.006$^{(+)}$
ARFIMA-L		-	0.000$^{(+)}$	0.000$^{(+)}$	0.010$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$	0.000$^{(+)}$
ARFIMA-L-IV			-	0.326	0.051$^{(+)}$	0.026$^{(+)}$	0.008$^{(+)}$	0.320	0.042$^{(+)}$	0.023$^{(+)}$	0.005$^{(+)}$
HAR				-	0.002$^{(+)}$	0.025$^{(+)}$	0.008$^{(+)}$	0.646	0.004$^{(+)}$	0.015$^{(+)}$	0.003$^{(+)}$
HAR-IV					-	0.253	0.060$^{(+)}$	0.003$^{(-)}$	0.496	0.276	0.035$^{(+)}$
HAR-L						-	0.026$^{(+)}$	0.031$^{(-)}$	0.246	0.305	0.194
HAR-L-IV							-	0.010$^{(-)}$	0.069$^{(-)}$	0.023$^{(-)}$	0.432
HAR-G								-	0.001$^{(+)}$	0.018$^{(+)}$	0.004$^{(+)}$
HAR-IV-G									-	0.254	0.040$^{(+)}$
HAR-L-G										-	0.017$^{(+)}$

Note: The p-values of the conditional Giacomini-White test are reported. The null hypothesis is that the row model and column model perform equally well is tested in terms of squared forecast error. The superscripts $+$and $-$indicate rejection of the null hypothesis, with a positive (negative) sign denoting that the row (column) model is outperformed by the column (row) model.

Table 71: SPA test (QLIKE) under the rolling scheme

	SPA-QLIKE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA-log(RV)	0.006	0.004	0.001	0.000	0.001	0.010	0.002	0.006	0.001
ARFIMA-log(RV)-IV	0.045	0.043	0.042	0.040	0.044	0.356	0.027	0.000	0.078
ARFIMAX-log(RV)-alt2	0.000	0.000	0.002	0.000	0.000	0.008	0.028	0.017	0.005
ARFIMAX-log(RV)-IV-alt2	0.045	0.037	0.030	0.038	0.026	0.440	0.015	0.000	0.001
HAR-log(RV)	0.000	0.000	0.000	0.000	0.002	0.000	0.012	0.000	0.000
HARX-log(RV)	0.128	0.380	0.247	0.162	0.073	0.171	0.038	0.009	0.313
AHAR-log(RV)-alt	0.000	0.000	0.013	0.000	0.003	0.000	0.537	0.003	0.000
AHARX-log(RV)-alt	0.051	0.141	**0.978**	**0.874**	**0.986**	**0.917**	**0.574**	**0.677**	**0.934**
HAR-log(RV)-GARCH	0.000	0.000	0.001	0.000	0.000	0.000	0.012	0.000	0.000
HARX-log(RV)-GARCH	0.584	**0.780**	0.195	0.002	0.039	0.163	0.023	0.018	0.151
AHAR-log(RV)-GARCH-alt	0.000	0.000	0.014	0.000	0.001	0.000	0.004	0.000	0.000
AHARX-log(RV)-GARCH-alt	**0.705**	0.695	0.164	0.052	0.096	0.481	0.012	0.019	0.242

Note: The p-values of the SPA test are reported. The null hypothesis is that the forecast under consideration is not inferior to any alternative forecast.

Table 72: SPA test (MAE) under the recursive scheme

		SPA-MAE								
		S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
1	ARFIMA-log(RV)	0.000	0.035	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	ARFIMA-log(RV)-IV	0.000	0.006	0.027	0.031	0.025	0.074	0.024	0.000	0.037
3	ARFIMAX-log(RV)-alt2	0.000	0.004	0.000	0.003	0.003	0.002	0.007	0.000	0.001
4	ARFIMAX-log(RV)-IV-alt2	0.000	0.105	0.024	0.046	0.017	0.103	0.021	0.000	0.003
5	HAR-log(RV)	0.000	0.000	0.002	0.002	0.000	0.000	0.001	0.014	0.002
6	HARX-log(RV)	0.205	0.375	0.028	0.088	0.048	0.053	0.060	0.024	0.054
7	AHAR-log(RV)-alt	0.005	0.000	0.092	0.004	0.000	0.003	0.004	**0.825**	0.004
8	AHARX-log(RV)-alt	0.414	0.646	**0.982**	**0.931**	**0.957**	**0.769**	**0.924**	0.398	**0.801**
9	HAR-log(RV)-GARCH	0.001	0.000	0.007	0.002	0.000	0.000	0.002	0.026	0.003
10	HARX-log(RV)-GARCH	0.316	0.560	0.034	0.103	0.058	0.137	0.071	0.037	0.097
11	AHAR-log(RV)-GARCH-alt	0.010	0.002	0.055	0.010	0.001	0.009	0.005	0.054	0.006
12	AHARX-log(RV)-GARCH-alt	**1.000**	**0.987**	0.280	0.182	0.096	0.851	0.119	0.804	0.559

Note: The p-values of the SPA test are reported. The null hypothesis is that the forecast under consideration is not inferior to any alternative forecast.

Table 73: SPA test (MSE) under the recursive scheme

	SPA-MSE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA-log(RV)	0.086	0.047	0.016	0.008	0.013	0.014	0.033	0.038	0.006
ARFIMA-log(RV)-IV	0.026	0.098	0.087	0.105	0.123	0.132	0.169	0.141	0.127
ARFIMAX-log(RV)-alt2	0.335	0.068	0.026	0.080	0.053	0.034	0.137	0.137	0.038
ARFIMAX-log(RV)-IV-alt2	0.123	0.323	0.072	0.136	0.104	0.171	0.154	0.179	0.123
HAR-log(RV)	0.056	0.033	0.063	0.008	0.026	0.027	0.112	0.068	0.064
HARX-log(RV)	0.511	0.570	0.208	0.096	0.064	0.121	0.108	0.075	0.068
AHAR-log(RV)-alt	0.090	0.067	0.369	0.566	0.583	0.642	0.286	**0.875**	0.198
AHARX-log(RV)-alt	0.843	0.847	**0.990**	**0.988**	**0.972**	**0.974**	**0.942**	0.423	**0.987**
HAR-log(RV)-GARCH	0.067	0.036	0.067	0.006	0.034	0.046	0.092	0.039	0.073
HARX-log(RV)-GARCH	0.419	0.313	0.167	0.062	0.088	0.130	0.146	0.101	0.085
AHAR-log(RV)-GARCH-alt	0.087	0.060	0.119	0.310	0.111	0.148	0.147	0.062	0.097
AHARX-log(RV)-GARCH-alt	**0.976**	**0.908**	0.256	0.129	0.122	0.400	0.244	0.647	0.233

Note: The p-values of the SPA test are reported. The null hypothesis is that the forecast under consideration is not inferior to any alternative forecast.

Table 74: SPA test (QLIKE) under the recursive scheme

	SPA-QLIKE								
	S&P500	DJIA	Nasdaq100	STOXX	CAC	DAX	AEX	SMI	FTSE100
ARFIMA-log(RV)	0.016	0.004	0.000	0.000	0.000	0.002	0.001	0.007	0.001
ARFIMA-log(RV)-IV	0.025	0.029	0.024	0.034	0.050	0.090	0.028	0.000	0.024
ARFIMAX-log(RV)-alt2	0.000	0.000	0.000	0.000	0.002	0.003	0.021	0.012	0.003
ARFIMAX-log(RV)-IV-alt2	0.051	0.035	0.024	0.038	0.036	0.255	0.019	0.000	0.000
HAR-log(RV)	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.000	0.000
HARX-log(RV)	0.356	0.240	0.206	0.172	0.078	0.199	0.036	0.004	0.236
AHAR-log(RV)-alt	0.001	0.000	0.042	0.000	0.000	0.000	0.450	0.001	0.000
AHARX-log(RV)-alt	0.184	0.163	**0.979**	**0.863**	**0.953**	**0.925**	**0.699**	0.148	**0.901**
HAR-log(RV)-GARCH	0.001	0.000	0.001	0.000	0.000	0.000	0.008	0.000	0.000
HARX-log(RV)-GARCH	0.508	**0.796**	0.116	0.001	0.046	0.197	0.024	0.015	0.124
AHAR-log(RV)-GARCH-alt	0.001	0.000	0.042	0.000	0.000	0.000	0.002	0.000	0.000
AHARX-log(RV)-GARCH-alt	**0.922**	0.687	0.155	0.001	0.026	0.406	0.010	**0.852**	0.301

Note: The p-values of the SPA test are reported. The null hypothesis is that the forecast under consideration is not inferior to any alternative forecast.

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