THE DECAY OF PROFITABILITY

AN ASPECT OF INDUSTRY PERFORMANCE

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CHAPTERI

INIRODUCTION

The raw material of this study is a record for a number of years of the rates of return earned by quoted companies in the United Kingdom. Frevious authors ${ }^{l}$ have identified a systematic component in such data: regression of profitability towards some central value. It is this systematic component in the inter-temporal behaviour of rates of return that is to be isolated, measured and interpreted in the following chapters.

In order to avoid confusion, two terms need to be introduced. Firstly, the regression of profitability towards a central level is hereafter called "the decay of profitability"; and secondly that central point towards which decay is directed is called the "decay origin".

The connecting theme of this study is the particular interpretation that it puts upon the decay of profitability. The main exposition of this interpretation is in the next chapter, but a brief sketch is presented at. this stage. In the model of the working of perfectly competitive markets, resources are allocated in order to eliminate supernormal and subnormal profits. It is this process that we are observing when we study the decay of profitability. Just as the speed at which resources are transferred and introduced will determine in rart how speedily the non-normal profits are eliminated, so we regard the rate of decay of profitability as a measure of the speed and efficiency with which resource allocation takes place. The aim of the theoretical work is to examine and develop that chain of argument and to consider how divergence from the competitive model will affect the decay of profitability.

[^0]
#### Abstract

In isolating and measuring the decay of profitability, a second aim of this study is fulfilled. This is to develop and demonstrate a statistical technique that has advantages over the direct application of regression analysis when a large body of data is available. This technique is based upon the transition matrix of the Markov stochastic process.


A third aim is to report a piece of research. This involves recording not only the finally selected sequence of analysis but also reporting when certain directions turned out to be unrewarding.

To return again to the main theme, the literature on allocative efficiency has, of necessity, mainly dealt with static questions. Both neo-classical and Walrasian general equilibrium systems are primarily concerned to develop the characteristics of an equilibrium state. More recent work has considered the (mathematical) existence of such an equilibrium. Where dynamic systems are developed, major simplifications are made and very simple types of change imposed on the resulting models. In observing the real world, change is a complex phenomenon; different variables shift in conflicting directions and shocks are overlaid one upon another. It is the process of compensation for shocks and of adjustment for once-and-forall changes that concerns us here and, in particular, the role of the firm in this process.

In the static general equilibrium system, the firm plays a very small part. This point is made by G C Archibald ${ }^{2}$ when, having sketched the allocation problem, he remarks:
2) Archibald G C (Ed), "The Theory of the Firm", Penguin London 1971, Editor's Introduction p 10

> "It will be noticed that the allocation problem was set out without any mention of 'firms'. This is because of its universality: it exists whether there are firms or not, and however they may be owned or organised. Yet firms exist, and must fit in somewhere. Formally we may think of them as intermediate a, ents, between resource owners and consumers, that perform certain organisational tasks. In neoclassical general equilibrium theory, firms are completely described by their production functions."

Without wishing to overstate the case it is not very far from the truth to regard the firm as essentially a creature of disequilibrium. ${ }^{3}$ In equilibrium, as Archibald says, the firm is merely a production plant combining inputs in specific proportions to produce a given set of outputs. If there is a change in prices then the firm will move along its production function and/or its product transformation frontier to a new equilibrium position. But it is in that process of movement from one point of equilibrium to another that the raison d'etre of the firm lies.

To talk of a "firm" is to refer to more than those "organisational tasks" involved in operating a production plant efficiently - managers do more than just stand guard over a production function. Our usual idea of a firm involves more than this because the firm operates in a world of disequilibrium and it is the aspects of its operations that are connected with disequilibrium and its companion, uncertainty, that receive predominant attention. It may be helpful to draw a distinction between those actions of firms that tend towards the restoration of equilibrium and those that are disequilibrating. No one category of actions can be fitted into this classification without error, but, for example, we generally expect investment decisions to be equilibrating and innovation to be disequilibrating. The intention in making this distinction is to emphasize

[^1]that only part of the economically relevant behaviour of firms tends to restore equilibrium and it is only this part of the role of the firm that

## is examined here.

In the real world, however closely pure competition is approached, change ensures disequilibrium. So it is at least as interesting to examine the strength of the tendency to restore equilibrium as it is to consider the extent to which structural conditions compatible with an optimal allocation are attained (particularly in a second best world). Knowledge of the structure of an industry is needed to assess whether equilibrium, should It be attained, will be optimal. But if the movement of that industry towards equilibrium is exceedingly slow, such information is of arguable relevance. 4

Such an industry may be more efficient in disequilibrium than another is in equilibrium but that is not easy to test and, indeed, may not be a meaningful question. ${ }^{5}$ The intention is not to dismiss measures of industry structure but to emphasize that amongst the important aspects of industry performance is the speed of adjustment of the industry to disturbances.

How can this speed of adjustment be observed? In the competition model, profits greater or less than normal only occur in disequilibrium. It is the existence of non-normal profits that motivates the shift of resources towards those products whose output is too low and away from those whose output is too high. This process eliminates the non-normal profits and
4) Svennilson I, "Monopoly, Efficiency and the Structure of Industry" in Chamberlin E H (Ed), "Monopoly and Competition and Their Regulation", Macmillan London 1954, p 275: "A cross-section of industrial structure at a given moment ... can only be regarded as a snapshot of an industry in perpetual change."
5) The characteristics of an industry in disequilibrium change from time period to time period. Therefore a comparison at time $t$ may be incorrect at time $t+1$.
we observe it as the decay of profitability. The performance measure that is required would seem therefore to be the rate of decay of profitability.

The aims of this study were set out at the start of this introduction and a brief sketch of the ideas underlying the primary theme has been given. It is this primary theme - the rate of decay of profitability as an aspect of industry performance - that is the main contribution of this study. The theoretical development, whilst directed to an unconventional goal, deviates from common practice only by recognising both the heterogeneity of industries and the multiproduct nature of firms. While it is not claimed that the statistical technique employed represents radical innovation, it is new and it does have some merits in work of the kind attempted here. Although the decay of profitability has been observed and measured before, the present study presents a fuller examination than has previously been made.

The three themes of the study are pursued in parallel. The economic ideas have been introduced in this chapter and their main theoretical development occurs in Chapter II. The conclusions of that chapter are given mathematical formulation in Chapter VI, Section 1. Chapter VII reports the estimation of the equations and the estimated coefficients are used in Chapter VIII to calculate a summary measure of the behaviour of rates of return within industries. The characteristics and behaviour of the measure are also investigated in that chapter. Finally, in Chapter IX this measure is compared with established measures of industry structure and performance.

The statistical theme originates in Chapter III where the technique is developed. The data are introduced in Chapter IV. Chapter V reports the direct application of the technique to the data. Chapter VI, Section 2,
discusses the econometric difficulties inherent in using the results of Chapter $V$ in the functional forms introduced in Chapter VI, jection 1. Then in Chapter VII the economic and statistical aspects come together at the estimation stage.

It may be noticed that the preceding description of the structure of this study made no mention of a literature survey. The view has been taken that there is little of precise relevance but much that relates to particular aspects of the development. The literature whose influence pervades many of the following chapters has been treated in one of two ways. Firstly two works ${ }^{6}$ particularly important in the theoretical development are discussed in Section 2.7. The other works 7 are empirical and deal with a broad range of questions, only some of which are relevant here. For these, the policy adopted has been to refer to them either textually or by footnote at the appropriate points in the argument.
6) Downie J, "The Competitive Process, Duckworth London 1958 Robinson J , "The Impossibility of Competition" in Chamberlin E (Ed) "Monopoly and Competition and their Regulation", Macmillan London 1954
7) Singh $A \& G$ Whittington, "Growth, Profitability and Valuation", Cambridge University Press 1968 Whittington G, "The Prediction of Profitability", Cambridge University Press 1971 Stigler G J, "Capital and Rates of Return in Manufacturing Industry", NBER 1963

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#### Abstract

In this chapter we are concerned to look in more detail into the process of resource allocation and in particular to consider the role played by the rate of return.


Section 2.1 introduces some basic ideas and assumptions. In Section 2.2 we consider the organizational means for resource transfer and the types of resources that are transferred. Section 2.3 introduces the way the firm decides upon the allocation of its resources. A model is proposed in which the firm constructs a preference ordering of the markets it either operates in or feels itself capable of entering. Then the influence of the expected rate of return on that ordering for various hypotheses of firms' objectives is analysed. Section 2.4 looks more closely into the expected rate of return and its relation to the present market rate of return, bearing in mind market structure and possible multiple entry. Section 2.5 summarizes the foregoing before in Section 2.6 the move is made from market to firm rates of return. Also in this penultimate section there is an examination of the problem of the point towards which convergence occurs and of the form that the relationship between rate of return in one period and in the next should take. Section 2.7 discusses the similarities and contrasts between the arguments of this chapter and those of Downie in his book "The Competitive Process". This leads it into an additional examination of the problem of over-capacity and industry contraction. Finally, Section 2.8 summarises the chapter.

The argument in this chapter will be based on a narrow nrecise idea of a commodity. This contrasts with the usual assmmption in industrial economics that commodity, market and industry are of similar extent. ${ }^{1}$ Therefore some clarification of the idea of a commodity as used here mast first be attempted.

Van Praag? gives a warning of the nroblems involved in attempting to define a commodity: "One of the vaguest concents emplnyed in economic theory is that of a homogeneous or basic commodity." This is reinforced by Samuelson's noinion that the mursuit of the narrowly defined or "basic" commodity is endless:

> ". . even if we confine our attention to what is ordinarily called a commodity, surh as 'wheat', we find ourselves dea, ing with a composite commodity made up of winter wheat, spring wheat, of varying grades. Fach of these in turn is a composite of heterogeneous comnonents and so forth in an infinite regression."?

Such views suggest, that the pursuit of a definition is not a task to be attompted here. The annroach will therefore be to regrard the basic commodity as a primitive idea and merely attempt some clarification.

1) e.e. Rain J, "Tndustrial Organisation", Tohn Wiley New York 1967, p 7 "We may define a market as including all the sellers in any individual industry and all the buyers to whom (in common) they sell."
2) Van Praag R M S, "Tndividual Welfare Functions and Sonsumer Rehaviour", North Holland Amsterdam 1968, Section 3.3
3) Samuelson $P A, \quad$ Foundations of Ficnnomic Analyajs", ('ambridge Mass. Harvard ITniversity Press 1947

We are concerned with the movement of resources between markets and we regard the firm as the allocating agent. Therefore our idea of a market need not, should not, be more specific than that which is distinguishable by the firm. Producers of soap powders and detergents distinguish between the markets for materials for standard washing, for fine fabrics and for machine washing. On the other hand they will not regard powder in red boxes as being sold in a different market from that in blue boxes. A single distinct commodity must be dealt with in a distinct market and for two markets to be operationally distinguishable by firms the rewards must vary independently in each of them. Thus at any one time there may be an excess supply of standard washing powder and an excess demand for power for machine washing. The profitability of the two markets will be different and firms will recognize and react to this. On the other hand, physical distinction may not imply a separate product either because consumers are indifferent to a particular variation or because there are so many variations of the product that only categories of the good can be distinguished for market purposes. This latter situation is well shown up by Stigler's example of hot rolled carbon steel sheets of which at least 135 million varieties can be distinguished. ${ }^{4}$ It would be exceedingly difficult to specify at what point a product distinction becomes too fine or too broad for our purposes. In the following market and commodity will be used in the narrowest sense consonant with resource allocation by firms between markets.
4) Stigler G J and Kindahl J K, "The Behaviour of Industrial Prices", NBER 1970 pp 4-5

With this sense of "commodity" established, it becomes unreasonable to limit analysis to the single product firm. Therefore multiproduct firms will be treated as the usual case, but this does not of course imply the assumption that there are no single product firms. It will be assumed that firms generally restrict their activities to one industry and that any firms that do operate in more than one industry have a divisional structure in which no division overlaps industry boundaries. But the single industry firm will be taken as the general pattern. 5 At various points these assumptions will be supported by the argument but nonetheless they are to be taken as prior to the ensuing discussion.

[^2]In this section there are two matters to consider. Firstly, what mechanisms are available to shift resources between uses, and secondy the nature of these resources.

In Chapter I one role of the firm in the capitalist economy was said to be to act as the agent by which adjustment to change is made. The firm is therefore the prime means of resource transfer and allocation. This function may be carried out in a number of ways. We may classify these in two dimensions: the activities of single or multiproduct firms and exit/entry or expansion/contraction. It is suggested that expansion and contraction by multiproduct firms is the most important of these methods. This means that the firm either transfers its existing resources or allocates new resources among those markets in which it is already operating. The other possibilities for the multiproduct firm are complete withdrawal from one of its markets or entry into a new market. The equivalent actions of the single product firm are expansion or contraction within its market, withdrawal from that market, which would usually imply the death of that firm, or the birth of a new firm into a market.

Of these devices, births and deaths of single product firms seem least likely to make any significant contribution to the adjustment of resource allocations. It is necessary that a birth involve the introduction of new capital and a death the withdrawal of existing capital. In these terms births and deaths amone any but the smallest companies are rare. If deviations from equilibrium are small, it may be that the marginal effect of births and deaths among the smallest members of an
industry is sufficient to push the whole industry back into equilibrium. There are activities for which births and deaths of small firms are the typical pattern but they are not major sectors in a developed economy. ${ }^{6}$ Therefore in the presentation of the argument the emphasis will be on multiproduct firms and on expansion and contraction rather than entry and exit; although the substance of the argument is not dependent upon such emphasis.

What are the resources to be transferred? In the short run the capital stock of the firm is given and therefore conventional analysis allows only variations in the labour input. More labour can be applied by shift working that inoreases the rate of utilisation of the existing capital, by using it to bring into operation capital equipment that was otherwise idle and, lastly, labour can be applied to increase the number of men operating the capital equipment at any one time. Labour may be newly recruited for the purpose or may be transferred from another product within the multiproduct firm.

Discretionary expenditure may be used to improve a firm's competitive position in a market by advertising, marketing or improved credit terms. These activities and their corresponding resources are of some importance in the process of eliminating extremes of profitability. They must usually be accompanied by a rise in production if they are to have an effect upon market profitability.

[^3]In the long run fixed capital may be adjusted and any major shifts in production must require changes in capital allocation. A different model will follow from the assumption that existing fixed capital can change its use than from a "clay" type assumption that existing capital is fixed in its use. In the latter model only the allocation of new capital can bring about the return to equilibrium. In the absence of any empirical studies of this, an assumption must be made. The most obvious is the moderate one that it is possible for some existing fixed capital to change its use. As a use of capital is intended to mean the production of a single commodity, the change of use of existing capital may involve only trivial alteration. For example, most machine tools can be quickly adapted to produce a considerable range of simple metal goods and this change of use is a fact of everyday life in the engineering industry. Of course, there are pieces of capital equipment for which a change of use is impossible. The general point is that change of use of existing capital can often occur.

While allowing for the possibility that capital equipment may be transferred between products, the major way of altering the resources committed to different markets is by applying new capital. A firm must choose how to employ its investible funds and it is the commitment of these to a particular product and thus to a particular market that provides the basic means of adjustment.

## Section 2. 3 : The Allocation Decision

In this section we first consider how a firm will order the various opportunities for employing its available resources. Then assuming that the firm formulates an expected rate of return for these opportunities we look at the influence of that expected value on the firm's preference ordering for a range of objective functions that misht characterise the firm. 'The process by which a firm decides on the allocation of its resources 7 involves firstly information and secondly criteria for assessing that information. The information required is first a selection of markets to be considered - the whole set of markets could not be scanned by one firm. Once a subset of markets has been selected, the data needed on each one can be decided. With this data on a subset of markets, the aim must be to construct a preference ordering of the markets.

The selection of a subset of markets is a necessary first step in any periodic appraisal of a firm's range of products. Clearly such an appraisal must involve those products which the firm is presently producing, although the firm must be expected to apply rather different standards from those used for potential products. It is not feasible for a firm to consider all potential markets, primarily becouse of the search cost but also because some markets will be so dissimilar from those the firm knows that it may judge itself incapable of a competent appraisal. ${ }^{8}$ For such
7) That this is an area of decision facing the firm is suggested by Wi.lliamson .J H, "Profit, Growth \& Sales Maximization", Economica Vol 33 1966, pp 1-16: "There are the decisions on input levels required to satisfy the efficiency conditions - the selection of least-cost input combinations, the optimal distribution of given investment funds between alternative projects, and the optimal distribution of sales effort."
8) Downie op cit $p$ 102: "... specialisation means that any point of time there will be what we may call a technological horizon, within which the firm will follow the lieht but beyond which it will not normally leap."
reasons, it is reasonable to expect that the vast majority of firms restrict their attention to markets within their own industry. The firm has regularly to decide upon the allocation of its scarce resources between the markets that it is operating in. The two problems of deciding the product range and deciding resource allocation will not be dealt with independently. The former is just as much part of the problem of allocating resources within the firm as is the latter. Perhaps it is worth inserting here a reminder of the narrowness of the idea of a product that is beine used here in order that this model of regular appraisal of potential products does not seem too far fetched. Where perhaps it does deviate from reality is in terminology; the firm will regard itself as looking for profitable opportunities rather than scanning potential markets, the substance is the same. The first component of the adjustment model is, then, the multiproduct firm with a resource allocation decision to make and a limited set of markets (some of which it is already encraged in) to consider - the set being limited by considerations of search costs and the firmfs own range of competence. 9 Such a situation must also describe the completely new firm (or independent entry in Downie's terms). The entrepreneur or embryonic management must consider a range of markets and they are at least equally constrained by the costs of search and range of competence.

One situation in which range of competence may not be immediately relevant is where a firm decides to enter a new industry and to buy the necessary skills. In general the purchase of skills will take the form of a takeover. Such a happening is not directly an entry as the

[^4]immediate consequence is not an expansion of capacity. ${ }^{10}$ Once the takeover has occurred, the range of competence again constrains the actions of the firm.

With a range of markets to consider, the next stage for the firm is to order them according to their attractiveness as uses of the firm's resources. This demands information upon each of these markets but not merely contemporaneous information but predictions of future conditions. Gux limited scone is to consider how the rate of return influences the preference ordering. The first stace is to assume that firms have an expected rate of return in these markets and to consider how that expected value influences the ordering.

The effect of expected rate of return on the preference ordering of the firm will depend on the firm's utility function. This is a controversial matter. There are three "families" of utility function: profit maximisine, growth maximising and satisficing. Fach major type of function has numerous variants. In this confusing situation, the one attitude that does not seem acceptable is to settle on any of these models as beine the theory of the firm, that is to say, the model which describes all firms. Indeed, a case might be made for suggesting that the utility function of a firm is so complex that all three models must be amalgamated to describe i.t. We must briefly consider how each of these behavioural patterns affects the decision unon which our attention is directed.

If a firm is a profit maximiser, its criterion for ranking markets will quite simply be the rate of return that it expects to earn in them.

[^5]We may assume that the higher the rate of return expected in a market, the higher that market will appear in the preference ordering of a profit maximising firm. The only difficulty that arises is whether profit maximising means lump sum maximisation or rate of return maximisation. Either interpretation may be single or multinerion. In the latter case the lump sum is a net present value and the rate of return is a yield rate ${ }^{l l}$ or internal rate of return. The ordering of projects should be invariant under these different methods but not, of course, invariant as between single and multiperiod assessments, or between different multiperiod horizons or for firms with different discount rates. The simple profit maximisation model is always formulated in lump sum terms ${ }^{12}$ but as it is a short run analysis the capital stock is unchanged and therefore lump sum and rate of return maximisation are completely equivalent. In the single period case, the rule then becomes that the firm invests until the return on the marginal project equals the cost of capital i.e. a rate of return argument is used. Again this is completely equivalent to lump sum maximisation. This model leads to a preference ordering based solely upon the expected rate of return.

If the firm is operating under a constraint which limits its expansion the lowest rate of return on a project undertaken may be considerably above the cost of capital. Considerations of risk may also lead to such
11) Merrett, $A$. \& Syken $\wedge$ : "Finance \& Analysis of Capital Projects", Longmans London 1963, p 36
12) Fenderson.$T$ M \& Ouandt, $R$ Fi, "Micrnenonomic Theory", Morraw-Hill New York, lst edition 1958; "The entrepreneur .... his ultimate aim is the maximisation of profit .... this profit is the difference between his total revenue and his total cost". p 53
a cut-off point. Even under these conditions the preference ordering will be rate of return determined. The reformulation of the discussion in terms of multi-period comparison of investment opportunities does not alter that conclusion, although the expected multiperiod retums may produce a different ordering from that based on single period assessment. The assumption up till now has been that the firm involves itself in projects starting from that for which it has highest preference and continuing to less preferred projects until it decides either that subsequent projects are not attractive or that it has used up its available resources. But most projects will involve a minimum size of resource commitment, i.e. there are indivisibilities. It may therefore be that selectine projects in order of preference leads to a residual resource amount that is too small to be employed on the next most preferred use. The most desirable project that can be attempted with those resources may offer a very low rate of return. In such a situation, there may be a different set of markets from that selected by a simple preference ordering that provides the highest rate of return or, equally, the highest joint lump sum.

We can conclude that for the profit maximising firm its preference ordering will be solely determined by the expected rate of return. It is possible to say that the higher the expected rate of return is, the more likely it is that that project will rank high. But the complications of multinerion assessmant, profect indivisibilities or conflicts, and constraints make it impossible to state that the preference ordering will exact,ly matoh the ordering by expected rate of return.

When we come to the family of firm utility functions that have been loosely called growth maximisers, the first task is to consider the variations in this group of functions. The first type is Baumol's static sales revenue maximisation and its dynamic counterpart is the
maximisation of the present value of future sales revenue. Then there is the simple growth rate maximiser. As in the profit maximisation case, the static theory is not anpropriate in the present context of resource allocation. In sales revenue maximisation more will be produced than in nrofit or growth maximisation ${ }^{13}$ subject to some minimum profit constraint. The present value of sales revenue is dependent unon the growth of sales revenue, therefore there is some similarity between maximising this variable and maximising the growth rate, but they are not formally identical. Our concern is how the expected rate of return will affect the preference ordering of markets for firms whose utility function is best described by this type. It is to be expected that such firms will have some minimum rate of return that they demand from any project. 14 Therefore for any market, the higher the expected rate of return, the more likely it is that it will be in the operational section of the firm's preference ordering. The secnnd way that the exnerted rate of return can have influence on the preference ordering is if the firm has a finance constraint. For the sales revenue maximiser, the importance of finance will depend upon the importance of future sales, which in its turn will denend unon the discount rate applied. Bint in any case, the experted ratie of return must. have some influence unon the preference ordering in this cese. A growth rate maximiser mast pursue a maximal investmont policy which meens maximising availeble funds if finance is a onnstraint, This implies the selection of profects ascording to their experted rate of return.
13) Willjaman ith, op oit, deala with the threa thenries and provea the basic reaulta.
14) This is usually explained as a manasement security device to prevent takeover by maintaining shareholder satisfaction.

If, on the other hand, finance is not the operational constraint but management capacity, for example, there is no need for the firm to take note of the expected rate of return in determining its preference ordering as long, that is, as the projects satisfy the minimum return requirement. On the other hand, a firm in this position, when faced with a multiplicity of directions for expansion, is more likely than not to be partially influenced by the expected rates of return.

Generally, however, the sales revenue maximiser will, in the absence of a finance constraint, look primarily at expected future sales in determining his resource allocation plan. The growth rate maximiser will pursue a policy which minitizes the effect of whatever constrains his growth. In the most plausible situation where management capacity restricts growth, the growth rate maximiser will be concerned to operate in markets which themselves permit considerable growth. This assumes that diversification is more costly in its use of management resources than is expansion within a market. It is therefore likely that both growth rate and sales revenue maximisation will lead to the selection of markets that offer greatest growth of sales within them.

The third way in which the rate of return mar have an important influence upon market selection is if there is a relationship between expected sales and expected rate of return. Whilst it is not possible to state a universal rule for the sales/rate of return relation, it is likely that a market rate of return that is high indicates a considerable discrepancy between demand and supply - price is well above marginal cost. The greater this discrepancy, the freater ceteris paribus the potential for increasing sales in the market without eliminating profits. Therefore
15) Penrose F T, "Theory of the Growth of the Firm", Oxford University Press 1959


#### Abstract

it is reasonable to presume that a highly profitable market will be attractive to both the sales revenue and the growth rate maximiser. On the other hand, markets may offer future increases in sales without presentiy displaying any supemormal profits. Such situations are those where either demand is expected to shift or costs to fall. Therefore we may conclude that high rates of return will be attractive to both types of firm, whether or not there is a finance constraint operating.


Before leaving this topic, the satisficing firm demands brief attention. It will have some minimum rate of return that it must attain for security reasons, but beyond this it is hard to develop any specific males to describe its resource allocation behaviour that would suggest a connection with the expected rate of return.

To summarize the impact of the expected rate of return on resource allocation: the profit maximising firm will be guided by expected rate of return, sales revenue and growth rate maximisers with a finance constraint will be primarily though not solely guided by the expected rate of return. Without a finance constraint, markets with a high rate of return will be attractive but others may be equally or more attractive. The relationship is thus weaker. For the satisficing firm, the expected rate of return will only generally have influence through the minimum requirement. In all cases the higher the minimum requirement, the more influence the rate of return will have on resource allocation.

Section 2.4 : Determination of the Expected Rat,e of Return

In assessing a market, the firm will consider a number of time periods. For these periods it will forecast resources and outgoings and it may then be assumed to follow conventional techniques of investment appraisal and discount these cash flows in order to get a single measure of the profitability of entering the market. Such a measure might be either a net present value or an internal rate of return. It is not material to this argument which is employed but for convenience the internal rate of return will be used in the following.

In forecasting the future of the market, three components may be identified: how the industry as a whole may be expected to perform, how that particular market will fare relative to the industry and how that firm would perform in the market. The first component leads us to the previously made assumption that while the majority of firms are taken to be multiproduct, spanning of more than one industry is rare. To support this state of affairs, firms will only in exceptional circumstances include a market outside their own industry in the set of markets they consider. So the expected profitability of the industry will not affect the allocation of resources by a firm but only their total amount. In other words, we are concerned with intra-industry equilibration and not with interindustry equilibration ${ }^{16}$ and so need only attend to profitabilities relative to the industry. That is with the latter two components listed above .

The market and firm effects cannot be completely separated. The situation is a firm considering a market in order to calculate how profitable the

Stigler G J, "Capital and Rates of Return in Manufacturing Industry", Princeton University Press for NBER 1963, Ch 3 looks at the process of movement towards equilibrium between industries.
firm would find it were that firm to join the market. It must therefore take into account its own effect on that market and it must distinguish between the situation whilst it is establishing membership of that market and that prevailing after entry is established.

Consider first the case where the market involves a large number of firms and entry can be made on a small scale. In addition the entrant may presume that he and any other entrants will have little or no effect on price by their entry, He may therefore expect that his revenue per unit will be the same as the present firms in the market but that while he is building up output, sales and expertise, both in the technique of production and in the approach to selling relevant to that market, he will have higher costs per unit than established firms. As his experience in the market increases so his cost will shift downwards. He may not expect to have identical costs even after adjustment is complete and he is an established member of the market. He may be using adapted capital equipment that is less efficient than that of other producers or he may be located further from the market and have higher transport costs. Therefore after becoming established he may expect a continued deviation from the market rate of return. Downie ${ }^{17}$ suggests that a firm will expect to lie in about the same relative position in a new market as it does in its present markets, thus if it is in the second quartile of rates of return at the present it will expect to occupy the same position in a new market.

If the usual approach is taken, the expected market rate of return in this case where the entrant has no significant impact upon it may be assumed to be represented adequately by the present market rate of return.

The firm's expected rate of retum (after adjustment) may therefore be presumed to be the present rate of return in the market times some factor unique to the firm and the market. But it follows that the higher the market rate of return and therefore the higher the expected market rate of return, the more likely it is that a given firm will find even with its unique multiplying factor that an attractive return is to be gained in that market, once the adjustment period is over. Similarly, the higher the profitability in the market the quicker the new entrant will get his costs below price and start earning profits. Also the higher the profits once adjustment is over, the more adjustment costs will be worth bearing for the longer term benefit. Therefore the higher is the present rate of return, the higher is the firm's expected overall rate of return in the case where the entrant assumes that entry will have no impact on price.

A second case is that where the market is atomistic but the firm considering entry expects sufficient other entrants for there to be an aggregate effect upon the market. This may be the way case one develops when the present rate of return is very high. The market's attractiveness and visibility is likely to induce a large amount of entry. If significant entry is to be expected, the potential entrant must exnect $a$ fall in rates of return. Should the expected rate of return react so that a rise in the present rate of return produces a fall - through the increased level of expected entry - then there would be a disequilibrating tendency. This is probably only possible where the market is on the margin of the atomistic category where there are only just sufficient firms and where the minimum efficient scale is just small enough. Although such a reversal of the effect of the present rate of return on the expected level may be rather unlikely, the expectation of there being other entrants will
reduce for a single firm the attractiveness of a given present rate of return and thereby moderate the strength of the equilibrating tendency.

Once we move onto the situation where a single entrant may have a marked effect on a market, we enter the realm of oligopoly with ite attendant problems. Considering first the post-adjustment state: the firm is assumed to have become established in the industry. The new entry may have precipitated a movement away from oligopolistic behaviour in the market to something more freely competitive. The likely result of this is a decline in price and profits and a rise in output. Should the entrant correctly forecast this occurence then the expected rate of return will still be influenced by the present rate of return as any competing away of excess profits earned under oligopoly conditions will take some time and therefore the average rate will bear some relation to the initial rate. The result holds more strongly if the firm fails to predict its effect on the conduct of the firms in the market. It will therefore expect to enjoy the hisher profits of an oligopolistic situation and regard present rete of return as a good proxy for future rates of return.

The preceding paragraph presumes the effect of oligopoly is higher than normal profits; while this may not be so in any time period under the assumptions of some oligopoly models, it will prevail under collusive joint maximisation or Cournot-type models. It is a reasonable assumption except in cases where the oligopolistic interdependence has generated considerable instability in the actions of the member firms.

So far the case where the entrant has no effect on the market and the case where his noticeable arrival in the market results in a reduction of the amount of oligopolistic interdependence have been considered.

This latter case was only considered once entry had been completed; before considering the problems of the adjustment process in such a case, there is a third possibility to be considered. It is not impossible for the new entrant to precipitate more collusion or more interdependence 18 though it is clearly a relatively unlikely occurence. Prediction of it by the entrant is sufficiently unlikely for it to be ignored in this discussion of the process by which firms form their rate of return expectations.

The question now is the adjustment process. If adjustment is quick, then the costs and revenues involved in it will not carry very much weight in the discounting process and ther fore unless the costs are for some reason very large, the expected steady rate of return may be taken as the overall expected rate of return. But most of the difficult problems arise in considering the process of entry and adjustment to a market. It is necessary to point out that in this section the concern so far has been to show that under most conditions the present rate of return will be the prime determinant of the expected rate of return. 'Purning as we are now to the adjustment costs, this is to consider factors that may influence the relation between the firm's expected rate of return in a market and the present rate of return.

Adjustment costs may be divided into three categories: Those costs that are Incurred in increasing productive capacity in a market even if the investment is made by a firm already established in the market; those costs experienced by any entrant to the market; and thirdly those unique to a particular entrant. The basic costs of investment are straightforward and, therefore, for the present purposes, the first category
18) C Cearly this dnes not, armen with Cournot's result, that for firms maximisins nrofit, hy output variftions the more firms there are in a market, the clnser that markot will he to pure nomnetition. Gee Menderson J M R Muandt R F, on cit. $n$ l?9
need not delay us. Ieaving temporarily the second category to one side, the third - factors unique to a particular firm - can next be dealt with. Entry to a new market involves in general the acquiring of new techniques, learning how to produce a different product and learning how to sell in a new market and, perhaps of lesser importance, learning how to buy new raw materials and intermediate goods and specialised factors of production. Fach potential entrant to a particular market will differ in the degree to which it is equipped to engage in that market, so the costs of learning will differ. The expected rate of return will consequently differ from one firm to another. Therefore the number of entrants to a market will, amongst other things, be affected by the number of firms employing similar skills to those relevant to that market. ${ }^{19}$

Turning now to the second category - those factors common to all entrants to a particular market. Such factors are of course those usually known as "barriers to entry". 20 They are costs that must be born by a new entrant but not usually by an established firm considering expansion. More correctly, of the three types of barrier suggested by Bain, one is definitely only a barrier to entrants and not to expansions of capacity, while the other two may affect all investments in the markets. Bain's three types of barrier are: product differentiation, absolute cost advantages and economies of scale. Product differentiation only affects new firms coming to a market as an established firm must have an established product. Established firms may decide that to expand they should launch a new product, but this is a result of weighing relative costs, whereas the new entrant cannot avoid the costs of launching and establishing a new product. The second barrier - absolute cost advantages conveys the possibility that established firms (or some of them) have
19) This might be represented in terms of the snace described in Footnote 10 as the density of firms in the area of the market.
20) The primary source is J S Bain's "Barriers to New Competition" op cit
control of superior production techniques and/or advantageous positions in factor or raw material markets. The third barrier - economies of scale - refers to the case where the minimum optimal scale of operation is a significant fraction of the total scale or capacity of the industry. If, in addition, unit costs are significantly raised at lower than minimum optimal scales, then entrants must either bear higher average costs than established firms or enter at the minimum optimal scale and thereby make a mariced increase in the total capacity of the industry. The existence of any type of barrier means that adjustment costs for the new entrant will be high and that the adjustment is likely to be lengthy. Therefore the expected rate of return will be well below the present market rate of return. ${ }^{21}$

So far the discussion has been of new entrants to markets and the way they formulate the rate of return expectations that they use in making diversification decisions, in particular about how this expectation will relate to the present rate of return being earned in that market. But a very considerable amount of resource allocation will be done by firms between the markets they already operate in. Again the formation of the expected rate of return plays a part in the decision process, a part whose importance depends upon the objective function of the firm. But the factors that suggest a divergence between the present rate of return and the expected rate will be, apart from the barriers to entry, the same as for new entrants.

Briefly and finally in this sectior, what about exits? As the resource allocation decision is based on the firm's assessment of the future of
21) Modigliani F, "New Developments on the Oligopoly Front", Journal of Political Economy June 1958 discusses Bain's and Sylos-Labini's assumptions about the likely reaction of established firms to new entrants. According to the view that the entering firm has of the policy that established firms will adopt, the expected to present market rate of return relationship will vary.
each market it considers, not all firms will jump the same way. Some will be entering or increasing their activity in a market while others are reducing their activity or actually leaving the market. Any change in the market will be the net effect of various actions by firms. ${ }^{22}$ Actual withdrawal from a market is probably a rare phenomenon, but given a certain degree of capital adaptability there may conceivably be occasions when the benefits of moving it to a new use outweighs the costs of that move and the profits to be earned in the original market. There it is profit relatives that decide the allocation of resources rather than levels of profit. Generally we may regard exit as determined by the same process as entry - in each case there are costs to be borne that may or may not be compensated for by later profits.

[^6]We have argued that allocation of resources between markets is achieved by firms either expanding and contracting or entering and leaving those markets, and that expansion and contraction by multiproduct firms within the markets they are already established in is the most important. The resources shifted (or newly applied) are labour, working capital (advertising, marketing, credit terms, etc), existing fixed capital and new capital.

It is suggested that the firm makes its resource allocation decisions after considering a number of markets - those in which it is already operating and a number of others within its horizon of technical, marketing etc, competence. The influence that the rate of return the firm expects to make in each of these markets upon the way in which it orders its preferences for increased (or new) activity in these markets is dependent upon the utility function of the firm. Fxpected rate of return will be the sole determinant of the ordering for the profit maximising firm, and it will be an important determinant for the growth rate or sales revenue maximiser if there is a finance constraint operative. It will still have some effect upon the ordering for these latter two groups even without the finance constraint. But then it is through the minimum nrofit constraint that rate of return will primarily have an influence, as it is solely for the satisficing firm.

The final stage of the argument is to link the expected rate of return with the present rate of return in the market. We find that the more atomistic the market and the fewer expected entrants, the closer the present rate of return to be expected. As the market becomes more oligopolistic, or as more entrants are exnected, so the rate of return
the firm expecte to earn in the market diverges from the present market rate of retum. Similarly the higher the barriers to entry, the greater is this divergence.

We may therefore expect that the speed with which resources are allocated towards the most profitable opening will be increased by the presence of broadly diversified firms that can reallocate internally. An industry of single product firms will be much slower. Secondly the more capital intensive is production, the less swift will be adjustment. For a given level of capital intensity, adaptability and short life of capital assets will lead to faster adjustment. These factors will aid both expansion and contraction.

The more similar are the markets of an industry, the easier firms will find it to move into new activities and so the swifter resource allocation within that industry. An industry of profit maximising firms or growth maximisers under a finance constraint will transfer resources towards profitable opportunities more quickly than one of growth maximisers without a finance constraint, or one of satisficers. Whatever the utility function, a factor that raises the level of the minimum profit constraint will speed the elimination of high rates of return.

Generally the closer the industry structure is to the purely competitive the more atomistic, the lower barriers to entry - then the faster high rates of return will be reduced. The reduction of high rates of return will be faster if firms expect few other entrants than if they expect many.

The nature of the production process - its capital intensity, capital adaptability and capital life - and the multiproduct or single product
nature of the firms within the industry must influence both exnansion and contraction. Althouch it ia perhaps less immediate a deductinn, the profit maximisers and growth maximisers with a finance constraint are likely to get out of low rate of return activities more quickly than those without a finance constraint and satisficers, and presumably none will tolerate persiatent returns below their minimum standard. So the hieher the minimum standard, the more ranidly will contraction of the market take place. On the other hand, market, stmot,ure and barriers to entry are likely to have a weaker influence unon the rate of onntractinn than unon the rate of expansion. Rut, any firms that contemplate the withdrawal of their resources from the unprofitable market and entrance to another market will be affected, as will any entrant, by the nature of that market. A much more general problem relating to the contraction rate is that capital assets wear nut slowly and whil.st low rates of return may kill firms, assets are more difficult to eliminate. This will be returned to in Section 2.7, but the general view taren by writers in this area is that contraction is much less speedy than expansion and therefore low rates of return may take more to eliminate than hiog rates of return.

## Section 2.6 : The Firm's Rate of Return

This section argues the connection between the firm's rate of return and the market's rate of return. It then discusses the implied point towards which profitability of firms tends, and finally specifies the basic requirement of the function relating the rate of return in one period with that in the ensuing period.

So far we have talked of firms allocating resources in response to market rates of return and of the consequences for the market rate of return of this transfer of resources. Simply we have said high (low) market rates bring in (drive out) resources that cause a fall (rise) in those market rates. But the observable variables are firms' rates of return and these are a.verages of the rates of return earned in each of the markets that a particular firm is engaged in. Conversely, the market rate of return must be the average of the returns earned in that market by all firms active in it. Therefore if the rate of return in a given market declines by a certain proportion, so on average must (by definition) the rates of return earned in that market by the firms operating in it. So the effect of resource transfer on the market rate of return must on average be reflected in the rates of return earned in that market by firms operating in it. This effect will be experienced in every market in the industry to a greater or lesser extent and therefore every firm will experience it on average for all the markets in which it continues to operate. Therefore in the absence of entry to markets, we may expect that, on average, high (low) firms' rates of return will be reduced (increased) through the transfer of resources. This continues to be true unless the rather unlikely situation occurs in which a very high proportion of industry resources are used in entry.

This pattern is an average one as some firms in a market whose rate of return is decreasing may achieve increasing profitability. Some firms may for various reasons be thus situated in a number of markets and so, despite high average profitability, experience a rise in profitability. Some firms may undertake entry on such a scale that their change in profitability is dominated by the effect of this. But all these possibilities notwithstanding, the average effect upon firms' rates of return will be as the markets' rates of return. Observing the average behaviour of all firms in an industry is to observe the average effect across all the markets of the industry.

So far we have spoken of high rimfitability inducing the inward movement of resources and low prof'itability outward movement. The consequence of this being a downward tendency for high profitability and an upward tendency for low. High and low need clarification. 'The movement of resources is motivated in a complex way - there will be markets where some firms are withdrawing resources while others are bringing them in. We are therefore concerned with net movements of resources within an industry. There will be some level of the rate of return below which there is net loss of resources and above which there is a net gain of resources.

The precise point at which this reversal occurs is, in what follows, called the "decay origin" and is assumed to be the industry mean rate of return. Whilst it is not possible to formulate a strong argument for any specific rate of return, it does seem unlikely that the decay origin will deviate far from the mean. It is also likely that there is quite a range of rates of return over which net flows are approximately zero, so any point within that range will serve. In general rates of return
in all that follows will be expressed as deviations from the mean.

We must now consider how we may describe this intertemporal behaviour of rates of return in mathematical form. We will speak of the tendency to convergence as the "decay of profitability". If we write the rate of return (expressed as a deviation) of firm $j$ at time $t$ as $r_{j t}$, then the function we are interested in is

$$
r_{j t}=f\left(r_{j t-1}\right)+u_{j t}
$$

where $u_{j t}$ is an error term with mean zero that encompasses all movements of profitability that counter its decay.

To put forward such a function is not to deny that more lagged rates of return would be relevant to a complete description of $r_{t}$. But we are concerned with the annual movement of rates of return and it is therefore this first order function that we must investigate.

For there to be decay of profitability, such a function must satisfy

$$
0<f^{\prime}\left(x_{j t-1}\right)<1
$$

This ensures that for high rates of return $r_{j t}<r_{j t-1}$ and for low rates of return (i.e. negative deviations) $r_{j t}>r_{j t-1}$. No attempt has so far been made to specify decay of profitability any further, and indeed a rigorous theoretical exercise would demand more than we know of the dynamics of micro resources allocation. What we should expect is that the faster resources may flow in, the faster will the decay of profitability occur, i.e. the smaller will be $f^{\prime}\left(r_{j t-1}\right)$. So the more quickly an industry moves towards competitive equilibrium, the lower will be the first derivative of the function. It is plausible to argue that the transfer of resources will tend to be faster into (out of) markets with very high (low) ratees of return than into (out of) markets with more moderate rates of return. We mf.ght therefore suspect
that the second derivative of the function would be negative (and certainly nonpositive):

$$
f^{\prime \prime}\left(r_{j t-1}\right) \leqslant 0
$$

Graphically we may represent this as in Diagram 2.1 in which the axes are rates of return measure as deviations. The 45 degree line is the locus of points for which $r_{j t}=r_{j t-1}$ and we expect the decay of profitability line to intersect it at the origin: there is no decay of average profitability.

## Diagram 2.1



The algebraic specification of the function is necessarily a matter related to empirical convenience and will be discussed in Chapter $V 1$. For the present, we conclude that the function should have positive slope of less than unity and, if not linear, the second derivative should be negative. If the point of convergence is specified correctly and all rates of return are expressed as deviations from it, then the function should pass through the origin.

The final empirical roint to be mentioned in this section involves the dispersion of the rates of return of firms within an industry. In order to eliminate the effect of variations in this on inter-industry comparisons, each industry's data have been expressed in standard deviation units as well as in deviations from the mean, before the decay of profitability function has been estimated. The precise details of this transformation of the data are described in Section 4.3

Section 2.7 : Downie's "Competitive Process" - A Comparison

This section looks at the way the preceeding arguments relate to those in the most relevant other work: that of Downie. ${ }^{23}$ This leads to Joan Robinson's paper "The Impossibility of Competition" 24 and from there to是 further consideration of the problems involved in the contraction of markets.

Although large portions of the industrial economics literatire is relevant to particular aspects of this study, only Downie's is pervasive in its connections. He is concerned with the competitive forces and defines two: the "transfer mechanism" and the "innovation mechanism". The former term is used to describe the process which transfers market shares from the less to the more efficient. This tendency towards concentration is countered by the "innovation mechanism", which is the process by which firms change their efficiency by innovating. His argument relies upon the idea that such efficiency-enhancine innovations are brought about by the pressures of competition. As these pressures bear more heavily upon the less efficient firms, they will be the main innovators. Thus the concentrating effect of the "transfer mechanism" is reduced by the "innovation mechanism" throwino up new leaders for the industry.

The first aspect demanding clarification is - what is meant by efficiency? Downie uses it in the sense of the difference between the value of inputs and outputs, constructing an expression whose numerator is the value of
23) Downie J, "The Competitive Process" op cit
24) Robinson J, op cit
inputs and the denominator the value of outputs. He tempers the ideal with his view of the practical and reaches an expression for efficiency which takes a final form:

$$
\epsilon=1-\beta(r-\bar{r})
$$

where $\beta$ is the capital output ratio, $r=$ rate of return on capital (pre-tax and post-depreciation) and $\bar{r}$ is the average rate of profit on the assets of the industry. Such a measure will generally relate in a simple way to the actual rate of return earned by firms unless the capital output ratio fluctuates considerably. "The efficiency ranking indicated by rates of return on capital ... will ... usually provide a fair guide." ${ }^{25}$ This measure of effiniency will therefore correlate very highly with the rate of return of the firm expressed as a deviation from the industry mean - the variable used in this present study.

The main distinction that must be drawn between Downie's analysis and the present study's is that Downie is concerned with a longer run process. Thus he states: "The plausibility of my account therefore rests upon the assumption that fundamental disequilibrium will be corrected fairly quickly." 26 on the other hand, this study is concerned primarily with the strength of forces working to correct fundamental disequilibrium. Downie's transfer mechanism operates to shift market shares to the most efficient who will be able to win this increase because they can expand capacity more quickly than their competitors. This is possible because their efficiency provides a greater supply of internal finance for investment. The general operating environment is one characterised by excess demand that must be met rather than by a need to work to create
25) Downie op cit p 48
26) op cit p 113
extra demand. 27 Downie therefore concerns himself with the relative
increases in capacity and the tendency of the transfer mechanism to
increase concentration.

The counter-force is changing relative efficiency through changes in technique. That this does counter the transfer mechanism is dependent upon the assumption that falling market shares will inspire such innovations and so will originate in the less efficient firms. The most efficient are too concerned to increase their capacity to get involved in innovation of this kind. Therefore the innovation mechanism will work to change the relative efficiencies that direct the workings of the transfer mechanism. Just as tin idea of the transfer mechanism ignores market creating activities, so the innovation mechanism is discussed in terms of technique rather than product innovations. This throws up the second main distinction between this and Downie's work: he talks in terms of industries rather than markets. He means by "industry", "a group of firms whose techniques of production are sufficiently alike for it to make sense to conceive of one as being able to do the business of another" and points out that this definition is "very close to that used by the authors of the standard Industrial Classification in the United Kingdom." 28 Whilst the weight he has put unon technique rather than product innovation and upon meeting rather than creating demand may be appropriate at this level of aggregation, the problems of different markets within an industry do cause him some difficulty.

[^7]He has a chapter (No VIII) in which he considers the effect of entry and exit upon his model. of competition. The entry that he considers important is that resulting from an existing firm deciding to diversify into another industry. 29 The statement that he makes about this action by a firm also reveals very clearly that his firms are solely motivated by growth: "The potential migrant becomes an actual srosser of industrial frontiers when it believes that its combined rate of erowth in two (or more) industries will be greater than that which it would achieve in only one." 30

So diversification is motivated by growth as are all other firm actions and it means crossing to a new industry. Before looking at the impact of this complication upon the two mechanisms of the competitive process, the point must be made that Downie's reliance upon the industry rather than the market means that it is only the rare "crosses of industrial frontiers" that are explicitly treated as multiproduct. He does discuss the firms' choice of "production objectives" 31 in a way that would permit the consideration of the multiproduct but one industry firm, but does not develop the point. It is important because he points out that once firms have diversified, industries will contain firms that are insensitive to the transfer mechanism as their losses may be financed by the parent from activities outside the industry. Once firms are thus shielded from the transfer mechanism, the pressures that bring about innovation will also be severely diminished.

[^8]Diversiffcation by the firm will thus realuce the strength of Downie's two forces and therefore "the tendency to ossification in the stmeture of concentrated industries will be all the stronger. "3? On the other hand, he argues that the working of the two forces will be accelerated in, what he terms, the "colonised industry". The transfer mechanism will he reinforced by the new entrants and this will, in its turn, enhance the operation of the innovation mechanism. Thus there are very different, consequences of diversification accordine to whether the industry is colonising or colonised. This distinction is very diffjeult to maintain once the parts of the firm lose thejr clear nerent-subsidiary relationship and become competing users of the resources available to the firm. At this stage we are faced with the multiproduct firm again and it has already been pointed out that Downie does not deal with this sase. 33. As long as moving into 9 new industry is rare and a.s long as the industry can be treated as homogeneous, this is not a serious omission.

Having pointed out these two main contrasts between Downie's and my approach, the connections should also be discussed. To deal first with the innovation mechanism: in so far as it is restricted to techniques, its main place in the present study is amongst those factors that counter the decay of profitability and maintain the disnersion of profitability. On occasion an innovation of technique may nermit one or a minority of firms to compete more effectively in particular market and therefore bring about the decay of profitability for the majority of firms in the market. It might also be argued that if it is falling profitability rather than falling market share that inspires innovatory efforts, the innovation mechanism may underlie some of the decay of profitahility from
32) op cit p 109
33) See preceding page
low rates of return. Probably rather more rare but perhaps important nonetheless is the role of innovation in overcoming barriers to entry, particular scale barriers and absolute cost, advantage barriers. Therefore the innovation mechanism, whilst playing mainly a disequilibrating role can, on occasion, contribute to the tendency towards equilibrium.

The transfer mechanism is a differential growth of capacity. It is the most efficient and therefore, in general, the most profitable who increase their capacity most rapidly. Within a model recognising industry heterogeneity, a proportion of firm profitability is explained by the profitability of the markets in which it operates. Therefore Downie's transfer mechanism in this context is equivalent to the allocation of resources towards the most profitable markets. It therefore induces the decay of profitability. It is Downie's emphasis on the effects of this process on industry structure rather than on the elimination of fundamental disequilibrium that leads to the different interpretations of the effect of this mechanism.

The pervading, although often implicit, assumption that growing demand is the usual situation means that Downie does not spend much time on the problems of excess capacity. He recognises the problem:

> "... what is needed to kill a firm is a period of negative gross profits, or a good takeover bid from another. But what is needed if capacity is to be scrapped is that reasonable men should believe that under no future conditions which it is reasonable to envisage will it be possible to earn any nositive grass profit by working the capacity. Such a view will usually be taken only if the capacity is either very decrepit or, technical innovation in the industry having been very rapid, very old fashioned. In other words, firms can be killed by prices, but capacity only by time." 34

The situation of excess capacity will interrupt the working of the transfer mechanism and because of low profits and need for the securjty of liquidity in an industry suffering from over-capacity, the innovation
mechanism will "tend to be suspended for the duration of the disequilibrium which will be longer in consequence." 35 Thus Jownie's mode] of the competitive process suggests that readjustment of excess capacity will be a slow process. He points out that the "saving grace of growing demand" 36 will usually deal with the problem and therefore believes persistent overcapacity to be rare.

A more pessimistic discussion of this problem is that of Joan Robinson's. 37 The problem as she expresses it is that: "Supernormal profits are usually wiped out by new investment more quickly than subnormal profits are raised by disinvestment." 38 The conclusions are much the same as Downie's. There clearly are examples of the over-wapacity continuing for extended periods but the industries where this is most likely to occur are suggested by the statement that: "We will confine the following argument to an industry producing a homogeneous commodity" 39 and by the remark that "most plant is highly specific" 40 . In the situation postulated as common in this present study - that is, multiproduct firms operating in industries encompassing many markets with capital permitting some degree of change of use - only quite extreme degrees of over-capacity or industry-wide over-capacity are likely to be particularly prolonged. Clearly this does occur: Cotton and Shipbuilding may be cited. These examples also have quite specialised and unadaptable capital. Therefore while the Downie and Robinson situation does occur, its frequency can be overstated due to the assumption of industry homogeneity and capital equipment specificity. The present study uses post-war data and therefore will deal with the full employment growing demand that, Downie says, makes over-capacity an abnormal situation.
35) op cit p 121
38) op cit p 247
36) op cit p 122
39) op cit 247
37) Robinson J op cit
40) op cit p 251

Section 2.8 : Summary

In Section 2.5 the various factors that may influence the speed of resource allocation were summarized. Here we may therefore merely state that resources will tend to be transferred from markets offering low profitability to markets offering high profitability. The nature of the industry will affect the strength of this process but is very unlikely to reverse it. It is net resource transfers that matter and there will be some rate of return (referred to as the "decay origin") at which net outward movement will change to net inward movement.

It is argued that this resource transfer will lead to a tendency for market rates of return to move toward the decay origin. We then conclude that firm rates of return will display a similar tendency and assume that the decay origin may be represented by the industry mean. We then suggest the basic form that the relationship between the rates of return at time t-l and at time $t$ should obey and point out that we will use standardised data.

Finally it is argued that in the most closely related study to this one there is an emphasis on the long run problem of changes in industry structure and that much of the divergence beiween conclusions follows from the present study's recognition of the heterogeneity of industries and the ubiquity of the multiproduct firm. In particular this leads to a differing view of the likely period involved in eliminating excess capacity.

CHAPTERIII

STATISTICAL TECENTQUE

In this chapter, the main statistical technique used in the study is developed. It is based on the ideas of Markov chains, so Section 3.1 presents the fundamentals of the Markov stochastic model. Section 3.2 goes on to develop the continuous analogue of the Markov transition matrix, continuous in the state rather than the time dimension. Section 3.3 introduces the method of using this device. The development of mathematical ideas here is intended to be heuristic rather than rigorous.

## Section 3.1: The Markov Process

The first order Markov process is a particular form of stochastic process in which the outcome of any trial depends only on the outcome of the preceding trial. So if there are a set of outcomes $E_{1}$, $E_{2}, \ldots, E_{n}$, and if $E_{j}$ is succeeded by $E_{k}$ at the next trial, we describe this transition by ( $E_{j}, E_{k}$ ) and ascribe a probability $p_{j k}$ to it. The outcome of the trial preceding that at which the outcome was $E_{j}$ does not affect the value of $p_{j k}$. An example of such a process is given by Howard ${ }^{1}$ where he introduces the usual terminology of Markov processes;
"As time goes by, the frog jumps from one lily pad to another according to his whim of the moment. The state of the system is the number of the pad currently occupied by the frog; the state transition is of course his leap."

Thus "state" is used rather than the usual "outcome" of probability theory and instead of referring to trials and pairs of trials, state transitions are used.

This study is concerned with the change in rates of return from one period to the next. It is therefore acceptable to use the first-order Markov process as a model. It is not to deny that, at least, a higher order process is necessary for a full description of the behaviour of profitability over time.

Consider a system to be in one of $N$ discrete states at time $t$ and in another state at time $t+1$. Then if we denote the initial state by i and the state after transition by $j$, the transition probability - the

1) Howard R A, "Dynamic Programming and Markov Processes", MIT Press, Cambridge, Mass, 1960, p3.
probability of that particular transition from ito $j$ - may be written $p_{i j}$. The behaviour of such a process may be summarised by a matrix of transition probabilities:

$$
\begin{equation*}
T=\left\{p_{i j}\right\} \quad i, j=1, \ldots \ldots, N \tag{1}
\end{equation*}
$$

Certain conditions can of course be imposed on these probabilities. Firstly the fundamental:

$$
\begin{equation*}
0 \leqslant p_{i j} \leqslant 1 \tag{2}
\end{equation*}
$$

Secondly for any $1, p_{i j}$ is a conditional probability - the probability of the system being in state $f$ next period given that it is in the state $i$ this period. If not moving is treated as a transition (i.e. $i=j$ is not ruled out) then clearly in the next period the system must be in one of the set of $N$ states. So the sum of the conditional probabilities must be unity:

$$
\begin{equation*}
\sum_{j=1}^{N} p_{i j}=1 \quad \text { for } i=1, \ldots, N \tag{3}
\end{equation*}
$$

With these two conditions, $T$ is a stochastic matrix.

At this stage a simple example may be useful. Let there be two states; above average profitability and below average profitability. There is a quite high chance that a firm will stay in the above average state next period and similarly a firm presently in the below average range will most probably stay there. Therefore the transition matrix will look something like:

$$
T=\left(\begin{array}{ll}
0.8 & 0.2 \\
0.3 & 0.7
\end{array}\right)
$$

The rows - the conditional distributions - add to unity as required. In auch a model, the firm is allowed no history. That is to say, if a below average firm lifts into the above average state it is then no more or less likely to stay above average than a firm that has been above average for some time. This is clearly a very sweeping assumption. It is of the kind fundamental to first order Markov processes. Generally Markovian analysis does have this implication that all that is relevant of the past is given when the state is specified.

Instead of treating a single firm by such a transition matrix it is equally possible to take a frequency distribution and apply the transition probabilities to that to get the next period distribution. So, If there are $x_{01}$ companies in state 1 and $x_{02}$ in state 2 at time zero, then the next period distribution is given in our example by:

$$
\left(\begin{array}{ll}
x_{01} & x_{02}
\end{array}\right) T
$$

which we may write:

$$
\left(\begin{array}{ll}
x_{11} & x_{12}
\end{array}\right)=\left(\begin{array}{ll}
x_{01} & x_{02}
\end{array}\right) \quad T
$$

Converting this into vector notation gives:

$$
\begin{equation*}
x_{1}=x_{0}^{T} \tag{4}
\end{equation*}
$$

where $x_{0}$ is the vector of initial state distribution and $x_{1}$ is the vector of the state distribution after transition. Note that the number of individuals in the state distribution is constant over transitions, i.e. if the sum of the elements in the vector $x_{0}$ is $M$, then so is the sum of the elements in $x_{1}$. Therefore we can divide both sides of equation (4) by $M$ and reduce the two $x$ vectors to probability
vectors, i.e.:

$$
x_{1} u=x_{0} u=1
$$

where $u$ is the sum vector.

The most interesting characteristic of the Markov process is its propensity to attain a steady state where the state probability distribution vector does not change between transitions. Assume that the transition probabilities are constant over time, then if $x_{2}$ is the vector at time 2

$$
\begin{equation*}
x_{2}=x_{1} T \tag{5}
\end{equation*}
$$

Substituting (4) in (5)

$$
x_{2}=x_{0} T^{2}
$$

or generally

$$
x_{n}=x_{0} T^{n}
$$

Now there is a common type of transition matrix for which after some number of transitions the state probability distribution becomes constant, i.e.:

$$
\begin{equation*}
x_{n+1}=x_{n} T \tag{6}
\end{equation*}
$$

or we may write this

$$
\begin{equation*}
x=x T \tag{7}
\end{equation*}
$$

It is not necessary that this should hold for any particular transition matrix. A Markov process in which this characteristic holds, that state probability distributions for a large number of transitions are independent of the starting distribution, is known as completely ergodic. I do not intend to go into the discussion of ergodic and non-ergodic states. ${ }^{2}$
2) Howard, op. cit., has a very elegant discussion of these aspects in his first chapter.

Returning now to the example concerning firms of above and below average profitability, let that matrix be completely ergodic and the steady state distribution is then $(0.6,0.4)$. That is, if the transition nrobabilities are unchanged, a stage will be rearhed when $60 \%$ are in state 1 and $40 \%$ in state 2. Once this has been reached these proportions will be constant over time. ${ }^{3}$ The main comment must, be that it is a very strong assumption that the transition probabilities are unchanged.

Any economic study which presented a steady state distribution as a. forecast would only be reasonable if the steady state distribution was quite similar to the prevailine observed distribution. This is not to deny the value of deriving a steady state distributinn for a transition matrix based on economic data, but that value lies not, except in excentional. circumstances, in accuracy as a predictor but rather in onnvenience as a description of tendencies inherent in gresent conditions and policies. That nressures of one kind or another are very likely to ensure that the steady state is not attained does not cancel the evidence on the desirability or undesirability of present tendencies. For example, if the steady state distribution of income is more inequitable than the present, one, it suggests that the process working to change the distribution of income is inconsistent with any desire to reduce inenuities. This is valuable information, but the steady state distribution is nontheless not to be regarded as a forecast of the future income distribution.

The idea of a steady state distribution does not imply stability for the individuals involved in the process. This is well shown by recourse

[^9]to Marshall's ${ }^{4}$ example of the "trees in the forest" where the number of trees or each height may remain constant but:
n.. one tree will last longer in full vigour and attain a greater size than another; but sooner or later old age tells on them all. Though the taller ones have better access to light and air then their rivals, they gradually lose their vitality, and one after another they give place to others, which, though of less material strength, have on their side the vigour of youth."

Marshall was talking about the growth of firms and it would be reading too much into his writing to clain that he was describing a complete steady state. But his metaphor applies to the situation of a stable frequency distribution describing a population within which individuals are all the time mobile. It is well described as a statistical equilibrium.

The idea of the steady state distribution has been used in empirical economics a number of times, for example, Vandome's investigation of the distribution of income and Adelman's stady of the distribution of firms by size within an industry. 6
4) Marshall A, "Principles of Economics" 8th edition, Macmillan, London, 1949 reprint, Bk IV Ch XIII para 1 p 263.
5) Vandome P, "Aspects of the Dynamics of Consumer Behaviour", Bulletin of the Oxford Institute of Economics \& Statistics, Vol 201958 pp 65-105.
6) Adelman I, "A Stochastic Analysis of the Size Distribution of Firms", Journal of the American Statistical Association, Dec $1958 \mathrm{pp} 893-904$.

## Section 3.2: Continuous Analogue of the Markov Chain

Much has been done to avoid the temporal discreteness of the Markov chain, but this is not a problem when accounting data is being used. On the other hand, the discrete states are inconvenient in economic work as continuous variables are usually employed. Unfortunately the work done on developing the Markov chain in this direction seems to lie in the more unapproachable realms of mathematical statistics. 7 The alternative is to seek some other stochastic model, but the simplicity of the transition matrix idea is valuable. Therefore in this section an attempt is made to develop a continuous analogue of the Markov chain, or to show how the Markov model relates to simple ideas of distributions and conditional probability.

For any particular state probability distribution $x_{0}$, let the probability of a particular state $i$ be written $P\left(i_{0}\right)$. That is, the vector $x_{0}$ is a probability vector and therefore each element is the probability of the corresponding state being occupied under that probability scheme. Similarly for the subsequent vector $x_{1}$ the $j$ th element may be written $P\left(j_{1}\right)$. A conditional probability is defined:
"Let $H$ be an event with positive probability. For an arbitrary event A we shall write

$$
P(A \mid H)=\frac{P(A, H)}{P(H)}
$$

The quantity so defined will be called the conditional probability of A on the hypothesis $H$ (or for given H)! $\mathbf{0}^{8}$

[^10]We can construct such a conditional probability statement: state $i$ is occupied at time 0 , what is the probability of $j$ being occupied at time 1 given this information?

$$
\begin{equation*}
P\left(j_{1} \mid i_{0}\right)=\frac{P\left(i_{0}, j_{1}\right)}{P\left(i_{0}\right)} \tag{8}
\end{equation*}
$$

rearranging:

$$
\begin{equation*}
P\left(j_{1} \mid i_{0}\right) . P\left(i_{0}\right)=P\left(i_{0}, j_{1}\right) \tag{9}
\end{equation*}
$$

summing over all the states at tine 0:

$$
\begin{equation*}
\sum_{i_{0}=1}^{n} P\left(j_{1} \mid i_{0}\right) \cdot P\left(i_{0}\right)=\sum_{i_{0}=1}^{n} P\left(i_{0}, j_{1}\right) \tag{10}
\end{equation*}
$$

The right hand side becomes the probability of $j$ and all possible states at time 0. This latter is unity, therefore the equation may be written:

$$
\sum_{i_{0}=1}^{n} P\left(j_{1} \mid i_{0}\right) \cdot P\left(i_{0}\right)=P\left(j_{1}\right)
$$

This is equivalent to the jth equation of the set given by the matrix equation (4).

Now let the system be described by a single variable so that the N states of the system can now be specified as intervals in the range of the variable. These intervals are not necessarily adjacent and the problem is still in a discrete form. Let this variable be $Z$ and let the ith state be defined by $z_{i} \leqslant z \leqslant z_{i}+\triangle Z$. Let the period be denoted by a superscript so $Z^{0}$ is the value of $Z$ at time 0 and $Z^{1}$ at time 1.

Equation (9) may now be rewritten in this notation:

$$
\begin{align*}
& P\left(z_{j} \leqslant z^{l} \leqslant z_{j}+\Delta z \mid z_{i} \leqslant z^{0} \leqslant z_{i}+\Delta z\right) \cdot P\left(z_{i} \leqslant z^{0} \leqslant z_{i}+\Delta z\right) \\
& =P\left(z_{j} \leqslant z^{1} \leqslant z_{j}+\Delta z, z_{i} \leqslant z^{0} \leqslant z_{i}+\Delta z\right) \tag{12}
\end{align*}
$$

It is now possible to introduce probability density functions into the relationship. We may write the conditional probability of being in the interval $Z_{j}$ to $Z_{j}+\Delta Z$, given the state at time 0 is the interval $Z_{i}$ to $Z_{i}+\Delta Z$ as $g\left(Z_{j} \mid Z_{i}\right) Z Z$, the function being defined for a standard value of Z. Similarly, the state probability distribution may be described by a function $f\left(Z_{i}\right) \Delta Z$. The time zero distribution may be written $f^{0}\left(Z_{i}\right) \Delta Z$ and the distribution at time $1: f^{l}\left(Z_{i}\right) \Delta Z$. Each of these functions is dependent upon a standard value of $\triangle \mathrm{Z}$.

At this point it is perhaps helpful to explain the steps so far taken. The aim is to show the relationship between the conventional Markov model of discrete states and a variant allowing a continuous variable to fulfill the function of the states of the system. The initial stage was to convert from the specific notation of Markov chains to standard probability notation. For this step equation (4) is shown to be derivable from the definition of conditional probability, once the elements of the transition matrix are recognised as conditional probabilities and the state vectors are converted to state probability distributions. As the exact meaning of the elements of the state probability distributions is perhaps not yet clear, they may be regarded as the probability of a particular state being occupied at a particular time by a particular individual. Thus in the example employed before, if there are a number of companies operating subject to the given transition matrix, then the probability of company $A$, about which one has no previous knowledge, being in state 1 at time 1 is given by $x_{11}$.

With the translation to probability notation, we may choose to define the states in any way we wish. The use of intervals in the range of a continuous variable is selected here as a useful step towards the aim of defining the states of the system as a continuum. The last notational ohange takes us further towards our end result where functions must replace vectors and matrices if continuity is to be achieved. In this newest notation it is important to emphasize that the functions are defined for a specific interval in the range of $Z$. Using new functions in equation (1l) we get:

$$
\sum_{i=1}^{n} g\left(z_{j} \mid z_{i}\right) \Delta z \cdot f^{0}\left(z_{i}\right) \wedge z=f^{1}\left(z_{j}\right) \Delta z
$$

If at this stage the intervals of $Z$ are assumed to be adjacent, then these variables may now be regarded as continuous and once $\Delta Z$ tends to zero the problem is converted to a straightforward continuous one and (13) may be written in integral form. One difficulty needs dealing with first; the summation is only over the states occupied in the first period, that is, $Z_{i}$ varies but $Z_{j}$ does not. This leads to a rather confusing notation, therefore let the final state be denoted by $W$ and the initial state by $Z$.

Then (13) becomes:

$$
\begin{equation*}
\int_{Z=a}^{Z=b} g(W \mid Z) f^{0}(Z) d Z d W=f^{1}(W) d W \tag{14}
\end{equation*}
$$

Where $a$ and $b$ are the linits of the range of the continuous state variable. Just as (13) is equivalent to the jth equation of the set summarised by (4) so is (14). The function of $g(W / Z)$ is the analogue
of the transition matrix. ${ }^{9}$ It is not therefore a bivariate joint distribution, but rather a set of conditional distributions of $W$ for given values of 2 . Therefore:

$$
\int_{W=\mathbf{a}}^{W=\mathbf{b}(W \mid z) d W=1}
$$

$g(W \mid Z)$ will be referred to as the transition function in what follows.

In the later empirical work it will be the transition function that we are investigating, summarising, as it does, all probability changes within an industry. But predominantly attention will be directed at one function that may be derived from it. This relates the mean of the conditional distribution for a given $Z$ to the value of $Z$. Other functions considered are those that relate the variance, skewness and kurtosis of the conditional distribution to the value of the prior variable. The results of this work are described in Chapter V. For the present the need is to clarify the empirical method and the meaning of such functions.

[^11]
## Seotion 3.3: The Transition Function in an Empirical Context

The transition function sumarises all the year to year changes in rates of return (or any other variable to which it is applied). To identify its functional form and estimate the parameters involved would clearly be the ideal. But this is a difficult job that is made more so by some of the characteristics of the transition process that will be described in Chapter 5. It is therefore likely that any function fitted directly would involve considerable compromise. For these reasons the problem is attacked by considering the relationships between various summary statistics of the conditional distributions and the value of the prior variable. For example, the relationship between $E(W \mid Z)$ and $Z-i . e$. the relationship between the mean of the conditional distribution for a given $Z$ and the value of $Z$ ( the prior variable). Clearly such an approach could still lead to an estimation of the transition function.

An example of how this might be done can easily be set out. Let the mean of the conditional distribution be given by the function $\mu(Z)$ for any Z and let the standard deviation be given by the function $\sigma(Z)$. Then, presuming that the distributions are symmetric and mesokurtic, the normal distribution may be taken to be a satisfactory approximation for the transition function. The normal distribution may be written in the form:

$$
f(x)=\frac{1}{\sigma \sqrt{2 \pi}} \quad \exp \left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}\right\}
$$

Substituting for $\mu$ and $\sigma$ we may write:

$$
g(W, Z)=\frac{1}{\sigma(Z) \sqrt{2 \pi}} \quad \exp \left\{-\frac{1}{2}\left(\frac{W-\mu(Z)}{\sigma(Z)}\right)^{2}\right\}
$$

Were the distribution function to involve extra parameters corresponding to higher moments of the distribution, then a similar approach could be adopted. In other words, the procedure consists in observing how various measures of the conditional distributions (the row distributions of the transition matrix) vary as one moves over the range of the prior variable. This information can then be inserted into a univariate distribution function which fits the conditional distributions.

In fact this step becomes less important when it is realised that it
is these very functions relating characteristics of the conditional
distributions that are of prime interest. Actually estimating the
transition function is unnecessary. The first function is that relating
$E(W \mid Z)$ to $Z$; this is the regression line of $W$ on $Z$, if we quote Noel: 10
"A theoretical regression curve is basically the graph of the mean of the conditional distribution $f(y \mid x) \ldots$ (or) the locus of such mean points, that is, the graph of $\mu_{\mathbf{y} \mid x}$ as a function of $x \ldots .$.

Thus we see that it is only study of the higher moments of the conditional distributions that give more information than would straightforward regression analysis. On the other hand, information on the whole transition function makes it less likely that an inapproprate form of function will be fitted to the means. 11

Turing now to the variance relation, its interpretation in regression analysis terms is again illuminating. If the mean relationship is the regression line of $W$ on $Z$, the variance relationship describes the
10) Hoel P G, "Introduction to Mathematical Statistics" 3rd edition, J Wiley \& Sons Inc New York 1962, p 194
11) Another advantage is that the demands on computer size can be much less than for a straightforward regression. This is relevant when, as in this study, there would be up to 3500 observations for a single regression. This point, with respect to grouping data, is made by Prais $S \mathrm{~J}$ and Aitchison J: "The Grouping of Observations in Regression Analysis". Review of the International Statistical Institute Vol 22, pl.
errors of the regression, and so if the variance is related to $Z$ the simple regression of $W$ on $f(Z)$ would suffer from heteroscedasticity. This, in

$$
W=f(Z)+\epsilon
$$

$\epsilon$ is not distributed with constant variance and its variance is not independent of 2 . What of the regression line relating the means of the conditional distributions to a function of $Z$ ? If there is heteroscedasticity in the straightforwardly estimated equation, then this equation will also have heteroscedastic errors. The errors in this equation are the result of sampling errors in the means of the conditional distributions and this will have a variance given by $\{\sigma(Z)\}^{2} /\{N(Z)-1\}$ where $\sigma(Z)$ is the standard derivation of the conditional distribution given $Z$, and $N(Z)$ is the number of observations in that distribution. So if there are heteroscedastic errors in the straightforward regression of $W$ on $f(Z)$ there will also be the same problem in the regression using the conditional distributions. That is, unless $N(Z)$ varies with $Z$ so as to compensate for the variation in $\sigma(Z)$. One final point is that the total sum of squares in the regression of the conditional distribution means will be much lower than in the straightforward regression and this will lead to a much higher $R^{2}$.

Finally, the skewness and kurtosis of the conditional distributions are primarily of interest as an indication of how far these distributions deviate from the normal. Major divergence from normality would indicate that the usual tests of significance on the estimates produced by a straightforward regression would be inexact. This applies more to skewness than kurtosis, as $t$ and $F$ tests are robust as long as the
distributions are unimodal and approximately symmetrical. On the other hand, the regression using the means of the conditional distribution will avoid this problem to a considerable extent as the skewness of the sampling distribution of the mean is much less than the skewness of the original distribution. ${ }^{12}$ A similar result applies to the kurtosis, the sampling distribution being more mesokurtic than the original distribution. ${ }^{13}$ In other words, the regression using the means of the conditional distributions will be closer to possessing the desirable properties of having normally distributed errors than a straightforward regression.
12) See Croxton Cowden \& Klein, "Applied General Statistics" 3rd edition, Pitman, London 1968, p 538-9
13) Croxton Cowden \& Klein op.cit., p 540-541

## Section 3.4: Summary

The Markov chain is a stochastic model that describes transitions from one state to another. In the usual first order model, the only factor influencing the probability of a transition to a particular state is the present state occupied. This simple model can be developed to permit the substitution of a continuous variable for the set of discrete states. Such a substitution leads to a transition function rather than a transition matrix.

The form of the transition function can be investigated by considering the relationships between the value of the prior variable and the characteristics of the conditional distributions produced by setting a value to the prior variable. In particular the relationship involving the mean of the conditional distribution is equivalent to the regression of the final variable on the prior variable. The functions involving higher moments provide information on the errors of that regression.

THE DATA

This chapter is concerned with the data used - their origin, nature and problems. In Section 4.1 the history of the company accounts data is briefly given and their overall scope described. Section 4.2 deals with the rate of return employed in this study, the reasons for selecting it and the way it is calculated from the company accounts data. Section 4.3 brings us to two problems of time; firstly the choice of period to be used in the analysis and secondly whether to correct for differences in accounting date, and if 80 , how. Section 4.4 describes the sample of companies for which the company accounts data is available and the classification of those companies first into industrial orders and then into more narrowly defined industry subgroups. Finally, in Section 4.5, the annual distributions of the rate of return by industry are examined.

## Section 4.1: The Data

The National Institute of Economic and Social Research started to collect and standardize the accounts of UK quoted companies after the passing of the 1948 Companies Act had set new standards for the information to be provided in published accounts. They continued this work for five years for all quoted UK companies other than those engaged mainly in financial activities, shipping and agriculture. 1,2 After the National Institute had ceased this work, the Board of Trade continued it. In 1961 the semple was considerably reduced, therefore the data 1948-1960 are conveniently used where long runs of observations for a large number of companies are required.

The Department of Applied Economics at Cambridge converted the data for this period on to magnetic tapes. The results of their use of the data are reported in "Growth, Profitability and Valuation" by A Singh and G Whittington, CUP 1968, "The Prediction of Profitability" by G Whittington, CUP 1971, and "Takeovers" by A Singh, CUP 1971. The first of these three books contains a useful account of the data in Appendix A. The Canbridge magnetic tapes were further organised at Stirling for convenience of use, but the company records are just as used at Cambridge.

Briefly there is for each company in the sample a record for each year that the company existed. ${ }^{3}$ This record consists firstly of indicative

1) NIESR, "Company Income and Finance 1949-1953" op.cit, summarises and describes the data.
2) Tew B \& Henderson R F (eds), "Studies in Company Finance", CUP 1959, does further analysis on these data.
3) A few were brought into and removed from the sample during the period, but generally companies appear when they gain a quotation and disappear upon death or merger.

\footnotetext{
data specifying, amongst other things, the accounting date, the industry and the industry sub-group to which the company belongs. The second and major part of the record is a set of standardized accounts for the company for that year. This comprises a Balance Sheet, Appropriation of Income Statement and a Sources and Uses of Funds Statement. The components of these accounts are listed in Table 4.1.

Table 4.1 : List of Standardized Variables in the Basic Accounting Data


Variable No. Title
Assets

Fixed Assets: tangible, net of depreciation
do : intangible
do : trade investments
Stocks and work in progress
Trade and other debtors
Marketable securities
Tax reserve certificates
Cash
Sumaxy
Total net assets
Sources of Funds
$\begin{array}{cl}\text { Issue of Shares } & \text { Ordinary } \\ \text { do } \quad: \text { Preference }\end{array}$
Increase in liability to minority interests
Issue of long term loans
Bank credit received
Trade and other credit received
Increase in dividend and interest liabilities
do current tax liabilities
do future tax reserves
Balance of profit : depreciation provision
do : provision for amortization
do : other provisions
do : retained in reserves
Other receipts


## Variable No. Title

Summary

60
61
62
63
64
65
66
67

Total capital and reserves (items 1 to 5)
Total liabilities (items 7 to 12)
Total fired assets, net of depreciation (items 14 to 16)
Total current assets (items 17 to 21)
Total sources (items 23 to 36)
Total uses (items 37 to 43)
Total profit (items 50 to 52)
Total balance of profit (items 32 to 35)

Taken from Singh \& Whittington op. cit., Appendix C.

Section 4.2 : The Rate of Return

The present analysis is concerned with the rate of return on net assets. The numerator is calculated gross of tax and net of depreciation. It is thus insulated from the immediate effects of changes in tax rates or the tax system. But in so far as accounting figures permit, capital consumption is deducted. Profits are also calculated before deduction of interest on long term debt, so that the effects of variations in capital structure are removed. ${ }^{4}$ Included in this profit figure is investment and other income. This is on the assumption that such income is usually derived primarily from activities within the same industry as that of the firm. In general it is small relative to operating profit so the choice of inclusion is unlikely to have any significant effect. In terms of the accounting quantities listed in Table 4.1, the profit figure used is the sum of operating profit (before depreciation) (variable number 50), dividends and interest received (gross of income tax) (51), other income (52) and prior year adjustments (general) (59), minus the three components of balance of profit - depreciation provision (32), amortization provision (33) and other provisions (34).

Such a quantity departs considerably from the economic concept of profit - including as it does income to be paid as interest explicitly. It is, of course, unavoidable that any reported profit figure bears rather a distant relation to economic profit: in many cases some component that is strictly management wages will be included and some part of the income accruing to the equity holders is strictly interest.
4) Some interest is deducted before the operating profit figure is presented - bank interest for example. The removal of capital structure effects is therefore not complete.

One has to trust that the relation with the pure economic concept is sufficiently olose for the analysis to be interpreted in terms of eoonomic theory.

The denominator of the rate of return is net assets. This is calculated as the sum of issued capital (ordinary and preference) (variable numbers 1 and 2), capital and revenue reserves (3), future tax reserves (5), interest of minority shareholders in subsidiaries (7) and long term liabilities (8). This encompasses what is usually known as Capital and Reserves. 5 From the balance sheet identity it can be deduced that it is equal to the sum of fixed and current assets minus current liabilities, which may be regarded as fixed capital plus working capital. The fixed capital component being net of depreciation. Note also that, just as the profit figure includes investment income, so the net assets figure includes trade investments (variable number 16 ). The choice of net assets rather than equity assets, or any other denominator, is based on the arguments of Chapter II in terms of resource allocation. Net assets being the sum of the two types of capital employed (fixed and working). On the other hand, what evidence there is suggests that the rate of return on equity assets behaves very sinilarly to the rate of return on net assets. ${ }^{6}$

This atudy is concerned with changes in rates of return - comparing values for two years for the same company - and the consequences of the weaknesses in the data are therefore not too serious. For example,
5) In Table 4.1 provisions are included and minority interests and long tern debts are excluded. This is to accord with the NIESR and Board of Trade treatment.
6) Singh \& Whittington op.cit. Ch 6.5. P E Hart (ed), "Studies in Profit, Business Saving and Investment", Vol 1, Allen \& Unwin 1965, Ch 8 "Alternative Measures of the Size of Firms" $p$ 149: "... in practice it does not seem to matter very much which measures (of aize) are used, since they are mostly highly correlated with each other."
any persistent undervaluation of assets will tend to be reduced
in importance through these year to year comparisons. Secondiy, all the analysis is within industries or more narrowly defined groups of firms and relative to the average of the industry or group. Common accounting practices are thereby allowed for. But finally there is no choice. Measures derived from the accounts have to be used and as such quantities are the information or part of the information used within industries to guide resource allocation, they are not inappropriate measures to employ here.
7) Hart P E (ed), "Studies in Profit, Business Saving and Investment", Vol II, Allen \& Unwin 1968, p 269: "Rates of return calculated from balance sheets of samples of companies may be used for this purpose because it was found .... that accounting data are a reliable guide to trends in rates of return, in spite of the well known objections to balance sheet figures."

## Section 4.3 : The Accounting Period and Accounting Date

There is nothing in economic theory to suggeat the correct interval at which rates of return should be calculated. On the other hand, the data constrain us to using a period of one year or a number of years. Fractions of years are ruled out by the convention of the annual account.

There are two main arguments for using a period of more than one year. The first is that any concern with resource allocation is concerned with the long run. But to this it may be said that in the case of many capital grods the long run is less than a year. Additionally, the reallocation of working capital and labour can usually be achieved (or partically achieved) in less than a year. Complete adjustment of the allocation of productive resources may be a lengthy process but partial adjustment - and some profitability effects - will generally be possible within the basic accounting period.

The second argument is that year to year changes in rates of return will include many random factors that would average out over a longer period. This is undeniable but the assumption must be that these weaken the postulated relation between rates of return in adjacent periods rather than biasing that relation. The method of statistical analysis described in Chapter III, in effect, averages over large numbers of observations and so reduces the impact of these random factors. Therefore it may be said that this second argument is one of statistical desirability and the results to be reported later will deal with it.

It seems fair to conclude that the case against using the single year as a basis is not strong. On the other hand, the empirical argument for not wasting observations seems very strong. Therefore the single
vear is hereafter used. The likely effect is to provide us with a. rother low ficure for tie rate of decay of profitability. ${ }^{8}$ It will be low because if resource allonation is slow, only a small amount will be comoleted within the year and therefore only a, small movement of rates of return brought about. But with each industry covering a number of markets and firms some reallocation will occur within any year and so therefore there will be some decay of profitability.

The other time period problem is that of accounting date. Firms' accounting years end at different times in the year, although this does not matter in so far as we are looking at year to year changes in rates of return. But it causes problems because we are using rates of return relative to the industry experience - the industry mean. The industry mean for the year ending March 31st is different to that for the year ending December 31st. Refore proposing a solution to this problem it is necessary to clarify the function of the industry average rate of return in this work. Primarily it is that rate of return to which the decay of profitability is assumed to ocour - the decar crigin. This implies that markets wjith rates of return below that level are likely to experience a net, withdrawal of assets, whereas markets with rates of return above that level are likely to experience a net addition of assets. Such a phenomenon will not follow the precise industry average but rather some idea of mean experience. 'This is likely to have a lagged resnonse to actual changes in the average. For this reason the slight variations from one quarter to anothor are likely to be unim. portant, therefore on this basis industine for accombiner dete would not be necessary.
8) Whittineton op cit estimaties a function like the linear recression used labar, but; on fisures that, are five year averames. remay therefore expect, that he firids a hishor mate of docaiv of mofit... ability. See Ohortor $X x$ sotion 3.

But the rate of return average does serve another purposes it
removes the variations in rates of return that are experienced by all firms in the industry. Effects of macroeconomic circumstances are thus dealt with. The process of inter-industry equilibration is also removed. ${ }^{9}$ Clearly netting out these effects would be done more completely if allowance were made for variations in accounting date. There is therefore a choice to be made and it has been decided that allowance should be made for accounting date. It is very difficult to see that this will lead to any consistent bias in the estimated rate of decay of profitability and it does have the empirical advantages outlined earlier in this paragraph.

Having taken this decision, the next stage is to describe how the allowance is made. Firstly the data ascribes each company to a quarter according to its accounting date. Each company whose financial year ends in a particular quarter is treated as having the same financial year. Given that the two most comon accounting dates are at the end of quarters December 31st and March 3lst, this assumption does not involve any serious approximation. Secondly the average rate of return is calculated for each industry for each year for each accounting date. This average is the average rate of return of companies in the sample, i.e. most quoted companies. Theoretically it should include all and not just quoted companies but the lack of data makes this impossible and as in most industries the quoted companies account for a very large portion of total assets or total tumover, the divergence is probably not serious. In a few industries the number of companies with a particular accounting date is small and the mean is therefore

[^12]liable to considerable error. The ameliorating factor is that where there are few companies involved their impact on the full body of data for that industry will also be small. Industry 8 is an exception to this but, throughout, the small number of companies for this industry makes its figures suspect.

Just as means by accounting date have been used, so the standardization has employed standard deviations by accounting date by year by industry. The caveat in the case of small numbers applies a fortiori to this.

Finally it must be mentioned that in calculating both means and standard deviations very extreme observations have been thrown out. In practice any rate of return with an absolute value greater than $150 \%$ has been rejected - there were twenty such observations. These are indicated in Table 4.2 which also gives the numbers of companies with each accounting date by industry.

In summary, the data are standardized, the means and standard deviations are calculated separately and applied separately according to both year and accounting quarter.

|  |  |  | No , of Co | mnanies |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Industixy | $\frac{6 \text { April }}{-5 \mathrm{JuI}}$ | $\frac{6 \mathrm{July}}{-5 \mathrm{oct}}$ | $\frac{60 c t}{-5 \mathrm{Jan}}$ | $\frac{6 \mathrm{Jan}}{-5 \mathrm{April}}$ |
| 1 | Bricks, pottery, glass \& cement | 14 | 16 | 58 | 33 |
| 4 | Shipbuilding \& non-electrical ongineoring | 42(1) | 59 | 136 | 84 |
| 5 | Electrical engineoring | 13 | 17(1) | 48 | 46 |
| 6 | Vehicles | 11 | 36 | 31 | 15 |
| 7 | Metal goods n.e.s. | 16 | 37 | 85(1) | 51 |
| 8 | Cotton \& man-made fibres | 4 | 5 | 9 | 21 |
| 9 | Woollen \& worsted | 7 | 8 | 31 | 26(2) |
| 11 | Clothing \& footwear | 15 | 15 | 51(3) | 17 |
| 12 | Food | 20 | 22 | 55 | 35 |
| 13 | Drink | 19 | 83 | 33 | 41 |
| 15 | Paper, printing \& publishing | 31 | 13 | 55 | 44 |
| 16 | Leather, leather goods \& fur, timber, furniture, other manufacturing | 21 | 33(1) | 84(3) | 50(3) |
| 17 | Construction | 5 | 6 | 33 | 13 |
| 18 | Wholesaling | 47(2) | 20 | 120(2) | 82 |
| 19 | Retailing | 14 | 31 | 37 | 106 |
| 20 | Entertainment \& sport | 13 | 15 | 39 | 17 |
| 21 | Transport \& communication Miscellaneous services | 32 | 35 | 70 | 36 |

Numbers are for 1954. Those in brackets indicate number of company years omitted from that industry and accounting date, not just for
1954 but for all years 1948-1960.

Section A. 4 : The Nature of the Sample

The analysis is mainly cone by industry although some smaller subdivisions - referred to as "industry smborouns" - are employed later. The industries and the number of comnanies in each is shown in Table 4.3. Also given in this tahle are the number of najrs of years observations, that is, the number of transitions. The industry classification is based on the 1948 Standard Industrial Classification. It has, of course, to be a little arbitrary as classifying financial units must involve more anomalies than classifying establishments. Nonetheless, an exeruise carried out by the Board of Trade revealed that $87 \%$ of employees worked in esteblishments belonging to the industry to which their employing firm was classified. ${ }^{10}$ Therefore the broad irdustry classification is probably satisfactory.

When we turn to the industiry subgroups the extent to which activities not relating to that groun become included must increase considerably. Only those subgrouns that have over 20 members have been used and these are show, together with their meaning in terms of 1948 S.I.G. minimum list headincs, in Table 4.4. Forty subgrouns have been used out of a possible 71. Agein the classification was done during the collection of the data by the NIESR and the Board of Trade.

One way in which the sample does differ from that used by Singh and Whittington is that it has not been restricted to continuing companies. Every transition from one rate of return to the noxt year's has been used. ${ }^{17}$ This means that nearly double the number of companies can be
10) See NTESR on. cit. Annendix A for the report of this work.
11) There is one renerat excention: tho nrofit ficure uses values from the Sources and llses statrement. As this is nrodnced by comnaring two Ralance Sheetis, it; is nevor availahle for the first, vear in which a. compny anneare in the rata. Thoreforn the firsti rate of return that is ushble mofers to the oomnanyis socond veate
used; of course, as the companies brought in in this way provide fewer years observations than continuing companies, the number of transitions is by no means doubled. The number of continuing companies, companies born and companies dying in the data is shown in Table 4.5.

Table 4.3 : Number of Companies and Transitions for Each Industry

| Code | Industry | $\begin{aligned} & \text { No. of } \\ & \text { Companies } \\ & \hline \end{aligned}$ | $\frac{\text { No. of }}{\text { Transitions }}$ |
| :---: | :---: | :---: | :---: |
| 1 | Bricks, pottery, glass \& cement | 146 | 1227 |
| 4 | Shipbuilding \& non-lectrical engineering | 370 | 3199 |
| 5 | Electrical engineering | 146 | 1261 |
| 6 | Vehicles | 107 | 916 |
| 7 | Metal goods not elsewhere specified | 217 | 1906 |
| 8 | Cotton \& man-made fibres ${ }^{+}$ | 44 | 376 |
| 9 | Woollen \& worsted ${ }^{+}$ | 73 | 729 |
| 11 | Clothing \& footwear | 118 | 994 |
| 12 | Food | 152 | 1299 |
| 13 | Drink | 206 | 1753 |
| 15 | Paper, printing \& publishing | 167 | 1470 |
| 16 | Leather, leather goods, fur Timber, furniture, Other manufacturing | 212 | 1863 |
| 17 | Construction | 75 | 586 |
| 18 | Wholesaling | 294 | 2620 |
| 19 | Retailing | 236 | 1841 |
| 20 | Entertainment \& sport | 95 | 872 |
| 21 | Transport \& communication Miscellaneous services | 333 | 2260 |

```
* After omitting all rates of return more than three standard
    deviations from the mean - see Section 5.1.
+ This is only a part of the industry - see Table 4.5.
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The industry codes and descriptions are taken from Whittington op. cit. p 6 Table l.1. Certain industries - Chemicals \& Allied Industries (2), Metal Manufacture (3), Hosiery, Carpets \& Other Textiles (10) - have been omitted as their data was not available. The Tohacen industry (14) has heen omittod as heing too small.

Table 4.4: Subgroup Definitions and the Number of Companies and Transitions for Each

| Code | Industry Subgroup SIC | $\frac{\text { SIC Minimum List }}{\text { Headings }}$ | $\frac{\text { No. of }}{\text { Companies }}$ | $\frac{\text { No. of }}{\text { Transitions }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1.2 | Building materials 461, | $\begin{gathered} 461,469,102,103, \\ 109(5) \&(3) \end{gathered}$ | 98 | 804 |
| 1.3 | Pottery | 462 | 27 | 220 |
| 4.1 | Shipbuilding | 370 | 38 | 351 |
| 4.2 | Machine tools | 332, 333 | 33 | 314 |
| 4.4 | Constructional engineering | 341 (2) | 23 | 206 |
| 4.5 | Other engineering 3 | $\begin{aligned} & 331,334,336-9, \\ & 341(1), 342,349 \end{aligned}$ | 258 | 2219 |
| 5.3 | Wireless etc. | 363, 364 | 28 | 268 |
| 5.4 | Other electrical manufactures | res 365, 369 | 77 | 615 |
| 6.4 | Vehicle components | 381, 382 | 38 | 340 |
| 7.1 | Other metal goods 36 | $\begin{gathered} 364(2), 391-6,399, \\ 499(1) \end{gathered}$ | , 176 | 1514 |
| 7.2 | Instruments etc. | 351, 352 | 43 | 418 |
| 8.1 | Cotton spinning | 412 | 31 | 255 |
| 9.1 | Wool | 414 | 66 | 658 |
| 11.1 | Clothing | 441-446, 449 | 86 | 725 |
| 11.2 | Footwear | 450, 888 | 32 | 278 |

Table 4.4 (continued)

| Code | Industry Subgroup | $\frac{\text { SIC Minimun List }}{\text { Headings }}$ | $\frac{\text { No. of }}{\text { Companies }}$ | $\begin{aligned} & \text { No. of } \\ & \text { Transitions } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 12.2 | Baking etc. | 212, 213 | 34 | 306 |
| 12.4 | Sweets | 217 | 33 | 257 |
| 12.6 | Other food | 215, 218, 219, 229 | 43 | 364 |
| 13.1 | Brewing | 231, 810(1) | 184 | 1572 |
| 15.1 | Paper | 481-483 | 79 | 674 |
| 15.2 | Newapapers | 486 | 33 | 322 |
| 15.3 | Printing etc. | 489 | 55 | 483 |
| 16.1 | Rubber | 491 | 33 | 316 |
| 16.2 | Timber | 471, 474-5, 479 | 43 | 365 |
| 16.3 | Furniture | 472, 473 | 43 | 373 |
| 16.4 | Leather | 492, 431, 432 | 32 | 307 |
| 16.5 | Other manufactures | 433, 493-496, 499(2) | 61 | 502 |
| 17.1 | Building | 500 | 68 | 531 |
| 18.1 | Food wholesale | 810(1) \& (2) | 57 | 526 |
| 18.2 | Building merchants etc. | 831 | 57 | 506 |
| 18.3 | Other wholesale | $831,832,810(3)-(8)$ | ) 150 | 1432 |
| 19.1 | Food retail | 820(1) \& (2) | 45 | 388 |
| 19.2 | Stores | 820 (6) \& (7) | 45 | 326 |
| 19.3 | Other retail | 820(3) \& (5), 831, 887 | 7145 | 1117 |
| 20.2 | Dog racing | 882-3 | 26 | 263 |
| 20.3 | Entertainment | 881 (2) | 52 | 461 |

Table 4.4 (continued)

| Code | Industyy Suberoup | $\frac{\text { SIC Minimum List }}{\text { Hoadings }}$ | $\frac{\text { No. of }}{\text { Companies }}$ | $\frac{\text { No. of }}{\text { Transitions }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 21.2 | Catering etc., hotels etc. | 884 | 74 | 677 |
| 21.3 | Laundries etc. | 885-6 | 28 | 244 |
| 21.4 | Storage | 709(2) | 28 | 245 |
| 21.5 | Transport \& communication | $\begin{gathered} 702,703,705-7, \\ 709(1) \&(3) \end{gathered}$ | 30 | 270 |
| 21.6 | Other services | 889, 899 | 42 | 342 |

Source of definitions: Board of Trade working paper.

Table 4.5 : Numbers of Births, Deaths and Continuing Companies by Industry

| Code | Industry $\quad \frac{\text { To }}{0}$ | $\begin{aligned} & \text { Total no. } \\ & \text { of cos. } \end{aligned}$ | $\frac{\text { No. of }}{\text { continuing }} \text { cos. }$ | $\frac{\text { No. of }}{\text { deaths }}$ | $\frac{\text { No. of }}{\text { births }}$ | $\text { Double }^{* *}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Bricks, pottery etc. | 145 | 81 | 26 | 42 | 4 |
| 4 | Non-electrical engineering | ng 369 | 214 | 70 | 102 | 17 |
| 5 | Electrical engineering | $147{ }^{+}$ | 84 | 33 | 35 | 6 |
| 6 | Vehicles | 107 | 57 | 30 | 23 | 3 |
| 7 | Metal goods n.e.s. | 217 | 128 | 38 | 60 | 9 |
| 8 | Cotton \& man-made fibres | $44^{*}$ | 18 | 16 | 15 | 5 |
| 9 | Woollen \& worsted | $73^{*}$ | 50 | 17 | 10 | 4 |
| 11 | Clothing \& footwear | 118 | 70 | 26 | 26 | 4 |
| 12 | Food | 152 | 75 | 51 | 35 | 9 |
| 13 | Drink | $208{ }^{+}$ | 103 | 83 | 28 | 8 |
| 15 | Paper, printing, publishing | $169+$ | 105 | 35 | 33 | 5 |
| 16 | Other manufacturing | 208 | 124 | 44 | 42 | 2 |
| 17 | Construction | 75 | 40 | 8 | 29 | 2 |
| 18 | Wholesaling | 294 | 170 | 65 | 67 | 8 |
| 19 | Retailing | $237{ }^{+}$ | 112 | 79 | 56 | 11 |
| 20 | Entertaiment etc. | 95 | 65 | 19 | 11 | 0 |
| 21 | Misc. services | $333^{+}$ | 125 | 182 | 28 | 7 |
|  | TOTAL | 3015 | 1624 | 832 | 651 | 102 |

** This allows for companies that were born after the start of the period and died before the end.

+ These rows do not add correctly, due to the presence in the data of a few companies for which some observations in the middle of the period are not available.
* These are smaller than the corresponding figures in Whittington on. cit. Table 1.2 because some companies were temporarily inaccessible on the Stirling magnetic tapes.


## Section 4.5 : Annual Distributions of Rates of Return

Before looking at the transition functions of rates of return from one year to the next, it is useful to look at the distributions of rates of return in any given year. Clearly the form of these distributions will have a profound influence upon the transition function, and in particular on the distribution of $r_{t}$ (rate of return at time $t$ ) for a given range of $\dot{r}_{t-1}$.

For each industry, for each year, the mean, variance, skewness and kurtosis have been calculated. Note that this has been done before the standardization of the data mentioned in Section 2.6. The results for the Shipbuilding \& Mechanical Engineering Industry (No. 4) are shown in Table 4.6 and for all industries in Appendix. A.

The measure of skewness used is that based on the third moment of the distribution. ${ }^{12}$ This measure is computationally the most convenient and there is a significance test available. ${ }^{13}$ The precise form of the measure $\left(\beta_{1}\right)$ is $\mu_{3}^{2} / \mu_{2}^{3}$ where $\mu_{2}$ and $\mu_{3}$ are second and third central moments of the distribution. The second moment providing a scale factor so that the measure is of relative skewness. A symmetrical distribution will have $\beta_{1}$ equal to zero and in particular this will be so for the normal distribution. The significance test is based on the null hypothesis that the distribution is normal. This measure, involving as it does the square of the third moment, does not indicate the direction of skewness. This is recorded in Table 4.6, from the sign of the third central moment.
12) This leads to considerable sensitivity to outlying observations.
13) Pearson E.S., "A Further Development of Test of Normality", Biometrika Vol XXII pp 239 ff. Tables reproduced in Croxton, Cowden \& Klein op. cit. Appendix 0.

Table 4.6: Annual Distributions for the Shipbuilding and Mechanical Engineoring Industry (No. 4)

| Year | Mean | Variance | Sign of <br> Skewness | Skewness | Kurtosis | $\frac{\text { No. of }}{\text { Firms }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1949 | 0.2090 | 0.01556 | - | 0.01760 | $5.694^{*}$ | 274 |
| 1950 | 0.2189 | 0.01748 | + | 0.005377 | $6.94^{*}$ | 283 |
| 1951 | 0.2394 | 0.01830 | + | $0.6859^{*}$ | $7.875^{*}$ | 292 |
| 1952 | 0.2200 | 0.01385 | + | $0.209^{*}$ | $4.288^{*}$ | 304 |
| 1953 | 0.2008 | 0.01532 | + | $0.1334^{*}$ | $7.705^{*}$ | 312 |
| 1954 | 0.2039 | 0.01211 | - | $0.1612^{*}$ | $5.816^{*}$ | 315 |
| 1955 | 0.2006 | 0.01015 | + | 0.02550 | 3.064 | 324 |
| 1956 | 0.1900 | 0.01121 | + | 0.02979 | $4.50^{*}$ | 332 |
| 1957 | 0.1845 | 0.01047 | - | 0.003957 | $3.935^{*}$ | 328 |
| 1958 | 0.1637 | 0.01158 | - | 0.01189 | $4.88^{*}$ | 330 |
| 1959 | 0.1605 | 0.01152 | + | 0.008471 | $4.96^{*}$ | 313 |
| 1960 | 0.1527 | 0.01356 | + | $0.1239^{*}$ | $6.833^{*}$ | 298 |

[^13]The varying number of decimal places is a result of the need to use a computer output format of four significant figures.

All rates of return greater than 100\% or less than - $100 \%$ have been excluded from the calculation.

The measure of kurtosis used is that known as $\beta_{2}$ : the ratio $\mu_{4} / \mu_{2}{ }^{2}$, that is, the fourth central moment divided by the second central moment squared. The denominator providing (as in $\beta_{l}$ ) a scale factor to ensure a relative measure. $\beta_{2}$ takes the value 3 for the normal distribution. A leptokurtic (peaked) distribution has a value greater than 3, whilst a platykurtic (flat) one has a value below 3. A test for whether the distribution is significantly non-mesokurtic is available. 14

Referring now to Table 4.6 , the means for this industry show the common pattern for the 1950s: a decline in the rate of return. As this is the pattern reported by such studies as that of Samuels \& Smyth ${ }^{15}$ and as the results will be required in Chapter TX, a regression of the average rate of return against time has been done for each industry. The results are reported in Table 4.7, columan 1, 2 and 3. Nine out of seventeen industries have a significant trend and of these only those of the Drink Industry (No. 13) and Miscellaneous Services (No. 21) are upward. Overall the slope coefficient is negative in 13 out of 17 cases.
'The main question of interest at this point is the stability of the average rate of return. Brief inspection shows that there are only rare cases of the average falling below $10 \%$ of exceeding $25 \%$ and a good number of these occur in the Textile Industries (Nos. 8 and 9) whose varied career in the 1950 s is notorious. The stability across years within industries, even without taking note of the trend, is considerable. The coefficient of variation (Table 4.7, column 7) ranges between 0.76968 for Industry 8 (Cotton) and 0.08077 for Industry 5 (Electrical
14) Pearson E S op. cit., also Croxton, Cowden \& Klein op. cit., Appendix $P$.
15) Samuels J M \& Smyth D J, "Profits, Variability of Profits and Firm Size", Economica Vol 35 pp 127-140.

# (7) <br> 0.0808 <br> 0.129 <br> oे 0 0 0 

 0.2160.120 0.770
0.371 0.264 0.107 0.0892 0.203
0.172
$(6)$
0.0126
$(6)$
0.0126
0.0252
0.0164
0.0377
0.0230
0.100
0.0377
0.0230
0.100
0.0377
0.0230
0.100
0.0619
0.0396
0.0176
0.0106 0.0337
0.0247
Table 4.7 Variability of the Industry Mean Standard Deviation
of Annual Mean Standard Deviation Errors ${ }^{\text {E }}$
$\frac{\text { Overall Mean }}{(5)}$
0.0757

0.490
0.196

$\begin{array}{cc}8 & 0 \\ -1 & 7 \\ -1 & 0\end{array}$

 | 0 |
| :---: |
| 0 |
| 0 |
| 0 |
| 4 |
| 4 |
| 4 |
| 4 |
| 0 | $(4)$

0.0119
0.0107
0.0148
0.0203
0.0163
0.0163
0.0639
0.0327 0.0327 ñ
$\stackrel{n}{n}$
0
0 0.0104 0.00493 $\begin{array}{ll}\text { n } & \text { or } \\ \\ 0 & 0 \\ 0 \\ 0 & 0\end{array}$

Slope
Coefficient
$\quad(2)$
0.00128
$(1.2)$
-0.00660 $(L \cdot 9)$
$09900 \cdot 0-$ عOZ00•0-$-0.00921$ 2L+00.0-
 (3.8) $(\tau \cdot S)$
$2 S T 0^{\circ} 0^{-}$ OZS00*0--0.00520
$(1.6)$
0.00413 -0.00413
$(4.4)$ 0.00272



Table 4.7 contd. Variability of the Industry Mean

 Coefficient of
Variation of An
 of Annual Mean $\qquad$






## + 7 4 0 0 0 0



\[

\]

Average (all industries)
Average (excluding industries 8 and 9)

Engineering), but the high value is exceptional: the next highest, again in Textiles, is 0.37072 for Woollen \& Worsted (No. 9). Over all industries, the average coefficient is 0.18810 , which reduces to 0.13715 if the two Textile industries (Nos. 8 and 9) are omitted. The industry average is thus quite a stable variable.

If note is taken of the trend, then we may discuss the stability in terms of the trend coefficient and the standard deviation of the errors (Table 4.7, colums 2 and 4). Only the two 'rextile industries have slope coefficients that indicate an annual change in average of more than one percentage point. The Vehicle Industry (No. 6) is close to this with 0.9. For those industries showing a downward trend, the average annual change is 0.6 , which falls to 0.4 percentage points if the Textile industries are omitted. To assess the variability about trend, the ratio of the standard deviation of the errors about trend to the mean rate of return has been calculated for each industry (column 5). This may be compared with the coefficient of variation in column 7. Allowance for the trend has a particular impact on the two extreme cases - the Textile industries - in each case bringing about an approximate halving in the coefficient. Apart from these two industries, the crude variability without allowance for trend was small. In these two cases, allowance for trend has eliminated a considerable proportion of the variability and the residual standard deviation is quite small relative to the mean.

For the present purposes, only one aspect of the behaviour of dispersion of the annual distributions need concern us. That aspect is the stability over time. In Table 4.8, the average standard deviation over the twelve years is shown in column 1. There is a considerable
degree of uniformity in this measure between industries, but it is the behaviour over the years within an industry that is of interest. Column 2 of the table therefore presents the standard deviation of the annual distribution standard deviations. The maximum value is for the Entertainment \& Sport Industry (No. 20) where the average standard deviation is approximately 10 percentage points and this has a dispersion over the twelve years of something less than 4.5 percentage points. Clearly, even this is not a great amount of fluctuation. A useful standard of comparison is to calculate the coefficient of variation of the standard deviation for each industry. This has been done and the results are shown in Table 4.8, column 3. Only four industries have a coefficient that exceeds 20\%: Vehicles (No. 6), Cotton (No. 8), Woollen \& Worsted (No. 9) and Entertainment \& Sport (No. 20). As usual, the Textile industries are distinguished for their extremely variable experience. In contrast, four industries have a coefficient well below 10\%: Building Materials (No. 1), Drink (No. 13), Retailing (No. 19) and Miscellaneous Services (No. 21). The conclusion is that the standard deviation of the annual distributions is a relatively stable quantity. This stability suggests that, in so far as we regard rates of return as an example of a first order Markov process, the distribution of rates of return approximates to a steady state solution to the process.

The evidence on the direction of skewness of the distribution of rates of return on net assets is inconclusive. In nine industries, there are more years in which the distribution is negatively rather than positively skewed. The converse is true for five industries and there are equal numbers skewed in each direction in the remaining three industries. When only significantly skewed distributions are used, the

Table 4.8 : Standard Deviations of the Annual Distributions

Ind. No. $\frac{\text { Average Annual }}{\frac{\text { Standard Deviation }}{\text { Of Annual Standard }}} \frac{\text { Coefficient of }}{\text { Deviations }}$

| 1 | 0.09654 | 0.00756 | 0.07826 |
| :--- | :--- | :--- | :--- |
| 4 | 0.11534 | 0.01108 | 0.09610 |
| 5 | 0.13070 | 0.01559 | 0.11927 |
| 6 | 0.12240 | 0.02617 | 0.21384 |
| 7 | 0.12781 | 0.01188 | 0.09296 |
| 8 | 0.11567 | 0.09952 | 0.03313 |

same results are achieved. ${ }^{16}$ Taking all distributions together, we find 90 positively and 114 negatively skewed, which is hardly sufficient to support a firm conclusion. ${ }^{17}$ Hart ${ }^{18}$ in his work concluded that there was a slight positive skewness.

It is of interest to see if this uncertain evidence can be strengthened by considering whether certain years are characterised by positive and others by negative skewness. If macroeconomic influences were producing such an effect, the average over a number of years would depend on the precise years chosen. In our case the years chosen appear approximately unskewed on average. The number of industries, positively and negatively, significantly and insignificantly skewed, are shown in Table 4.9 for each year.
16) Except that there are too few observations in Industry 8 to allow a test of significance. There are therefore 8 industries for which it can be said that the majority of significantly skewed distributions are negatively skewed.
17) There is a $12 \%$ chance of there being 90 positive signs out of 204 samples when the population is symmetric. The chance of this outcome when the population is positively skewed is, of course, less. It might be argued that rejection of all observations whose absolute value exceeds $100 \%$ will tend to produce spurious negative skewness in the remaining data. In fact, similar analysis employing all observations found more negative skewness, e.g. 120 distributions were negatively skewed and only 84 positively skewed.
18) Hart P E (ed) op. cit. Vol II p 263; "The arithmetic mean and median rates of return of the 1844 companies .... are 15.6 and 14.4, indicating a slight positive skewness."

Table 4.9 : Direction of Skewness by Year

|  | Year | 1949 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| POSI- | Significant | 10 | 5 | 9 | 4 | 6 | 3 | 3 | 2 | 6 | 4 | 6 | 5 |
| TIVE | Insigni- <br> ficant | - | 4 | 1 | 2 | 3 | 3 | 3 | 4 | 1 | 0 | 2 | 2 |
| NEGA- | Insignificant | 3 | 3 | 1 | 3 | 1 | 3 | 5 | 3 | 3 | 4 | 2 | 3 |
| TIVE | Significant | 3 | 4 | 5 | 7 | 6 | 7 | 5 | 7 | 6 | 8 | 6 | 6 |

Industry 8 has been onitted from this analysis, as the significance test cannot be applied, there being insufficient observations.

A majority of years are negatively skewed: 7 against 4 if all values are used, 6 against 3 if only significant ones are counted. The four years that are predominantly positively skewed occur at the beginning of the period: 1949, 1950, 1951 and 1953. 1953 is lost from this list if only significant skewness is considered. If we further restrict attention to those years in which there is a marked difference in the numbers displaying significant skewness in each direction, we are left with 1949 and 1951 being positive and 1952, 1954, 1956 and 1958 being negative. There does seem to be some support for the proposition that the direction of skewness varies between years. It is not the intention here to pursue this matter much further. It is worth interpreting this preliminary result: positive skewness means that the lengthy tail of rates of return points towards the higher values, whereas negative skewness means that the distribution tail points to lower values. Or, positive skewness means that we find more observations at a given large
nositive deviation from the mean than at an enua negative doviation. So in the early years of the period vory high rates of return were mone common than very low nes. The distribution of rates of return might, be regarded as having a partial constraint at zero - fimn will try very hard to avoid renorting losses. Thorefore positive skewness misht be the result of Jow averace profitability and this nartial constrant, But in the early period rates of return were hich relative to their values in the 1950s (see above). Therefore this tupe of explanation does not look very promising given that it is the later years of lower averape rates of return that displey the negative skew. $T t$ is tempting to conclude that negative skewness is the usual situation and that the positive skew of the early years is a consequence of special conditions then prevailing. Particularly one might point to the age of the carital stock at that time and its resultant low net book value.

Having pursued the ambiguities of the skewness a little, the behaviour of the kurtosis is satisfactorily straightforward. In every industry, more years heve leptokurtic than platykurtic distributions, in fact only 14 out of 204 distributions are platykurtic and none is significantly so. We may therefore conclude that rates of return are leptokurtically distributed.

## Section 4.6 : Sumary

The data used in this study are the NIESR - Board of Trade collection of standardised accounts for quoted companies as organised for machine processing by Singh and Whittington at Cambridge. The period covered is 1948-1960. Within this body of data companies are arranged by industrial orders (1948 S.I.C.) and within these into more homogeneous groups whose meaning in terms of Minimum List Headings is given in Table 4.4.
'The variable used is the rate of retum on net assets calculated before tax and interest payments but after depreciation. The time period used is one year and the rates of return are standardised according to year and accounting quarter for each industry (and for each of the smaller groups where these are used).

In the examination of the annual distributions of rates of return, it was found that, within industries, the means and standard deviations are quite stable. There was slight evidence that the distributions are usually negatively skewed but it would appear that positive skewness predominated at the beginning of the period. There was strong evidence that the distributions were leptokurtic.

## TRANSITION MANRICES

In this chapter certain characteristics of the transition matrices are considered. The intention is twofold: to obtain an initial pointer to the consistency of the decay of profitability and secondly to get some guidance on the form of the conditional distributions.

For the decay of profitability aspect we must first ask whether there is decay at all and, if it does occur, is it general, within an industry for all rates of return, for all industries and for all subgroups. Then it is possible that certain parts of the range of profitability display more consistent decay than others. The main tool used for this is comparison of the final and prior means - if there is decay then the prior mean should have a greater absolute value than the final mean. The investigation of the form of the conditional distributions is pursued in terms of the skewness and kurtosis.

These topics are discussed for one industry in Section 5.1 , then for all industries in Section 5.2 and for all subgroups in Section 5.3 .

Finally, in Section 5.4 the relationship between the variance and the prior mean is explored. This is preparatory to the consideration of the heteroscedasticity of the decay function in Section 6.2.

Section 5.1 : Transition Matrices for Industry 1
At an early stage in the statistical work here described, conventional discrete transition matrices were prepared, an example of which is shown in Table 5.1. Note that the class intervals relate to deviations from the mean but are in rate of return percentage point units rather than standard deviation units. The general pattern is immediately apparent: the mode of the row distribution lies on the main diagonal while the mean lies to the right of the mode for rows above and to the left for those below the industry average. That is, as each row is a conditional distribution for a given rate of return interval in the initial period, the expected value in the next period is lower than in the initial period. It appears that there is regression towards the mean. Th $\geqslant$ transitions on which this matrix is based are all pairs of rates of return for adjacent years in the period 1948-1960 for Industry 1 (Building Materials, Pottery \& Glass). This pooling of 12 years data is discussed in Section 6.3.

As described in Chapter III, the analysis is carried out using discrete intervals for the initial states but calculating summary statistics (without grouping) for the row or contingent distributions. It will be seen that the initial impressions from the discrete transition matrix are confirmed when this technique is used.

The process by which these transition matrices are produced is as follows: the data is read into the computer on paper tape and standardised. Observations more than three standard deviations from the mean are rejected. Then for each year up to the penultimate one the rates of return are ordered. This set of rates of return is the set of all the first members of the pairs of rates of return that make up transitions.
Finel Cless (oercent pts)

|  | 0 | 0 | 0 | 0 | 0 | 0 | $0$ | $\bigcirc$ | $\bigcirc$ |  | 0 | - | N | $\pm$ | $m$ | $\pm$ | 0 | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | $\bigcirc$ | $\bigcirc$ | 0 | H | 0 | $\bigcirc$ | - | - | ${ }^{-1}$ | N | $m$ | $\pm$ | -1 | H | $\cdots$ |
| $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \sim \\ & H \end{aligned}$ | 'o' | 0 | O | O | 0 | 0 | $\bigcirc$ | 0 | 0 | H | 0 | $\pm$ | $\sim$ | $\checkmark$ | N | in | c | H |
|  | 0 | 0 | O | - | 0 | 0 | 0 | 0 | 0 | N | - | $m$ | in | - | $\infty$ | in | $\sim$ | N! |
| $\begin{array}{ccc} \hline 8 & 9 \\ 0 & 0 \\ 0 & 0 \\ 1 & \underset{1}{1} \\ \hline \end{array}$ | - | 0 | 0 | 0 | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | 0 | $m$ | in | $a$ | in | $\stackrel{-1}{-1}$ | $a$ | $m$ | 6 | $\sim$ |
| ¢0\% | $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | 0 | $\cdots$ | $\bigcirc$ | 0 | N | m | $\infty$ | $\stackrel{-}{-1}$ | $\ln _{\substack{\text { n }}}$ | $\xrightarrow{-1}$ | $m$ | H | N | $\pm$ |
| 8 | 0 | 0 | 0 | 0 | 0 | - | © | 6 | 0. | N | $\stackrel{\rightharpoonup}{\sim}$ | $\cdots$ | ¢ | $m$ | $\sim$ | $c$ | O | 0 |
| cras | 0 | $r$ | 0 | 0 | c | $\sim$ | - | in | $\xrightarrow{\sim}$ | ¢ | $\underset{\sim}{\sim}$ | N | 6 | $\cdots$ | . H | c | Cl | H |
| 8.89 | 0 | 0 | $\bigcirc$ | $r 1$ | N | in | 5 | $\cdots$ | c | ${ }^{-1}$ | ç | $\overrightarrow{\mathrm{r}}$ | 6 | N | $\cdots$ | H | c | $\cdots$ |
| $\underset{\sim}{\text { O }}$ | -1 | $\bigcirc$ | 0 | $\bigcirc$ | $\pm$ | 6 | $\underset{\sim}{\sim}$ | m | $\pm$ | さ | $\xrightarrow{-}$ | n | v | $\cdots$ | $\bigcirc$ | $\bigcirc$ | $r$ | 0 |
|  | 0 | 0 | -1 | N | in | $\xrightarrow{\text { H }}$ | $\stackrel{\sim}{\sim}$ | ก | N | $\cdots$ | $\infty$ | $\pm$ | H | $\cdots$ | $c$ | C | $c$ | 0 |
| \% | 0 | 0 | $\pm$ | $m$ | $\xrightarrow{\sim}$ | त | ®̀ | $\stackrel{7}{N}$ | $\xrightarrow{\sim}$ | in | $\pm$ | -1 | - | $\sim$ | C | H | $\bigcirc$ | c |
|  | ${ }^{-1}$ | $m$ | $=$ | $N$ $\sim$ | $\xrightarrow{N}$ | $\stackrel{0}{\square}$ | $\stackrel{-1}{-1}$ | $\stackrel{\sim}{\sim}$ | in | H | H | c | c | c | c | $c$ | $\bigcirc$ | c |
|  | 0 | 0 | 0 | $\infty$ | $\stackrel{ \pm}{7}$ | न <br>  <br>  | $\checkmark$ | 5 | N | $\bigcirc$ | 0 | H | $\bigcirc$ | c | $-1$ | $c$ | c | 0 |
|  | $m$ | $-$ | in | $\infty$ | $\pm$ | $\omega$ | $\pm$ | H | H | 0 | - | 0 | 0 | c | c | $c$ | $\bigcirc$ | c |
| (1) | in |  | H | $m$ | $\cdots$ | $\pm$ | in | 0 | 0 | $\bigcirc$ | c | 0 | $c$ | O | 0 | 0 | 0 | c |
| 8 $80 \begin{gathered}\text { c } \\ 0 \\ 8\end{gathered}$ | $\pm$ | $\sim$ | $m$ | 0 | N | N | H | $\bigcirc$ | $\bigcirc$ | $c$ | $\bigcirc$ | - | $\bigcirc$ | 0 | c | 0 | 0 | c |
| $\begin{array}{lll}0 & 0 \\ 0 & 5 \\ \text { of } \\ \text { \& } \\ \text { on }\end{array}$ | $\xrightarrow{N}$ | $\pm$ | $\vec{\square}$ | c | -1 | $\sigma$ | $\bigcirc$ | - | 0 | 0 | 0 | 0 | 0 | 0 | $c$ |  | 0 | c |
|  | $\begin{aligned} & \text { ry } \\ & \text { g } \\ & 8 \\ & 8 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 8 \\ & +2 \\ & 8 \\ & 0 \\ & 0 \\ & 8 \end{aligned}$ | $\begin{aligned} & 0 \\ & +3 \\ & 0 \\ & 0 \\ & 0 \\ & \cdots \\ & \cdots \end{aligned}$ | $\begin{aligned} & 8 \\ & 8 \\ & 8 \\ & \text { n } \\ & 4 \end{aligned}$ | $\left\|\begin{array}{cc} 0 & \\ 1 & \\ 0 & 0 \\ 0 & 0 \\ 9 & 0 \\ \hline 1 & 0 \end{array}\right\|$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \end{gathered}$ |  | $\left\{\begin{array}{l} 0 \\ 0 \\ n^{-1} \\ n^{\circ} \end{array}\right.$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ N \\ \infty \\ \infty \end{array}\right\|$ | $\begin{array}{ll} 0 & \\ + & 0 \\ 8 & = \\ 8 & n i \end{array}$ | $\left.\begin{gathered} o \\ + \\ 0 \\ م \\ n \\ n \\ n \\ i \end{gathered} \right\rvert\,$ | $\begin{aligned} & 0 \\ & +8 \\ & 8 \\ & 0 \\ & 10 \\ & 1 \\ & 1 \end{aligned}$ |  | $\left[\begin{array}{ll} 0 & \\ 1 & \\ 0 & 2 \\ 0 & 9 \\ 0 & 0 \\ 1 & 0-1 \end{array}\right.$ | $\left\|\begin{array}{l} 0 \\ + \\ 0 \\ 0 \\ 0 \\ 2 \\ n \\ n \\ \\ 1 \\ 1 \end{array}\right\|$ | $\left\|\begin{array}{cc} 0 & \\ 9 & \\ 8 & -9 \\ 8 & - \\ 10 & - \\ -1 & -1 \end{array}\right\|$ |  |  |

Now for a given number of transitions for each row distribution, the class intervals can be fixed using this ordering of rates of return. Thus the class intervals are determined in each case to maximise the number of rows subject to a constraint on the minimum number of transitions necessary per row. With the data standardised and the class intervals set, the statistics of each row distribution can be calculated. Mean, variance, skewness and kurtosis are produced, as is the mean of the prior variable. ${ }^{l}$ The measures of skewness and kurtosis are those described in Section 4.5. As before, the sign of the third moment is recorded to indicate the direction of skewness.

The industry transition matrices are given in Appendix $B$ but for convenience that for the Building Materials Industry (No. 1) is presented in Table 5.2. The pattern observed in the discrete matrix is repeated. The final mean is in nearly every case nearer to the industry average rate of retum than the prior mean. So companies with an above average rate of return in one year do, on average, experience a decline in profitability in the subsequent period. For those companies below the industry mean, the corresponding effect is an improvement in profitability, but even casual inspection of columns 2 and 3 suggests that the process is less consistent for firms with below average profitability. There are more cases ${ }^{2}$ where the absolute value of the final mean is greater than that of the prior mean in those classes below the industry average (referred to in the ensuing text as the "negative range") than in

1) The calculation of the prior mean rather than the assumption that it is the midpoint of the class avoids the usual need in the grouping of data to apply Sheppard's correction.
2) Classes $11,15,19$ above average; classes 23, 25, 31, 35, 38 below average; and class 22 about on the average value.

No. of transitions $=1227$
No. of rejected observations $=5$
$\frac{\text { Class }}{\text { No. }} \frac{\text { Lower }}{\text { Limit }} \frac{\text { Prior }}{\text { Mean }}$ Mean Variance $\frac{\text { Sign of }}{\text { Skewness }}$ Skewness Kurtosis No.

|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.714 | 2.145 | 1.737 | 0.359 | - | 0.296 | 3.008 | 43 |
| 2 | 1.489 | 1.604 | 1.349 | 0.234 | + | 0.041 | 2.800 | 30 |
| 3 | 1.321 | 1.410 | 1.176 | 0.251 | + | 0.438 | 3.449 | 30 |
| 4 | 1.183 | 1.244 | 0.880 | 0.272 | - | 0.252 | 2.785 | 30 |
| 5 | 1.068 | 1.127 | 0.727 | 0.384 | - | 0.066 | 2.497 | 30 |
| 6 | 0.961 | 1.010 | 0.775 | 0.291 | - | 0.124 | 2.324 | 30 |
| 7 | 0.872 | 0.917 | 0.819 | 0.309 | + | 0.325 | 3.361 | 30 |
| 8 | 0.802 | 0.843 | 0.710 | 0.212 | + | 0.539 | 4.479 | 30 |
| 9 | 0.705 | 0.759 | 0.674 | 0.393 | - | 0.318 | 3.403 | 30 |
| 10 | 0.638 | 0.674 | 0.662 | 0.293 | - | 0.009 | 2.639 | 30 |
| 11 | 0.581 | 0.607 | 0.611 | 0.189 | - | 1.094 | 5.042 | 30 |
| 12 | 0.522 | 0.555 | 0.522 | 0.215 | - | 0.392 | 2.652 | 30 |
| 13 | 0.476 | 0.498 | 0.445 | 0.264 | - | 0.250 | 3.278 | 30 |
| 14 | 0.424 | 0.454 | 0.214 | 0.234 | + | 0.035 | 2.542 | 30 |
| 15 | 0.357 | 0.396 | 0.412 | 0.382 | - | 1.900 | 7.343 | 30 |
| 16 | 0.290 | 0.327 | 0.207 | 0.131 | $+$ | 0.127 | 2.387 | 30 |
| 17 | 0.241 | 0.266 | 0.219 | 0.094 | + | 0.311 | 2.457 | 30 |
| 18 | 0.175 | 0.212 | 0.187 | 0.263 | + | 0.205 | 3.330 | 30 |
| 19 | 0.115 | 0.143 | 0.169 | 0.111 | + | 0.003 | 2.125 | 30 |
| 20 | 0.052 | 0.083 | 0.021 | 0.181 | - | 0.137 | 6.139 | 30 |
| 21 | -0.017 | 0.024 | 0.010 | 0.249 | - | 0.104 | 2.198 | 30 |
| 22 | -0.069 | -0.044 | 0.055 | 0.356 | + | 3.826 | 8.788 | 30 |
| 23 | -0.116 | -0.091 | -0.144 | 0.159 | - | 0.086 | 2.750 | 30 |
| 24 | -0.173 | -0.144 | -0.122 | 0.308 | - | 0.769 | 6.334 | 30 |
| 25 | -0.233 | -0.208 | -0.435 | 0.202 | + | 0.006 | 2.624 | 29 |
| 26 | -0.274 | -0.252 | -0.212 | 0.083 | - | 0.008 | 2.313 | 31 |
| 27 | -0.323 | -0.299 | -0.292 | 0.447 | + | 0.271 | 3.533 | 30 |
| 28 | -0.395 | -0.360 | -0.303 | 0.230 | + | 0.172 | 4.425 | 30 |
| 29 | -0.467 | -0.425 | -0.386 | 0.297 | - | 2.707 | 7.962 | 30 |
| 30 | -0.535 | -0.498 | -0.462 | 0.112 | - | 0.124 | 3.334 | 30 |
| 31 | -0.592 | -0.562 | -0.603 | 0.277 | + | 0.101 | 2.394 | 30 |
| 32 | -0.642 | -0.618 | -0.461 | 0.235 | $+$ | 0.214 | 3.665 | 30 |
| 33 | -0.723 | -0.687 | -0.547 | 0.190 | + | 0.294 | 2.623 | 30 |
| 34 | -0.821 | -0.768 | -0.692 | 0.181 | + | 0.002 | 5.232 | 30 |
| 35 | -0.927 | -0.882 | -0.943 | 0.273 | - | 0.363 | 3.952 | 30 |
| 36 | -1.025 | -0.969 | -0.880 | 0.535 | + | 0.722 | 3.181 | 30 |
| 37 | -1.152 | -1.086 | -0.933 | 0.241 | - | 0.016 | 2.822 | 30 |
| 38 | -1.317 | -1.242 | -1.249 | 0.379 | + | 0.304 | 4.441 | 30 |
| 39 | -1.576 | -1.445 | -1.268 | 0.620 | + | 0.129 | 3.537 | 30 |
| 40 | - | -2.075 | -1.529 | 0.475 | - | 0.035 | 2.592 | 44 |

those classes above the industry average (the "positive range"). Such an occurence means that the average rate of return in time $t+1$ of firms whose time $t$ standardised rate of return lies in that class is further from the mean than their average in time $t$. Profitability relative to the industry has moved against the general regression. This is perhaps an appropriate point to emphasize that measurement is now in terms of standardised rates of return, whereas the discrete matrix was in terms of rates of return expressed as straightforward deviations from the mean.

Another simple indicator of the stability of the regression towards the mean is to count the number of cases where the final mean of class $N$ is greater than that of class $N-1$. As the prior means take successively lower values, an inversion in the size of final means means a disturbance in the regression. There are eight such inversions out of 40 classes evenly split between the positive and negative ranges. Finally in the discussion of these two columns, it is to be noted that two of the cases where the prior mean is less divergent than the final mean occur at the industry origin, as does one of the inversions in the final mean column.

Moving onto the variance column, there appears to be little regularity in the behaviour of this statistic; a few classes have very large values, in particular the last two, the result no doubt of extreme values. It would not be surprising if both extreme classes had very high variance because of the much wider range of values encompassed by them. While this effect is observable, it is perhaps less marked than might be expected, especially at the upper end. Although later analysis will reveal a relationship between the variance and the initial mean, there is little evidence on inspection of this. The standard deviation of column (4) is 0.1154 , which when linked with a mean of 0.2727 suggests a reasonably stable variable. The variance is considered further in Section 5.4.

Moving now from the question of the consistency of the decay process to the form of the conditional distribution, we must first consider the direction of skewness. Neither appears dominant - 20 classes have negative and 20 positive skewness. Neither the positive range nor the negative range shows any divergence from this even split. But this ignores whether the skewness is signjficant or not. In testing for significant skewness the problem arises that Pearson's tables ${ }^{3}$ are only for more than 50 observations, at which the $10 \%$ level is 0.285 . It is evident from the tables that the value of $\beta_{1}$ corresponding to a $10 \%$ chance that a sample from a normal population may exceed that value rises rapidly as the number of observations diminish. It would clearly for this purpose have been preferable to use larger class sizes, but given the more iuportant need to have a good number of classes this had to be foregone. Therefore some approximate way of distinguishing the seriously skewed distributions had to be used. It was considered. that error should be of Type I rather than Type II, that is, the null hypothesis of normality should be rejected mistakenly rather than accepted mistakenly. Then the procedure adopted is to use the $10 \%$ significance level of 50 observations as the standard. This might roughly be a $20 \%$ significance level. On this crude basis, 16 distributions are seriously skewed, 9 of these being positively skewed and 7 negatively skewed; these being evenly spread between the positive range of classes and the negative range, and wi.thin the ranges evenly spread between directions of skewness. This even balance continues even if a more stringent cut-off level is used. It may, therefore, reasonably be concluded that the conditional distributions for this industry are not skewed. Although such a result is awlwardly doubtful, given the
3) See Croxton, Cowden \& Klein on. cit., Arpendix 0.
inability to properly test for significance, it is reassuring to record that some earlier results on a transition matrix for this industry, with acceptable numbers in each class, produce supporting evidence. 4

Turning now to the kuxtosis of the distributions, the same significance testing problem arises. The lowest number of observations in the published table 5 for $\beta_{2}$ is 100. Again a crude test must be used and again the preference is for Type I rather than Type II errors. So the $5 \%$ limits for the 100 observations are used. But first the simple count of leptokurtic and platykurtic distributions: 23 distributions have $\beta_{2}>3.0$, i.e. are leptokurtic, and 17 are platykurtic. Each range of classes shows a similar balance. Using $\beta_{2}>3.77$ as a test of serious leptokurtosis, 11 distributions exceed this value of which 7 are in the negative range of classes. Only 4 distributions are platykurtic $\left(\beta_{2}<2.35\right)$ with 2 in the positive range and 2 in the negative range. Thus there is some evidence of leptokurtosis.

Before going onto the other industries, a little more consideration of the crude tests used on the measures of skewness and kurtosis is in order. The main reason for Pearson restricting his tables to large numbers of observations lies in the numerical approximation that he was employing ${ }^{6}$ but undoubtedly the sampling error of both $\beta_{1}$ and $\beta_{2}$ increase very rapidly as the sample size diminishes. Therefore the tests of significance employed are certainly equivalent to a high probability of Type I errors. It then becomes likely that with the
4) In that work 6 out of 17 classes had skewness significant at the $10 \%$ level and 2 of these were positive and 4 negative.
5) See Croxton, Cowden \& Klein op. cit., Appendix P.
6) Pearson E S op. cit., p 244 et seq.
number of distributions for each industry, even with normal populations, a number of the distributions will reveal values of $\beta_{1}$ and $\beta_{2}$ that lie outside the significance limits. On the other hand, some cut-off point must be employed and in a context where the desirable (and convenient) result is that the population is normally distributed, it is only proper that the cut-off point should err against the desired result. On the other hand, it is not really possible to conclude anything about the distributions by the application of such methods to one industry. It is to be hoped that the accumalated evidence of all the industries will enable a more positive conclusion to be attained.

Rather than present transition matrices for each industry in this chapter, a table sumarising their characteristics is given - Table 5.3. The full data are given in Appendix B.

Column 4 of the table gives the number of classes used in each industry. Only two industries provide less than 20 classes; these are No. 8 (Cotton) for which the data used are not complete, and No. 17 (Construction). The maximum is not suxprisingly provided by the Shipbuilding and Mechanical Fngineering Industry (No. 4) with 106. Of more interest is the way in which the classes divide between above and below the industry average (column 5). The only industry in which the numbers in the two groups diverge to any great extent is Miscellaneous Services (No. 21) for which there are 33 classes in the positive range (above average) and 42 in the negative range (below average). This means that the median is well below the mean and therefore the distribution of rates of return appears positively skewed. The distribution here is the sum of the annual distributions after they have been standardised. Of the annual distributions for Industry 21 (see Appendix A) 8 are positively skewed and it is therefore to be expected that the aggregate standardised distribution would be positively skewed. This similarly explains why Industry 13 (Drink) has 27 classes in the positive range to 31 in the negative, and Industry 20 (Entertainment and Sport) has 12 and 17 respectively. Overall in column 5 eleven industries show n negative skew and 5 a positive skew, whilst one (No. 5, Electrical Engineering) appears symmetrical. It is probably a fair conclusion that in the two positively skewed Service industries, low asset to turnover situations generate a tail of very high rate of return of firms. The explanation of the

Table 5.3: Summary of Industry Transition Matrices

| Ind No. | No. of Transitions | No. of Rejected Observations | No. of Classes |  |  | Rinal Mean $>$ <br> Prior Mean |  | $\begin{gathered} \text { Nth Mean } \\ > \\ (\mathrm{N}+1) \text { th Mean } \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Total | Pos. Range | Neg. Range |  |  |  |  |
|  |  |  |  |  |  | Pos. Range | Neg. Range | Pos. Range | Neg. Range |
| (1) | (2) | (3) | (4) | (5) |  | (6) |  | (7) |  |
| 1 | 1277 | 5 | 40 | 21 | 19 | 3 | 6 | 4 | 5 |
| 4 | 3199 | 44 | 106 | 54 | 52 | 10 | 20 | 21 | 19 |
| 5 | 1261 | 16 | 42 | 21 | 21 | 2 | 8 | 5 | 6 |
| 6 | 916 | 11 | 30 | 17 | 13 | 3 | 1 | 3 | 4 |
| 7 | 1906 | 33 | 63 | 31 | 32 | 3 | 7 | 10 | 11 |
| 8 | 376 | 3 | 12 | 7 | 5 | 1 | 2 | 1 | 0 |
| 9 | 729 | 8 | 24 | 13 | 11 | 2 | 3 | 5 | 2 |
| 11 | 994 | 24 | 33 | 18 | 15 | 1 | 8 | 6 | 4 |
| 12 | 1299 | 12 | 43 | 22 | 21 | 5 | 8 | 6 | 5 |
| 13 | 1753 | 34 | 58 | 27 | 31 | 7 | 15 | 10 | 13 |
| 15 | 1470 | 18 | 49 | 26 | 23 | 4 | 4 | 9 | 7 |
| 16 | 1863 | 36 | 62 | 32 | 30 | 2 | 9 | 9 | 12 |
| 17 | 586 | 10 | 19 | 9 | 10 | 2 | 3 | 2 | 2 |
| 18 | 2620 | 43 | 87 | 45 | 42 | 3 | 16 | 20 | 19 |
| 19 | 1841 | 16 | 61 | 31 | 30 | 2 | 10 | 10 | 9 |
| 20 | 872 | 15 | 29 | 12 | 17 | 1 | 3 | 3 | 4 |
| 21 | 2260 | 37 | 75 | 33 | 42 | 2 | 14 | 8 | 19 |
|  |  |  |  |  |  |  |  |  |  |

positive skew of the Drink industry is less obvious.

As an indicator of the consistency of the decay process, column (6) gives the number of cases where a final mean is further from the industry mean than the onrresnonding nrior mean. Numbers are shown for each industry and separately within industring for the nositive and nogative ranges. In total, the highest number relative to the number of classes for that industry is for the Drink industry, where it is just over a third, the usual value being about one quarter. But generally the total number of such cases is of less interest, once we have seen they are a small minority, than the distribution of them between the positive and negative ranges. The total number relative to the number of classes is likely to be reflected in the later regression analysis. 'The number of instances of the final mean being more divergent than the prior mean in the negative range exceed the number in the positive range in all industries but two. The one case where the opposite is true is Industry 6 (Vehicles) and in that there are only 4 instances, 3 in the positive and 1 in the negative range. So the effect found for Industry 1 is supported by the evidence of other industries: this is the perhaps unsurprising result that the regression towards the mean (which requires that the final mean be closer to the average than the prior mean) is a less even process for firms of below average profitability. In other words, transfer of resources from unprofitable markets is less straightforward than the movement of resources into profitable ones.

The next column also cives a guide to the consistency of the decay process as it records the number of cases where the final mean of class $N$ is greater than that of class $N-1$. That is, cases where the
expected profitability for a group of firms whose prior mean is $r_{1}$ is less than for a number of firms whose prior mean $r_{2}$ is more than $r_{1}$. It is another indication of how steady the regression is towards the mean and again the overall number shows what will be better shown by the goodness-of-fit of the regression of the final mean on the prior mean. Considering the distribution between the positive and negative ranges, the number of industries with a majority of sunh inversions in the nositive range is halanced by the number that displays the converse, Only Miscellaneous Services (No. 2l) displays a marked disparjty in the numbers in each range, there being eight in the positive and 19 in the negative.

Coming to the evidence on the skewness of the conditional distributions, shown in Table 5.4, the first stage is to count the number of positive and negatively skewed distributions without consideration of significance. In 13 out of 17 industries there are more negatively skewed than positively skewed distributions. When only significantly skewed (in the sense described above) distributions are counted, the predominance of negative skew increases: 16 industries have more negatively than positively skewed distributions. In many cases there is a considerable divergence between the numbers of each type. This seems reasonably to establish that the general form of the contingent distribution shows a negative skewness, that is, the lone tail of the distribution stretches towards the low rate of return end. This means that a large fall in profitability is more likely than an equally large rise. This is observed despite the rejection of very extreme observations - more than 3 standard deviations from the mean.

It is of interest to question whether this holds equally both for above industry average and below industry average distributions. The analysis

I'able 5.4 : Skewness of the Industry Conditional Distributions

senarately for these two sets of distributions for each industry is also shown in Tabie 5.4. The conclusion has to be that nerative skewness predominates in both renges, al though there are mose industriea for which the dominant skewness is nositive in the negntive ranme than tiere are for which the same is true in the nositive range. There is a hint that the Service industries (Nos. 18-21) are slightly different in this resnect as the nonconforming cases seem primarily to be from these four industries,

The conclusion on the kurtosis of the distributions is more definite lentokixtosis is shown to be predominant in Table 5.5. Traking no acoount, of the significance of deviations from mesokurtosis, every industry has more distributions that are leptokurtic than ones that are nlatykurtic. This is equally tme inr the negative range, in the nositive range 3 industries show more nlat,ykurtic distributions but the difference in numbers in these industries is smajl. When the nrevionsly describer test of siem nificance is used, the rosilt is urichanged for all distributions teken together. Tr the negative rance, the only chenge is that one industry (Wonllen \& Worsted, No. 9) with only two sienificantly nonmesnkurtic distributions has one lentokurtic and one platykurtic. Th the nositive range only one industry (Construction, No, 17), again with very few significant values, has more platykurtic than leptokurtic distributions. Generaily it may be concluded that the conditional distributions are lentokurtic both in the positive and negative ranges.

So far we heve considered the conditional distributions together and separately for the positive and negative ranges. Now we senarate out; those distributions whose prior means are close to the inciustry mean. Tt is not surnrising that it is heire that most of the cases where the final mean is further from the irnustry average than the instial moan ocemr, on the other henc inversions in the declining order of the firat

Table 5.5: Kurtiosis of the Industry Conditionsl nistributions
$1=$ leptokurtic $\quad p=$ platykurtic

| $\begin{aligned} & \text { Ind } \\ & \text { No. } \end{aligned}$ | All Values |  |  |  | Significant Values Only |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Positive Range |  | Negative Range |  | Positive Range |  | Negative Range |  |
|  | No.of 1 | No.of $p$ | No.of 1 | No. of p | No.of 1 | No. of p | No.of I | No. of p |
| 1 | 10 | 11 | 12 | 7 | 3 | 0 | 7 | 0 |
| 4 | 40 | 14 | 38 | 14 | 26 | 5 | 23 | 5 |
| 5 | 15 | 6 | 17 | 4 | 13 | 0 | 14 | 0 |
| 6 | 12 | 5 | 11 | 2 | 7 | 1 | 8 | 2 |
| 7 | 18 | 13 | 24 | 8 | 10 | 1 | 16 | 3 |
| 8 | 4 | 3 | 3 | 2 | 2 | 0 | 1 | 0 |
| 9 | 9 | 4 | 8 | 3 | 3 | 0 | 1 | 1 |
| 11 | 14 | 4 | 9 | 6 | 9 | 1 | 6 | 1 |
| 12 | 15 | 7 | 16 | 5 | 13 | 2 | 13 | 2 |
| 13 | 23 | 4 | 24 | 7 | 15 | 1 | 22 | 0 |
| 15 | 19 | 7 | 17 | 6 | 15 | 1 | 9 | 4 |
| 16 | 21 | 11 | 23 | 7 | 14 | 3 | 11 | 0 |
| 17 | 4 | 5 | 7 | 3 | 1 | 2 | 3 | 0 |
| 18 | 39 | 6 | 36 | 6 | 30 | 3 | 2.5 | 1 |
| 19 | 19 | 12 | 25 | 5 | 8 | 2 | 18 | 0 |
| 20 | 5 | 7 | 14 | 3 | 3 | 3 | 11 | 1 |
| 21 | 25 | 8 | 37 | 5 | 13 | 2 | 22 | 0 |
|  |  |  |  |  |  |  |  |  |

means appear to be evenly spread throughout the range. The central group has been separately analysed for 4 industries $(7,11,12,16)$ and the results are presented in Table 5.6 with those for the whole of these industries for comparison. About one quarter of the classes symmetrically arranged about the industry mean have been used as this central group. Apart from the final prior mean observation, this group is not distinguishable from the overall set of distributions in any way. Although in one industry (No. 7) the predominant direction of skewness is positive, which is opposite to that for the whole distribution of that industry, this phenomenon is not repeated in any of the other industries. In none of the four industries does the nature of the kurtosis for the central group differ in any way from that for the whole industry.

The few classes at the industry mean do show up one other matter. As is to be expected in some cases, the sign of the final mean differs from that of the corresponding prior mean. If such occurrences are random, they are of no interest, but if there is any pattern it is a guide to the suitability of the industry average as a proxy for the decay origin. There is such a regularity: in nearly every case the sign errors are of the kind where the prior mean is positive and the final mean negative. They are also, usually, together nearest the zero class. There would seem to be a suggestion from this that the industry mean is higher than the point towards which the regression of profits is directed.

The salient points that emerge from this examination of the transition matrices are five; firstly the regression of profits towards some central value does certainly occur although there is some evidence to suggest that the central value is lower than the mean. Secondly, this regression appears more regular above the industry average than below.


Thirdly, the conditional distributions are best represented with negative skew. Fourthly, they are leptokurtic. Fifthly, these two characteristics prevail throughout the profitability range.

In this section the variance has not been examined beyond the very earliest stage as this will be pursued with regression analysis in the next section.

What can be concluded is that an assumption of normality would be unjustified by the evidence that the conditional distributions are assymetric and more peaked than the normal curve. On the other hand, the evidence of the leptokurtic form of the distribution suggests that the lognormal curve might fit; it would require some transformation to produce negative skewness and would introduce the problem of negative values. In the absence of an immediately applicable distribution, this line of development will be pursued no further 7 and consolation must be found in the remark ${ }^{8}$ that: "Phe fitting of distributions to observational data has a certain intrinsic interest which is apt to outrun its statistical usefulness." The rest of this study will therefore be concerned with relations between the prior mean and the final mean.

[^14]
## Section 5.3 : Transition Matrices - Subsroups

The only need in this discussion is to comment on any ways in which the results for subgroups deviate from those for industries. The results for each subgroup are shown in Appendix $C$ and a summary is given in "Thale 5.7.

The only difference in their calculation from those for the industries is that the generally fever companies meant that using means and standard dewiations by accounting date was usually not possible, in fact only 11 subgroups out of 41 were large enough to allow this. For the rest anmal averages and standard deviations were used. Overall the small maber of observations led to a small number of classes in each transition matrix - the minimum number was 12 which occurred in a number of subgroups.

Thrmine now to the specific results, just as for the industries, the majority of subgroups had more classes in the positive range than in the negative, indicating that the median is greater than the mean and there is therefore negative skewness. 9 Only Industry 7 Subgroup ? (Other Metal Goods - Instruments etc) shows any marked difference between the numbers in the ranges with 14 positive and 6 negative range classes.

Again the industry pattern is repeated when we turn to enumerating the classes where the final mean exceeds the prior mean: 28 out of 41 have more such occurrences in the negative range. Similarly the overall
negative skewness of the conditional distributions is again found, as
9) $S k=\frac{3(x-M e d)}{s}$ Pearson's measure

See Croxton, Cowden \& Klein p 202.

Table 5.7 : Summary of Subgroup Transition Matrices
$1=$ leptokurtic $\quad p=$ platykurtic

| $\begin{aligned} & \text { Ind } \\ & \text { No. } \end{aligned}$ | SubGroup No. | No. of Transitions | No. of Re jected Observations | No. of Classes |  |  | Final Mean Prior Mean |  | Skewness |  | Kurtogis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Total | Pos. Range | Neg. Range | Pos. Range | Neg. Range | No. Ров. Skewed | No. Neg. Skewed | No. of 1 | No. of $p$ |
| 1 | 2* | 804 | 4 | 26 | 14 | 12 | 1 | 3 | 14 | 12 | 17 | 9 |
|  | 3 | 220 | - | 14 | 6 | 5 | 0 | 3 | 4 | 7 | 5 | 6 |
| 4 | 1 | 351 | 4 | 17 | 8 | 9 | 1 | 3 | 10 | 7 | 9 | 7 |
|  | 2 | 314 | 2 | 15 | 7 | 8 | 1 | 2 | 5 | 10 | 9 | 6 |
|  | 4 | 206 | - | 10 | 5 | 5 | 1 | 2 | 4 | 6 | 6 | 4 |
|  | 5* | 2219 | 29 | 73 | 38 | 35 | 9 | 12 | 23 | 50 | 57 | 16 |
| 5 | 3 | 268 | 9 | 13 | 7 | 6 | 0 | 2 | 3 | 10 | 6 | 7 |
|  | 4* | 615 | 5 | 20 | 10 | 10 | 2 | 4 | 6 | 14 | 20 | 0 |
| 6 | 4 | 340 | 2 | 17 | 9 | 8 | 0 | 2 | 7 | 10 | 14 | 3 |
| 7 | 1* | 1514 | 27 | 50 | 24 | 26 | 4 | 10 | 17 | 33 | 34 | 16 |
|  | 2 | 418 | 7 | 20 | 14 | 6 | 4 | 0 | 13 | 7 | 11 | 9 |
| 8 | 1 | 255 | 4 | 12 | 7 | 5 | 2 | 3 | 5 | 7 | 4 | 8 |
| 9 | 1* | 658 | 10 | 21 | 12 | 9 | 4 | 2 | 8 | 13 | 14 | 1 |
| 11 | 1* | 725 | 16 | 24 | 13 | 11 | 2 | 5 | 5 | 19 | 19 | 5 |
|  | 2 | 278 | 2 | 13 | 7 | 6 | 1 | 2 | 4 | 9 | 8 | 5 |
| 12 | 2 | 306 | 4 | 15 | 7 |  | 2 | 4 | 6 | 9 | 11 | 4 |
|  | 4 | 257 | 3 | 12 | 7 | 5 | 2 | 0 | 4 | 8 | 9 | 3 |
|  | 6 | 364 | 4 | 18 | 9 | 9 | 3 | 2 | 8 | 10 | 11 | 7 |
| 13 | 1* | 1572 | 32 | 52 | 26 | 26 | 4 | 10 | 24 | 28 | 40 | 12 |
| 15 | 1* | 674 | 2 | 22 | 12 | 10 | 3 | 3 | 7 | 15 | 15 | 7 |
|  | 2 | 322 | 4 | 16 | 8 | 8 | 1 | 2 | 6 | 10 | 11 | 5 |
|  | 3 | 483 | 6 | 24 | 13 | 11 | 4 | 5 | 12 | 12 | 12 | 12 |
| 16 | 1 | 316 | 4 | 15 | 8 | 7 | 2 | 2 | 6 | 9 | 6 | 9 |
|  | 2 | 365 | 11 | 18 | 10 | 8 | 2 | 1 | 7 | 11. | 12 | 6 |
|  | 3 | 373 | 9 | 18 | 10 | 8 | 3 | 3 | 9 | 9 | 7 | 11 |
|  | 4 | 307 | 4 | 15 | 8 | 7 | 2 | 1 | 6 | 9 | 4 | 12 |
|  | 5 | 502 | 9 | 16 | 7 | 9 | 0 | 6 | 10 | 6 | 11 | 5 |
| 17 | 1 | 531 | 13 | 17 | 9 | 8 | 2 | 1 | 7 | 10 | 11 | 6 |
| 18 | 1 | 526 | 7 | 26 | 13 | 13 | 2 | 5 | 13 | 13 | 16 | 10 |
|  | 2 | 506 | 14 | 25 | 13 | 12 | 4 | 4 | 10 | 15 | 12 | 13 |
|  | 3* | 1432 | 21 | 47 | 23 | 24 | 6 | 9 | 24 | 23 | 38 | 9 |
| 19 | 1 | 388 | 3 | 19 | 9 | 10 | 1 | 2 | 9 | 10 | 7 | 12 |
|  | 2 | 326 | 4 | 16 | 8 | 8 | 1 | 6 | 6 | 10 | 10 | 6 |
|  | 3* | 1117 | 7 | 37 | 19 | 18 | 4 | 7 | 15 | $2 ?$ | 26 | 11 |
| 20 | 2 | 263 | 1 | 13 | 6 | 7 | 1 | 1 | 5 | 8 | 7 | 6 |
|  | 3 | 461 | 11 | 23 | 9 | 14 | 2 | 6 | 12 | 11 | 12 | 11 |
| 21 | >* | 677 | 10 | 22 | 10 | 12 | 1 | 5 | 9 | 3 | 14 | 8 |
|  | 3 | 244 | 4 | 12 | 5 | 7 | 1 | 3 | 5 | 7 | 6 | 6 |
|  | 4 | 245 | 1 | 12 | 6 | 6 | 2 | 2 | 7 | 5 | 4 | 8 |
|  | 5 | 270 | 8 | 13 | 6 | 7 | 0 | 4 | 4 | 9 | 5 | 8 |
|  | 6 | 342 | 1 | 17 | 8 | 9 | 1 | 3 | 13 | 4 | 14 | 3 |

[^15]Section 5.4: The Variance of the Conditional Distribution

In Section 5.1 it was remarked that there appeared to be no regularity in the pattern of the variance except that the extreme classes had rather high variance. It is necessary to look a little more closely into the relation between the variances of the conditional distributions and their prior means.

There are 3 factors working to bring about a positive relationship between the variance and the absolute value of the prine mean. It has to be the absolute value both because of the way these factors work and the nonnegative nature of the variance. The first two of these factors derive from the bell-shaped prior distribution of rates of return. Assume that there is a nonstochastic linear relationship between rate of return at time $t-1$ and at time $t$ (i.e. $r_{j t}=f\left(r_{j t-1}\right)$. The distribution of $r_{t}$ within a given class will be set by the distribution of $r_{t-1}$ under this assumption. Consider class intervals of equal size imposed upon a bell-shaped distribution; the distribution within each class will be approximately shaped:


The variance of this distribution is a decreasing function ${ }^{10}$ of ( $d-c$ ), so beyond the point of inflexion of the bell-shaped distribution the class variances will increase.
10) See the Appendix to this chapter for proof of this and a further investigation of how this problem may influence the conditional distributions.

The second factor is that the analysis has been performed with cinsses containing equal numbers of members, not with classes covering equa? interval.s. Therefore moving from the origin invoives increasine the interval ('a' in the nreceding diegram) and thus again increasing the variance. Althongh this effect aonlies as one moves from the oriein while the first anplies only from the point of inflexion, the interval effect is probably the stronger and therefore we mar expect that the variance within classes containing equal numbers of members from a bellshaned distribution will increase as one moves away from the origin.

Now if we relax the assumption of 2 nonstochastic relationship between $r_{t}$ and $r_{t-1}$, the exror term in the relationshjp may, denending unon its nature, provide the third of the factors. If the variance of the error term rises as $r_{t-1}$ deviates further from the moan, then this heteroscedasticity will add to the strength of the variance-prior mean relationship. If the errors are homoscedastic then we are left with the effect of the first two factors. Were they to be heteroscedastic but decreasing with increases in $r_{t-1}$, then there is a theoretical possibility that the end result could be constant variance of the conditional distributions (excepting, of course, the open-ended extreme distributions).

It seems most likely that it is the former kind of heteroscedasticity that apnlies, for some of the firms earning very high profits owe their position to very short term factors and are likely to experience a very ranid return to more modest profitabjlity. Also there are some firms which continually strive to reach high profit nositions where others are ouietily contient with average returns. 11 It is reasnnable to exnect that,
17) Tor exampe sep Politicol and ooncmio phannine: "Thmoters and

of firms with high rates of return, an excentional pronortion are suoh strivers. More attempts will therefore be made to co against the competi.tive nressure on profitability by firms with high rates of return. There is thus likely to be more variability of exporience and behariour anong high nrofit earners.

A similar pair of arguments for high variability may be mede for the case of firms with rates of return well below average. Some will be there through short term random influences and will rise quickly back to more acceptable level.s of profitability. Althoush there may be few striving companies amone those with low profitability, the threat of bankrunticy or takeover may alter the behaviour of firms and inspire them to great efforts to raise their rate of retum. 'Therefore it seems at least nlausjble that the variability of profit movements will increase as the absolute value of the prior moan increases and that there are three factors all working to strengthen this relationshin.

A linear resression of the variance on the prior mean for the full ranop of classes would not be helpful. Rather than try a parabolic form (say), strajght lines have been separately fitted to each range. The MorbinWatson statistics indicate that this method does not lead to any serial correlation. The exnected sjgns were found in evory case: a positive slone coefficient for the nositive range and negetive for the negative range. In the majority of cases the slone noefficient was significant ${ }^{1 ?}$ but generaliy the exnlanatory nower of the equation was guite low. But overall there was sufficient evidence to sunnort the predietions ahout the verience-prior mean relation.

[^16]It is clear that the transition matrices for the industries and for the subgroups do not display any different characteristics. We can therefore report our findings as applying at both levels of aggregation. These findings are that profit decay is displayed in every industry and subgroup, and that it is a less steady process in the negative range than in the positive range, i.e. above average profitability is more steadily (this does not imply more quickly) eroded than below average profitability is built up. Apart from this we find no other differences between the ranges.

The conditional distributions are predominantly negatively skewed and leptokurtic. These last two conclusions rule out the normality assumption for the conditional distribution and lead to the decision that an attempt to fit a distribution function to them would be difficult, quite possibly unsatisfactory and certainly of doubtful value in the present context. Therefore the prior mean to final mean relation will be the sole aspect of the transition function to be further developed.

Separate analysis of those conditional distributions whose prior means were close to zero revealed only one way in which they differed from the whole set: the majority of cases where the final mean was further from zero than the prior mean occurred in these central distributions. Perhaps unsurprisingly this indicates that the profit decay process is more disturbed near the industry average. Also the cases where the final mean had the opposite sign to the prior mean were mainly with the final mean negative and suggested that the industry mean was perbaps higher than the point of convergence.

Finally, we found that the variance of the conditional distribution is an increasing function of the absolute value of the prior mean.

The Influence of the Distribution of $x_{t-1}$ on the Conditional Distributions

The $r_{t-1}$ are distributed according to an approximately bell-shaped distribution. Deviations from normality with respect to kurtosis or skewness are not important in the following. The distribution is divided into intervals and it is intended to consider the characteristics of the distribution within an interval. This interval distribution may be illustrated:


Such a distribution has a density function:

$$
\left.f(x)=\frac{(c+d)}{2}+\frac{(d-0}{2 a}\right) x
$$

We require that:

$$
\begin{aligned}
\int_{-a}^{+a} f(x) d x & =1 \\
\text { i.e. } a(c+d) & =1
\end{aligned}
$$

The mean is given by:

$$
\mu_{1}^{\prime}=\int_{-a}^{+a} x f(x) d x=\frac{(d-c) a^{2}}{3}
$$

The second origin moment:

$$
\mu_{2}^{\prime}=\int_{-a}^{+a} x^{2} f(x) d x=\frac{a^{2}}{3}
$$

The variance $=\frac{a^{2}}{3}-\frac{(d-c)^{2}}{9} a^{4}$

This deoreases as the difference between $d$ and $c$ increases, or as the absolute value of the slope of the bell-shaped distribution decreases. Therefore maintaining a constant (i.e. equal class intervals) and moving away from the centre of the distribution will first produce decreasing variance until the point of inflexion on the distribution is reached, whereafter the variance will increase.

In practice the class intervals are not kept, constant but are chosen to provide equal numbers in each class. This necessarily means that a increases an one moves away from the centre of the distribution and therefore so does the variance.

It is an obvious development to consider the skewness and kurtosis of these interval distributions. It is immediately apparent that the sign of the skewness will depend upon (c-d) and that interval distributions to the left of mean of the bell-shaped distribution will be negatively and those to the right positively skewed.

For the kurtosis the 4 th origin moment:

$$
\mu_{4}^{\prime}=\frac{a^{4}}{5}
$$

and the 4 th central moment:

$$
\begin{aligned}
\mu_{4} & =\mu_{4}^{\prime}-4 \mu_{1}^{\prime} \mu_{3}+6\left(\mu_{1}^{\prime}\right)^{2} \mu_{2}^{\prime}-3\left(\mu_{1}^{\prime}\right)^{4} \\
& =\frac{a^{4}}{5}-\frac{2}{45}(d-c)^{2} a^{6}-\frac{1}{27}(d-c)^{4} a^{8}
\end{aligned}
$$

It can easily be shown that the ratio $\mu_{4} / \mu_{2}{ }^{2}$ cannot, take a value as large as 3 and therefore that the interval diatributions are platykurtic.

So far we have considered the diatributions of $r_{t-1}$ within the class intervals and we have found these distributions to bave increasing varimnce as the deviation of the class from the moan $r_{t-1}$ increases. It has also heer shown that these interval distributions will be necatively skewed on one side of the mean $x_{t-1}$ and positively skewed on the other. They will be nlatykurtic.

Now were the relation hetween $x_{t}$ and $r_{t-1}$ to lack a stochastio term, these charactieristics wonid be carried streicht over into the conditional distributions of $r_{t}$ and the form of these distributions would be nurely a. consemuence of taking intervals in the rance of $r_{t-1}$. Fiven if the equation linking $r_{t, \ldots}$ to $r_{t}$ is stochastic, the effect of the stochastio term will he overlaid on the distributions desoribed above.

Therefore we may conclude that some of the increase in veriance thet poonmnanies increases in the absolute value of the prion mean is exnlained in this wey. On the other hand the similer skewness nattorn both abore and below the mean that, we have found means that the stochastic term dominates in this resnoct. The Jentokurtosis found similarly must indicate the relative importance of the stochestic term.

THE FORM OF THE DECAY FUNCTION AND THE PROBLEMS OF ITS ESTIMATION :

In this chapter we are concerned with the regression of the rate of return at time $t$ on a function of the rate of return in the previous period - the decay function. The actual observations to be used are those derived from the transition functions described in the preceding chapter. So we are concerned with standardized rates of return and the data are the means of the conditional distributions and the means of the initial period values falling within a particular class - in the terminology of Chapter III, the final means an 1 the prior means. This chapter falls into three main sections; inrstly an attempt to isolate functional forms that fulfill the criteria set out in Chapter II for the behaviour of the decay of profitability (Section 6.1); secondly, a consideration of the problems of estimating these functions (Section 6.?). The third section considers the pooling of the data (Section 6.3).

## Section 6.1: The Form of the Function

For brevity, the final mean will be written as $r_{s t}$ and the prior mean as $r_{s t-1}$, each referring to the sth class of the transition matrix. Our topic is therefore:

$$
\begin{equation*}
r_{s t}=f\left(r_{s t-1}\right) \tag{1}
\end{equation*}
$$

In Chapter II we argued that the first derivative of this function should only take values between 0 and 1:

$$
\begin{equation*}
0 \leqslant f^{\prime}\left(r_{s t-1}\right) \leqslant 1 \tag{2}
\end{equation*}
$$

and that the second derivative should be nonpositive above the mean and nonnegative below. The first condition (2) is to ensure that there is decay of profitability.

Before developing any more constraints upon the functional form, it is appropriate to indicate how the concept of regression towards the mean that is employed here differs from that of Galton's. Hart and Prais ${ }^{1}$ summarise it and emphasize the reduction in dispersion thus: if variable $x$ at time $t$ is related to variable $x$ in the previous period by a simple linear relationship:

$$
x_{t+1}-\bar{x}_{t+1}=\beta\left(x_{t}-\bar{x}_{t}\right)+\epsilon
$$

then writing the variance of $x_{t}$ as $V\left(x_{t}\right)$ :

$$
v\left(x_{t+1}\right)=\beta^{2} v\left(x_{t}\right)+\sigma_{\epsilon}^{2}
$$

so the change in variance will depend upon $\beta^{2}$ and $\sigma_{t}^{2}$.

1) Hart P E \& Prais S J, "The Analysis of Business Concentration", Journal of the Royal Statistical Society Series A 1956 口 172

Changing $\sigma_{c}^{2}$ to the other side and dividing through by $V\left(x_{t+1}\right)$ :

$$
\frac{V\left(x_{t+1}\right)-\sigma_{\epsilon}^{2}}{\nabla\left(x_{t+1}\right)}=\frac{\beta^{2} v\left(x_{t}\right)}{\nabla\left(x_{t+1}\right)}
$$

The left hand side is the ratio of explained to total variance, and is thus the square of the correlation coefficient of $x_{t+1}$ to $x_{t}$,

$$
\therefore \frac{v\left(x_{t+1}\right)}{v\left(x_{t}\right)}=\frac{\beta^{2}}{e^{2}}
$$

There is thus a reduction in variance if $\beta<e$.

There was no attempt in Chapter II to claim anything about the behaviour of the variance of rates of return from year to year; the postulate was that the dominant systematic movement of individual rates of return was towards the mean (or some approximately central value). For this, the expected value of $\mathbf{r}_{s t}$ must be closer to this central value than was $\mathbf{r}_{\text {st-1 }}$. Thus in the equivalent simple linear relationship to that postulated by Hart and Prais we are only putting a constraint upon $\beta$. We are saying something about the average year to year pattern of movement of individuals in the population but nothing about the year to year movement of the dispersion of the population. In fact we found in Section 4.5 considerable stability in the annual dispersion of rates of return which would suggest that on average over a number of years $\beta^{2} \simeq e^{2}$. The original use of Galton's concept was with a characteristic fixed for any individual but with a changing population. Here we are considering a characteristic which changes for any individual from a population whose membership varies little. Galton's regression is the regression of a population, the regression considered here is the regression of individuals.

To continue with the main topic, although there is not any certainty that the regression is towards the mean rather than mome other point, we will develop the functional form on the basis of this assumption. We may therefore state a third condition: that the function should pass through the origin:

$$
f(0)=0
$$

That is, at the mean $r_{s t}=r_{s t-1}$ there is no profit decay. A necessary consequence of this condition and condition (2) is that the function ming always take the sign of its arguaent or, to put that another way, that function can only lie in the first and third quadrants (see Diagram 2.1).

The second derivative condition restricts the range of curvilinear shapes and permits a straight line relationship. If a linear relationship is used, it may be argued that the working of the competitive resource allocation does bring increased pressure on very high rather than moderately high rates of return as the absolute fall in rate of return will be greater in the former case. If it is felt that this increased competitive pressure should bring about an increased proportional fall in the rate of return, then a nonlinear relationship is required. The form shown in Diagram 2.1 would meet this latter requirement.

This means that the slope diminishes with increasing positive values of $r_{s t-1}$ :

$$
f^{\prime \prime}\left(r_{s t-1}\right)<0 \quad \text { for } \quad r_{s t-1}>0
$$

and should increase with increases in $r_{s t-1}$ while $r_{s t-1}$ is negative:

$$
f^{\prime \prime}\left(r_{s t-1}\right)>0 \quad \text { for } \quad r_{s t-1}<0
$$

Therefore we must use:

$$
\begin{equation*}
f^{\prime \prime}\left(r_{s t-1}\right)=\eta r_{s t-1} \quad \eta<0 \tag{3}
\end{equation*}
$$

We will develop this case first. Integrating (3) gives:

$$
f^{\prime}\left(r_{s t-1}\right)=\frac{1}{2} \eta r_{s t-1}^{2}+\beta
$$

Now the limits within which this slope must fall give:

$$
0<\frac{1}{2} \eta r_{s t-1}^{2}+\beta<1
$$

It is apparent that any such restrictions mean that the function is only suitable within some range of $r_{s t-1}$. The limits of this range are:

$$
\frac{2(1-\beta)}{\eta}<r^{2} \text { st-1 }<-\frac{2 \beta}{\eta} \quad \text { for } \eta<0
$$

These may be expressed as:

$$
r_{s t-1}^{2}<-\frac{2 \beta}{\eta}
$$

The second ensures that the lower limit on $r^{2}$ at-1 is negative and the third that the upper limit is positive.

Integrating for the second time we get the function itself:

$$
\begin{equation*}
r_{s t}=\frac{1}{6} \eta r_{s t-1}^{3}+\beta r_{s t-1} \tag{4}
\end{equation*}
$$

The constant of integration must be zero in order to ensure that the curve passes through the origin. Redefining the coefficients we have:

$$
\begin{equation*}
r_{s t}=\beta r_{s t-1}-\eta r_{s t-1}^{3} \tag{5}
\end{equation*}
$$

The limit on $r_{s t-1}$ now becomes:

$$
r_{s t-1}^{2}<-\frac{\beta}{3 \eta}
$$

If $n$ is zero then we get the linear form:

$$
\begin{equation*}
\mathbf{r}_{s t}=\beta \mathbf{r}_{s t-1} \tag{6}
\end{equation*}
$$

and

$$
f^{\prime \prime}\left(r_{g t-1}\right)=0
$$

The range of $r_{s t-1}$ is now unrestricted and the only requirement for the parameter values is:

$$
0<\beta<1
$$

If we now bring into question the location of the decay origin, we derive two other forms. Let us assume that if our rates of return are expressed as deviations from the decay origin, then one of the preceding pair of functional forms will fit. Let the true decay origin be on average $\theta$ standard deviations below the mean. Then we must substitute $\mathbf{r}_{\mathrm{st}-1}+\theta$ and $\mathbf{r}_{\mathrm{st}}+\theta$ into the equations. So if a linear form is appropriate and if the rates of return are expressed as deviations from the mean, then:

$$
\left(\mathbf{r}_{\mathbf{s t}}+\theta\right)=\beta\left(\mathbf{r}_{\mathbf{s t - 1}}+\theta\right)
$$

is the equation. This may be written:

$$
\begin{equation*}
\mathbf{x}_{\mathbf{s t}}=\theta(\beta-1)+\beta \mathbf{x}_{\mathbf{s t - 1}} \tag{7}
\end{equation*}
$$

Therefore we may calculate from the coefficients of an equation:

$$
\begin{equation*}
r_{s t}=a+b r_{s t-1} \tag{8}
\end{equation*}
$$

the values of $\beta$ and $\theta$. Given $0<\beta<1$, a negative' a' implies a positive $\theta$ and therefore that the decay origin is below the mean. 'a'may be insignificantly different from zero without implying that the mean is a good approximation for the decay origin if $\beta$ is insignificantly
different from 1. The important point is that $b$ is an unbiased estimate of $\beta$.

Should a nonlinear form be appropriate, then under the same assumptions we must substitute:

$$
\left(r_{s t}+\theta\right)=\beta\left(r_{s t-1}+\theta\right)+\eta\left(r_{s t-1}+\theta\right)^{3}
$$

which may be written:

$$
\begin{equation*}
r_{s t}=\theta\left(\beta-1+\eta \theta^{2}\right)+\left(\beta+3 \eta \theta^{2}\right) r_{s t-1}+3 \eta \theta r_{s t-1}^{2}+\eta r_{s t-1}^{3} \tag{9}
\end{equation*}
$$

Therefore we may estimate:

$$
\begin{equation*}
r_{s t}=a+b r_{s t-1}+c r_{s t-1}^{2}+d r_{s t-1}^{3} \tag{10}
\end{equation*}
$$

Now calculating $\beta, \eta$ and $\theta$ from this equation runs into the problem of overidentification - we may solve for these unknowns in more than one way and we may expect to get different numerical values for each method of solution. The usual method in this situation is to assume a value for one of the structural parameters - as we are doing when we assume the mean is the decay origin. We might regard it in the light; are $b$ and $\alpha$ good estimates of $\beta$ and $\eta$ ? The answer would be that $d$ is a good estimate of $\eta$ and as long as $\theta$ is small, $b$ will be close but below $\beta$. We will deal with these matters again when we discuss the actual estimated equations in Section 7.1.

The objection to these power function forms is their limited range. The other family of functions that appear applicable are those based on exponentials. They have an immediate limitation in that estimation has to be done in terms of the logarithmic transforms and then we are limited to positive values of the variables. This is not insuperable
in that except for a very small number of observations:

$$
\operatorname{sign}\left(r_{j t}\right)=\operatorname{sign}\left(r_{j t-1}\right)
$$

and the function can be moved into the positive quadrant by multiplying both variables by -1. But it is necessary to estimate the function separately for the data above the industry mean and for the data below the industry mean.

A possible form will be briefly discussed below although it will ultimately be rejected due to the difficulty of estimating it. The function:

$$
\begin{equation*}
r_{j t-1}+a=a e^{b r} j t \tag{11}
\end{equation*}
$$

has the correct characteristics, taking logarithms:

$$
\begin{align*}
& r_{j t}=\frac{1}{b} \log \left(r_{j t-1}+a\right)-\frac{1}{b} \log a  \tag{12}\\
& \frac{d r_{j t}}{d r_{j t-1}}=\frac{1}{b\left(r_{j t-1}+a\right)}
\end{align*}
$$

This is greater than zero for $a, b>0$ as $r_{j t-1}>0$. It will reach $a$ maximum value when $r_{j t-1}$ is at a minimum. This is $r_{j t-1}=0$ when the slope is ${ }^{l} / a b$. This is less than 1 if $a b>1$.

Taking the second derivative:

$$
\frac{d^{2} r_{j t}}{d x^{2}}=-\frac{1}{b\left(r_{j t-1}+a\right)^{2}}
$$

which is negative given the prior restriction upon b.

Finally, it goes through the origin in the form given in (11). If the constant term does not equal the coefficient of the exponential
term then this is not so. It is exactly this term which causes the estimation problems - if it is assumed that the line does go through the origin, then an iterative method ${ }^{3}$ suggests itself. Otherwise the problem looks intractible. But even in the simple case an iterative method is not a realistic proposition when a separate function for each industry has to be estimated. The third form to be considered is:

$$
r_{j t}=\alpha r_{j t-1}^{\beta}
$$

This assumes that the mean is the correct decay origin but otherwise the function is quite convenient. The first derivative:

$$
\frac{d r_{j t}}{d r_{j t-1}}=\alpha \beta r_{j t-1}^{\beta-1}
$$

is greater than zero for $r_{j t-1}>0$ if $\alpha$ and $\beta$ have the same sign. But we require that the sign of $r_{j t}$ equal the sign of $r_{j t-1}$, therefore $\alpha>0$ and so therefore $\beta>0$.

The restrictions upon the coefficients can more efficiently be dealt with once the second derivative has been examined:

$$
\frac{d^{2} r_{j t}}{d r_{j t-1}^{2}}=\alpha \beta(\beta-1) r_{j t}^{\beta-2}
$$

which is only negative as required if $\beta<1$, given that we require the product $\alpha \beta$ to be positive in order that the first derivative be positive. Returning now to the slope, we require:

$$
\alpha \beta_{j t-1}^{\beta-1}<1
$$

3) A first pass on $r_{j t}=\frac{1}{b} \log \left(r_{j t-1}+1\right)-\frac{1}{b} \log$ a gives a value of $a$, say $\hat{a}$, which can be substituted in: $r_{j t}=\frac{1}{b} \log \left(\hat{a} r_{j t-1}+\hat{a}\right)-\frac{1}{b} \log a$ This equation can be estimated and the cycle repeated.

Now as $\beta<1$, the left-hand expression tends to infinity as $\mathbf{r}_{j t-1}$ tends to zero. Therefore there must be some value $\mathbf{r}$ of $\mathbf{r}_{j t-I}$ below which the slope exceeds one. This is given by:

$$
(\alpha \beta)^{\frac{1}{1-\beta}}
$$

It is desirable that this be as small as possible in order that the range of $r_{j t-1}$ for which the first derivative condition is violated be as small as possible.

The estimation of this form is done by a logarithmic transformation:

$$
\begin{equation*}
\log r_{j t}=\log \alpha+\beta \log r_{j t-1} \tag{13}
\end{equation*}
$$

There is not an ideal function and, having rejected the exponential because of the difficulty of estimating it, we are left with the power functions or the log linear function. We may summarise the power functions into three forms:
(a)
linear $\quad r_{s t}=a+b r_{s t-1}$
where $b$ is an estimate of $\beta$ and $a$ of $\theta(\beta-1)$. The only restriction is $0<\beta<1$.
(b) cubic without squared form - in future referred to as linear-cubic

$$
r_{s t}=b r_{s t-1}-d r_{s t-1}^{3}
$$

where $b$ is an estimate of $\beta$ and $d$ of $\eta$. The restrictions are that $0<\beta<1, \eta<0$ and $r_{\text {st-1 }}^{2}<-\beta / 3 \eta$. This form assumes that the mean is the decay origin.
(c) cubic $\quad r_{s t}=a+b r_{s t-1}+c r^{2}{ }_{s t-1}+d r_{s t-1}^{3}$
where $a$ is an estimate of $\theta\left(\beta-1+\eta \theta^{2}\right)$, $b$ of $\left(\beta+3, \theta^{2}\right)$, $c$ of
$3 \eta \theta$ and $d$ of $n$. The constraints upon $\beta, \eta$ are as in the preceding form. There is a similar restriction upon the range of $r_{s t-1}$.

The log-linear form is:
(d) $\quad \log r_{s t}=\log a+b \log r_{s t-1}$
where $a$ and $b$ are estimates of $\alpha$ and $\beta$ respectively. The constraints are that $\alpha>0,0<\beta<1$ and $(\alpha \beta)^{\frac{1}{1-\beta}}$ should be close to zero. This form assumes that the mean is the decay origin.

## Section 6.2 : Estimation of the Decay Function

We now have four possible forms of the decay function and some expectations about the values their coefficients should take. Estimation of these functions is a little out of the ordinary because of the amount of knowledge we have about the process. This information comes from the examination of the transition matrices in Chapter 5. If we had estimated the decay function from the raw data we can see that the equation would have heteroscedastic and nonnormal errors. This is apparent because we may regard the conditional distribution of $\mathbf{r}_{t}$ given $r_{t-1}$ as the conditional distribution of the error term in the decay function given $\mathbf{r}_{t-1}$ - once the mean is set to zero. We therefore discover that the variance of this conditional distribution varies with the value of the independent variable and that this distribution appears negatively skewed and leptokurtic.

Neither problem is very serious for ordinary least squares regression (OLS) but they are nontheless undesirable. Their consequences are heteroscedasticity - the OLS estimator is not the minimum variance estimator but is unbiased. Nonnormally distributed errors mean that the OLS estimators are not maximum likelihood estimators and that small sample tests of significance are not exact: but $t$ and $F$ tests are robust as long as the distribution is unimodal and not seriously assymetrical. As was pointed out at the end of Section 3.3, the method of handling the data results in errors whose distribution is closer to the normal than would be the case of a straightforward regression on the raw data.

The problem of heteroscedasticity, on the other hand, does demand more attention. Its consideration requires clarification of the nature of the
equation that we are attempting to estimate. In Section 2.6 the model for an individual firm at time $t$ is given:

$$
\begin{equation*}
r_{j t}=f\left(r_{j t-1}\right)+u_{j t} \tag{1}
\end{equation*}
$$

where the effect of the transfer of resources is represented by the first term on the right hand side, and random factors both internal and external by the second. Fcr simplicity we will use the linear form and write:

$$
\begin{equation*}
r_{j t}=\beta r_{j t-1}+u_{j t} \tag{2}
\end{equation*}
$$

We will assume:

$$
E\left(u_{j t \cdot}\right)=0
$$

and

$$
\mathbb{E}\left(r_{j t-1} u_{j t .}\right)=0
$$

$\qquad$

Now we may assume that $u_{j t}$. is homoscedastic or that its variance is dependent upon $r_{j t-1}$ - the heteroscedastic assumption.

The estimation process uses groups of ciservations, so summing over a set of firms $S=\left\{j \mid a<r_{j t-1}<b\right\}$, (2) becomes:

$$
\begin{equation*}
\sum_{j \in s} r_{j t}=\sum_{j \in s}\left(\beta r_{j t-1}+u_{j t}\right) \tag{3}
\end{equation*}
$$

Dividing through by $N(S)$ - the number of members of the set $S$ - gives the means of the variables over the set S :

$$
\begin{equation*}
\bar{r}_{s t}=\beta \bar{x}_{s t-1}+\bar{u}_{s t} \tag{4}
\end{equation*}
$$

Now what are the terms of this equation? We have a fixed known set of $r_{j t-1}$, a subset of the set of rates of return of all firms in the
industry at time $t-1$. Now with each firm's rate of return ( $r_{j t-1}$ ) at a particular time there will be associated a random drawing from the population of errors. Therefore $\bar{u}_{s t}$ will be the mean of a random sample and therefore a stochastic variable. As the population mean is zero and it is distributed independently of $r_{j t-1}$, it follows that $E\left(\bar{u}_{s t}\right)=0$ for all sets $S$. The regression of $\bar{r}_{s t}$ on $\bar{r}_{s t-1}$ with error term $\bar{u}_{\text {st }}$ is well behaved ${ }^{4}$ by our previous assumptions except for its possible heteroscedasticity and serial correlation. The latter we will assume only arises through a misspecification of the resource transfer expression, and so we may assume for the present exercise that the linear form is appropriate and therefore that $E\left(u_{r t} u_{s t}\right)=0 \quad s \neq r$.

We must therefore consider the variance of the error term. If we take the variances of equation (2) we get:

$$
\begin{equation*}
\operatorname{Var}_{s}\left(r_{j t}\right)=\beta^{2} \operatorname{Var}_{s}\left(r_{j t-1}\right)+\operatorname{Var}_{s}\left(u_{j t}\right) \tag{5}
\end{equation*}
$$

where $\operatorname{Var}_{s}\left(r_{j t}\right)$ indicates the variance of the rates of return at time $t$ for those firms in set $S$ at time $t-1$. Ihe assumption of independent errors means that there is no covariance term. We have observed in Section 5.4 that:

$$
\begin{equation*}
\operatorname{Var}_{s}\left(r_{j t}\right)=f\left(\bar{r}_{s t-1}\right)+\varepsilon_{s t} \text { with } f^{\prime}>0 \tag{6}
\end{equation*}
$$

$\varepsilon_{\text {st }}$ has mean zero and is distributed independently of $\bar{r}_{\text {st-1 }}$.
In the Appendix to Ch. V

$$
\begin{equation*}
\operatorname{Var}_{s}\left(r_{j t-1}\right)=g\left(\bar{r}_{s t-1}\right) \text { with } g^{\prime}>0 \tag{7}
\end{equation*}
$$

This is a norstochastic equation describing the behaviour of the variance of $r_{j t-1}$ within the groups used in classifying the data. The
4) Note that we are not faced with the problem of lagged dependent. variables as we are looking at a cross-section in one time period.
way in which this variance is related to $\bar{r}_{s t-1}$ is purely a consequence of the grouping procedure.

Equation (7) may be substituted into (5):

$$
\begin{equation*}
\operatorname{Var}_{s}\left(r_{j t}\right)=\beta^{2} g\left(\bar{r}_{s t-1}\right)+\operatorname{Var}_{s}\left(u_{j t}\right) \tag{8}
\end{equation*}
$$

Now whether or not equation (4) has heteroscedastic erros depends upon whether $\operatorname{Var}_{s}\left(u_{j t}\right)$ is or is not a function of $\bar{r}_{s t-1}$. If $\operatorname{Var}{ }_{s}\left(u_{j t}\right)$ is not dependent upon $\bar{r}_{s t-I}$, then the only cause of covariation between $\operatorname{Var}_{s}\left(r_{j t}\right)$ and $\bar{r}_{s t-1}$ is the grouping procedure that underlies (7). So heteroscedastic errors in (4) involve some systematic variation of $\operatorname{Var}{ }_{s}\left(r_{j t}\right)$ with $\bar{r}_{s t-1}$ that is not covered by $\beta^{2} g\left(\bar{r}_{s t-1}\right)$.

As estimates of (6) using a linear functional form show no evidence of misspecification, it may be presumed that that function represents all the relationship between $\bar{r}_{s t-1}$ and $\operatorname{Var}_{s}\left(r_{j t}\right)$. In other words, the $\operatorname{Var}_{s}\left(u_{j t}\right)$ may be considered by comparing $f\left(\bar{r}_{s t-1}\right)$ and $\beta^{2} g\left(\bar{r}_{s t-1}\right)$. This is made clear if (6) is substituted into (8):

$$
\begin{equation*}
\operatorname{Var}_{s}\left(u_{j t}\right)=f\left(\bar{r}_{s t-1}\right)-\beta^{2} g\left(\bar{r}_{s t-1}\right)+\varepsilon_{s t} \tag{9}
\end{equation*}
$$

Both $f$ and $g$ are increaring functions of $\vec{r}_{s t-1}$. It is arguea in Section 5.4 that if $\operatorname{Var}_{s}\left(u_{j t}\right)$ does vary with $\bar{r}_{s t-1}$, it also will be an increasing function.

Therefore taking the derivative of (9) with respect to $\bar{r}_{\text {st-1 }}$ will give us an expression which, if zero, will indicate that Var: ${ }_{\mathrm{s}}{ }^{(u}{ }_{j \mathrm{jt}}$ ) is not dependent upon $\bar{r}_{\text {st-1 }}$ and equation (4) has homoscedastic errors. So we nust consider the nature of

$$
\begin{equation*}
f^{\prime}-\beta^{?} g^{\prime} \tag{10}
\end{equation*}
$$

The function $f$ and the coefricient $\beta$ have been estiratod. The function
g has not been estimated or specified theoretically. As f is linear, any nonlinearity in $g$ will lead to (10) being non zero and the errors heteroscedastic. It is apparent from the reasoning in the Appendix ,
to Ch. V that g is nearly certainly nonlinear; on the other hand it is not clear whether the coefficients of any nonlinear terms are large enough to cause serious concern in this context.

We will have a good indication that (10) is positive, if it takes that sign when the slope coefficient of linear approximation to $g$ is inserted. This conclusion can be drawn because g is convex downwards and any linear approximation will overestimate the coefficient of the linear term in a polynomial expression for $g$.

This rough estimate can be gained by graphing ${ }^{a 2} / 3$ against $\bar{r}_{\text {st-1 }}$, and fitting a line to those points. As the full expression of the variance is not $a^{2} / 3$ but $a^{2} / 3-(d-c)^{2} a^{4} / 9$, the graph exaggerates the variances and therefore exaggerates the slope of the function.

This exercise has been carried out on Industry 5 wihich has not untypical values for the estimated coefficients of $f$ or for $\beta$. The value of $g^{\prime}$ found is 0.010 in the positive and 0.015 in the negative range. The slope coefficients of $f$ are 0.270 and 0.180 and the estimated values of $\beta$ are 0.871 and 0.743 respectively. It is evident that (10) is positive. Inspection of the other industries does not suggest that any other conclusion will apply there. So we find that $\operatorname{Var}_{s}\left(u_{j t}\right)$ seems to depend positively upon $\bar{r}_{\text {st-1 }}$ and consequently equation (4) has heteroscedastic errors. The estimation method appropriate to an equation with heteroscedastic errors is weighted least squares (WLS), which is a particular case of generalized
least squares. ${ }^{5}$ Let the variance-covariance matrix be:

$$
E\left(u_{\text {st. }} \quad u_{s t}\right)=\sigma^{2}\left(\begin{array}{ccccc}
v_{11} & 0 & \cdots & \cdots & 0 \\
0 & v_{22} & & \vdots \\
\vdots & & & \vdots \\
\vdots & & & & 0 \\
0 & \cdots & \cdots & 0 & v_{n n}
\end{array}\right)
$$

then the weights to be applied to the regression are given by the matrix:

$$
-M=\left(\begin{array}{cccc}
1 / \sqrt{\overline{1}_{11}} & 0 & & 0 \\
0 & 1 / \sqrt{v_{22}} & & \vdots \\
\vdots & & \ddots & 0 \\
\dot{0} & \cdots & 0 & 1 / \sqrt{v_{1 n}}
\end{array}\right)
$$

Instead of the regression:

$$
\mathrm{Y}=\mathrm{X} \beta+\mathrm{u}
$$

WLS uses MY $=$ MK $\beta+$ Mu
and $\operatorname{Var}(\ddot{\beta})=\sigma^{2}\left(X^{\prime} M^{\prime} M X\right)^{-1}$
This method gives minimum variance estimators.

But we are faced with inadequate knowledge of the variance-covariance matrix, so let us consider the effect of applying incorrect weights:

$$
\begin{aligned}
N & =\left(\begin{array}{ccccc}
1 / \sqrt{e_{1}^{111}} & 0 & \cdots & \cdots & 0 \\
0 & \sqrt[1]{\sqrt{e_{2} v_{22}}} & & & \vdots \\
\vdots & & \ddots & 0 \\
0 & \cdots & \cdots & 0 & 1 / \sqrt{e_{n} v_{n n}}
\end{array}\right) \\
& =\left(\begin{array}{ccccc}
1 / \sqrt{e_{1}} & 0 & 1 / \sqrt{P_{2}} & & \\
0 & & & \vdots \\
\vdots & & & 0 \\
0 & \cdots & \cdots & 0 & 1 / \sqrt{e_{11}}
\end{array}\right) \\
& =\text { RM }
\end{aligned}
$$

5) Johnston J, "Econometric Methods", McGraw Hill 1963, p 207 et seq.

Using these woights gives an equation.

$$
R M Y=R M X \beta+R M u
$$

and $E\left(u^{\prime} M^{\prime} R^{\prime} R M u\right)$ will not have the form $\sigma^{2} I$, in other words there will still be heteroscedasticity. In the simple case where $\rho_{1}=\rho_{2} \ldots=\rho_{n}$ then $\quad E\left(u^{\prime} M^{\prime} R^{\prime} R M u\right)=\rho^{2} E\left(u^{\prime} M M u\right)=\rho^{2} \sigma^{2} I$
and

$$
\begin{aligned}
\operatorname{Var}(\hat{\beta}) & =\rho^{2} \sigma^{2}\left(X^{\prime} M^{\prime} \rho^{2} M X\right)^{-1} \\
& =\sigma^{2}\left(X^{\prime} M^{\prime} M X\right)^{-1}
\end{aligned}
$$

Thus a constant proportional error is of no concern. But generally whether it is preferable to use WLS with inexact weights or OLS will depend upon the particular $R$ matrix.

In the present case, the choice is between OLS or WLS using the $\operatorname{Var}_{s}\left(r_{j t}\right)$ as approximate values (or some adjustment of them). Our main concern is with the values of the estimated coefficients, and therefore the unbiasedness of the OLS estimators makes the choice less crucial. It is also made more difficult as the standard formula for calculating the standard error of estimate is biased downwards when OLS is used in the presence of heteroscedastic errors. ${ }^{6}$

WLS was tried for Industry 16 using the reciprocals of the standard deviations of the conditional distributions as weights. There was little consistent pattern to the differences in standard errors produced by this procedure and by OLS. The differences were also very small, none exceeding $10 \%$ and most being less than $5 \%$. Therefore with the weights employed WLs does not seem to offer any marked improvement in efficiency. It does, on the other hand, take us into an area of some difficulty - the effects of using weights that probably overestimate the amount of variation in the error variance. We could have regarded this as less serious if the standard errors of estimate were consistently improved when we used WIS, but this not being so, OLS will be used. We
must therefore keep in mind their inefficiency.

A problem of estimation that arises in using the linear-cubic and cubic power functions is multicollinearity: There is very high correlation between $r_{\text {st-1 }}$ and $r_{\text {st-1 }}^{3}$ over the whole range of data, both negative and positive, and between $r_{s t-1}, r_{s t-1}^{2}$ and $r_{s t-1}^{3}$ for data all of one sign. We must therefore expect high standard errors for the coefficients for this reason as well as because of the heteroscedasticity previously discussed.

In Section 5.1 the transition matrices are introduced as calculated on 12 years data of profitability. The decay functions are then estimated upon statistics calculated from these matrices. These functions and the derived statistics developed in Chapter VIII therefore bear something of the characters of 12 year averages. The question arises of whether much interesting and relevant variation is being lost by pooling such a long run of data. A complete answer can only be provided by attempting to investigate year-to-year variations in the decay function, a study that demands a separate and major exercise. A partial answer may be provided by three routes: firstly, are there any economic arguments that might suggest that the decay of profitability is quite stable. Secondly, is there any available statistical evidence already produced in this study that might suggest the answer to this problem. Thirdly, what relation will our observed functions have to the arnual functions if these latter do vary from year to year.

With respect to the first: it has been argued (in Chapter I especially) that the decay of profitability measures an aspect of industry performance. In the short run, we expect the chain of causation to run from structure to performance and this linkage has been discussed in Chapter 2. Whilst it would not be sensible to argue that only changes in structure vary performance, it would not be excessive to take the view that only structural changes would bring about persistent changes in performance. Of course, changes in the general operating environment of all companjes as well as those particular to the industry must be taken into account in this. Over any time period, trade cycle changes will occur and affect profitability and there will also be changes in legai and
tax positionc of companies. But note that in this study, factors varying the dispersion of profitability or mean profitability may be disregarded. Also profitability is calculated gross of tax and therefore straightforward changes in tax rates should have little effect. Further the one radical change in company taxation since the war - corporation tax - occurs outside our period. Therefore it seems safe to conclude that changes in the general operating environment do not have a major impact on the decay of profitability. In addition, the use to which we put the rates of decay is one based upon inter-industry relativities, so common effects upon all industries do not in practice cause difficulties.

The structure of an industry is a stable parameter. Indeed Adelman ${ }^{7}$ was led to the use of the phrase "glacial drift" to suggest the slowness of the change in the concentration of U.S. manufacturing industry. A quantitative estimate may be obtained for our period from Shepherd's work. 8 He found 73 industries that could be compared between 1251 and 1958 Censures of Production. Of these 73 , 61 showed a change in concentration of less than 10 percentage points and 42 of less than 5 points. Nearly $90 \%$ of the changes in concentration were increases. There is also evidence to suggest that relative concentration is stable over time. So in so far as the structure to performance relation predominates it is not to be expected that performance characteristics were at all volatile during our period.
7) M. A. Adelman 'The Measurement of Industrial Concentration' Review of Economics and Statistics Vol. 33 pp 269-296
8) W.G. Shepherd 'Changes in British Industrial Concentration' Oxford Economic Papers Vol. 181966 pp 126-132

With respect to the second question, the results of Section 4.5 may be helpful. In that section, the annual distributions for each industry of the rates of return of firms were presented and considered. The conclusions were: mean rates of return showed gentle downwards trend movements. The annual variances showed marked stability. The skewness seemed to change its predominant direction during the" period. The kurtosis was uniformly leptokurtic. It is first necessary to emphasize that changes in annual distributions may be brought about by the continuing effect of a stable Markov process. On the other hand apparent stability of annual distributions does not inevitably lead to the conclusion that a stable: process is operating, though it must raise the probability that such is the case. This is the basis on which we may conclude from Section 4.5 that pooling the data for the period probably does not involve conflating markedly differing processes.

Our first two approaches have provided some degree of confidence that the rate of decay of profitability did not'vary greatly during the period for which the data is pooled. But we cannot eliminate the possibility that there are some variations. Therefore we must consider the third approach to the problem. ${ }^{9}$ This is to ask how we may, if we so wish, interpret the coefficients of the decay functions estimated upon pooled data in terms of their annual equivalents. The ideal situation would be if the pooled coefficients are arithmetic means of the annual coefficients. We will in fact find this to be so for the linear form coefficients but not for the cubed term coefficients.
9) Justifying the pooling because of the completeness of the model would in some circumstances - but not these - be possible, If all those factors that influence the dependent variable are included in the model, then pooling would be appropriate because the model would apply to all time periods and all groups of data.

If we first investigate the linear decay function, the assumption is that the mean of the row distribution of the transition matrix is a linear function of the prior mean. Should the slope of this function vary from year to year, we must write the relationship for a particular firm $j$ at time $t:$

$$
r_{j t}=\beta_{t} r_{j t-1}+u_{j t}
$$

The linearity assumption implies that $\beta_{t}$ is uncorrelated with $\mathbf{r}_{j t-1}$.

Each year's set of rates of return is separately expressed in standard deviation units from the mean. Therefore unless the form of the distribution (skewness or kurtosis, not variance) varies systematically over the estimation period, we may expect that each of the classes into which the pooled data is grouped will have equal representation from each year. The change from positive to negative skewness that is suggested by the data of Section 4.5 would lead to a slight over-representation of observations from early years in the negative (below average) range and of later years' observations in the pesitive (above average) range.

The data is averaged over each class. Therefore for the class $S=\left\{j \mid a<r_{j t-1}<b\right\}$ with $N(S)$ members, summing over the individual decay relations:

$$
\begin{equation*}
\frac{1}{N(s)} \sum_{j \in s} r_{j t}=\frac{1}{N(s)} \sum_{j \in s} \beta_{t} r_{j t-1}+\frac{1}{N(s)} \sum_{j \in s} u_{j t} \tag{I}
\end{equation*}
$$

Using the notation $\bar{r}_{s}$ as the mean over the set $S$ of final rates of return and similarly for the other terms, we may write (I) as

$$
\begin{equation*}
\bar{r}_{S}=\bar{\beta}_{S} \cdot \bar{r}_{S,-1}+\bar{u}_{S} \tag{2}
\end{equation*}
$$

The question we wish to consider is the relation between $\bar{\beta}_{s}$ and the annual $\beta_{t}$ values. Assuming that each class $S$ has no disproportionate
representation of observations from any one year, and further that the expected representation is of equal numbers ${ }^{10}$ from each year, we may demonstrate, by using the zero correlation between $\beta_{t}$ and $\mathbf{r}_{j t-1}$, that $\bar{\beta}_{s}$ must have an expected value equal to the arithmetic mean of the $\beta_{t}{ }^{\prime} s^{l l}$

With $\bar{r}_{S}$ and $\bar{r}_{S,-1}$ calculated for each class, a regression line is fitted to the resultant data. The estimated slope coefficient

$$
\hat{\beta}=\frac{\sum \bar{r}_{s} \cdot \bar{r}_{s,-1}}{\sum \bar{r}_{s,-1}^{2}}
$$

(the summation being over all the sets $S$ ).
This formulation follows because both $\bar{r}_{S}$ and $\bar{r}_{S,-1}$ have zero means. Substituting from (2)

$$
\hat{\beta}=\frac{\sum\left(\bar{\beta}_{s} \bar{r}_{s,-1}^{2}+\bar{u}_{s,} \bar{r}_{s,-1}\right)}{\sum r_{s,-1}^{2}}
$$

Assuming independent errors ice. $E\left(\sum_{-} \bar{u}_{j} \cdot \bar{r}_{j_{i-i}}\right)=0$

$$
E(\hat{\beta})=\frac{\sum \bar{\beta}_{s} \bar{r}_{s_{s},-1}^{2}}{\sum \bar{r}_{s,-1}^{2}}
$$

Given the nature of $\bar{\beta}_{s}$ previously established, the squared deviations $\left(\bar{r}_{s,-1}{ }^{2}\right)$ will induce no particular and persistent direction of bias to $\mathfrak{F}(\hat{\beta})$. Therefore we may regard the calculated value $\beta$ as an unbiased estimate of the arithmetic mean of the $\beta_{t}{ }^{\prime} s$.

A similar argument may be developed for the nonlinear function with the same conclusion for the coefficient of the linear term. But there is, on the other hand, demonstrable bias in the coefficient of the cubed term.
10) This is a simplification as the number of observations varies somewhat between years.
11)

$$
\begin{aligned}
& \operatorname{Cos}\left(P_{6}, r_{x-1}\right)=0 \\
& \left.\frac{1}{N(5)}\right\rangle(0,-B)\left(n, x-1-r_{1},-1\right)=0 \\
& \frac{1}{N(s)} \sum_{j \in=} \beta_{t} r_{j t-1}=\bar{\beta} \cdot \bar{r}_{s,-1} \\
& \bar{b}=\bar{\beta}
\end{aligned}
$$

Equivalent to (2) above we get

$$
\bar{r}_{s}=\bar{\beta}_{s} \cdot \bar{r}_{s,-1}+\sum_{j \in s} \eta_{t} r_{j t-1}^{3}+\bar{u}_{s}
$$

$\bar{\beta}_{s}$ has the same property as in the linear form. If we assume independence between $\eta_{t}$ and $r_{j t-1}^{3}$ we may write the cubic term as the product of the arithmetic mean of the $\eta_{t}$ 's over the set $S$ and the arithmetic mean of the $r_{j t-1}^{3}$ over the same set. Again there being no reason to expect anything other than equal representation of all years in each class, the expected value of the arithmetic mean of the $\eta_{t}^{\prime}$ 's over the set $S$ is the arithmetic mean of the $\eta_{t}^{\prime} s$ over the whole period of 12 years. 12 Therefore we may write the cubic term:

$$
\frac{1}{N(s)} \sum_{j \in s} r_{j t-1}^{3}
$$

But the form of the cubed variable used in the regression differs from this. It is:

$$
\left(\frac{1}{N(s)} \sum_{j \in S} r_{j t-1}\right)^{3}
$$

i.e. instead of using the mean of the cubed values, the cube of the mean value has been employed. The relationship between these two quantities may be investigating by expanding the discrete expression for the third central moment of a distribution. ${ }^{13}$ We find that

$$
\frac{1}{N(S)} \sum_{j \in s} r_{j t-1}^{3}-\left(\frac{1}{N(s)} \sum_{j \in S} r_{j t-1}\right)^{3}=\frac{1}{N(s)} \sum_{j \in S}\left(r_{j t-1}-\bar{r}_{s,-1}\right)^{3}+3 \sigma^{2} \frac{1}{N(S)} \sum_{j \in S} r_{j t-1}
$$

where $\sigma^{2}$ is the variance of $r_{j t-1}$ taken over the set $S$.
Let this be abbreviated to:

$$
z=\mu_{3}-3 \sigma^{2} \bar{r}_{s,-1}
$$

where $Z$ is the discrepancy between the two measures with which we are concerned.
12) Again making the simplification thät there are equal numbers of observations in each year.
13) I am indebted to Robin Ruffell. for suggesting this approach.

Now $z \sum 0$ according as $\mu_{3}+30^{2} \bar{r}_{s,-1} \geq 0$
For all classes excepting that overlapping the mean, all the $r_{j t-1}$
in any one class have the same sign.
Case (i) $\bar{r}_{\text {st-1 }}>0$, then $\mu_{3}>0^{I .4}$ and therefore $z>0$
Case (i.i) $\bar{r}_{\text {st-1 }}<0$, then $\mu_{3}<0^{14}$ and therefore $z<0$

As the sign of the cube of the mean and the mean of the cubes will be the same (on our previous reasoning), these two cases may be summarised as: the absolute value of the mean of cubes always exceeds the absolute value of the cube of the means.

This result is only sufficient to indicate a possibie direction of bias in the intercept on the $\bar{r}_{\text {st }}$ axis. To establish anything about the bias of $n$ demands evidence about the way the discrepancy varies with $\bar{r}_{\text {st-I }}$. But we know that $Z$ depends upon $\bar{r}_{\text {st-I }}$ and takes the same sign as $\bar{r}_{\text {st-1 }}$. Therefore, unless the other elements in the expression ( $\mu_{3}$ and $\sigma^{2}$ ) counteract this, we do find that $|z|$ varies positively with $\left|\bar{r}_{\text {st-1 }}\right|$. In fact $\sigma^{2}$ strengthens this inter-relationship. ${ }^{15}$ It was observed in Section 5.2 that the skewness does not seem to vary with $\left|\overline{\mathrm{r}}_{\mathrm{st}-\mathrm{I}}\right|$. As skewness was measured as $\mu_{3}{ }^{2} / \mu_{2}{ }^{3}$ and $\sigma^{2}=\mu_{2}-\left(\mu_{1}\right)^{2}$, it seems likely that $\mu_{3}$ also contributes to the strength of the relationship. So the variable actually used shows an increasing discrepancy from the correct variable as $\left|\stackrel{\rightharpoonup}{r}_{\text {st-1 }}\right|$ increases. Therefore we must expect an upward bias in the coefficient of the cubed term. That is, upward bias if we wish to interpret it in terms of its annual equivalents.

The conclusions of this section are, firstly that pooling does not appear to involve lumping very disparate processes together. Secondly that the estimated coefficients may be interpreted as the arithmetic means of their
14) See Appendix to Chapter V.
15) See Section 5.4.
annual equivalents. The exception to this second result is as a consequence of the form of variable used and not of the pooling.

## CHAPTER VII

## THE ESTTMMATED EQUATIONS

This chapter has two functions: firstly, in Section 7.1 there is a brief report of the results of estimating the various equation forms both on industry and subgroup data. The full results are in Appendix $D$. Secondly, in Section 7.2, an attempt is made to decide upon the best equation for each industry and subgroup. In this section, the principles employed are set out, a few examples of their application described and the selections tabulated. In each of these sections the industry cases are dealt with before the suberoups. Section 7.3 summarizes the chapter.

Before reporting the results of the estimation, mention must first be made of a data problem. It will be recalled that observations involving rates of return of over three standard deviations from the mean have been rejected. Inspection reveals that a firm which earns such a rate of return in one year will usually have a rate of return in the preceding year that falls in the extreme class of the accontable range. Therefore when we look at the averaee rate of return in year $t$ of firms that occunied an extreme class in year $t-1$ we have a biased statistic that indicates very rapid decay of profitability. This is because a number of the adverse moves have been rejected from the sample. ${ }^{1}$ For this reason the extreme classes at each end of the range have been rejected. If there is no bias in the extreme class, its omission should have no systematic effect upon the estimated equations. But as the nonlinearity, if any, will be mainly detectable well away from the mean, there is a strone possibility that dropping these observations lowers the amount of nonlinearity found. On the other hand, of course, the likely bias in those data points may induce a spurious nonlinearity.

## 7.1(a): Industries

The four functional forms have been estimated for each industry. This has been done first for the data relating to above averace profitability, then for that relating to below averaee profitability and then to the full body of data for that industry. These three sets of data are referred to as the "positive range", the "negative ranee" and the "full range". The forms estimated differ in only one respect from those summarised at the end of section 6.1: the linear-cubic is used with a constant term. This is done so that the usual measures of condness-of-fit such

[^17]as $R^{2}$ and Durbin-Watson statistic can be used. If the constant is found to be significant then the equation form is inappropriate.

Although $R^{2}$ measured on these equations is not a measure of the relationship between the rate of return at time $t$ and at time $t-1$ for a set of firms, it is a measure of the goodness-of-fit of the final mean $r_{s t}$ on the prior mean $r_{s t-1}$ for the set of conditional distributions. It is therefore one atandard to employ in judging between the functions proposed. The corrected $R^{2}$ is nearly always above 0.9 for the linear form and this in a majority of cases is bettered when we move to the linear cubic form. The addition of a squared term only makes a worthwhile contribution in a handful of cases. The log-linear is less good than any of the power functions on this criterion although it still attains quite respectable levels.

The second indicator of the goodness-of-fit is the Durbin-Watson statistic. This measure of serial correlation may be regarded as an indicator of how satisfactory the functional form is in cross-section analysis such as this. For example, Diagram 7.1 illustrates the fitting of a straight line to data displaying the form of non-linearity we expect.

## Diagram 7.1



The relationship between successive residuals is immediately obvious. Seven linear equations have some evidence (i.e. the Durbin-Watson statistic lies below the unper bound $d_{u}$ at the $5 \%$ level of significance) of serial correlation, the number falls in the linearmelobic and the cubic having just a single case. It would seem that there is little evidence of nonlinearity provided by the consideration of serial correlation. ${ }^{2}$

The goodness-of-fit of the log-linear equation is in all but a handful of cases less good than any of the power functions. As it also has rather a large number of examples where one of the coefficient requirements is violated, this form was not developed any further. Our attention from now on will be limited to the three power function forms.

Coming now to the coefficients of the equations, we find for the linear form that every slope coefficient lies between 0 and 1 . Thus we have further confirmation of the general occurrence of decay of profitability. For a majority of industries, the negative range slope is less steep than that of the positive range - this means that profit decay is faster below than above the mean. It is apparent from Diegram 7.1 that any nonlinearity in the data will lead to a spurious value for the constant term of a linear equation fitted to that data. Therefore we must delay consideration of this term until appropriate equation forms for particular ranges in industries have been selected.

In the linear-cubic form, there are only a few cases where the requirements of Section 6.1 are not fulfilled by the coefficients. The commonest deviation from those requirements is a. signjficant constant term, and this
2) When the extreme observations were included the linear form had some evidence of serial correlation in one third of cases.
is commonest in the full range equations. Although the coefficient of the cubed term is only significant (a.t the $5 \%$ level) in about one fifth of cases (of which only one occurs in the positive range), it is negative in 39 out of 51 equations. There are only two cases, both in the negative range, where the coefficient of the first degree term is greater than 1 . In summary, the linear-cubic has acceptable coefficients but the nonlinearity in the data is insufficient in most cases to justify a nonlinear form. In so far as differences between the ranges are detectable, the positive range shows less nonlinearity than the other ranges. ${ }^{3}$

When we come to the cubic we find that multicollinearity has become quite a problem for the positive and negative ranee equations and the pattern of significance and size of coefficients is very confusing. Discussion of this will be left to the section on selecting appropriate forms for the various ranges and industries. The full range case is rather different as there is much less inter-correlation between the independent variables. This is because when $r_{s t-1}$ varies above and below zero, $r^{2}{ }_{s t-1}$ is not correlated with $r_{s t-1}$ or $r_{s t-1}{ }^{3}$. In the separate range equations, $r_{s t-1}$ takes only one sign and all three terms are highly correlated. Therefore it is only in the full range that the cubic seems appropriate. As it appears that the decay process is different above and below the mean (see Section 8.4), it is also to be expected that in some cases a cubic form will be needed to describe the decay process over the full range where there is in fact no error in using the mean as the decay origin.

## 7.1(b): Subgroups

Only two functional forms have been applied to the industry subgroup data: the linear and the linear-cubic, each with a constant term. Experience
3) An effect that is stronger if the extreme classes are not rejected.
of the industry data suegested that the log-linear form was not worth pursuing, while the degrees of freedom problem meant that the cubic form would be inappropriate for a considerable number of subgroups. For fifteen industry subgroups, 3 equations have been estimated: one each for the positive, negative and full ranges. For the other 26 subgroups only a full range equation has been estimated as there are insufficient data to allow the separate treatment of above and below average values. As for the industries, the goodness-of-fit is high. There is very little evidence of serial correlation. Generally the coefficients satisfy the requirements although there are a few cases where the slope coefficient of the linear form is greater but not significantly greater than one. The cubed term of the linear-cubic is only significant in a minority of cases.

## Section 7.2 : Choice of Appropriate Equation

## Industries

In attempting to select a single equation form for each range of each industry, there are two aims in view. Firstly to find the form that best characterises the decay process in that case and thereby conclude something about the decay process. Secondly, at a later stage, we will be calculating summary measures of the rate of decay in each case and for this we need, where possible, a single best form of function.

Choice of equation must take into account both the statistical aspects of the equations and the suitability of their parameter values. This has been done in two steps. First an equation was chosen on the basis of its goodness-of-fit and parameter significance, then with this initial allocation a few cases were reconsidered because of inconvenient parameter values. Final choices are shown in Table 7.1.

Ignoring temporarily the difficulties of deciding upon parameter significance, we might set up a selection scheme based on the cubic form. To show this, equation 9 of Section 6.1 is reproduced:-

$$
\mathbf{r}_{\mathbf{s t}}=\theta\left(\beta-1+n \theta^{2}\right)+(\beta+3 v \theta) r_{s t-1}+3 \eta \theta r_{s t-1}^{2}+\eta r_{s t-1}^{3}
$$

If the cubed term is insignificant, then the coefficient of $r_{\text {st-1 }}^{3}$ in the true relation may be taken as zero and therefore the true relation must be linear and we may go straight to that form. If, on the other hand, the cubed term is significantly different from zero but the squared term is not, then this implies

$$
\eta \theta=c \quad \text { but } \quad \eta \neq 0
$$

We may then conclude that $\theta=0$, that is, the mean is the decay origin and the correct curvilinear form is the linear cubic. If in the cubic
both the squared and cubed terms are significantly different from zero, then we conclude that the relationship is nonlinear and the decay origin diverges significantly from the mean. The cubic is therefore the appropriate form. If in this case the constant is insignificantly different from zero, it merely means that $\left(\beta-1+\eta \theta^{2}\right)$ is insignificantly different zero. If the linear form is selected and it has an insignificant constant, this does not necessarily show that the mean is a good choice for the decay origin. This is because the constant ${ }^{4}$ is $\theta(\beta-1)$ and its insignificance may be a consequence of $\beta$ being insignificantly different from 1. On the other hand, a significant constant does necessarily imply a significant value for the divergence of the decay origin from the mean, if the linear equation is the correct choice.

The multicollinearity and heteroscedasticity from which the equations suffer both imply exaggerated standard errors and some care must therefore be taken with the judgement of coefficient significance. Multicollinearity is only a problem in the cubic form where the intercorrelation of $r_{s t-1}^{2}$ and $r_{s t-1}^{3}$ may make them both insignificant despite there being nonlinearity in the data. The correlation between $r_{s t-1}$ and $r_{s t-1}^{3}$ in the linear cubic is not a problem in judging significance. This is primarily because the t-statistic of the linear term is always very high and therefore there is never a problem in deciding its significance. If, on the other hand, the cubed term is insignificant, we may take this as an indication that its distinctive contribution is not required, i.e. that the data are not nonlinear. To cope with the multicollinearity in
4) Section 6.1 equation 7
the cubic we can call upon the linear cubic as supporting evidence for or against nonlinearity in the data. If the nonlinear terms are insignificant in both equations, then the linear form is chosen. An example of this is shown in Table 7.2 for Industry 1 negative range. In order to avoid incorrect rejections due not to multicollinearity but to heteroscedasticity, insignificance is only decided where the t-statistic is well below the critical 5 per cent value. 5

Acceptance of the cubic form is the easiest choice to make, as the likely problems of the equations all tend towards spurious insignificance. So if all the coefficients of the cubic are significant, no interpretation is needed. If we permit cases where one coefficient has a t-statistic slightly below the 5 per cent significance level, then all but two of the cubic choices are explained. An example is shown in Table 7.2 - Industry 5 full range.

Choosing the linear-cubic is the most difficult of the three as we are making a decision on the basis of misleading standard errors without any other supporting evidence. The erucial indicator according to the basic selection scheme is the significance of the squared term in the cubic, by only allowing very low t-statistics for this term to guide rejection of the cubic form. In practice the very few linear-cubic forms selected (5 in all) came from the problem cases: such as those where although the linear form was indicated, it, suffered from serial correlation. In this situation the linear cubic was chosen if the t-statistic of the cubed term was reasonably close to the critical value. In all, six cases did not fit well with the selection scheme of which five were in the separate ranges where multicollinearity in the cubic made its use as a basis for

[^18]Table 7.1 : Choice of Equation Forms for Industries
$1=$ linear, l.c. $=$ linear-cubic, $c=$ cubic

| Industry No. | Positive Range | Negative Range | Full Range |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 |
| 4 | 1 | 1.c.* | c |
| 5 | 1 | 1 | c |
| 6 | 1 | 1.c.* | 1 |
| 7 | 1 | 1 | 1 |
| 8 | 1 | 1 | 1 |
| 9 | 1 | 1 | 1 |
| 11 | 1 | 1 | c |
| 12 | 1 | 1 | c |
| 13 | 1 | 1 | c |
| 15 | 1.c.* | 1 | 1 |
| 16 | 1* | 1.c. | c* |
| 17 | 1 | 1 | 1 |
| 18 | 1.c.* | 1 | c |
| 19 | 1 | 1 | 1 |
| 20 | 1 | 1 | 1 |
| 21 | 1 | 1 | 1 |

* indicates case where selection scheme did not provide direct choice.
the scheme awkward. The 6 cases are marked in Table 7.1. Reference to the goodness-of-fit of the equations to aid selection would have led to little if any change in the chosen forms. In nearly every case the change in corrected $R^{?}$ from one form to another was extremely small. Further the lack of serial correlation meant that this aspect of goodness-of-fit would only have been appropriately considered in a very few cases it did influence the choice on three occasions.

The separate ranges are predominantly linear while the full range has some nonlinearity. For the separate ranges, the cubic makes hardly any showing, despite the efforts made to allow for its multicollinearity. This hints that the mean may not be unsuitable as a decar origin. In the full range, by contrast, seven of the industries are best fitted by the cubic. Given the difference above and below the decay origin, the choice of the cubic for the full range cannot be taken as evidence of the need for the decay origin to be different from the mean. Rather it lends support to the feeling that a single function for the full range has weaknesses.

## Subgroups

As only the linear and linear-cubic forms have been estimated for the subgroups, the choice is more restricted and the selection scheme appropriate is simpler. Because of the omission of the cubic form we do not have the problem of multicollinearity that made choice of the industry equations particularly difficult. On the other hand, heteroscedasticity is still present to make coefficient significance a problem. The scheme can nonetheless be expressed simply: if the coefficient of the cubed term of the linear-cubic form is significant then that form is chosen. Otherwise the linear is selected. In order to deal with the likely effect of heteroscedasticity, the linear is only chosen if the
nonlinear term of the linear-cubic has a t-statistic well below the 5 per cent critical value.

In 58 out of 71 cases, this leads to the choice of the linear form. Most of the linear-cubic choices are straightforward but 5 have significant constant terms which suggest that the cubic would have been the appropriate equation form. There are also 5 where the coefficient of the first degree is greater than one, and in 2 of these it is significantly so. This implies that close to the mean the tendency to decay of profitability is outweighed by factors working to increase the dispersion of profitability. There are also two chosen linear equations for which the slope coefficient is greater than one (but not significantly). In the event, there is no subgroup for which the choice between linear or linearcubic is ambiguous.

Industry 1 negative rance

| linear | $\begin{array}{r} 0.0183 \\ (0.517) \end{array}$ | $\begin{gathered} 0.792 \\ (11.716) \end{gathered}$ |  |  | 0.943 | 1.518 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| linear-cubic | $\begin{aligned} & -0.0182 \\ & (0.350) \end{aligned}$ | $\begin{gathered} 0.904 \\ (7.172) \end{gathered}$ |  | $\begin{aligned} & -0.00208 \\ & (0.033) \end{aligned}$ | 0.938 | 2.374 | 20 |
| cubic | $\begin{aligned} & -0.0247 \\ & (0.292) \end{aligned}$ | $\begin{gathered} 0.854 \\ (1.635) \end{gathered}$ | $\begin{aligned} & -0.0843 \\ & (0.099) \end{aligned}$ | $\begin{aligned} & -0.0397 \\ & (0.103) \end{aligned}$ | 0.933 | 2.369 | 20 |

Industry 5 full range

| linear | -0.0452 | 0.835 |  |  | 0.984 | 2.049 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (3.503) | (48.597) |  |  |  |  |  |
| linear-cubic | -0.0422 | 0.865 |  | -0.0193 | 0.984 | 2.098 | 40 |
|  | (3.21.7) | (27.930) |  | (1.163) |  |  |  |
| cubic | -0.0700 | 0.890 | 0.0591 | -0.0484 | 0.987 | 2.570 | 40 |
|  | (4.652) | (30.459) | (3.012) | (2.707) |  |  |  |

## Table 7.3: Choice of Fquation for Subgroups

|  | Positive Range |  |  | Negative Range |  |  | Full Range |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Linear | $1 / 2$ | 4/5 | 5/4 | 1/2 | 4/5 | 5/4 | 1/2 | 1/3 | 4/1 |
|  | 7/1 | $9 / 1$ | 13/1 | 7/1 | 9/1 | 11/1 | 4/2 | 4/4 | 5/1 |
|  | 15/1 | 18/2 | 19/3 | 13/1 | 15/1 | 15/3 | 5/4 | 6/4 | 7/1 |
|  | 20/3 | 21/2 |  | 18/1 | 18/2 | 18/3 | 7/2 | 8/1 | 9/1 |
|  |  |  |  | 19/3 | 21/2 |  | 11/1 | 11/2 | 12/2 |
|  |  |  |  |  |  |  | 12/4 | 12/6 | 15/1 |
|  |  |  |  |  |  |  | 15/2 | 16/1 | 16/2 |
|  |  |  |  |  |  |  | 16/3 | 16/4 | 17/1 |
|  |  |  |  |  |  |  | 18/1 | 19/1 | 19/2 |
|  |  |  |  |  |  |  | 20/2 | 20/3 | 21/2 |
|  |  |  |  |  |  |  | 21/3 | 21/4 | 21/5 |
|  |  |  |  |  |  |  | 21/6 |  |  |
| Linear-Cubic | 11/1 | 15/3 |  | 20/3 |  |  | 4/5 | 13/1 | 15/3 |
|  | 18/1 | 18/3 |  |  |  |  | 16/5 | 18/2 | 18/3 |
|  |  |  |  |  |  |  | 19/3 |  |  |

## Section 7.3: Summary

The results of this examination of the estimated equations support the concept of decay of profitability. The basic evidence for this comes from the linear form - in every case at the industry level and nearly every one at the subgroup level, the slope coefficient is less than one. This means that the expected value of the rate of return at time $t$ is closer to the decay oricin than at time ( $\mathrm{t}-1$ ). When we look at curvilinear forms we find a small number where this decay does not appear to operate in the immediate vicinity of the mean, but it is only a sure result in a few instances. Therefore it may be taken that decay of profitability occurs.

In the majority of cases, the linear form proves sufficient although there are 7 full range industry cases where the cubic is needed. The subgroup full range results show much less need for nonlinear decay functions than this. Overall violations of the requirements for the coefficients were rare and the goodness-of-fit very high.

CHAPTERVIII

THE DEGAY OF PROTITARTETTY - TTS MEASUREMENT

Tn this chapter the decay functions of the previous chapter are nut to use. Section 8.1 is concerned with developing a summary statistic of the rate of decay of profitability implicit, in a decay function. Then in Section 8.2 the measure is calculated, the values given and their precision evaluated. Section 8.3 considers whether the differences in rates of decay between industries are statistically significant. Section 8.4 looks at differences between the ranges: Finally, Section 8.5 looks at the results for snecific industries and subgroups and introduces an interpretation of the rate of decay in terms of a vears-equivalent.

In this section, the aim is to derive from the decay function a statistic sumarising the rate of decay of profitability for an industry range. For this purnose we wiil first presume that we hare a decay function:

$$
r_{t}=f\left(r_{t-1}\right)
$$

and the rates of return are measured as deviations from the decay origin.

The rate of decay of profitability may be defined as the ratio of the rate of return at time to the rate of return at time $t-1$. An alternative would be the first derivative of the decay function, but it is the ratio that will be used. This choice is motivated by interest in comparing annual levels, that $j s$, in the proportionate decay in the rate of return from one year to another towards the decay origin. The rate of decay (that we will denote by $D$ ) is therefore:

$$
D=\frac{r_{t}}{r_{t-1}}
$$

and, given our decay function, by

$$
D=\frac{f\left(r_{t-1}\right)}{r_{t-1}}
$$

Substituting specific functional forms for $f\left(r_{t-1}\right)$ will give various measures of D. Still taking the rates of retum as measured from the decay origin gives two measures:

$$
I D=\beta
$$

from the linear form and :

$$
1 c D=\beta+\eta r_{t-1}^{2}
$$

from the linear-cubic form.

The lcD measure presents a problem in that it is dependent upon the value of $r_{t-1}$ chosen. It is quite conceivable that for one such value industry $A$ has a higher rate of decay than industry $B$, while another value reverses such an ordering. This is illustrated in Diagram 8.1.

## Diagram 8.1




There is clearly some interest in its value at particular values of $\boldsymbol{r}_{t-1}$ but it is also desirable to have a measure not so dependent for industries with nonlinear decay functions. Such a measure would be an
average of the point measure over some range of $r_{t-1}$. 'This is provided by the integral:

$$
\frac{1}{(t-a)} \int_{a}^{i t} \frac{f\left(r_{t-1}\right)}{r_{t-1}} d r_{t-1}
$$

which averages over the range:

$$
a \leqslant r_{t-1} \leqslant t
$$

We may now redefine the linear-cubic measure:

$$
\begin{aligned}
t(1) & =\frac{1}{(b-a)} \int_{a}^{b}\left(\beta+\eta^{r_{t-1}^{2}}\right) d r_{t \cdot 1} \\
& =\beta+\frac{1}{3} \eta\left(b^{2}+a b+a^{2}\right)
\end{aligned}
$$

This formulation permits the direct comparison of profit, decay in the positive and negative ranges as the limits of integration all appear as second degree terms. If the limits of integration are:

$$
0 \leqslant r_{t-1} \leqslant R
$$

where $R$ is the extreme permitted value, then the positive value is:

$$
l C D^{+}=\beta+\frac{1}{3} \eta R^{2}
$$

If the limits are:

$$
-R \leqslant r_{t-1} \leqslant C
$$

then the negative range value is:

$$
l C D=\beta+\frac{1}{3} \eta R^{2}
$$

although $\beta_{1}$ will take different values from those for the positive range.

If the limits are:

$$
-R \leqslant r_{t-1} \leqslant R
$$

then the full range value is:

$$
P(D)^{3}=\beta+\frac{1}{3} \eta R^{2} .
$$

again $\beta_{i} \eta$ taking different, values.

This definition in terms of the integral can he used in the linear case also. Tba value will always be $\beta$ whatever the limits of the integral.

In practice all the equation forms have constant terms and both the linear and the cubic imply (or may imply) that the mean is not the decay origin. So we must use the coefficients of these forms to derive the coefficients of the true relationship. For the linear this canses no problems, the slope coefficient being an unbiased estimate of the slope of the true relation. This coefficient is therefore the value of 1.D. The cubic is more difficult and using it returns us to the nroblem of overidentification previously mentioned. It will be recalled that, the cubic form is postulated to occur where the tme relation is linearcubic but the decay origin is not the mean. So we wish to obtain from the cubic estimates of the coefficients of the tme linear oubic. The cubed term poses no problems in that the coefficient in the cubic is an unbiased estimate of the corresnonding coefficient in the true linearcubic. Rut given the overidentification, the structural coefficients may be calculated in more than one way and thus produce more than one set of values. The imprecision of the coefficients of the nonlinear terms in the separate range equations suggests that any involved sequence of calculation is going to produce estimates with a very low level of precision. Therefore no attempt is made to deal with the problem by such means. Rather the coefficient of the linear term is taken as a direct estimate of the equivalent coefficient in the true linear-cubic relationship. This estimete is biased, as the cubic coefficiont is (in the notation of section 6.1) ( $\beta+3 \eta(2)$. As y is of the orier 0.01 and $\theta$ ia nearly certainly less than 0.5 , the hiag is liknly to be less than 0.01 which is considerably less than the standard error of the linear coefficient.

The symbols used for the various measures of profit decay will abide by the following conventions: the prefix letter(s) will denote the equation form used - 1 for linear, lc for linear-cobio and cor cubic. The need to distinequish between linear-cubic and cubic is not to distinguish the form of the measure - which is the same - but, the source of the estimates of $\beta$ and $\eta$ used. $A$ superscript, + , or $f$ will denote which range is being referred to.

Choosing the limits of integration poses a problem and the solution must be to some extent arbitrary. I.t seems undesirable to employ in the measure any portion of the decay function beyond the extrome noints used in estimation. That is, extranolation is to be avoided. Having rejected the extreme classes, the outer values actually used rarely exceed an absolute value of 2 standard deviations. Therefore this has been chosen as the limit of integration and its justification is purely empirical.

With this value of $R$ we have:

$$
l c D=\beta+1.333 n
$$

as the decay measure for each range.

## Section 8.2 : The Decay Measures - Selection and Standard Errors

In this section a set of decay measures is produced for each range and for each industry and subgroup. The definition of the measure of decay was formulated in the preceding section. We wish to find a single measure for each industry-range and it is to this end that we attempted to choose the best equation for each. Tnevitably there were a number of cases where such a choice was difficult. Therefore one of our concerns in this section is to ask whether, in these cases, equation choice is critical. The second question that will be considered is the reliability of these measures.

The firgt task then is to examine in peneral how sensitive the choice of equation is for the decay measure and in particular whether the choice is crucial for those industries and subgroups which do not allow an unambiguous selection. The main tool to be employed in this is Spearman's rank correlation coefficient. 'The choice of this particular statistic is primarily motivated by the recognition that we cannot make judgements about the desirability of narticular levels of the decay measure but only relative judgements: that industry $A$ has a faster rate of decay than industry B. Secondly, whilst we will consider the statistical significance of differences in the decay measure, we are not able to discuss the economic importance of such differences. Therefore our prime concern will be with the ranking of rates of decay.

Tn Table 8.1 the rank correlations are reported for comnarisons between equation forms within ranges.

Table $8.1:$ Kank Correlation of Decay Measures Derived From Different
Equation Forms

## 1D against lcD $1 D$ against $C D$ lcDagainst $C D$

Industry

| positive range | 0.77 |  |  |
| :--- | :--- | :--- | :--- |
| negative range | 0.93 |  |  |
| full range | 0.97 | 0.96 |  |

Subgroups

| positive range | 0.73 |
| :--- | :--- |
| negative range | 0.91 |
| full range | 0.82 |

Because of the unreliability of the cubic form coefficients in the separate range cases, the measure was only calculated for this equation form in the full range case. The values of the decay coefficients are given in Appendix E. In general the rank correlations are satisfactorily high and the lower values can be attributed to one or two particular industries (or subgroups) whose measures differ very markedly between one equation form and another. It is concluded that these statistics do not point to any great sensitivity to equation form.

This is not sufficient for two reasons. Firstly, it may be that the cases of problematic equation selection are the ones whose rankine changes drastically between equation forms. Secondly, we are not choosing one form for all industries (or subgroups) but rather the best form separately for each industry (subgroup). A very high correlation between two sets of measures may obscure very great differences between the numerical values attached to particuler individual industries or suberouns. A new set of values taken partly from one of the original sets and partly
from the other may hardly correlate at all with the original sets. We could clearly check on this by looking at the means and standard deviations of the original sets. But a. simpler method is to assemble our composite set and calculate how it correlates with the originals. So we next compile the vector of the measures for each industry that are derived from the best equations and correlate this with the vectors of measures relating to the original equations. These best measures will be denoted by $D$ with the appropriate superscript to denote the range. The results are shown in Table 8.2.

Table 8.2 : Rank Correlation of the Best Measures with those from Specific Equation Forms
D against 1D D against ICD D against CD

| Industry |  |  |  |
| :--- | :--- | :--- | :--- |
| positive range | 0.93 | 0.81 |  |
| negative range | 0.99 | 0.94 |  |
| full range | 0.97 | 0.98 | 0.99 |

Subgroups

| positive range | 0.80 | 0.88 |
| :--- | :--- | :--- |
| negative range | 0.93 | 0.98 |
| full range | 0.97 | 0.84 |

It is apparent that the best set correlates very highly wi.th the others and therefore in general the choice of equation form for a particular industry is not crucial. Nonetheless those cases where the choice is not obvious must be examined one by one to see whether the choice in these particular cases makes an important difference to the ranking of these particular industries (or suberoups) in the best set. There are 6 such cases and 4 involve a ranking change in the best set of 2 places
or less Where are then left; $?$ cones, both in the industry positive range. These are noted in Table 8.3 where the best measures for each range for industries are given. The measures for subgrouns are presenters in Table R.A. Apart from these three exceptions $i t$ seems safe to conclude that, the ranking of industries given by the best measures ( $n$ ) is unlikely to be seriously affected by any errors in the selection of the hest equations.

We must next consider the calculation of the standard error of the decay measure. Tn the case of a measure derived from the linear form we have:

$$
I n=B
$$

and therefore the standard error of $I D$ equals the standard error of the slope coefficient in the linear form. But in the case of the linearcubic or cubic measure we have:

$$
10 D=c D=\beta+1.333 \eta
$$

Taking the variance of these:

$$
\begin{aligned}
\operatorname{Var}(1 c D) & =\operatorname{var}(\beta+1.333 \eta) \\
& =E\left[\{(\beta+1.333 \eta)-(\bar{\beta}+1.333 \bar{\eta})\}^{2}\right] \\
& =E\left[(\beta-\bar{B})^{2}\right]+2.666 E[(\beta-\bar{\beta})(\eta-\bar{\eta})]+1.777 E\left[\left(r^{\prime}-\bar{\eta}\right)^{2}\right] \\
& =\operatorname{Var}(\beta)+2.666 \operatorname{Cov}(\beta, \eta)+1.777 \operatorname{Var}(\eta)
\end{aligned}
$$

$\therefore \quad$ Standard $\quad \begin{aligned} \text { error of } \operatorname{IcD}\end{aligned}=\sqrt{[\operatorname{Uar}(\beta)+2.666 \operatorname{Cov}(\beta, \eta)+1.777 \operatorname{Uar}(n)]}$ This expression can be calculated from the results for the relevant regression.

The standard errors are given for each of the chosen measures in Tables 8.3 and 8.4. Their interpretation is held over to the next section.

## Table 8.3 : Values of $D$ for Industries

## (a) Positive Range

| Ind No. | $\mathrm{D}^{+}$ | rank | standard error | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.792 | 12 | 0.045 | 18 |
| 4 | 0.760 | 13 | 0.025 | 51 |
| 5 | 0.871 | 5 | 0.038 | 18 |
| 6 | 0.710 | 15 | 0.057 | 14 |
| 7 | 0.851 | 7 | 0.028 | 28 |
| 8 | 0.630 | 17 | 0.124 | 4 |
| 9 | 0.706 | 16 | 0.09 ? | 10 |
| 11 | 0.843 | 9 | 0.042 | 15 |
| 12 | 0.751 | 14 | 0.034 | 19 |
| 13 | 0.836 | 10 | 0.035 | 24 |
| 15* | 0.807 | 11 | 0.062 | 22 |
| 16 | 0.87 ? | 4 | 0.041 | 29 |
| 17 | 0.844 | 8 | 0.102 | 6 |
| 18* | 0.865 | 6 | 0.038 | 41 |
| 19 | 0.874 | 3 | 0.024 | 28 |
| 20 | 0.918 | 1 | 0.068 | 9 |
| 21 | 0.883 | 2 | 0.028 | 30 |

* Choice of equation makes more than two places change in ranking.

Table 8.3: Values of $D$ for Industries
(b) Negative Range

| Ind No. | $\mathrm{D}^{-}$ | rank | standard error | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.901 | 2 | 0.054 | 16 |
| 4 | 0.801 | 9 | 0.033 | 49 |
| 5 | 0.743 | 12 | 0.043 | 18 |
| 6 | 0.755 | 11 | 0.077 | 9 |
| 7 | 0.759 | 10 | 0.042 | 29 |
| 8 | 0.578 | 15 | 0.075 | 3 |
| 9 | 0.439 | 17 | 0.092 | 8 |
| 11 | 0.551 | 16 | 0.076 | 12 |
| 12 | 0.870 | 5 | 0.038 | 18 |
| 13 | 0.884 | 3 | 0.037 | 28 |
| 15 | 0.638 | 14 | 0.046 | 20 |
| 16 | 0.838 | 8 | 0.046 | 2f |
| 17 | 0.850 | 7 | 0.094 | 7 |
| 18 | 0.705 | 13 | 0.041 | 39 |
| 19 | 0.939 | 1 | 0.033 | 27 |
| 20 | 0.862 | 6 | 0.041 | 14 |
| 21 | 0.875 | 4 | 0.031 | 39 |

Table 8.3: Values of $D$ for Industries
(c) Full Range

| Ind. No. | $\mathrm{D}^{\text {f }}$ | rank | standard error | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.863 | 4 | 0.020 | 36 |
| 4 | 0.830 | 8 | 0.012 | 100 |
| 5 | 0.825 | 9 | 0.017 | 36 |
| 6 | 0.745 | 14 | 0.028 | 26 |
| 7 | 0.803 | 11 | 0.015 | 59 |
| 8 | 0.642 | 17 | 0.039 | 9 |
| 9 | 0.677 | 16 | 0.041 | 20 |
| 11 | 0.761 | 13 | 0.030 | 27 |
| 12 | 0.851 | 6 | 0.016 | 37 |
| 13 | 0.907 | 1 | 0.016 | 53 |
| 15 | 0.737 | 15 | 0.021 | 45 |
| 16 | 0.797 | 12 | 0.019 | 56 |
| 17 | 0.855 | 5 | 0.037 | 15 |
| 18 | 0.813 | 10 | 0.015 | 81 |
| 19 | 0.897 | 2 | 0.011 | 57 |
| 20 | 0.847 | 7 | 0.022 | 25 |
| 21 | 0.881 | 3 | 0.012 | 71 |

Table 8.4: Values of D for Subgroups
(a) Positive Range $-D^{+}$

| Ind No. | $\frac{\text { Subgroup }}{\text { No. }}$ | D $^{+}$ | rank | $\frac{\text { standard }}{\text { error }}$ | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 0.798 | 9 | 0.088 | 11 |
| 4 | 5 | 0.761 | 13 | 0.034 | 35 |
| 5 | 4 | 0.957 | 4 | 0.072 | 7 |
| 7 | 1 | 0.881 | 6 | 0.035 | 21 |
| 9 | 1 | 0.703 | 15 | 0.137 | 8 |
| 11 | 1 | 1.029 | 1 | 0.064 | 9 |
| 13 | 1 | 0.876 | 7 | 0.029 | 23 |
| 15 | 1 | 0.784 | 11 | 0.089 | 9 |
| 15 | 3 | 0.945 | 5 | 0.093 | 9 |
| 18 | 1 | 0.758 | 14 | 0.055 | 9 |
| 18 | 2 | 0.676 | $1 ?$ | 0.065 | 10 |
| 18 | 3 | 0.875 | 8 | 0.074 | 19 |
| 19 | 3 | 0.794 | 10 | 0.043 | 16 |
| 20 | 3 | 1.029 | 1 | 0.117 | 6 |
| 21 | 2 | 1.011 | 3 | 0.061 | 7 |

Table 8.4: Values of $D$ for Subgroups
(b) Negative Range - $\mathrm{D}^{-}$

| Ind No. | $\frac{\text { Subgroup }}{\text { No. }}$ | $\mathrm{D}^{-}$ | rank | $\frac{\text { standard }}{\text { error }}$ | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 0.916 | 2 | 0.068 | 9 |
| 4 | 5 | 0.819 | 6 | 0.038 | 32 |
| 5 | 4 | 0.803 | 7 | 0.148 | 7 |
| 7 | 1 | 0.709 | 11 | 0.049 | 23 |
| 9 | 1 | 0.557 | 14 | 0.139 | 7 |
| 11 | 1 | 0.401 | 15 | 0.082 | 8 |
| 13 | 1 | 0.866 | 5 | 0.029 | 23 |
| 15 | 1 | 0.638 | 12 | 0.032 | 7 |
| 15 | 3 | 0.799 | 8 | 0.108 | 8 |
| 18 | 1 | 0.886 | 3 | 0.070 | 10 |
| 18 | 2 | 0.632 | 13 | 0.078 | 9 |
| 18 | 3 | 0.712 | 10 | 0.062 | 21 |
| 19 | 3 | 0.881 | 4 | 0.035 | 15 |
| 20 | 3 | 0.740 | 9 | 0.087 | 10 |
| 21 | 2 | 0.998 | 1 | 0.078 | 9 |

Table 8.4 : Values of D for Subgrouns
(c) Full Range - $\mathrm{D}^{\mathrm{f}}$

| Ind No. | $\frac{\text { Subgroup }}{\text { No. }}$ | $\mathrm{D}^{\text {f }}$ | rank | $\frac{\text { standard }}{\text { error }}$ | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 0.843 | 18 | 0.032 | 22 |
| 1 | 3 | 0.972 | 2 | 0.065 | 7 |
| 4 | 1 | 0.812 | 24 | 0.035 | 13 |
| 4 | 2 | 0.759 | 33 | 0.044 | 12 |
| 4 | 4 | 0.888 | 8 | 0.084 | 6 |
| 4 | 5 | 0.847 | 17 | 0.015 | 68 |
| 5 | 3 | 0.746 | 35 | 0.059 | 8 |
| 5 | 4 | 0.858 | 15 | 0.040 | 16 |
| 6 | 4 | 0.802 | 28 | 0.029 | 13 |
| 7 | 1 | 0.808 | 25 | 0.019 | 46 |
| 7 | 2 | 0.737 | 38 | 0.042 | 16 |
| 8 | 1 | 0.636 | 41 | 0.082 | 8 |
| 9 | 1 | 0.665 | 40 | 0.052 | 17 |
| 11 | 1 | 0.800 | 29 | 0.045 | 20 |
| 11 | 2 | 0.907 | 6 | 0.063 | 9 |
| 12 | 2 | 0.940 | 4 | 0.049 | 11 |
| 12 | 4 | 0.794 | 30 | 0.057 | 8 |
| 12 | 6 | 0.807 | 24 | 0.042 | 14 |
| 13 | 1 | 0.901 | 7 | 0.012 | 47 |
| 15 | 1 | 0.760 | 32 | 0.029 | 18 |
| 15 | 2 | 0.850 | 16 | 0.042 | 12 |
| 15 | 3 | 0.816 | 22. | 0.051 | 19 |

(c) Full Flange $-\mathrm{D}^{\mathrm{f}}$ (cont'd)

| Ind No. | $\frac{\text { Subgroup }}{\text { No. }}$ | $\underline{D}^{\text {f }}$ | rank | $\frac{\text { standard }}{\text { error }}$ | $\frac{\text { degrees of }}{\text { freedom }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 1 | 0.823 | 21 | 0.060 | 11 |
| 16 | 2 | 0.750 | 34. | 0.055 | 14 |
| 16 | 3 | 0.859 | 14 | 0.041 | 14 |
| 16 | 4 | 0.690 | 39 | 0.055 | 11 |
| 16 | 5 | 0.823 | 21 | 0.052 | 11 |
| 17 | 1 | 0.802 | 28 | 0.044 | 13 |
| 18 | 1 | 0.880 | 10 | 0.027 | 22 |
| 18 | 2 | 0.739 | 37 | 0.040 | 20 |
| 18 | 3 | 0.781 | 31 | 0.027 | 42 |
| 19 | 1 | 0.887 | 9 | 0.034 | 15 |
| 19 | 2 | 0.980 | 1 | 0.032 | 12 |
| 19 | 3 | 0.864 | 13 | 0.015 | 32 |
| 20 | 2 | 0.840 | 19 | 0.048 | 9 |
| 20 | 3 | 0.868 | 12 | 0.044 | 19 |
| 21 | 2 | 0.958 | 3 | 0.028 | 18 |
| 21 | 3 | 0.871 | 11 | 0.045 | 8 |
| 21 | 4 | 0.812 | 24 | 0.074 | 8 |
| 21 | 5 | 0.741 | 36 | 0.047 | 9 |
| 21 | 6 | 0.910 | 5 | 0.038 | 13 |




In this section the question to be considered is that of the differences between industries (and subgroups) in the rate of decay of profitability. The last section presented the results and their standard errors. To test for the significance of the differences in rates of decay between industries poses a slight, statistical problem: there is no reason to expect that the sampling errors of the decay measure have the same variance in different industries. Therefore the most conventional tests are not appropriate. Fortunately there is a suitable test - the Welch-Aspin test. ${ }^{1}$ This is designed:

> "for use when the precision of an estimate ..... of a population parameter ..... depends linearly on two population variances."?

It has been used to test the difference within ranges between each pair of industries. The pattern of the results may be briefly summarised as showing that the industry ranked $n$ is generally not significantly different from the industry ranked $n+1$, but is significantly different from industries ranked $n+r$ where $r>1$. For example, the industry ranked 6th is not significantly different from that ranked 7 th, but is significantly different from those ranked 8 th and below. There are, of course, few industries whose decay measures have laree atandard errors and break this nattern. Whe insignificant differences (at the 5, level) are shown in Table 8.5. They amount to somewhat less than $10 \%$ of all nairwise comparisons in the negative and nositive ranses and less than $2 \%$ for the full

[^19]range. The testing of significance has not been done for all the subgroup results but the behaviour of those tested is similar to that found for the industries.

The results enable us to conclude that the decay rate does vary significantly between industries and therefore that it is a dimension of industry performance by which industries may be distinguished.

## Table 8.5: Differences Between Industries in their Rates of Decay

Pairs of industries for which differences are insignifiont at the 5\% level.

## (a) Positive Range


(b) Negative Range


## (c) Full Range

| 1 | $\because$ | 17 |
| ---: | ---: | ---: |
| 4 |  | 5 |
| 6 |  | 15 |
| 12 | $\cdots$ | 17,20 |
| 17 |  | 20 |

See text for test of significance used.

## Section 8.4 : Comparison Retween Ranges

The results for the positive and negative rances differ and it is relevant to ask whether these differences may or may not have arisen by chance. Having considered whether $D^{+}$is significantly different from $D^{-}$for each industry, we must turn to consider the similarity in the ordering of industries according to the decay rates in the two ranges.

Although there might be more justification for assumine a similar distribution of errors in each range for a given industry, this has not been done and so the Welch-Aspin test has again been used. Twelve out of the 17 industries have significantly ( $5 \%$ level) different decay measures for the positive and negative ranges. ${ }^{3}$. It is therefore reasonable to conclude that the decay rate does generally differ above and below the mean. Whereas 12 of the 17 industries showed $D^{-}<D^{+}$, i.e. that profitability decays faster below the mean, only 7 out of the 12 with significant differences show the same inequality. This weak support for the thesis that $D^{+}$has a tendency to exceed $D^{-}$is reinforced by the subgroup figures. These show that 12 out of 15 subgroups ${ }^{4}$ for which separate range figures have been calculated have $D^{+}$significantly different from $D^{-}$and 8 out of that 12 have $D^{+}$greater than $D^{-}$. It seems safe to conclude that the general pattern is of faster decay below the mean. 5
3) The five with insignificant differences are Tndustries 6, 8, 9, 17 and 21.
4) The three with insignificant differences are Industry 13 subgroup 1 , Industry 18 subgroup 2 and Industry 21 subgroup 3.
5) It might be argued that revaluations raise the rate of decay more in one range than the other. Whittington op cit pp 64-65 finds that in 1948-1954 "we cannot say whether revaluine companies would be more or less profitable than average ..." while in 1954-1960 it appears "companies which revalued were rather more profitable than the averape." Therefore if revaluation introduces any bias i.t should lower $\mathrm{D}^{+}$relative to $\mathrm{D}^{-}$.

Looking now at the ordering of industries for the two ranges, Table 8.6 shows the rank correlation coefficients between the different ranges. The correlation between the positive and negative ranges is moderate at the industry level and negligible at the subgroup level. This has important implicatiors for any attempt to identify factors that explain the rate of decay of profitability. The process that brings about a fast ratc of decay of high profitability must differ from that which brings low rates of return quickly back towards normal levels. This question will be dealt with in Chapter IX where the factors affecting the decay process receive some preliminary investigation.

Table 8.6: Rank Correlations Between Decay Measures for the Different Ranges

|  | $\frac{D^{+} \text {against } D^{-}}{}$ | $\frac{D^{+} \text {against } D^{f}}{}$ | $\frac{D^{-} \text {against } D^{f}}{}$ |
| :--- | :---: | :---: | :---: |
| Industry | 0.41 | 0.48 | 0.91 |
| Subgroup | -0.02 | 0.32 | 0.83 |

When we move on to consider the rank correlations between the separate and full range measures, moderate correlation coefficients are found between $D^{+}$and $D^{f}$ while the correlation between $D^{-}$and $D^{f}$ is strong at both levels of aggregation. It would appear that the below average observations have more influence upon the full range equations than the above average observations. This may wholely or partly be explained by the fact that the negative skewness of the distributions of rates of return 6 leads to larger absolute deviations from the mean in the negative range classes.

[^20]Section 8.5: Discussion of Some Tndividual Cases

In this chapter we have decided unon a measure of decay and determined that there are only a handful of cases where the choice of decay function is critical. With a chosen set of measures for each range we have considered the statistical significance of differences between industries and concluded that we can distinguish between most industries according to their rate of decay. Finally, in the nevious section we found that the positive and negative ranges must be regarded separately, both because of the actual values of the decay rates and because of their ordering.

Here the intention is to look a little more closely at the actual results at the industry level. The first point to be made js that we have no absolute standard by which to decide whether a decay rate is too slow or too fast. It is only in cases where great importance is placed upon sufficient incentives to technical innovation that the decay rate might be regarded as too fast. The view that will be taken here is that industries err towards the laggardly rate of decay. The causes of such impeded decay were discussed in Chapter IT and will be pursued empirically in the next chapter.

Without a standard by which to discuss rates of decay, the only basis of evaluative judgements must be the performance of other industries in this respect. The question at its most basic must be: does this industry have a particularly slow rate of decay by comparison with the remaining 16? The question of whether to use the mean or median as the reference point is unimportant a.s their locations are close and fine distinctions cannot. he drawn in this context. As a starting noint in a comperison of industries, Diagrams 8.2 to R.A show for the throe ranges the relative positions of each industry, the standard errors being
also graphically represented. With these diagrams and the information about significant differences provided in Table 8.6. various patterns of grouping may be attempted. Unfortunately the positive and negative ranges differ sufficiently to prevent any similar grouning being employed in both. Even if a pattern of grouning is chosen arbitrarily and imposed upon the ranges, the dissimilarity in ordering is freat enough to mean that there are few industries that fall in the same group in each range.

Therefore the method of procedure adopted is to first look at the few industries that have very slow rates of decay in both positive and negative ranges and then at those with very fast rates in both ranges. Two industries only have extremely slow rates of decay in both ranes; they are both in the Service sector - Retailing (no. 19) and Miscellaneous Services (no. 21). A third Service industry - Fintertainment (no. 20) is not quite as consistent as the other two but does have slower than averace decay in both ranges, and the slowest of all in the positive range. There is no immediate explanation for this distinctive behaviour of 3 out of the 4 service industries. One reason micht be the relative size of quoted companies to all comnanies in these industries. If, as seems plausible, quoted companies in the Service sector are very much larger than the average service company, their market nower followine from that size differential may normit the maintenance of rates of roturn at a stable level. The approximately avera, behaviour of the wholesaling industry (no. 18) suits the argument as it is characterised by larger units than the other Service industries. The excentinnal behaviour of Retailing is also partly exnlained by the conditions of local mononoly which often nrevail and by the constraints unon mareins that apnlied to retail traders during the period under examination. $T$ t is initial y
surprising that the Service sector, which is usually regarded as particularly competitive should have a 31 ow rate of prosion of high profits, especjally as it, has a small ratio of fixed to working capital and rather unsnecialised fixed canital - both factors that should lead to fast rates of decay. Of course the earlier point about the size of quoted companies in the Service sector may mean not only excentinna, market nower but also that in other respect,s the quoted companies may be uncharacteristic of the industries as a whole.

Tumine now to industries with a fast rate of decay, two industries are consistent: Cotton (No 8) and Woollen and Worsted (No 9). These are perhaps predictable occupants of this particular place. The experience of the Textile industries since the early 1950's has been one of fierce, mainly foreign, comnetition and thus a ranid decay of ahove average profitability is unsurprising. There has also been a continuing policy of encouraging the scrapping of old machinery and therefore the main obstacle to the rapid restoration of normal profitability has been, at least, lowered in these industries.

Whilst no other industries clearly stand as having fast or slow decay in both ranges, the evidence of the separate ranges and the full range results does suegest two more industries are worth examination. The slow decay one is Industry 13 (Drink). This is ompletely dominated numerically by the Brewers, as a comparison of Tables 4.3 and 4.4 shows. Although the consolidation of the Brewing industry was only just beginning in the period under consideration, competition at, the local level was not preat, prices and market shares being reasonab]y stable. Th such an environment a slow rate of decay is to be expected. The Paper, Printing and Publishing industry (No 15) is on balance a fast decay industry, but
to treat it as one entity is difficult given its heterogenej.ty. Looking at the subgroup results reveals that whilst the Newsnaper subgroup has slightly below average decay, the Printing group is very nearly average and the Paner group has quite a fast rate of decay. As the Newsnaper croup is the smallest and Paper the largest, the aggregrate result is fast decay. Tt is compatible with the ideas of this study that the difficult entry and differentiater nature of the product of the Newspaper subgroup should lead to slow decay. The fast decay of the Paper industry tallies with the pressure of foreign comnetition in this industry.

It is appropriate at this point to look directly at the subgroun results and consider the extreme cases, as has just been done at the industry level. The subgroup results are notable first for the very great variety of rates of decay within one industry: Industry 4 , for example, has subgroups ranked $8,17,24$ and 33. The industry value is therefore very much influenced by the proportions of firms in each of its constituent subgroups and general comments about the industry (excent in terms of similar summary statistics) are difficult to make. Takine as an examnle the previously cited Jndustry 4 and bearing in mind that full range decay rates reflect more of the negrative than the nositive range nerformances, we find Machine Tools $(4 / 2)$ with a fast, rate of decay and Constructional Fingineering (4/4) with a very slow rate, whilst, Shinbuilding (4/1) and Other Fngineering (4/5) are around the average. Another very marked contrast occurs in the Clothing and Fontwear industry (No 11) where Clothing ( $11 / 1$ ) has a fast and Footwpar (11/2) a slow rate of decay for the full range. This difference fits with basic knowledue ahout these two industries. As 86 out of 118 comnanies in this industry are in the Clothing subgroun, we find that the full range rankine for the induatry
if fifth frastest. Th the separate ranger, $n$ is only avifiahte for the Glathing smberoin and this behaves pecmiarly. Tt has the slowest, rate of decay amongst the subgroup results for the nositive range and the fastest for the negetive range. The offect of this unon the industry results lowers the ranking from nint, alnwest in the nositive to seennd fastest in the necretive range.

Of the ton 5 suhgroups showing the slowest rate of decey, the Misnellaneous Servinos industry nrovides twn: Cateriner etc (21/2) and Other Services $(21 / 6)$. Retailine is represented by Stores $(19 / 2)$. The Raking subgroup $(12 / 2)$ also appears with Pottory (1/3) makins un the list. At. the other end of the list, fast decay is displayed by Cotton Sninnins (8/7) and Wool (9/1). Ruildine Merchants $(18 / 2)$ and Teather ( $16 / 4$ ) also anpear at this end of the list, together with one subgroup from Miscellaneous Services; vin, Transport and Inmmmication (21/5).

Without bringing additional quantitative information on industry characteristics we can only conclune from this brief discussion that to some extent, decay rates accord with exnectations but, that the very great differences between ranges makes internretation difficult.

Much of the difficulty in discussing rates of decay follows from the lack of any economic standard by which to ad judge their desirability. The present study cannot hope to provide this, but it can heln a little by proffering an equivalent measure to the rate of decay that is perhans more intuitive.

The rate of decay tells us what proportion of the abnormal profitability is eliminated in one year. We could as well ask ahout, the half-life of the abnormal profitability: how many years does it take to eliminate
a given proportion of these abnormal rates of return? Taking the given proportion a.s 50 per cent we get an equation:

$$
J^{N}=0.5
$$

where $N=$ the half-life, i.e. the numher of years to eliminate half the excess (or deficient) rates of return.

Such an equation may be evaluated by taking logs:

$$
N=\frac{\log (0 . s)}{\log (D)}
$$

The values of this measure for the three ranges for the industry level are given in Table 8.7. The measure has the useful characteristic of throwing un more clearly than D the extreme cases. In the positive range, Tndustry 20 has a half-life 50 per cent longer than any other industry, whilst in the negative range Tndustry 19 has a half-life almost double that of any other industry. Omittine these extreme cases, the span of values in the positive range is from 1.5 to 5.6 years and in the negative ranse from 0.8 to 6.6 years. Such a. snread of values as these micht well be regarded as accentahle, leaviner only the previously mentioned extreme cases representing undesirable situations; although perhaps a half-life of under one year might err on the ranid side. There is still no standard by which to judge these resplts ahsolutely, so such statements as have just heen made can only he suggested anpraisals. Before leaving this measure of decay, it is worth pointine out that for all three ranges the averace half-life is nearly 4 years. To present a numerical example; this means that a firm earning $25 \%$ in an industry whose average is $15 \%$ would on average be earning $20 \%$ after 4 years and $17 \frac{1}{5} \%$ after 8 years. Such a rate of adjuctment surely cannot be regarded as over-rapid and therefore the initial premise that decay rates enerally err on the slow side does not seem unjustified.

Table 8.7 : Half-Life Equivalents to Rates of necay

## Measured in years

| Industry No. | Positive Rance | Negative Range | Full Range |
| :---: | :---: | :---: | :---: |
| 1 | 2.97 | 6.64 | 4.70 |
| 4 | 2.53 | $3.1 ?$ | 3.72 |
| 5 | 5.02 | 2.33 | 3.60 |
| 6 | 2.02 | 2.47 | 2.36 |
| 7 | 4.29 | 2.51 | 3.16 |
| 8 | 1.50 | 1.26 | 1.56 |
| 9 | 1.99 | 0.84 | 1.78 |
| 11 | 4.06 | 1.16 | 2.54 |
| 12 | 2.42 | 4.98 | 4.29 |
| 13 | 3.87 | 5.63 | 7.10 |
| 15 | 3.23 | 1.54 | 2.27 |
| 16 | 5.06 | 3.92 | 3.06 |
| 17 | 4.08 | 3.86 | 4.43 |
| 18 | 4.78 | 1.98 | 3.35 |
| 19 | 5.15 | 11.03 | 6.38 |
| 20 | 8.09 | 4.67 | 4.17 |
| 21 | 5.57 | 5.19 | 5.47 |

The general conclusions of this section are rather negative. It is clear that decay is not simply or atrongly related to other characteristics of industries - thjs will be pursued further in the next chapter. Secondly, only a few industries disnlay distinctive and similar decay characteristics in hoth ranges. This clearly raises problems in evaluating industries unless one or the other range is regarded as the more important. Thirdly, the constituent subgroups of some industries have considerable differences in their rates of decay leading to problems in performing analysis of decay at, the industry level.

## Section 8.6 : Summary

In this chapter a decay statistic has been formulated. It is the average ratio of $r_{t}$ to $r_{t-1}$ over the relevant range and its form depends upon the decay function. For this purnose, the coefficients of the decay functions are regarded as estimates of the parameters of the true decay function with the decay origin properly snecified. It was found that the decay statistics based on one equation form correlated highly with those from the other forms. Fharther, there were only a few instances of the equation choice makine more than a minor imnact unon the ranking of the particular industry or subgroup. Therefore i.t was concluded that equation choice did not have a critical influence unon the overall ranking of industries according to their rates of decay.

Consideration of the precision of the decay coefficients revealed that most pairs of industries differed significantly in this resnect and therefore that decay of profitability is a dimension of industry performance which does separate and distinguish industries (and suberoups). In the last section certain industries displaying rather extreme rates of decay were briefly examined.

The positive and negative rances were found to differ very greatly, the ordering of industries in the positive range bearing approximately no relation to the ordering in the negative range. Some evidence was also found to support the view derived from the inspection of the estimated equations, that the rate of decay is faster in the negative than the positive range.

The Decay Origin

So far the assumption has been made that the mean industry rate of return is a good approximation for the decay origin, that is, the point towards which the decay of profitability is directed. It is now possible to consider the validity of this assumption.

In the first part of this Appendix, the calculated values of the decay origin are presented. In the second section we directly consider the assumption that the decay origin and the mean coincide. Thirdly the evidence for differing decay origins for the positive and negative ranges is examined. Fourthly the possibility of relationships between decay origins and decay coefficients is investigated. The last two sections attempt statistical and economic explanations respectively of the foregoing observations: -

8Al Calculation of the Decay Origin
In Section IV.I where various forms of decay function were developed, the possibility and effects of a deviation of the mean from the decay origin were considered. In the case of the linear form, if the deviation of the decay origin from the mean is $\theta$, then the constant term equals $\theta(\beta-1)$ where $\beta$ is the slope coefficient and $\theta$ is positive when the mean is greater than the decay origin. If the linear cubic form is found to be appropriate, then the mean must coincide with the decay origin. If this is not so and there is nonlinearity, the cubic form become appropriate.

It was found in Section 8.4 that, for most industries, a different decay rate prevailed in the positive range than in the negative range. This indicates that the full range function wijl be an untrustworthy guide to the decay origin. Therefore this appendix will restrict its attention to the separate range functions. This has a beneficial side-effect: the
cubic was never chosen for a separate range and so we are not faced with the problems of estimating $\theta$ from that functional form.

Table 7.1 shows five cases ${ }^{1}$ where the linear cubic form was found appropriate for the separate ranges. For these therefore $\theta$ may be taken to be zero - the mean coincides with the decay origin. There are another five such cases amongst the subgroups ${ }^{2}$. The remaining cases are all linear. To calculate $\theta$ from the linear function, we take the ratio ${ }^{\text {a }} /(\mathrm{b}-1$ ) where a is the estimatea constant and b the estimated slope coefficient. The results at the industry level are shown in Table BAl in stanaard deviation units in columns $\perp$ and 4 and in percentage point units in columns 3 and 6 . $\theta$ takes predominantly positive values and in both ranges has an average value of between 2 and 3 percentage points. In other words, the decay origin seems to lie a small amount below the industry mean. This is more consistently demonstrated in the negative range than the positive. This contrast is also found at the subgroup level.

8A2 Does the Decay Origin Differ Significantly from the Mean?
Further discussion of the value of $\theta$ murt depend upon the confidence limits that can be assigned to the calculated values. The standard error of the ratio of two stochastic quantities poses considerable problems. In what follows, reliance will be placed upon the result presented by O'Brien and Hilton ${ }^{3}$ that gives (asymptotic) $95 \%$ confidence intervals:

1 Positive range, industries 15, 18. Negative range, industries 4, 6, 16.
2 Positive range, $11 / 1,15 / 3,18 / 1,18 / 3$. Negative range, 20/3.
3 O'Brien, R.J. and Hilton, K., 'The Significance of Structural Coefficients in Economic Models', unpublished.

|  | Positive | Range |
| :---: | :---: | :---: |
| Industry No | $\frac{\frac{\text { Decay }}{\text { Origin }}()}{\left(\begin{array}{l} \text { (s.d. units) }) \\ \text { (1) } \end{array},\right.}$ | $\frac{\text { Standard }}{\frac{\text { Error }}{\text { (s.d. units) }}}$ <br> (2) |
| 1 | -0.0880 | 0.1399 |
| 4 | -0.1517* | 0.0699 |
| 5 | 0.5054 | 0.3296 |
| 6 | -0.0897 | 0.1816 |
| 7 | 0.3470 | 0.1776 |
| 8 | -0.0617 | 0.1308 |
| 9 | 0.1211 | 0.8424 |
| 11 | 0.2191 | 0.2150 |
| 12 | -0.1558 | 0.1457 |
| 13 | -0.1359 | 0.1339 |
| 15 |  |  |
| 16 | 0.6484 | 0.4197 |
| 17 | 0.0491 | 0.3559 |
| 18 |  |  |
| 19 | 0.1190 | 0.1778 |
| 20 | 1.1829* | 0.1154 |
| 21 | 0.3496 | 0.2658 |
| Average |  |  |





$$
\frac{a}{(b-1)} \pm 1.96 /\left(\left\{s_{a}^{2}-2 S_{a} S_{t} \cdot \rho \cdot \frac{a}{(b-1)}+s_{b}^{2}\left(\frac{a}{b-1}\right)^{2}\right\} /(b-1)^{2}\right)
$$

where $S_{a}^{2}$ is the estimated variance of $a$
$S_{b}^{2}$ is the estimated variance of $b$
$\rho \quad$ is the correlation coefficient between $a$ and $b$

It is the square root portjon of the above formula that is given as the standard error in Table 8Al.

At the industry level, the hypothesis that $\theta$ is zero is rejecced in 8 instances by the above test. This will tend to reject the null hypothesis incorrectly rather than accept it because the confidence interval is asymptotic. In only 2 subgroup cases is the hypothesis that $\theta$ is zero rejected. Overall therefore the use of the mean as the decay origin does not seem to have involved very much approximation.

As the subgroup results reveal only 2 out of 30 cases where $\theta$ is significantly different from zero - a number that might well occur by chance with this test.- attention will from now on be resiricted to the industry level results. It is worth noting that the contrast between industry and subgroup results might be used to argue that the deviations of the industry level decay origins from the mean are a consequence of aggregating over subgroups. But inspection of the component subgroups of those industries with significant values of $\theta$ does not suggest more heterogeneity of average rates of return than usual, nor more reterogeneity of decay rates. Therefore whilst it may stand as a general explanation of the industry/ subgroup contrast, it does not seem to assist in explaining differences between particular industries in this respect.

Of the 8 values of $\theta$ that differ significantly from zero, all but one are positive. Thus the evidence that, if the decay origin lies away from the mean, it lies below the mean is strengthened. Of the 12 industries with
$\theta$ having the same sign both for positive and negative ranges, 10 have $\theta$ non-negatjve and 2 have it non-positive. Of the remaining industries (with contradictory signs) only one has a significant value for $\theta$ : industry 13 for the negative range. So the significant results and the consistent results point to a decay origin at or below the mean. But as only 8 out of 34 estimates of $\theta$ differ significantly from zero and as the test used underestimates the standard error, the evidence against the mean is not strong.

8A3 Does the Decay Origin Differ Significantly between Ranges?
For every industry the decay function has been estimated separately for those observations relating to above average profitability and for those relating to below average profitability. Consequently there are two estimates of $\theta$ for each industry $\left(\theta^{+} \& \theta^{-}\right)$. Inevitably these will differ, the question is whether the differences can or cannot be attributed to chance.

As this question is taken after that of the preceding section it must take note of the results there reported. So where both $\theta^{+}$and $\theta^{-}$were there found insignificantly different from zero and of the same sign, it must be concluded that they do not differ significantly one fron another. Where one of the ranges has had the linear cubic form of decay function fitted, the exercise is more awkward. There is no standard error estimated for $\theta$ in such cases. The uncertainty relating to the value of $\theta$ is primarily derived from the fact that selection of a particular equation form is never sure. Such uncertainty is not amenable to standard statistical techniques. Therefore in the present context, $\theta$ has been taken as known with certainty to equal zero where the linear cubic form has been selected. 4 The remaining.

4 This will lead to incorrect rejection rather than incorrect acceptance of the null hypothesis of no difference. But only one industry (6) might thus be misclassified.
cases have been dealt with using the Welch-Aspin test. Apart from those involving the Iincar-cubic form, tho tost of significance has bocin a for one.

The results are evenly balanced: 8 industries ${ }^{5}$ have $\theta^{+}$significantly different from $\theta^{-}$and 9 do not show a significant difference. It therefore must be concluded that for some industries, either for statistical or economic reasons, the estimates of the location of the decay origin calculated from the separate ranges do diverge.

## 8A4 Decay Origins and Decay Coefficients

It proves interesting to look at the connection between $\theta$ and the slope of the decay functions. The results are summarized in Table 8 A 2.

Table 8A2 Decay Origin and D-Numbers of Industries

$$
D^{+} \geqslant D^{-} \quad D^{+}<D^{-}
$$

| $\theta^{+}, \theta^{-} \geqslant 0$ | $9(7)$ | $2(1)$ |
| :--- | :--- | :--- |
| $\theta^{+}<0, \theta^{-} \geqslant 0$. | $1(0)$ | $5(4)$ |

Bracketed values give the number of industries where $D^{+}$differs significantily from $D^{-}$see Section 8.4 footnote 3 .

In the majority of cases, the sign of $\theta^{+}$is the same as the sign of $\left(D^{+}-D^{-}\right)$. This result is not altered if the industries where $D^{+}$does not differ significantly from $D^{-}$are rejected. Of the two industries where $\theta^{+}$is significantly different from zero, both lie in cells on the principal. diagonal of Table 8 A 2 . If we restrict ourselves to those industries where

5 Industries $1,4,8,12,13,15,18,20$.
$\theta^{+}$and $\theta^{-}$are significantly different, then we find 7 out of 8 on the main diagonal. ( 3 in the top left hand cell and 4 in the lower right hand cell). Before attempting explanation of this result, it can be reported that the sign of $\left(\theta^{+}-\theta^{-}\right)$shows no relationship with either the sign of $\theta^{+}$or the $\operatorname{sign}\left(D^{+}-D^{-}\right)$.

## 8A5 Statistical Explanation of the Relationship between $D$ and $\theta$

Explanations based on linear decay relationships lead to the requirement that $\theta^{+}$and $\theta^{-}$should have the same sign. The results of Table 8 A 2 clearly rule out that as a general explanation, though it would suffice for the upper left hand cell of that table. But in the lower right hand cell, the decay origin appropriate to the positive range function lies above the mean whilst that for the negative range lies below.

If it is assumed that there is some nonlinearity of the form illustratod in Diagram 2.1, then fitting linear functions to the separate ranges would lead to a negative value of $\theta$ in the positive range and a positive value in the negative range. This holds in the case when the decay origin is correctly located at the mean. If the true decay origin lies below ine mean, then the sign prodiction for $\theta^{-}$is reinforced. But the sign of $\theta^{+}$now depends upon the actual shape of the curve in the positive range and the size of the deviation of the decay origin from the mean.

This argument based on nonlinearity now provides the link between the sign of $\theta^{+}$and the sign of $\left(D^{+}-D^{-}\right)$, or rather the size of $D^{+}$. For it implies that the smaller $\mathrm{D}^{+}$, all other things being equal, the larger $\theta^{+}$will be and therefore the more likely $\theta^{+}$is to be negative despite the decay origin lying below the mean. In fact every industry for which $\theta^{+}$is positive has a below average value of $\mathrm{D}^{+}$and conversely of those industries for which
$0^{+}$is nonpositive 7 out of 9 have abuve avelage values of $\mathrm{D}^{+}$. Given that $D^{+}$and $D^{-}$are only moderately correlated; it is not surprising that we have detected a relationship with the sign of $\left(D^{+}-D^{-}\right)$.

Therefore we may fit the results into a consistent pattern with the decay origin lying below the mean. This is not to reject the possibility that there are industries where it is above the mean. But the evidence that we have suggests that the converse predominates.

8A6 The Economics of the Decay Origin
It is now possible to turn to the economics of the decay origin and consider whether it is reasonable for the decay origin to lie below the mean. In addition it would be desirable to see whether there are economic argumerits for the link with the decay rate, for which we have so far only provided a possibie statistical explanation.

If the decay origin lies below the mean, then the interpretation in terms of the analysis of Chapter 2 would be that; there is a net inflow of resources to markets even when rates of return in those markets are below the industry average. The decay origin is the point of reversal in the direction of net resource movement.

If the positive range decay origin exceeds the negative one ${ }^{6}$ then there would appear to be a range of rates of return where resource in and outflows are in balance. The expected change in the rate of return in the next period for a firm whose rate of return in the present period is in that range would be zero. It is important to note that any such behaviour of the decay of profitability would lead to nonlinearity in the decay function.

In a situation with no capital rationing and firms investing down to the project whose rate of return equals the cost of capital, we would expect the decay origin to be the cost of capital. It is to be expected that average
6. 12 and 17 industries share this and 7 out of 8 for which $\theta^{+}$sipnificantiy different from $0^{-}$.
rates of return are above the cost of capital. Therefore the docay origin will be below the mean. Factors that deter firms from investing right down to the margin will raise the decay origin above the cost of capital but may very well leave it below the industry mean. ${ }^{7}$

A second framework for explanation can be found in the behavioural theory of the firm. The model of Chapter 2 was based upon the resource allocation decisions of a multi-product firm. We may interpret those decisions behaviourally. In Chapter 2 a periodic search of markets in which the firm was already operating and of markets the firm felt capable of entering was posited.

In a behavioural context we might expect some standard resource allocation procedure to be generally employed and for search to occur only when certain stimuli were experienced. It might be more appropriate to suggest a three level process: firstly a standard allocation procedure is employed; secondly, that procedure is adjusted but no change in the set of markets is considered; thirdly, the search for new markets is initiated.

Cyert and March have discussed a situation very much akin to that presentiy under examination:
"... on each dimension of organisational goals there are a number of
critical values - critical that is from the point of view of shifts in search strategy" 8 .

The goal of profitability is our concern and the critical values are levels of profitability that trigger off changes in the resource allocation mrocess. Because we are talking of multi-product firms, there are two types of critical profitabilities: those for individual products and those for the whole firm. If profitability of any one product falls below a critical value, some
7. It might be argued that this would lead to a larger divergence of the decay origin from the mean if the mean is high. But no such relationship is detectable in the results.
8. Cyert, R.M. arid March J.G. 'A Behavioural Theory of the Firm' Prentice Hall, Englewood Clifis, 1963 p. 123.
assessment of that product's share of available resources is likely. This may weil occur whatever the firm's overall profitability. Although some connection between that overall value and the critical value for individual products would seem probable.

It is likely therefore that change in the standard allocation scheme will occur because of experience in individual markets and this will happen whilst the firm's overall profitability is above its critical value. This in itself may induce improvements in profitability i.e. decay of profitability from below origin levels, Once the firm's overall profitability falls below the critical level, it may be that efforts are contained within the range of possibilities defined by reallocation within the existing set of markets. But it is more likely that the third level - the full search is embarked upon. As we move upwards away from the critical value, the likelihood of search or even reappraisal of standard allocations becomes decreasingly likely. So overlaid upon the process of resource allocation developed in Chapter 2 is this behavioural process.

$$
y
$$

The other behavioural effect that must ve incorporated is the accretion or erosion of organisational slack. When the firm is well clear of its critical profitability such costs increase and play some part in the decay of high profitability. Conversely once the critical value lies above actual profitability, strenuous efforts will be made to reduce slack.

Now turn to the observation that $\mathrm{D}^{+}$generally exceeds $\mathrm{D}^{-}$. This means that the rate at which profitability returns towards central values is usually faster for low than for high profitability: firms recover from bad periods more rapidly than they slip from-successful situations. Tn the present analysis high profitability is eroded by the basic resource transfer process of Ch. 2 and by the accretion of organisational slack. Whereas low profitability is corrected by the resource transfer process, by the climination slack and by the activation of the second and third stages.
of the firm's decision process. It is the behaviuxal components that seem likely to contribute the observed asymmetry of decay. As the second and third stages of the decision process are activated, so rates of improvement of profitability are likely to be increased. The precise rate of return at which the extra factor will come into operation depends upon the whole constellation of critical values. But it may be presumed that many more firms in the negative than the positive range are engaged in the latter stages of the decision process. It is also at least plausible to expect that the sloughing off of management slack will occur more rapidly than its accumulation. So $D^{+}$, if it differs from $D^{-}$should exceed it. This we have found.

If our behavioural arguments lead to the expectation that $D^{+}$and $D^{-}$will differ, they also lead us to expect that'the decay origin will tend to lie below the mean. We have seen the role that may be made out for the critical values of profitability. It is important to bear in mind that these values are adaptive: one of the effects of failure to achieve is an adjustment of the standerd. Indeed it may well be that failure to attain only affects the decision process once there is little room for their further downward adjustment. The critical value will become less flexible as it falls towards that level of profitability regarded by management as the minimum safe level. It is probably not too cavalier to ignore the role of critical values until we get near the minimum safe levels and these - playing a part rather akin to the minimum profit constraint of such models of the firm as sales revenue maximisation - may be presumed to lie well below the industry mean in all except the most troubled industries. Therefore the behavioural factors involved in the decay of profitability will tend to produce a decay origin below rather than above the mean.

The qualitative difference between the processes bringing about the decay of high and Low prof'itabilities cannot be presumed to be separated at a point profitability. There is likely to be a range of profitabilities where some firms are, for example, eliminating management slack whilst others are accumulating i.t. There will be some overlap. In this range the slope of the decay function will differ from both $D^{+}$and $D^{-}$. There will therefore be some nonlinearity about the decay origin if there is an asymmetry in the decay process. This supports the explanation of the discrepancy between $\theta^{+}$and $\theta^{-}$. The lines fitted to the separate ranges will hardly be affected by a small interval of different slope such as is suggested here. Therefore they will become inaccurate very close to the decay origin and bring about the effect suggested in 8A5.

Finally, can the behavioural factors provide an explanation of the relationship between sign ( $\mathrm{D}^{+} \rightarrow \mathrm{D}^{-}$) and sign $\left(\theta^{+}\right)$? An argument with some plausibility may be constructed: the stranger the part played by behavioural factors, the more likely it becomes that $D^{+}$exceeds $D^{-}$. Another effect of ine vehavioural factors is to lower the decay origin. The further the decay origin lies below the mean, the more likely it becomes that the observed $\theta^{+}$is positive despite the negative bias in this figure because of nonlinearity at the decay origin. So we may regard both the items in the relationship under consideration as reflecting the overall strength of behavioural factors.

In summary, we have seen that the use of the mean as an estimate of the decay origin is not seriously amiss. But any error is one of overestimation. Secondly we have seen that estimates of the location of the decay origin do differ between ranges and that this is probably a consequence of some nonlinearity in the decay function near to the decay origin. Thirdiy we have found a relationship between the deviation of the decay origin from
the mean (as estimatiod from the positive renge) end the direngoncc between positive and negative range decay coefficients. Explanation of this is probably primarily statistical. Finally by introducing ideas from the behavioural theory of the firm it has been possible to explain both the difference between $\mathrm{D}^{+}$and $\mathrm{D}^{-}$and the location of the decay origin.

## DFEAY OF PROFTTPARTITTYY AND OTPHFR INDUSTRY CHARACTERTSTICS

The main purpose of this study is achieved with the final measures of the decay of profitability. But in Chapter I it was argued that this measure contributed extra information about the performance of an industry, in particular giving a measure of the speed at which the equilibrating forces in the industry could bring about competitive equilibrium. The ranking of industries according to their decay of profitability gives us a comparison between industries alone this dimension. The question to be investigated in this chapter is: "How do other measures of industry stmoture, performance and experience relate to this measure?".

We may divide this question in n number of ways. But the main diatinction must be between studying relationshins because of a belief in a causal connection, that is, where we believe some factor has influence unon the rate of decay of profitability, and studying relationships between variables which may each be influenced by i third factor and therefore are likely to vary together. This annroximately conincines with the stmacture/performance distinction. It is reasnnable to nresume that the stmucture of an industry will influence nerformance in general and therefore, in particular, the rate of decay of profitability. Some arguments to this effect were presenter in Chanter IT and will he reviewed below. Comparing the rate of decay with other measures of performance will only pertly involve the idea of direct causal connection between the two performance measures. But it is worth looking at in onder to see to what extent comparison between industries on the bagis of other

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performance measures has, through close correlation, involved
implicitly comparison of rates of decay.
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The statistical method of this chanter is very basio and rank correlations will be the main tool. The limitations of this are recognised but what multivariate analysis was attemnted did not, yield qualitatively different, or indeed stronger, results. In the event, the rank oorrelations that are found are very weak and only a minority attain even $5 \%$ significance. Although in a few situations the consistent behaviour of rank correlations is felt to strengthen the probability that the results are not chance ones, in ceneral we are adducinc evidence in support of hypotheses rather than providinc tests of those hypotheses.

The second limitation of this work relates to the data emnloyed. It would be a senarate and considerable study to prepare the usual measures of stmpeture and nerformance for the industry classification used here. Therefore attention has been mestricted to published figmres in a convenient form, and to measures that can be derived from the data uad in this study. Measures of the latter kind have the serinus disadvantace that they are based nurely unon quoted companies and therefore are fanlty representations of the industry as a whole. Recause of this data problem, fewer variables are available at the subgroun level than at the industry level. All the date used, and where necessary explanations of their derivation, are given in Appendix $F$.

The remainder of the chanter is divided into three portions. The first, deals with the relation between the rate of decay of profitability and five measures of industry stmacture. The second section lonks at the rate of decay and its relation to various performance measures. The third
section reports the comparison of Whittington's decay cnefficients with those produced in the present study. A summary concludes the chapter.

Section 9.7: The Trocar of Profitability and Foasmren ot structimes

Two forms of concentration ration are west: the onnerentional A-firm ooncentration hased on fill industry date? and a measure mal minated hy Whittingtion, mamely the uronortion of the tontel net ascots of muted comnarios controlled by fims owning over f. 4 million net assots in 1954. The former is only available for the manufacturing industries and constmuction but, at, both industry and subgroup level, whilst, the latter is available only at, the industry level but for the service sector as well as manufacturine and construction.

The third mein concentration measme emioyed is the Variance of the Logarithms of sire (Net, Assets) of the firms in the sample Hart ${ }^{2}$ says:

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"If the underlying size distribution of firms is log normal,
    then (the variance of the logarithms of size) is the
    annronriate measure of concentration."
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It also has the feature that calculating it only on guoteri comnanies
is likely to lead to a downard bias, whereas the concentration ratio of Whittington is hiased marards by the omission of the unguoted sector. The remainins two structural measures used are average size (net, assets) and the variance of net assets.


The levels of correlation between CrA and the other measures in Table
9.1 sugrests that the latter are reasonably robust and not too severely $\therefore$.
distorted by the omission of unguoted comnanies.

1) Sawyer M $C$, "Concentratiou in Rritish Manufacturing Tndustry"; Oxford Economic Paners Vol. 23 1971, nn 352-383
2) Hart $P$ R, "Pntrony and Other Massumes of Concentration", Tournal. of the Royal Statistical Fociety Series A Vol 1734,1971 , pn 73-85

Table 9.1 : Rank Correlations Petween the Structural Measures

## At the Tndustry Teve?



Sawyer 4-firm
Concentration Ratio (CR4)* 0.830 .730 .83
Whittineton
Concentration Ratio (WhCR) $0.70 .0 .79 \quad 0.75$
Variance of the
Tincarithm of Size
$0.64 \quad 0.63$
Varjance of lize 0.8 ?

* This measure is only available for Manufacturing and Construntion and therefore 13 ohservations, not. 77, are used in caloulating these rank correlation coefficients.

Tn Chapter TT, fiection 2.5: it is stated that: Whe closer the industry stmoture is to the nurely competitive ...... the fastor hich rates of return will he reduced." The initial presurntion is therefore that rates of recay will fall an the induatry stmucture deviates further from the competitive ideal. Thalking firat snecifically of the concentration measure, it is to be expected that hish concentrotions are maintained by devices to restrict new competition. These devices, of which barriers to entry are nrobahly the most, imnortant, will obstmuct the pillonation of regnurces anonrding ton reties of rotiurn and therefore will. slow down the rate of decay (and tihns raise $n$ ). Twn factors may act, to weaken this relationshin. In the first nloce, the measurement, of the decay of profitahility is not weinhted hy the size of firm. a highly concentrated market structiure
may be composed of a few large and many small firms. The operating environment of the small firms may appear highly competitive and their decay rates may be correspondingly high. Because of their numerical dominance, the industry decay rate may be relatively high. If such cases arise, the variance measures may show a stronger relationship with $D$ than the concentration measures. The second factor relates to firm diversification: a high industry concentration measure may not necessarily imply hishly concentrated markets if the firms of the industry are all well diversified.

The average size of firm in an industry is inserted to provide a proxy (probably weak) for the capital cost barrier to entry; as such we may expect it to correlate negatively with the rate of decay for reasons already given with reference to barriers to entry in general. 3

There is another reason for expecting a negative correlation between average size of firm and the rate of decay of profitability. It is cenerally found ${ }^{4}$ that there is a negative relationship between size and the variability of rates of return. It is reasonable to expect that high rates of decay accompany highly volatile rates of return. Therefore an industry of large firms might be expected to have a higher value of $D$, all other things being equal, than an industry of low average size. This line of reasoning and that based upon barriers to entry may not be independent - the scale barrier to entry by obstructing the reallocation of resources reduces the variability of profit experience.
3) Whittington $G$, op cit $p 72$, and Samuels \& Smyth op cit
4) Shepherd ' $W$ G, "Elements of Market Structure", Keview of Economics \& Statistics Feb 1972 p 29, uses lof (net assets) to catch more effectively the capital-cost aspect of barriers. Such a transformation is irrelevant for rank correlation purposes.

Table 9.2 shows how these structural measures relate to the rate of decay of profitability for the three ranges. The first point is that the correlations are weak with only one attaining a $5 \%$ level of significance. On the other hand, the majority of the correlations are, as expected, positive. That is, structures that would be regarded as

Table 9.2 : Rank Correlations Between $D$ and Measures of Structure

|  | CRA $^{+}$ | WhCR | Var <br> Industry <br> $D^{+}$ | -0.04 | 0.10 |
| :---: | :---: | :---: | :---: | :---: | :---: |

+ Using only Manufacturing industries and Construction.
* Significant at the 5\% level. For test see T Yamane "Statistics" 2nd Edition, Harper Row New York 1960, p 470
more divergent from the competitive ideal than others do tend in the present sample of industries to be accompanied by slower rates of decay of profitability. But the structural measures have been shown to be intercorrelated (Table 9.1), and therefore the correlations with the rates of decay are not independent. That is, we cannot regard the results as five separate tests and take comfort from the similarity of the results despite the general insignificance. The intercorrelations between the structural measures were not so high that similar results for each of them provides no extra evidence over and above that given by one, but it supplies considerably less than would be provided by five separate tests. As the positive and negative range decay rates
show very low correlation with one another, their similar results for three structural measures might be taken as extra support for the existence of positive relationship between $D$ and non-competitive industry structures. As was stated in the introduction to this chapter, hypothesis testing is rarely possible. In the case of structure/decay relationships, the evidence we have obtained lends support to our expectations.

Whilst the general insignificance makes any more detailed examination rather dangerous, it may just be permissible to look at the difference between the ranges at the industry level. For each structural measure the correlation is lowest with the positive range decay measure. The negative range results are in every case the strongest. This consistent pattern suggests that the industry structure has more influence upon the decay of low profitability than of high profitability. The intermediate rank correlations for $D^{f}$ follow from its nature as a form of average of the separate range decay rates.

The subgroup results are disappointingly weak and contrary. Phis is most likely a consequence of the data - the criticisms of the measures of structure already made apply with added strength at the industry subgroup level. Also, the reliability of the rates of decay is lower at the subgroup level. The range of rates of decay is very nearly the same for industries and for subgroups, but in the subgroup case 41 observations fall within this range while only 17 industries have to be fitted in. Even if industry and subgroup rates of decay were equally well determined, more random disturbance of the ordering would be likely for the subgroups. When the subgroups are less well determined, the ordering becomes even less reliable. Therefore it is not too disturbing to find rank correlations for the subgroups are lower than for the industries.

[^21]
## Section 9.2 : The Relation of Decay and Other Performance Measures

The definition of performance measure is rather broad in this section. First there are two conventional measures: growth of net assets and industry average profitability; then three measures relating to the inter-temporal behaviour of average profitability. These are the standard deviation of the annual averages, the trend in the annual average and the standard deviation of the residnal error of the trend equation. The final measure is the averase annual dispersion of rates of return within the industry. The nrecise definitions of these various measures are given in Appendix $F$, topether with their values.

The reasons for looking at these various aspects of industry norformance (or, more generally, behaviour) will emeres as this section proceeds, but the second group relatine to the hehaviour of the indiustry average rate of return over the period 1948-1960 needs some initial explanation. In Whittington's book "The Prediction of Profitability" he finds that part of the variation from industry to industry of the rate of decay is expleined hy variations in the industry averace rate of return. It will he recalled that he uses two 6-year neriods in his analysis. He therefore takes the difference in industry averae between thone two periods as the indenendent variable in an equation whose denendent variahle is the industry rate of decay. ${ }^{5}$ The strength of the results that he erets makes it essential to perform similar analysis with the decay rates presented in the precedinc chapter. Whereas Whittingtion had only one available measure of the variahility of industry average nrofit, because of his use of only two norions, here the choice is wider with twelve perjods. Two senarate argments are available that

[^22]are compatible with whittincton's results but suggest different measures for the present analysis. The first is that it, is the rate at, which industry averave profitability falls that influences the internal rate of decay of profitability: this leads to using the trend. The other argument says that it is the volatility of the industry averase that affects the rate of decay: this wonld imnly the use of the otandard deviation of the industry averace or, removing the trend, the standerd deviation of the variations about the trend line.

With a range of nerformance measures to consider it is necessary to clarify their inter-relations before proeressine to an examination of how each of them is related to the rate of decay. A key ton the interrelation of three of the measures - growth, profitability and trend in profitability - may be found in the inter-industry aquilibrium nrocess. So far intra-industry adjustments have been lonked at, but the same arguments lead to an analogous process botween industries. In reality it is not a separate process but another facet of the overall adjustment of resource allonation. Tt was argued in Chapter IT that most entry and exit will occur within the bounds of a sinkle industry, so the interindustry equilibration will primarily result from differentigl rates of accumulation of assets in different industries. There will be some movements of firms between industries but this is of lesser importance. If there is such a process of inter-industry equilihration, resources will accumulate fastest in the most nrofitable industrias and this will. tend to reduce nrofitability most quickly in these industries. ${ }^{6}$ 'Phis
6) As Whittineton points out, " no industry exnerionced a substantial increase in profitability." op cit $p 91$. Therefore it is relative rates of decrease of profitability that are appropriatie ton the argment in this context.
latter conclusion is supported by Whittington, who in his empirical conclusions finds: ".. a tendency for the average profitability ... of industries to regress towards the mean for all industries by an amount proportionate to their initial distance from the mean." 7 This tendency for inter-industry profitability differences to be eroded has also been examined by stigler. ${ }^{8}$ Tt therefoce seems reasnnable to expect that average profitability will be positively correlated with growth ${ }^{9}$ and that both of these variables will be correlated with the trend in average rates of return. This latter correlation will be negative as the fastest growing industries will have the steepest (most negative) trends. There is a possibility that takiner $1 \geqslant$ year averages of these variables will obscure the postulated relationships because inter-industry differentials are eliminated well within that period. Stigler, for example, considers that there is no correlation between annual hierarchies of industry rates of return after 6 or 7 years. 10 Whittington, on the other hand, has al ready been mentioned as finding considerable persistency of inter-industry differences in profitability using 6 year averages. Such inter-correlations between growth and profitability would tie in with wellestablished links between growth and profitability at the firm level and with the importance of internal financing of investment. An influx of resources will have a tendency to lower profitability and the greater the influx the stronger that tendency.
7) op cit $p 104$
8) Stigler $r \mathrm{~J}$, on cit
9) Whittington op cit $p$ 25, finds his evidence sunnorts the view that: "those industries which have the most profitable comnanies have the faster prowing companies."
10) Stigler G J, op cit p 5

Referring to Table 9.3 where these rank correlations are nresented, we see that expectations are confirmed. The growth to profitability correlation is quite the hishest, while the correlations with the trend in averape nrofitability are lower but of the correct sign. The weaker relations in this latter case probably reflect both the length of the period and the more complex relationships involved.

Table 9.3: Rank Correlations Retween Frowth of Net Assets, Average Profitability and the Trend in Average Profitability

| $\frac{\text { Growth in Net }}{\text { Assets }}$ | $\frac{\text { Averare }}{\text { Profitability }}$ |
| :--- | :---: |
| Industry | Industry |
| $0.94^{*}$ | $*$ |
| -0.30 | -0.23 |

* Significant at the $5 \%$ level - see Table 9.2 for test used.

The next question is: how dnes the inter-industry adjustment process affect the intra-industry adjustment? If resources flowin. into the industry were evenly spread through all markets of that industry, any effect would be upon the industry average rate of return rather than on rates of decay. The same result would hold if the allocation of these resources to particular marketis was indenendent of the profitability of the markets. Rut it is assumed throughout that the profitability of a market has some influence upon the allocation of rescurces within an industry. This must apnly eqnally to these additional resomrens. Therefore an influx of resources to an industry means an influx predominantly to the
11) Whittington on cit $p 97$ finds "the inter-fj morsistency of profitability ... negatively onrelated with noofitability". For this he uses the averase industry rate of return in the eartion of his two periods - 1948-1954.
ahove averacely profitahle markets. This will nartly affect the indugt,ry average rate of return but the whole effect will not, be absorbed in that way becalise of the concentration of the extra, resources in the above averace markets. The remainder of the effect will appear as an increase in the rate of decay in the positive rance.

This can best be demonstrated by a simple example. Jot the influx of resources be divided into two parts, the first enenmnaseing an even spread through all markets. It, will only affect, the industry average rate of return. The second nart is that which is concentrated in the above average markets. Tet this, for simnlicity, be evenly distributed throidgh all the above average markets. with simjar markets, this extra influx may be assumed to lower all rates of roturn by an equal amount: $\Delta r$. If M markets are of above and $N$ of below averase profitiability, the industry average will fall by $\frac{M}{M+N} \Delta r$ and each above average market will move towards the mean by an amount $\frac{N}{M+N} \Delta r$. Therefore the positive range rate of decay will be thus inflated.

The simple model may be extended to demonstrate the effect unon the negative range rate of decay. The extra influx of resnurces does not impinge unon below average markets but the industry average rate of return is lowered by $\frac{M}{M+N} \Delta r$ and therefore the negative range of rate of decay is also increased because resources flow more strongly into the above averace markets of the industry. Just as in chanter TT, the argument has been developed in terms of markets, the step to firms is direct as was explained in Section ?.6.

Therefore once we move from consideration of the process that noves firms towards normal profitability relative to the industry, the the factors and way industries move towards ernilibrium, we find that: a net,
influx of resources, unless distributed without resnect to profitability, will tend to mase the rates of decay of bonth ranses.

Unless the proportion of incomine resources that gh to the more profitable markets diminishes quite markedly with increases in the volume of resources flowing in, the effect upon rates of decay will be an increasing function of the growth rate of the industry. Therefore it seems reasonable to exnect that an industry with high averase profitability will on average have a hich rate of decay. l? From this we may derive expectations that growth will heve a negative oorre?ation with D and the trend in average profitability will have a nositive correlation with D. The results are shown in Table 9.4.

Table 9.4 : Rank Correlations of Decay with Averace Profitability, Growth of Net Assets, and Prend in Averase Profitability

|  | $\frac{\text { Trowth in }}{\text { Net Assets }}$ | Pro Average | Trend in Average Profitability |
| :---: | :---: | :---: | :---: |
| Industry |  |  |  |
| $\mathrm{D}^{+}$ | -0.14 | -0. 27 | 0.65* |
| $\mathrm{D}^{-}$ | -0.12 | -0.26 | 0.78* |
| $D^{\text {f }}$ | -0.05 | -0.25 | 0.83* |
| ${\underset{D}{\text { Subgroup }}}$ | - | -0.09 | 0.28* |

* Significant at the $5 \%$ level. See Table 9.? for tost used.

With the very notable exception of the trend variable, the correlations are weak, but in every case the sjen predictions are fulfilled. The argument in terms of the inter-action of the equilibration processes at industry and firm level is therefore nrovided with some sunnort.
12) Most researchers find a nositive correlation between concentration and averape profitability, and therehy link hack ton the preceding section ( 9.1 ). See Weiss T, "Guantitative Stucios of Tndustrial Organisation" in M Intripisenter (Bd): "Wrontinre of mantitative Fconomics", North Holland Amsterdam 1971, nn 363-3f,6 review tinis research.

In Table 9.3 trend was only weakly comelated with growth and averace protitability, yet in Table 9.4 , respite the weak corcelations of these variables with the rate of decay, trend is very strongly correlated with decay. Therefore an explanation is not sufficient that reljes unon growth as the prime mover of both the trend in industry averace profita ability and the internal rate of decay. The results do not conflict with the idea of a causal chain working from growth to trend and influencing decay en route, but they do susgest, strongly thet this is not the whole explanation of the hieh comelation between trend and decay.

There seem to be two possibilities. The first is that the other prescures bringing about downard trends in average nrofitability are, like the one already discussed, more effective in the more profitable market: This would bring about a rise in rate of decay as previously argued, The other possibility is that variability of average profitability is related to the rate of decay: "in a less stable industry individual compenies micht have more onnortinnty to chance their relative nrofitability: for better or for worse, $"^{13}$ Before considering this line of reasonine, the correlations between the three measures of the inter-temnoral behaviour of average nrofitability must he examined. 14 They are shown in Table 9.5.

The correlation between the trend and the standard deviation of averape profitability is high, as was to be expected, because a large trend coefficient will tend to mean a high dispersion. The negative sign of the correlation is explained by the general pattern of downard sloring
13) Whittington on citi 991
14) See Sryth $D \mathrm{~T}_{\text {, }} \mathrm{G}$ Rrisone \& J M Samuels: "The Variability of Industry Profit Rates", Anplied Fonomics 1969 Vol l, pp 137-149. They report industrial. concentration is insicnificantiy correlated with trend and variance of industry averare protitabjlity but sionificantly (5\%) rank correlated with the residual varjance abont trend.

Toble 9.5: Bank Correlatione of Measuros of the Thter-Temnoral. Rehavinur of Averace Profitability

$$
\frac{\text { Standard Deviation of }}{\text { Average Profitahilityy }}
$$

Tnduatry : Suberoun
$-0.80 *$
Averace Profitabilityr
St,andard Deviation of Average Profitability
$\frac{\text { Hesidual iarror }}{\text { ahout Prend }}$
Tndustiry Gubernino
$-0.70 * \quad-0.51^{*}$
$-0.64^{*}$
$0.80 *$
$0.74^{*}$

* Sionjficant at the 5\% level.
trends. On the other hand, the correlation hotween the trend and the
standard deviation of the errors about, the trend line ia unexnected. It,
implies that, the faster averave profitability declines, the more irregnlar its behaviour. A steep decline over the nerion will tend tio be assoniated with year to year volatility.

Tt is annarent, from the hisch correlations between these three measures that it, will not he nossihle to distinguish between the trend effert, and the volatility effect, This is borne out, hy the very similar corrolations Refore these lest two sets of results were introntued, two possible betiween f
explanations of the strone correlation between trend and decar were The strer
sucgested. Now we are faced with the more ceneral nroblem of an axmlanthe amir
regressation of the strong correlation hetwen docay and all three measures of ability. He found an $R^{2}$ of 0,31 using ? observations whereas the resulta ヵ. in Table 9.6, using 17 observationa, apnroximate to on $R^{?}$ of nver 0.6. in Thable 9.6 , usinc 17 observations, apnroximate to an $K$ nt nver u.n.

Refore these lest two sets of results were introduced, two possible explanations of the strong enrelation between trend and decay were sucgested. Now we are faced with the more ceneral nroblem of an exnlanation of the strong correlation hetween docay and all three maasures of

Table 9.6: Rank Correlations Between D and Measures of the InterTemporal Rehaviour of Average Profitability
$\frac{\text { Trend in }}{\text { Average Profitability }} \frac{\text { Standard Deviation of }}{\text { Average Profitability }} \frac{\text { Residual Frror }}{\text { Abolat Trend }}$

Industry

| $D^{+}$ | $0.75^{*}$ | $-0.59^{*}$ | -0.17 |
| :--- | :--- | :--- | :--- |
| $D^{-}$ | $0.78^{*}$ | $-0.87^{*}$ | $-0.70^{*}$ |
| $n^{f}$ | $0.83^{*}$ | $-0.91^{*}$ | $-0.76^{*}$ |
| $n^{f}$ | $0.28^{*}$ | $-0.36^{*}$ | $-0.36^{*}$ |

* Sienificant at the 5\% level. See Table 9.2 for test, used.
inter-temnoral variations in industry averace profitability. The link
between trend and decay followins from the process of inter-industry equilibration has not been rejected but has been found insufficient as a


## fill explanation.

The first possibility refers back to the process whereby movements of profitabilitv exnerienced more strongly by grouns of firms with ahove this be generalised to exnlain the link botweon the volatility of industry : average profitability and the rate of decay, or ia it nnly relevant to 1 the trend-decar relationship? The initial sten in considering this question is to aprly the argument used for the case just stated th the other nossibilities. So far we have lnokon at, a fall in nrofitahility of above averace companies. The case of a rise in profitability or the more nrofitable comnanies implies a fall in the rate of decay both above and belnw the mean. Tf it is below averase comnaries that experience a chance in nonfitability relative ton the rest of the industry, the conclusions are reversed: a rise in profitahility laars to a rise in the motin af ammer ond a fall in mofitabilit,y to a fall in the rate of decay. chusions are reversed: a rise in profitahility learis to a rise in the rater of decay and a fall in menfitability to a fall in the rate nf decay.

These contradictory effects make it unlikely that volatility of industry profitability experience would. lead to an overall inflinence unon the rate of decay in either cirection. The result, we are trying to explain is a negative correlation between $n$ and volatility, i.e. that volatility raises the rate of decay. In so far as volati.lity is mainly produced by the movement, of above (below) average firms, itis effect over a mumber of years should be annroximatoly nentral on the rate of decay. A rise in the rate of decay would only he nroduced if falls in profitability were mainly experienced hy the more profitahle firms and rises in nrofitability by firms with below averase noritability. This would involve a contraction in industry disnersion which is att least, noit, evident, (see Appendix A). Further, it is hard to think of an explanation for such a continuine phenomenon.

Wherefore two nossihilities are left. $\operatorname{sither}$ it is the trent that is nrodmeine the relationshin or volatility dnes reflect conditions within the industry conducive to the rapid adjustment of resource allocations. The first argumont relies unon the statiatical effect, nreviously smployed, but says that clearly the inter-industry equilibration process is not, the sole cause and that some other factor(s) must he onerating to lower the relative profitabjutity of the more nrofitahle firms in the industry. The nthar arerment is stated by whittinctor:
"Instability in the environment of the industry, as reflected in the chance in its averate nonfitability, was assnnioted with greater intormal mobility in terms of relative nolit,ahility of the individual memher comnanies". 1.5

Stigler establishes a link between the instability of industry nrofitability ard the competitiveness of the industry. He arenes that:


#### Abstract

"Competitive industries will have a volatile nattern of rates of return, for the movements into high profit industries and out of low profit industries will - together with the flow of new disturbances of equilibrimm - lead to a constantily changing hierarchy of rates of return. In the mononolistic industries, on the other hand, the unusually nrofitable industries will be able to preserve their preferential nosition for considerable neriods of time." 16


Such an argument may be extended to decav rates via the structure-decay relationships investigated in the previous section. Thus the association between industry volat,ility and internal mobility suggested by Whittington may be explained (or partly explained) in terms of the influence of industry structure upon the inter-industry equilibration process.

The last of the performance measures to be considered is the averace anmual disnersion of profitability within an industry. The standard deviation of the rates of return of all companies in the industry has been calculated for each year of the twelve year neriod and then the standard deviations have been averased over these years. $T$ t, is possible that in all industries extreme absolute (rather than relative) rates of return decay faster than moderate rates of return. If this were so a high dispersion industry would have faster rates of decay because it, contained more firms with such extreme rates of return. This leads to $a$ predicted megative correlation hetween $\pi$ and disporsion. 17 Table 9.7 shows this prediction to be fulfilled in the negative and full ranges while there is annerently no relationshin in the positive range. So in the nositive range, the rate of decay apnears unaffected by the disnersion
16) Sticyler on sit n 70
17) Whittington op cit $p 23$ observes a positive correlation betwen the averase profitability of an industry and its intor-company disnersion of nofitahility. stiger op citt $n 63$ finds such a relotionshin but an insignificant one. lis we find a nespative correlation between averaep profitability and D , these results would lead to an exnecter negative correlation het, ween disnersion and $n$. A contrary imdication is criven by stigler's findine (on oit. $n$ 6) that disnerainn is laroor in concentrated industries which, from Section ?.1, Taads to a positive correlation het,wer $n$ and disnersion.

Table 9.7 : Rank Correlations of D with Averace Disnersion of Profitability

Industry

| $D^{+}$ | -0.01 |
| :--- | :--- |
| $D^{-}$ | $-0.51^{*}$ |
| $0^{f}$ | $-0.54^{*}$ |

* Sienificant at the 5\% level. See Table 9.? for test used.
of profitability. On the other hand, in the negative range (and onnsequently in the full range), wide dispersion leads ton fast decay.

In his book "Whe Prediction of Profitahility", Whittincton considers the persistence of profitability. 78 Whis is, in fret, the same as the rate of decay examined in this study. He only considers a lineat resmession equation of rate of retrm at time ton thet at time t-1. His envation differs from the ones used here in employing six yeer averages of relative profitability. So his indenendent variahle is the averace rate of rotum earned by firm $i$ over the six vears $1948-7954$ expressed as a deviation from the industry average. Fis denendent variable is similar but relates to 1954-1960. A further difference is that he did not use standardised data. 19 The nurnose here is to comnare the decay measures of this study with those of imittington.?

The average levels of the decay coefficienis are show in Table 9.8, and it is found that the decay rate of six year averaces is faster than for year to yeer comperisons. If the simple first order Markov process Table 9.8: Averare Rates of Decay

| Whittington* | 0.54 |
| :--- | :--- |
| Full rance linear $\left(1 . D^{f}\right)$ | 0.81 |
| Full range $\left(D^{f}\right)$ | 0.81 |

* Usine only the 17 industries of the present study.

18) Whittington op cit Chanter 4
19) This is immaterial for a comparison of linear forms.
20) Whittinerton only nrovides industry level figures.
nged in this atudy wore a full descrintion of the intor-temnoral linkages of nrofitability, Whit,tington's rate of decay chould he the sixth nower of $\mathrm{D}^{f}$. So the average of whittington'e measure should he 0.28 , or conversely the averace of $\pi^{f}$ should be 0.90 . This discrevancy is a valuahle reminder that, the decay function has a very limited purpose. It is not, intended to provide the best, possible explanation of its dependent variable but merely to clarify the relationshin of relative nrofitahilities oncuring in a.d jacent years. For the former ohjertive it would, at least, be necessary to use a hiwher order markov nroness? Turnine now to the rank correlations hetween rates of decay, it can he seen from Table 9.9 that they pre quite low. One industry is resnonsible for a considerable mroportion of the arparent lack of correlation. Phis

Table 9.9: Rank Correlations Between Whittincton's Rate of Decav and $\mathrm{Di}^{\mathrm{f}}$ and $1 \mathrm{D}^{\mathrm{f}}$

$$
\begin{array}{cc}
n^{f} & 0.49 \\
10^{f} & 0.44
\end{array}
$$

is the Food industry (no 12): Whittincton's decay measure is 0.27 and $D^{f}$ is 0.85. For this industry, choice of equation is not critical - the range of nossible decay coefficients is 0.84 to 0.85 . Therefore a wrong choice sannot explain the difference. A possible solintion ifes in the idiosyncratin behaviour of avecare nrofitability in this industry. Tt has a. steen downward trend (ranked 5th stepenest) and a amall standard deviation (ranked 15th). As was remarked in the nrevious section, steron trend usually acommanies a high standard deviation. The falt between two six

[^23]year averages will he much greater than that observed in year to yoar comparisons where there is a steep trend. Tn most, cases this effect is moderated by the hierh volatility that accomnanies the trend.

Apart from the Food industry, the ranking is not too dissimilar and may be ascribed to the various differences in technigue mentioned ato the beginning of this section.

## Section 9.4 : Summary

This chapter has dealt with the relationshins between the rate of decay and other industry chararteristics. 'Phese charncteristios have been divided into two broad grouns: structural measures and nerformance measures. Rank correlation coefficients have been used to investigrate these relationships which have generally been found to be weak. For that reason, the approach has not been to test hypotheses but rather to nresent evidence sugeestive of the nature of the relationships.

Chanter TT showed that obstiacles to competition could be exnected to slow the rate of decay. This was smpnorted by the correlations found between the rate of decay and the structural measures. So the evidence found does not contradict the hynothesis that on averaep high levels of concentration are associated with slow rates of decay of profitability.

The performance measures were of three tynes: firstly, two orthodox nerformance measures, averace profitability and prowth. Secondiy, there were three measures of the inter-temporal hehaviour of average nrofitability. Finally there was the averace disnorsion of profitability within an industry. It was argued in Section 9.3 that inter-industry equilibration would link exowth of assets to averasp nrofitability and to the trend in average profitability. The reasoning is analocous to that used when considering the intra-industry process of equilibration. This inter-industry process is not distinct from that oneratine within the industry and the two interact. A highly nrofitable industry will experience a faster rate of decay because the industry as a whole is under strong pressure to force it toward more normal (economy-wide) tevels of profitability. The correlations suggested by this morlel were found.

These wore: high growth with high profitabjlity and steep trend, high growth with fast decay and the various other relations correspondingly.

Irend was too stroncly correlater with the rate of decar for the preceding argument to provjde the whole explanation. Hieh correlations were found between a.1. three inter-temporal measures and the rate of decay. It seemed necessary to conclude that this was not a statistical effect but rather that volatility of industry exnerience implies the conditions for a fast rate of decay of profitability.

The final section of this chanter compares Whittington's estimated measures of the decree of nersistence of profitability with the rates of decay calculated in this study. Whittington's ficures, being based on six year averaces, should be lower than the single year figures derived here, but they are not as low as direct calculation would suggest. This is because a full description of chanes in profitability demands a more complex model. then that used here. Anert from one industry, the ordering of the two measures is about as high as could be exnected given the differences in methods used.

## CHAPTERX

## CONCLIJSTONS

Refore turnine to the nrimary theme - the rate of decay of profitability and its interpretation as an indicator of the sneed of resnurce allocation the secondary aim of the study will be reviewed.

The statistical technique does relax the discreteness of the Markov transition matrix: the conditional or row distrihutions can be treat,ed continumonsly. Considerine various summary statiatics of these sonditional distributions, and in particular their relationshins ton the nrior variable (that unon which the distributions are onntingent, , does reveal eonsiderably more about the nrocess than strajehtforward reoressinn aralysis. Tn fact, it, mav be regarded as information about the error variance-covariance matrix of the equivalent regression equation. Tinfortunately the synthesis of the conditional distributions into a sinule continuons mathematical statement, ahont the whole hivariate transition matrix nroved ton comnlex an exercise tin he whth its nraction (if not. aesthetic) rewardse

Whe theoretical develonment, of the nrimary theme was in torms of narrowly defined nroivots and markets and conseruentily multigroduct firms were taken to he typical. Th this nontext, a major nortion of resomrces is allonater hy manarement decision and the allonation ir hetween the items of the fimm's nooduct ranos. lintry and exit are taken the be oommon bitt ton be gredominantily the resulta of exnansions and onntractions of established
 Resource alloestion deridiong will danond menn tho oh jocetivas of manacramont, hut, whatever these are, nenfitahility nf antivities under ennsideration will he melevant to some extent. The weight given ton remfitehility wil?
depend unon the nerticular ohiective function and therefore the latter will influence the sneed with whinh resnurces are shifter in resnnnse to variations in profitability. The speed 27 sn deponds unon the efficacy of nresent profitability as a proxy for expected nrofitability; as it, is the exnected value which directs management, derisions. The speed of rescurce movement denends also unon the flexihility of canital stock and many nther technological factors. Finally, it denends unon thoso ohstantes ton comprotitive antivity in an industry such as harriers ton entry and olignnotistice interdenendencies.

The allocation of resources tnwards merket, offering hiwh nenfitahility and away from those offering low profitahility leads to the decar of markot. profitahility. Firms will, on averace, axnorience chanses in their rates of return that reflect those nccurrine in the markets in which they oporate. Therefore it is to be experted that the rafitability of firms will denay and that those factors previously mentioned as influencing the sneed of resource allocation will alsn influence the firms' matee of decay of profitability.

The emnirical work does establish that decar oncurs from year to year in all industries and nearly every subgroup - the few excentions are such that. littile weight need he rixt unon them. Whittinaton has grevinualy shown that decay occurs in all industries when rates of return are six-year averages, that is, he finds a relatively lone noriod decay. The nresent, results confirm that, desnite all the disturbances, profit, decay is detectable in short period year by year analysis. The mbininity of decay indicates that, with the grouning of firms used, comnetition is working to brine about the ennvergence of nrofitability. The evidence shows that
those factors which obstmut the competitive process are never strong enough to obliterate this underlying tendency. It is important to emphasize that this evidence relates to the size of groups used and does not rule out individual markets or groups of markets where the competjtive forces are completely negated. But there are neither industries nor subgrouns where such markets predominate.

The theoretical development led to the view that there might be differerces between the rate of erosion of hish profitability and the rate of restoration of low rates of return. For this reason, the rate of decay was measured senarately for above average and below average nrofjtabjlityo The two rates of decay were significantly different in a majorjty of cases with the general pattern being a faster rate of decay below than above the mean. This rather weak evidence disagrees with the genorally held view that it is harder to eliminate excess canacity than sunernormal profits. The general growth in demand during the perjod studied may well explain this apparent conflict.

Allowance was made for a nonlinear relationshin between relative nrofitiability at time $t$ and at time $t-1$. Any nonlinearity was expected to take the form of an increasing rate of decay accomnanying increases in the absolute value of deviations of profitability from the decay orisin. A nonlinear function was in bractice only required in a minority of cases and in these the exnected form of nonlinearjty was found. the linear form involves increasing comnetitive pressure at extreme rates of return because its constant proportional decay means increasine absolute decay as rates of returm deviate further from the decay orisin. The nonlinear form demonstrates this increasino competitive nressure more stronyly as it involves increasing proportional decay.

Perhaps, after the establishment of the fact of profitability decav, the most important conclusion is thet industries do differ significantly with respect to their rates of decay. T. is, of course, statistically significant differences, not economically sisnificant ones, that have been established. This latter problem brines us to the question of the place of the rate of decav of profitability in determining nublic nolicy on industry stmoture, conduct and perfommance. Tt was emphasjzed in the first chanter that the rate of decay shows how effective the forces working to return the industry to equilibrium are; and that, this contrasts with static measures of market structure which relate to the nature of an equilibrium should it be attained. Firstly, it, is necessary to query whether fast decay is preferable to slow decay and secondly, whether market, structure is a good indicator of rate of decay, i.e. whether both are needed.

If the equilibrium towards which the system is tendine is optimal (in the Paretian sense), then unless fast decar has direct disadvantages, it is preferable to slow decay. Fast. decay may be disadvantageous if it hrines uncertainty and disorder to the industry - none of the docay rates measured would seem fast, enough to raise this problem. The second way in which fast, decay may be disadvantagemus is if it acts to doter valuable activities. Tn narticular, too ranid erosion of hich nrofitahility may deter investment in research and develorment; as competitive advantases smed from these Aotivities may be ton shortijved.

Tn the case where the notential enuilibrium stato is not Pareto ontimal. the question of whether fast decay is preferable heonmes more difficult. Profit docar shnws the transfer of resmumes from the unnonfjtahle to the profitahle activities. This must he a shift thwards the notimm although the induatry may he one that wonld reach an eguitibrinm situation hefore
an optimum is attained. Abstracting from the problems of $a$ second best, situation, there must be a nresumption that decay brings about an improvement and therefore fast decay is to be preferred to slow as long as the disadvantages of fast growth mentioned previously can be ignored.

The second asnect, of the relevance of the rate of decay to public policy is the way that the newly suggested variable relates to other commonly employed measures of industry characteristios. These questions were examined in Chanter IX. Theory led us to expect that the rate of decay would be slower in concentrated than unconcentrated industrios. The evidence did not contradict this. Therefore it seems that, on average, taking note of the rate of decay in an industry appraisal would not conflict with the evidence of industry structure. On the other hand, the correlation between decay and concentration was low and suggests that decay can only be taken eccount of by its explicit inclusion and not by reliance on concentration as a proxy.

The relationships between decay and various performance characteristics were less simple. They were consistent with an interaction between interand intre-industry allocation processes. That is to say, an industry of high profitability would be expected to grow quickly and to have a relatively rapid decay of industry average profitability towards the economy-wide normal level. The process involved would affect profitability relativities within the industry and lead to a high rate of decay of firm profitability. This now leads to a conflict between the average profitability variable and the rate of decay. Preferred values of one tend to accompany less preferred values of the other.

The whole equilibratine process between and within industries lends some attractiveness to a passive industry poliny. Intervention then hangs
unon whether the process is working quickly enough and this is a problem area that the economist is ill-equipped to handle. This has already been seen at the intra-industry level where no minimum acceptable value of the rate of decay could be put forward. But the question of intervention is not restricted solely to timing. The relationships between the various performance measures are average relationships - there will be cases where there is little evidence of self-equilibration. The intra-industry process applies on average over the markets of that industry and there may well be markets where no profitability decay takes place, A laissez-faire conclusion is not necessarily appropriate.

In summary, the decay of profitability has been found in nearly every instance examined. Because the rate of decay of profitability measures the speed of resource allocation, it is a relevant variable to include in any assessment of industry performance and, because industries may be distinguished statistically in this respect, it is a practical variable for such use.

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[^24]See Manpter IV Section 5 for definitions and summary.


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## TRANSITION MATRICES - TNDUSTRIES

See Chapter V Section I for definitions and explanations and
Section 2 for discussion of these data.

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| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.1450 | 1.7370 | 0.3592 | - | 0.2957 | 3.0080 |
| 1. 6040 | 1. 3.3490 | 0.2336 | + | 0.0410 | 2.8000 |
| 1.4100 | 1.1 .760 | 0.2512 | + | 0.4378 | 3.4490 |
| 1.2440 | 0.6304 | 0.2717 | - | 0.251 .9 | 2.7850 |
| 1.1270 | 0.7273 | 0.3837 | - | 0.0661 | 2.4970 |
| 1.0100 | 0.7745 | 0.2912 | - | 0.1236 | 2.3240 |
| 0.9174 | 0.8190 | 0.3090 | + | 0.3248 | 3.3610 |
| 0.3432 | 0.7103 | 0.2119 | + | 0.5380 | 4.4790 |
| 0.7589 | 0.6743 | 0.3932 | - | 0.3179 | 3.4030 |
| 0.6740 | 0.6616 | 0.2932 | - | 0.0087 | 2.6390 |
| 0.6971 | 0.6109 | 0.1893 | * | 1.0940 | 5.0420 |
| 0.5646 | 0.5215 | 0.2153 | - | 0.3922 | $2.65 \% 0$ |
| 0.4978 | 0.4447 | 0.2639 | - | 0.2500 | 3.2750 |
| 0.4540 | 0.2136 | 0.2335 | + | 0.0349 | 2.5420 |
| 0.3956 | 0.4122 | 0.3820 | $\cdots$ | 1.9000 | 7.3430 |
| 0.3269 | 0.2072 | 0.1306 | 4 | 0.1268 | 2.3870 |
| 0.2657 | 0.2189 | 0.0939 | + | 0.31 .08 | 2.4570 |
| 0.2117 | 0.1866 | 0.2630 | + | 0.2052 | 3.3390 |
| 0.3 .433 | 0.1688 | 0.1110 | + | 0.0026 | 2.1250 |
| 0.0834 | 0.0211 | 0.1807 | - | 0.1372 | 6.1390 |
| 0.0235 | 0.0095 | 0.2491 | - | 0.1040 | 2.1980 |
| -0.0440 | 0.0555 | 0.3555 | + | 3.8260 | 8.7880 |
| -0.0905 | -0.1.44 | 0.1592 | - | 0.0863 | 2.7500 |
| -0.1437 | -0.1216 | 0.3075 | - | 0.7690 | 6.3340 |
| -0.2077 | -0.4348 | 0.2021 | + | 0.0057 | 2.6240 |
| -0.2524 | -0.2115 | 0.0827 | - | 0.0081 | 2.3130 |
| -0.2995 | -0.2921 | 0.4467 | + | 0.2709 | 3.5330 |
| -0.3599 | -0.3030 | 0.2302 | + | 0.1723 | 4.4250 |
| -0.4252 | -0.3958 | 0.2973 | " | 2.7070 | 7.9620 |
| - -0.4983 | -0.4623 | 0.1122 | - | 0.1242 | 3.3340 |
| -0.5624 | -0.6027 | 0.2767 | + | 0.1007 | 2.3940 |
| -0.0177 | -0.4609 | 0.2347 | + | 0.21 .40 | 3.6650 |
| -0.6868 | -0.5473 | 0.1904 | + | 0.2943 | 2.6230 |
| -0.7584 | -0.6923 | 0.1812 | + | 0.0023 | 5.2320 |
| -0.8321 | -0.9433 | 0.2730 | - | 0.3630 | 3.9520 |
| -0.9690 | -0.8799 | 0.5346 | + | 0.7224 | 3.1810 |
| -1.0260 | $-0.9330$ | 0.2411 | " | 0.0162 | 2.8220 |
| -1.2420 | -1.2490 | 0.3792 | + | 0.3044 | 4.4410 |
| -1.4450 | -1.2680 | 0.6195 | + | 0.1292 | 3.5370 |
| $-2.0750$ | -1.5290 | 0.4747 | $\cdots$ | 0.0350 | 2.5920 |


| ClASS MID-PT | MEAN | variance | SIGN | SkEuness | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.5320 | 1.8530 | 0.4157 | " | 0.1200 | 3.5940 |
| 1.9990 | 1.5390 | 0.5223 | - | 0.6797 | 4.9510 |
| 1.7970 | 1.3210 | 0.3956 | - | 0.2012 | 4.6890 |
| 1.6730 | 1.2730 | 0.5578 | + | 0.0001 | 2.1520 |
| 1.5530 | 1.1720 | 0.4068 | - | 0.7370 | 3.6500 |
| 1.4700 | 1.1670 | 0.2136 | * | 0.3096 | 2.6710 |
| 1.3320 | 1.0590 | 0.1668 | - | 0.0971 | 3.3290 |
| 1.3100 | 1.0670 | 0.4741. | - | 0.0754 | 3.6820 |
| 1.2570 | 1.2170 | 0.3250 | + | 0.3914 | 3.2620 |
| 1.2090 | 1.0510 | 0.4111 | - | 0.0016 | 2,3440 |
| 1.1550 | 0.9832 | 0.1636 | - | 0.1448 | 3.6930 |
| 1.1040 | 0.6252 | 0.3085 | + | 0.2808 | 4.1380 |
| 1.0640 | 0.8667 | 0.3272 | - | 0.0000 | 3.9290 |
| 1.0090 | 0.8933 | 0.3764 | - | 0.3725 | 4.8860 |
| 0.9517 | 0.6775 | 0.2410 | - | 0.7428 | 3.4970 |
| 0.9138 | 0.693 ? | 0.2900 | - | 1.0220 | 4.7900 |
| 0.8778 | 0.7218 | 0.4013 | + | 0.1450 | 3.2710 |
| 0.8406 | 0.5881 | 0.2912 | - | 0.8392 | 6.0610 |
| 0.8043 | 0.5994 | 0.3041 | + | 0.01 .12 | 3.2010 |
| 0.7799 | 0.4742 | 0.6474 | - | 0.2054 | 4.2720 |
| 0.7462 | 0.7077 | 0.2161 | + | 0.1805 | 2.1370 |
| 0.71 .13 | 0.7455 | 0.1888 | + | 0.0091 | 2.3350 |
| 0.6661 | 0.6889 | 0.4737 | - | 0.3678 | 5.1900 |
| 0.6349 | 0.3660 | 0.3172 | + | 0.0511 | 2.5060 |
| 0.6102 | 0.5907 | 0.1299 | - | 0.6766 | 5.4440 |
| 0.5891 | 0.5157 | 0.2035 | - | 4.6550 | 10.3500 |
| 0.5721 | 0.5371 | 0.1085 | * | 0.41 .87 | 3.1490 |
| 0.5498 | 0.5206 | 0.1770 | + | 0.0036 | 3.0870 |
| 0.5260 | 0.4619 | 0.2992 | - | 2.2970 | 7.6530 |
| 0.5008 | 0.3744 | 0.1927 | - | 1.0640 | 4.1070 |
| 0.4794 | 0.3469 | 0.2471 | + | 0.1811 | 4.8120 |
| 0.4614 | 0.3773 | 0.1398 | - | 1.2780 | 3.7850 |
| 0.4406 | 0.2199 | 0.4265 | - | 13.1600 | 18.3200 |
| 0.4224 | 0.3247 | 0.4478 | - | 6.7840 | 11.7700 |
| 0.4075 | 0.3971 | 0.1435 | - | 0.1847 | 2.6770 |
| 0.3392 | 0.4018 | 0.2127 | + | 0.3640 | 2.9360 |
| 0.3652 | 0.3023 | 0.1636 | - | 0.0374 | 5.7210 |
| 0.3395 | 0.3182 | 0.2710 | - | 0.5858 | 3.3660 |
| 0.3204 | 0.2850 | 0.19?2 | + | 0.7058 | 4.2160 |
| 0.2981. | 0.3167 | 0.1865 | + | 0.5643 | 2.8230 |
| 0.2726 | 0.3446 | 0.3707 | + | 1.2710 | 4.5020 |
| 0.2485 | 0.2090 | 0.2817 | + | 0.0178 | 3.0130 |
| 0.2309 | 0.3307 | 0.2171 | - | 0.6472 | 5.0600 |
| 0.2100 | 0.1618 | 0.1165 | - | 0.0073 | 3.2420 |
| 0.1882 | 0.1719 | 0.1041 | + | 0.0123 | 2.5800 |
| 0.1677 | 0.0945 | 0.1471 | + | 0.0106 | 4.5970 |
| 0.1447 | 0.1971 | 0.2815 | + | 0.11 .77 | 3,7710 |
| 0.1233 | 0.1828 | 0.2458 | - | 0.2965 | 3.3230 |
| 0.1012 | 0.0458 | 0.1822 | - | 0.5041 | 3.5060 |
| 0.0821 | 0.0303 | 0.1533 | - | 0.1822 | 2.0930 |
| 0.0639 | 0.0870 | 0.1612 | $\bullet$ | 0.0293 | 2.7180 |
| 0.0451 | 0.0507 | 0.2538 | - | 2.7640 | 9.9480 |


| CLASS MIDEPT | MEAN | VARIANCE | SIGN | SKEWNESS | KuRTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0260 | 0.0218 | 0.1409 | + | 0.1275 | 2.7770 |
| 0.0099 | -0.0894 | 0.1667 | - | 1.7660 | 4.7480 |
| -0.0043 | -0.0398 | $0.396 \%$ | - | 1.6260 | 5.0910 |
| -0.0226 | -0.0885 | 0.1529 | * | 0.0050 | 3,2400 |
| -0.0337 | -0.0957 | 0.2148 | - | 0.8041 | 3.3900 |
| -0.0548 | -0.11957 | 0.1864 | - | 0.0235 | 3.2200 |
| -0.0746 | -0.1380 | 0.1011 | + | 0.5933 | 4.1700 |
| -0.0950 | -0.1920 | 0.2278 | - | 2.1200 | 5.0350 |
| -0.1149 | 0.0439 | 0.1630 | + | 0.1147 | 2.7570 |
| -0.135? | -0.0724 | 0.1579 | + | 0.0610 | 3.7120 |
| - -0.1572 | -0.1122 | 0.1271 | - | 0.1188 | 2.0530 |
| -0.1757 | -0,1097 | 0.1816 | + | 0.0197 | 1.7550 |
| -0.1955 | -0.1618 | 0.1533 | - | 0.0556 | 2.6829 |
| -0.2136 | -0. 2730 | 0.1339 | $\cdots$ | 0.3050 | 2.2340 |
| -0.2291 | -0.2528 | 0.2169 | * | 7.3370 | 12.9700 |
| -0.2480 | -0.2873 | 0.1324 | - | 0.5023 | 3.6550 |
| -0.2681 | -0.1926 | 0.3936 | - | 0.0027 | 4.0460 |
| -0.2512 | -0.3743 | 0.11 .39 | - | 1.5040 | 6.4860 |
| -0.3098 | -0.4480 | 0.3154 | - | 2.2300 | 6.7350 |
| -0, 0.329 | -0.2781 | 0.1097 | + | 0.0014 | 2.6990 |
| -0.3496 | -0.4396 | 0.1244 | - | 0.0327 | 3.3600 |
| -0.3636 | -0.3299 | 0.2237 | + | 0.0269 | 5.7230 |
| -0.3913 | -0.2764 | 0.0928 | + | 0.0406 | 2.9730 |
| -0.4133 | -0.2428 | 0.1732 | + | 0.0475 | 3.1760 |
| -0.4417 | -0.4265 | 0.3112 | + | 2.2810 | 7.9450 |
| -0.4523 | -0.6499 | 0.2041 | - | 0.1178 | 2.4340 |
| -0.4342 | -0.5473 | 0.4361 | - | 1.1080 | 4.0210 |
| -0.5081 | -0.5284 | 0.2429 | - | 0.6466 | 4.2180 |
| -0.5283 | -0.5320 | 0.2063 | * | 3.8560 | 8.9070 |
| -0.55? | -0.4656 | 0.1908 | + | 0.1080 | 2.7880 |
| -0.0.573 | -0.5435 | 0.2237 | - | 0.0766 | 3.6500 |
| -0.5989 | -0.5334 | 0.1089 | + | 0.0044 | 1.9030 |
| -0.6215 | -0.4484 | 0.1808 | - | 0.0032 | 3.4190 |
| -0.6509 | -0.5788 | 0.2853 | - | 8.2390 | 13.7900 |
| -0.08843 | -0.5153 | 0.2795 | + | 1.6720 | 5.5950 |
| -0.7126 | -0.7130 | 0.4215 | + | 0.0986 | 5.8470 |
| -0.7391 | -0.0013 | 0.3074 | - | 1.5410 | 5.1430 |
| -0.7567 | -0.7776 | 0.2226 | $\cdots$ | 0.7051 | 4.9550 |
| -0.7955 | -0.5599 | 0.2731 | + | 0.4097 | 2.7940 |
| -0.8237 | -0.7838 | 0.1452 | * | 0.5881 | 3.0500 |
| -0.8489 | -0.7651 | 0.1073 | - | 0.0106 | 2.7510 |
| -0.0862 | -0.6718 | 0.1040 | + | 1.5670 | 5.5180 |
| -0.9158 | -0.8034 | 0.4185 | - | 0.0955 | 5.06 .30 |
| -0.9407 | -0.9558 | 0.3436 | - | 3.3370 | 6.7660 |
| -0.9740 | -0.7962 | 0.2418 | + | 0.0467 | 4.9450 |
| -1.0160 | -0.7822 | 0.2063 | + | 0.771 .0 | 3.3310 |
| -1.0520 | -0.8649 | 0.1569 | + | 0.0136 | 2.6330 |
| -1.1140 | -0.8762 | 0.2096 | + | 0.2338 | 3.8460 |
| -1.1.790 | -0.9721 | 0.1093 | - | 0.6428 | 4.8920 |
| -1.2480 | -1.0530 | 0.5512 | + | 0.1290 | 3.1910 |
| -1. 3.340 | -1.0330 | 0.31 .87 | - | 0.4129 | 2.5360 |
| $\because 1.4720$ | $-1.1340$ | 0.6506 | " | 0.0517 | 2.5460 |

INDUSTK゙ \& (GONTD)

| CLASS MID-FT | MEAN | VARIANCE SIGN | SKEWIESS | KURTOSIS |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -1.6820 | -1.3220 | 0.3060 |  | 0.0624 | 3.1400 |
| -2.3170 | -1.5100 | 0.4411 | - | 0.1596 | 2.2410 |


| CLASS MIDMPT | MEAM | variande | SIGN | SKENESS | kuRTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.3550 | 1.5440 | 0.9707 | - | 9.3320 | 14.7500 |
| 1.8510 | 1.4790 | 0.5793 | - | 1.4200 | 5.2990 |
| 1.3320 | 1.3160 | 0.5722 | - | 0.1479 | 2.6290 |
| 1. 3500 | 1.0210 | 0.41060 | - | 0.0008 | 2.4800 |
| 1.1340 | 0.9786 | 0.3655 | + | 0.2128 | 2.7390 |
| 1.0110 | 0,9320 | 0.3934 | - | 0.0004 | 3.0750 |
| 0.3114 | 0.6490 | 0.2597 | - | 0.3041 | 2,7010 |
| 0.3176 | 0.7676 | 0.1851 | + | 0.1336 | 3.2140 |
| 0.7258 | 0.5553 | 0.1850 | + | 0.0000 | 4.8410 |
| 0.6349 | 0.5289 | 0.3162 | + | 0.8827 | 5.9310 |
| 0.5551 | 0.4101 | 0.3226 | - | 1.4150 | 4.2100 |
| 0.4839 | 0.5275 | 0.2130 | + | 1.0120 | 5.5630 |
| 0.4179 | 0.1743 | 0.2959 | - | 0.0158 | 2.5190 |
| 0.3671 | 0.2237 | 0.0898 | * | 0.1056 | 2.8190 |
| 0.3026 | 0.1740 | 0.2304 | - | 2.1400 | 6.91 .00 |
| 0.2427 | 0.2884 | 0.2498 | * | 0.0579 | 5.21 .80 |
| 0.1957 | 0.0496 | 0.1870 | - | 1.2230 | 5.1750 |
| 0.1518 | -0.0166 | 0.1736 | - | 2.1400 | 5.6490 |
| 0.1036 | 0.0197 | 0.2188 | + | 1.2230 | 7.9200 |
| 0.0546 | -0.0361 | 0.1148 | + | 0.9609 | 5.0160 |
| 0.0020 | -0.1135 | 0.3363 | + | 0.5660 | 4.1680 |
| -0.0491. | -0.0909 | 0.1415 | + | 0.9557 | 4.4090 |
| -0.0966 | -0.1535 | 0.5003 | - | 2.8320 | 6.6640 |
| -0.1414 | -0.2385 | 0.2024 | - | 0.8657 | 6.0590 |
| -0.1923 | -0.1522 | 0.2414 | + | 2.1290 | 4.6390 |
| -0.2434 | -0.2902 | 0.3250 | - | 1. 5690 | 5.6230 |
| -0.2851. | -0.2785 | 0.2477 | * | 0.0361 | 4.7100 |
| -0.3309 | -0.3240 | 0.1747 | + | 0.0146 | 3.4800 |
| -0.3731 | -0.3415 | 0.1586 | + | 0.0901 | 2.6360 |
| -0.4272 | -0.4336 | 0.2473 | - | 3.0520 | 8. 4750 |
| -0.4824 | -0.5424 | 0.2362 | - | 0.3179 | 3.8020 |
| -0.5330 | -0.5629 | 0.4769 | + | 0.0017 | 6.4580 |
| -0.5897 | -0.51.32 | 0.2428 | * | 0.2823 | 3.4340 |
| -0.6293 | -0.5423 | 0.2370 | - | 0.7594 | 4.3370 |
| -0.6913 | -0.5959 | 0.2951 | + | 0.0794 | 2.7620 |
| -0.7468 | -0.8237 | 0.4288 | - | 0.1696 | 2.9920 |
| -0.8260 | -0.8048 | 0.2377 | " | 0.2063 | 2.81 .40 |
| -0.91.45 | -0.7466 | 0.2246 | + | 1.2890 | 5.1130 |
| -1.0310 | -0.7702 | 0.3401 | + | 1.4710 | 5.3360 |
| -1.1540 | -1.0480 | 0.4479 | " | 0.7251 | 3.6190 |
| -1.4310 | -1.061.0 | 0.381 .0 | - | 0.9662 | 3.9510 |
| -2.0670 | -0.8600 | 0.7063 | + | 0.1803 | 4.1740 |


| CLASS MIDMFT | MEAN | VARIARCE | SIGN | SKEWMESS | KURTOS 15 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0560 | 1.4550 | 0.7035 | - | 1.4680 | 4.9490 |
| 1.5540 | 1. 2170 | 0.4893 | - | 1.5570 | 6.0950 |
| 1.2330 | 0.7941 | 0.3600 | m' | 0.1819 | 3.2500 |
| 1.0520 | 0.7437 | 0.2923 | + | 0.1414 | 3.2760 |
| 0.9243 | 0.6902 | 0.2324 | " | 0.1625 | 3.3400 |
| 0.6102 | 0.6883 | 0.2316 | + | 0.0890 | 3.7240 |
| 0.7192 | 0.6623 | 0.1727 | + | 0.0006 | 2. 3070 |
| 0.6326 | 0. 351.9 | 0.2185 | - | 0.4451 | 2.5900 |
| 0.5550 | 0.5134 | 0.1944 | + | 0.2737 | 2.9620 |
| 0.4813 | 0.4194 | 0.2301 | + | 0.8064 | 5.3040 |
| 0.4080 | 0.2774 | 0.1680 | - | 2.6630 | 7.6130 |
| 0.3345 | 0.1384 | 0.1490 | 4 | 0.6465 | 4.0900 |
| 0.2515 | . $0.1 .5 \pm 4$ | 0.3475 | + | 0.0204 | 2.9240 |
| 0.1795 | 0.2771 | 0.2835 | - | 0.1262 | 4.2450 |
| 0.1231 | -0.0220 | 0.2111 | - | 0.4408 | 2.6650 |
| 0.0611 | 0.0900 | 0.1337 | - | 1.7320 | 4.9790 |
| 0.0065 | 0.1034 | 0.3964 | - | 0.6617 | 3,2630 |
| -0.0516 | $-0.0037$ | 0.3408 | \% | 1.5320 | 5.5850 |
| -0.1331 | -0.1256 | 0.2155 | + | 0.3008 | 3.4470 |
| -0.2175 | -0.1761 | 0.3269 | - | 1.2930 | 4.8290 |
| -0.3053 | -0.1434 | 0.3558 | + | 0.4271 | 3.8760 |
| -0.3850 | -0.3600 | 0.4382 | - | 0.8511 | 4.9280 |
| -0.4542 | -0.2606 | 0.2765 | + | 1.4460 | 4.4630 |
| -0.5478 | -0.5876 | 0.3964 | - | 0.6301 | 4.0890 |
| -0.6523 | -0.5838 | 0.4420 | - | 0.3307 | 3.0100 |
| -0.7361 | -0.7298 | 0.1911 | $\div$ | 0.0612 | 2.0090 |
| -0.9272 | -0.8375 | 0.2597 | $\cdots$ | 0.2509 | 3.1170 |
| -1.0550 | -0.6413 | 0.2866 | 4 | 2.2260 | 6.6440 |
| -1.2840 | -0.8120 | 0.7473 | + | 0.8074 | 5.6890 |
| -1.7640 | $-1.5170$ | 0.5477 | - | 0.0116 | 1.8770 |

CLASS MID-PT
MEAN
2.3310

| 1.8860 | 0.3215 | $\cdots$ | 0.0961 | 2.3760 |
| :---: | :---: | :---: | :---: | :---: |
| 1.3110 | 0.4435 | " | 1.0400 | 5.1520 |
| 1.4280 | 0.1701 | $\cdots$ | 0.4571 | 2.6640 |
| 1.0610 | 0.2340 | - | 0.0045 | 2.6590 |
| 1.0750 | 0.2458 | + | 0.0044 | 3.0760 |
| 0.7798 | 0.2090 | $+$ | 0.0014 | 1.7660 |
| 0.7199 | 0.2094 | - | 1.1330 | 4.6490 |
| 0.6963 | 0.1686 | - | 2.6440 | 7.9990 |
| 0.7022 | 0.3663 | - | 0.1295 | 2.9210 |
| 0.5532 | 0.2265 | - | 0.0047 | 3.1200 |
| 0.4817 | 0.1046 | * | 0.0218 | 2.3740 |
| 0.4540 | 0.2653 | - | \%.8820 | \%.3870 |
| 0.4997 | 0.1541 | " | 0.0130 | 2.3900 |
| 0.4206 | 0.1219 | - | 0.7193 | 3.2850 |
| 0.3244 | 0.1061 | - | 0.1501 | 2.3800 |
| 0.3702 | 0.0970 | - | 0.4260 | 3.6030 |
| 0.3187 | 0.1130 | + | 0.0309 | 2.6930 |
| 0.2275 | 0.1259 | - | 0.1552 | 2.2730 |
| 0.2223 | 0.2223 | + | 0.4483 | 5.0530 |
| 0.2111 | 0.1336 | - | 0.0020 | 2.6100 |
| 0.3098 | 0.0723 | - | 0.4539 | 3.4020 |
| 0.1960 | 0.0718 | + | 0.0032 | 2.8000 |
| 0.2217 | 0.1795 | $+$ | 3.4170 | 8.3950 |
| 0.1995 | 0.1386 | + | 0.0109 | 3.1730 |
| 0.0625 | 0.1910 | - | 2.6160 | 3.8060 |
| 0.0209 | 0.1444 | $\square$ | 8, 0050 | 13.4300 |
| 0.0522 | 0.1427 | + | 0.0463 | 3.5590 |
| -0.0894 | 0.2155 | - | 1.6240 | 4.1630 |
| 0.0120 | 0.1066 | + | 0.3572 | 5.0210 |
| -0.0750 | 0.0933 | + | 0.2027 | 2.3680 |
| 0.0167 | 0.1342 | + | 0.3048 | 3.6490 |
| -0.0076 | 0.0531 | + | 0.9070 | 4.3010 |
| -0.0390 | 0.1034 | - | 0.5252 | 5.9510 |
| -0.0454 | 0.0365 | m | 0.52 .42 | 2.9350 |
| -0.0.657 | 0.0790 | $\cdots$ | 0. 0.6832 | 3.8650 |
| 0.0391 | 0.1431 | + | 1.3180 | 3.7250 |
| -0.0566 | 0.1330 | + | 0.9634 | 3.9660 |
| -0.2826 | 0.1743 | - | 1.0880 | 3.8080 |
| -0.1848 | 0.0615 | 4 | 0.0007 | 2.1.210 |
| -0.2981 | 0.2015 | - | 3.5750 | 7.5820 |
| -0.2249 | 0.1484 | + | 0.4509 | 3.7170 |
| -0.1374 | 0.061 .9 | + | 0.0639 | 2.5570 |
| -0.2648 | 0.0685 | - | 1. 21.30 | 4.1330 |
| -0.4015 | 0.1112 | + | 0.5070 | 3.4140 |
| -0.2710 | 0.0632 | + | 0.0239 | 4.4820 |
| -0.3716 | 0.1333 | - | 0.1436 | 3.3130 |
| -0.4273 | 0.4040 | - | 1.3350 | 5.6420 |
| -0.2286 | 0.1160 | + | 0.0239 | 2.5770 |
| -0.4042 | 0.2362 | - | 3.6430 | 9.7850 |
| -0.4121 | 0.1966 | - | 1.6400 | 9.1 .260 |
| -0.4769 | 0.3878 | + | 5.3037 | 12.7000 |
| -0.4730 | 0.1443 | * | 0.0103 | 2.2690 |

IMTSTに 7 (CONTB)

| CLASS UD-PT | MEAN | VARIANCE | SIGN | SGTMNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.616? | -0.4644 | 0.3150 | + | 0.0068 | 3.1210 |
| $-0.6565$ | -0.4601 | 0.0956 | - | 0.0181 | 1.9660 |
| -0.7059 | $-0.6855$ | 0.1134 | + | 0.1562 | 3.2110 |
| -0.776? | -0.4992 | 0.3111 | - | 0.0006 | 3.0000 |
| -0.8436 | -0.6105 | 0.2197 | - | 0.0232 | 4.6920 |
| $-0.9060$ | -0.6508 | 0.2923 | - | 1.2150 | 4.4420 |
| -0.9716 | -0.7199 | 0.1660 | + | 0.1999 | 2.5310 |
| -1.0720 | -0.7328 | 0.2332 | + | 0.1349 | 5.2790 |
| -1.1980 | $-1.0420$ | 0.3261 | - | 0.8974 | 3.7340 |
| -1.3750 | -1.0880 | 0.5699 | + | 0.0323 | 3.2940 |
| -2.0580 | $-1.4930$ | 0.6326 | + | 0.2222 | 3.8100 |

MEAN

| 1.6420 | 1.1040 |
| ---: | ---: |
| 1.1200 | 0.7107 |
| 0.7978 | 0.6572 |
| 0.5650 | 0.2752 |
| 0.4007 | 0.2604 |
| 0.2210 | 0.0620 |
| 0.0102 | 0.1336 |
| -0.1361 | -0.1506 |
| -0.2636 | -0.2922 |
| -0.4697 | -0.2940 |
| -0.8457 | -0.4380 |
| -1.5530 | -1.0070 |

VARIANCE
516 N
SkEVMESS
KURTOSIS

| 0.6572 | - |
| :--- | :--- |
| 0.4302 | - |
| 0.5318 | - |
| 0.3401 | - |
| 0.3159 | - |
| 0.6743 | - |
| 0.5222 | - |
| 0.6393 | - |
| 0.4336 | - |
| 0.6865 | - |
| 0.4687 | + |
| 0.6336 | + |

2.9090
2.9270
2.5910
3.1730
3.5000
7.4540
5.6660
3.4260
3.4390
2.9920
2.8050
4.4590

IMUSTAY 9

| CLASS MIDWPT | MEAN | VARIANCE | SIGU | SKEWNESS | K:RTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6920 | 1.4390 | 0.4878 | - | 0.5490 | 3.4110 |
| 1. 4740 | 1.0670 | 0.3491 | - | 0.1073 | 2.4830 |
| 1.1940 | 0.7781 | 0.5065 | '7 | 0.5975 | 3.2240 |
| 1.0030 | 0.6824 | 0.5105 | - | 0.5764 | 3.0980 |
| 0.8487 | 0.3847 | 0.9714 | - | 1.5560 | 4.7650 |
| 0.6882 | 0.3927 | 0.4710 | - | 0.5639 | 3.1300 |
| 0.5030 | 0.6336 | 0.2212 | $\stackrel{+}{+}$ | 0.0220 | 3.1430 |
| 0.4654 | 0.3792 | 0.3967 | \% | 0.0840 | 2.5460 |
| 0.3559 | 0.0791 | 0.5888 | - | 2.5610 | 5.8500 |
| 0.2788 | 0.2443 | 0.4803 | - | 0.4663 | 3.0240 |
| 0.1920 | -0.1097 | 0.3365 | - | 0.0232 | 2.5250 |
| 0.0942 | 0.0436 | 0.3028 | \% | $0.16 \pm 5$ | 2.4360 |
| 0.0180 | 0.0756 | 0.5377 | * | 0.4043 | 4.5470 |
| -0.0679 | -0.2028 | 0.3156 | - | 1.0550 | 3.9790 |
| -0.1447 | -0.2649 | 0.8997 | * | 0.7274 | 3.4610 |
| -0.2985 | $-0.1197$ | 0.5233 | + | 0.4697 | 3.4010 |
| -0.3538 | -0.5743 | 0.5252 | + | 0.1238 | 2.4910 |
| -0.4521 | -0.3608 | 0.3524 | " | 0.1624 | 2.1760 |
| -0. 0.5546 | -0.4077 | 0.8502 | + | 0.011 .4 | 3.4250 |
| -0.6728 | -0.5177 | 0.3965 | - | 1.0490 | 3.6140 |
| -0.8253 | -0.5785 | 0.3414 | + | 0.0095 | 3.1700 |
| -0.9853 | -0.5830 | 0.3114 | - | 0.2098 | 3.4170 |
| -1.2700 | -0.7344 | 0.3966 | - | 0.6220 | 3.6450 |
| -1.8790 | -0.9239 | 0.8471 | * | 0.0314 | 2.9450 |


| CLASS MIDMPT | MEAN | varlance | SIGN | SkEMMESS | KuRTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.1770 | 1.3620 | 0.7028 | * | 1.3730 | 5.4500 |
| 1.5730 | 1.2080 | 0.3041 | + | 0.0010 | 2.8940 |
| 1. 31.80 | 1.0920 | 0.4152 | - | 0.4117 | 5.7360 |
| 1.1440 | 0.8826 | 0.2976 | - | 0.1745 | 3.3240 |
| 0.9345 | 0.9016 | 0.3957 | - | 0.6516 | 4.1240 |
| 0.8459 | 0.5931 | 0.3601 | $\stackrel{\square}{*}$ | 0.0129 | 2.7480 |
| 0.7198 | 0.6233 | 0.3151 | - | 0.2621 | 4,3320 |
| 0.61 .77 | 0.6054 | 0.2133 | - | 0.4220 | 3.7080 |
| 0.5374 | 0.4447 | 0.1120 | * | 0.0136 | 2.6670 |
| 0.4346 | 0.4932 | 0.1145 | + | 0.0598 | 3.3510 |
| 0.4260 | 0.3142 | 0.3056 | - | 1.0970 | 5.4250 |
| 0.3625 | 0.1670 | 0.3056 | - | 2.3500 | 5.1710 |
| 0.2905 | 0.2676 | 0.2105 | + | 0.6048 | 5.2890 |
| 0.2283 | 0.0551 | 0.1630 | * | 0.9491 | 4.4050 |
| 0.1760 | 0.0860 | 0.1818 | + | 0.0196 | 3.1510 |
| 0.1669 | 0.0900 | 0.0714 | - | 0.0000 | 2.1520 |
| 0.0471. | -0.0345 | 0.2519 | - | 4.0620 | 10.4100 |
| 0.0026 | -0.0619 | 0.1102 | - | 0.2394 | 3,1690 |
| -0,0432 | -0.2447 | 0.2180 | - | 3.5010 | 7.0160 |
| -0.0857 | -0.1591. | 0.1671 | $\cdots$ | 0.0190 | 2.5500 |
| -0.1354 | -0.4549 | 0.3944 | - | 3.7320 | 6.5960 |
| -0.1373 | -0.1997 | 0.1593 | - | 0.0921 | 2.8350 |
| -0.2330 | -0.2601 | 0.1603 | - | 1.6910 | 4.3820 |
| -0.2960 | -0.31.51 | 0.4249 | - | 1.2050 | 4.8370 |
| -0.3613 | -0.5951 | 0.4953 | + | 0.2186 | 5.1520 |
| -0.4295 | -0.4684 | 0.4596 | - | 0.5630 | 2.8750 |
| -0.51.65 | -0.2737 | 0.1341 | " | 0.0262 | 3.5250 |
| -0.591.9 | -0.4033 | 0.2402 | - | 0.2277 | 3.6010 |
| -0.682? | -0.5469 | 0.1065 | - | 0.4043 | 2.9040 |
| -0.8124 | -0.7174 | 0.4513 | * | 0.5678 | 3.2820 |
| -0.9678 | -0.7101 | 0.2738 | - | 0.0292 | 2.6130 |
| -1.2520 | -0.8899 | 0.4777 | + | 0.0165 | 4.2720 |
| -1.9040 | -1.0590 | 0.6117 | - | 0.0385 | 2,2500 |


| CLASS M10-HT | MEAM | VARIANCE | SIGN | SKEWMESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.4900 | 1.9020 | 0.5479 | - | 1.4340 | 5.2850 |
| 1.9090 | 1. 3280 | 0.5401 | - | 0.3246 | 2.8700 |
| 1.5590 | 1.3950 | 0.7612 | - | 1.8090 | 6.8200 |
| 1.3580 | 1.1140 | 0.4365 | - | 0.9314 | 5.2530 |
| 1.1820 | 0.9439 | 0.5227 | - | 1.2280 | 5.5340 |
| 1.1570 | 0.7741 | 0.9429 | - | 0.6658 | 4, 3130 |
| 0.9449 | 0. 7146 | 0.4700 | - | 3.3420 | 8.7170 |
| 0.8199 | 0.6349 | 0.2756 | * | 0.0008 | 2.3460 |
| 0.7320 | 0.6917 | 0.1726 | - | 0.2021 | 4.4060 |
| 0.6850 | 0.5102 | 0.2383 | - | 0.0066 | 2.7930 |
| 0.6026 | 0.4382 | 0.2250 | * | 0.0009 | 2.2730 |
| 0. 5476 | 0.4559 | 0.2095 | $\dagger$ | 0.0056 | 6.6140 |
| 0.4332 | 0.3952 | 0.2243 | $\cdots$ | 2.9020 | 7.5020 |
| 0.4344 | 0.2175 | 0.2310 | " | 0.1895 | 3.2460 |
| 0.3673 | 0.3525 | 0.2470 | + | 1.6590 | 5.91 .90 |
| 0.3150 | 0.3421 | 0.1084 | + | 0.0540 | 2.5770 |
| 0.2627 | 0.21 .70 | 0.1218 | + | 0.8550 | 3.7620 |
| 0.2195 | 0.2796 | 0.1317 | $\stackrel{ }{*}$ | 0.9119 | 5.8910 |
| 0.1717 | 0.1238 | 0.1284 | $\cdots$ | 0.0246 | 3.4550 |
| 0.1249 | 0.1403 | 0.1523 | + | J. 4190 | 8.9540 |
| 0.0765 | 0.1123 | 0.3177 | + | 0.1134 | 8.0990 |
| 0.0326 | 0.0477 | 0.1365 | - | 0.0069 | 2.5350 |
| -0.0113 | 0.0365 | 0.2224 | - | 0.8699 | 4.8210 |
| -0.0553 | -0.0344 | 0.1550 | - | 2.3980 | 7.6950 |
| -0.0956 | -0.1479 | 0.2687 | - | 1.7460 | 5.1540 |
| -0.1370 | -0.1465 | 0.1340 | + | 0.0366 | 2.3470 |
| -0.1894 | -0.1261 | 0.1408 | - | 0.0011 | 2.1820 |
| -0.2606 | -0.2800 | 0.1142 | - | 0.5486 | 3.4390 |
| -0.3172 | -0.1641 | 0.1406 | - | 0.0329 | 3.5200 |
| -0.3775 | $-0.3577$ | 0.1520 | $\cdots$ | 0.4231 | 3.9630 |
| -0.4335 | -0.4533 | 0.1986 | - | 0.1887 | 2.9330 |
| -0.4933 | -0.5807 | 0.2693 | - | 0.1917 | 4.0340 |
| -0.5577 | -0.5467 | 0.2983 | - | 0.8855 | 5.5560 |
| -0.6077 | -0.6176 | 0.4778 | - | 0.1747 | 5.1470 |
| -0.6. 675 | -0.7511 | 0.2164 | - | 0.0194 | 2.4870 |
| -0.7511 | $-0.5785$ | 0.2225 | + | 2.3850 | 6.0550 |
| -0.8366 | $-0.6755$ | 0.2853 | + | 0.4528 | 5.0020 |
| -0.9250 | -0.8209 | 0.1411 | - | 0.9296 | 4.6140 |
| -1.0140 | -0.96.13 | 0.1916 | + | 0.8993 | 4.4620 |
| -1.1530 | -0.9726 | 0.3835 | + | 1.2710 | 4.8730 |
| -1.3420 | -1.2140 | 0.4563 | + | 0.0263 | 2.8320 |
| -1.5610 | -1. 3230 | 0.4415 | - | 0.0426 | 3.4610 |
| -2.0790 | $-1.3290$ | 0.5844 | + | 1.0470 | 4.4770 |

!1m!tryy I.3
CLASSMID-PT MEAN

| 2.5290 | 2.0660 | 0.3535 | - | 0.5468 | 3.0940 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.9220 | 1. 5450 | 0.4556 | - | 0.6277 | 3.3860 |
| 1. 6480 | 1.4060 | 0.2195 | - | 0.9654 | 5.4540 |
| 1.4110 | 1.1160 | 0.3133 | - | 2.8070 | 0.2560 |
| 1.2150 | 0.9646 | 0.1661 | * | 0.0013 | 2.2360 |
| 1.1090 | 0.9796 | 0.1945 | 4 | 0.0440 | 3.1650 |
| 0.9876 | 1.0460 | 0.1691 | + | 0.4653 | 3.2150 |
| 0.9050 | 0.7575 | 0.1924 | * | 2.7060 | 8.8100 |
| 0.8000 | 0.8403 | 0.1887 | - | 0.3939 | 6.3290 |
| 0.7252 | 0.7611 | 0.2203 | - | 0.0012 | 3.7930 |
| 0.6549 | 0.6338 | 0.4438 | + | 0.0857 | 7.2890 |
| 0.5989 | 0.5395 | 0.1410 | + | 1.0490 | 5.5580 |
| 0.5493 | 0. 4846 | 0.1093 | + | 0.0747 | 2.6290 |
| 0.4936 | 0.4097 | 0. 1697 | " | 0.7107 | 4.2100 |
| 0.4830 | 0.2346 | 0.3505 | $\cdots$ | 4.5920 | 8.7190 |
| 0.3878 | 0.3506 | 0.2362 | * | 0.2540 | 6.1150 |
| 0.3471 | 0.3678 | 0.1266 | + | 0.2944 | 3.1380 |
| 0.3215 | 0.2296 | 0.1074 | " | 0.5451 | 4.3030 |
| 0.2792 | 0.1927 | 0.3332 | - | 0.1 .928 | 6.2320 |
| 0.2315 | 0.1605 | 0.0649 | * | 0.7514 | 3.7300 |
| 0.1397 | 0.2554 | 0.1779 | * | 2. 5120 | 8.4350 |
| 0.1628 | 0.1460 | 0.0488 | + | 0.5247 | 2.6760 |
| 0.1246 | -0.0276 | 0.2297 | - | 16.2000 | 20.5700 |
| 0.0975 | 0.0266 | 0.1061 | * | 0.0621 | 4.6320 |
| $0.066 \%$ | 0.0556 | 0.0357 | + | 0.0384 | 2.9850 |
| 0.0376 | 0.1084 | 0.0871 | + | 0.1367 | 3.4920 |
| 0.0135 | 0.1261 | 0.0953 | + | 0.9214 | 3.0890 |
| -0.0131 | -0.0612 | 0.0804 | * | 0.5805 | 4.0630 |
| -0.0350 | -0.1160 | 0.0590 | - | 0.1076 | 2.9320 |
| -0.0622 | -0.0771 | 0.1497 | + | 0.4017 | 6.7560 |
| -0.0929 | -0.2090 | 0.0647 | - | 0.1230 | 2.8820 |
| -0.1220 | -0.1535 | 0.1439 | - | 4.4940 | 1.0.0600 |
| -0.1569 | -0.2547 | 0.2086 | - | 4.8760 | 7.2650 |
| -0.1859 | -0.1608 | 0.0481 | + | 0.0278 | 2.8290 |
| -0.2113 | -0.0565 | 0.1355 | + | 6.0740 | 11.7100 |
| -0.2376 | -0.3274 | 0.2553 | - | 8.4300 | 13.9800 |
| -0.2534 | -0.2954 | 0.0637 | - | 1.9480 | 5.5150 |
| -0.2046 | -0.3511 | 0.0682 | + | 0.8833 | 4.9070 |
| -0.3305 | -0.2404 | 0.0462 | - | 0.1967 | -2.8490 |
| -0.3706 | -0.4135 | 0.1156 | + | 0.8103 | 6.1860 |
| -0.4058 | -0.3972 | 0.0330 | - | 3.8440 | 9,9440 |
| -0.4423 | -0.4616 | 0.0933 | - | 0.7934 | 4.2590 |
| -0.4732 | -0.6193 | 0.1535 | - | 2.2000 | 6.3000 |
| -0.5918 | -0.4449 | 0.0898 | + | 0.9841 | 5.7020 |
| -0.5329 | -0.5792 | 0.1881 | - | 7.2040 | 13.5400 |
| -0.5729 | -0.5481 | 0.1238 | + | 0.9959 | 4.3910 |
| -0.6210 | -0.6125 | 0.0645 | - | 0.4394 | 3.9400 |
| -0.6599 | -0.6836 | 0.0385 | - | 0.0000 | 2.8240 |
| $-0.7165$ | -0.6799 | 0.2247 | " | 0.0010 | 4.3150 |
| -0.7520 | -0.7552 | 0.0384 | - | 0.3515 | 2.9740 |
| -0.8112 | -0.6771 | 0.0941. | $+$ | 1.8510 | 5.5960 3.5600 |
| -0.8747 | -0.7822 | 0.0761 | + | 0.4571 | 3.5600 |

IMDUSTPY 13 (CONTD)

| CLASS MIDmPT | MEAN | VARIANCE SIGN | SKEWNESS | KURTOSIS |  |
| :---: | :---: | :---: | :---: | :---: | ---: |
| -0.3382 | -0.7743 | 0.1548 | + | 0.9014 | 4.2130 |
| -1.0140 | -0.9838 | 0.1066 | + | 0.2446 | 4.54 .50 |
| -1.0920 | -0.8979 | 0.0334 | + | 0.0200 | 2.3490 |
| -1.2120 | -1.2140 | 0.2159 | - | 1.3160 | 5.6870 |
| -1.3650 | -1.3290 | 0.1212 | + | 0.0141 | 3,6700 |
| -1.9220 | -1.4210 | 0.5941 | + | 4.5050 | 11.5000 |


| CLASS MID-FT | MEAS | VARIANCE | SIGN | SKEANESS | Kuptosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.3550 | 1.6480 | 0.8152 | - | 0.2124 | 1.8680 |
| 1.7310 | 1.2900 | 0.5929 | * | 0.3058 | 2.4330 |
| 1.4520 | 1.0250 | 0.4856 | + | 0.0450 | 2.3690 |
| 1.2700 | 0.7312 | 0.6394 | - | 1.7060 | 4.6410 |
| 1.1350 | 0.8428 | 0.4475 | - | 0.2269 | 5.6900 |
| 1.0260 | 0.8375 | 11.2721 | - | 0.661 .3 | 5.2620 |
| 0.9356 | 0.7890 | 0.4325 | - | 0.5926 | 4.7000 |
| 0.3365 | 0.5904 | 0.2852 | $\cdots$ | 0.8123 | 5.4910 |
| 0.7696 | 0.6171 | 0.4757 | - | 0.2443 | 6.9510 |
| 0.7113 | 0.5620 | 0.3441 | - | 1.4840 | 5.0920 |
| 0.6564 | 0.5953 | 0.4326 | - | 0.0000 | 4.5730 |
| 0.6056 | 0.4536 | 0.2735 | - | 0.1051 | 3.4050 |
| 0.5493 | 0.5064 | 0.1662 | + | 0.0003 | 3.6300 |
| 0.5054 | 0.5759 | 0.2784 | + | 1.1350 | 4.2020 |
| 0.4666 | 0.3134 | 0.2484 | * | 0.0021 | 2.7790 |
| 0.4109 | 0.3046 | 0.2539 | - | 0.0107 | 3.8410 |
| 0.3534 | 0.5099 | 0.2657 | * | 1.1260 | 5.4070 |
| 0.3539 | 0.2901 | 0.4275 | - | 0.1446 | 2.6400 |
| 0.2646 | -0.0062 | 0.3581 | - | 0.0216 | 2.7220 |
| 0.2340 | 0.0793 | 0.1762 | - | 1.8720 | 6.6150 |
| 0.1970 | 0.2019 | 0.2321 | - | 0.0286 | 4.2660 |
| 0.1629 | 0.2322 | 0.2188 | + | 1.5730 | 4.5080 |
| 0.1183 | 0.0184 | 0.1309 | + | 0.1428 | 2.6120 |
| 0.0850 | -0.0086 | 0.1703 | + | 0.1303 | 3.1170 |
| 0.0443 | -0.0627 | 0.3791 | + | 0.5855 | 3.9170 |
| 0.0127 | -0.1415 | 0.1975 | - | 0.2853 | 3.2100 |
| -0.0206 | -0.1630 | 0.3777 | - | 0.1028 | 3.3520 |
| -0.0634 | -0.1733 | 0.4726 | - | 0.6475 | 4.2290 |
| -0.1121 | -0.0362 | 0.2145 | - | 0.4538 | 4.6430 |
| -0.1.468 | -0.3179 | 0.3038 | - | 0.5380 | 3.6440 |
| -0.1901 | -0.0245 | 0.3396 | + | 1.4290 | 5.9740 |
| -0.2331 | -0.2194 | 0.2353 | - | 0.6907 | 4.2330 |
| -0.2805 | -0.2697 | 0.3198 | + | 1.0670 | 7.4530 |
| -0.3321. | -0.3348 | 0.3473 | - | 0.0235 | 4.7430 |
| -0.3778 | -0.1.742 | 0.2225 | + | 0.4538 | 3.5090 |
| -0.4197 | -0.3423 | 0.1996 | - | 0.0485 | 2.2550 |
| -0.4574 | -0.3284 | 0.2608 | + | 0.0237 | 1.9410 |
| -0.5216 | -0.5095 | 0.1763 | - | 0.3417 | 2.9230 |
| -0.5818 | -0.3556 | 0.2067 | - | 0.5272 | 3.5010 |
| -0.6489 | -0.5710 | 0.2701 | - | 0.6344 | 3.2090 |
| -0.7190 | -0.6350 | 0.3047 | - | 0.0284 | 3.0220 |
| -0.7933 | -0.6271 | 0.4517 | + | 2.0330 | 6.1480 |
| -0.8831 | -0.5273 | 0.4733 | - | 0.5736 | 4.7780 |
| -0.9575 | -0.5672 | 0.5560 | + | 0.6469 | 4.0630 |
| -1.0410 | -0.8383 | 0.4848 | - | 0.0039 | 3.7230 |
| -1.1310 | -0.9021 | 0.4289 | - | 0.0313 | 2.2600 |
| -1.3760 | -0.9167 | 0.5626 | * | 0.0201 | 2.5850 |
| -1.6990 | -1.1570 | 0.4831. | + | 0.0214 | 3.0910 |
| -2.2420 | -1.1700 | 1.1720 | + | 0.1309 | 2.2960 |


| CLASS MID-PT | MEAN | VARIANCE | SIGN | SkETNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.2630 | 1.7250 | 0.2813 | - | 1.5980 | 5.1140 |
| 1.9340 | 1.6270 | 0.2811 | - | 0.1244 | 2.3350 |
| 1.6810 | 1.3810 | 0.3370 | - | 0.0195 | 2.1989 |
| 1.4790 | 1.1990 | 0.5710 | - | 2.5960 | 6.7240 |
| 1.3530 | 0.9253 | 0.3075 | - | 0.0293 | 3.2890 |
| 1. 420 | 0.93380 | 0.5322 | - | 1.6920 | 4.5450 |
| 1.1550 | 0.9990 | 0.2608 | + | 1.6990 | 5.4850 |
| 1.0740 | 0.8140 | 0.3259 | + | 0.1561 | 3.7040 |
| 0.9999 | 0.7962 | 0.2560 | - | 0.0443 | 2.8770 |
| 0.9369 | 0. .6780 | 0.2826 | - | 3.1720 | 6.6790 |
| 0.8792 | 0.8704 | 0.1256 | " | 0.5446 | 3,3910 |
| 0.8182 | 0.7057 | 0.22 ec | - | 0.0000 | 3.7230 |
| 0.7699 | 0.6072 | 0.2249 | + | 0. 0.0437 | 2.6660 |
| 0.7198 | 0.5920 | 0.1532 | - | 0.0000 | 2.7610 |
| 0.6701 | 0.6344 | 0.2702 | $\cdots$ | 1.6630 | 5.6010 |
| 0.6273 | 0.1133 | 0.3352 | - | 0.0061 | 2.6110 |
| 0.5615 | 0.3030 | 0.5082 | - | 0.4620 | 4.4300 |
| 0.5381 | 0.3858 | 0.3859 | - | 1.1770 | 4,3890 |
| 0.4882 | 0.4573 | 0.2257 | + | 0.1872 | 3.7120 |
| 0.4448 | 0.5321 | 0.1649 | * | 0.0245 | 2.4040 |
| 0.3948 | 0.2359 | 0.1902 | + | 0.1440 | 2.6750 |
| 0.3548 | 0.3390 | 0.1557 | + | 0.4175 | 2.4780 |
| 0.3313 | 0.2832 | 0.1782 | - | 0.6712 | 4.8800 |
| 0.2962 | 0.2390 | 0.2324 | - | 0.8857 | 3.5990 |
| 0.2534 | 0.1143 | 0.1454 | - | 0.1531 | 2.0750 |
| 0.2182 | -0.1370 | 0.4563 | - | 2.2330 | 5.3170 |
| 0.1784 | 0.1835 | 0.0964 | * | 0.4551 | 2.7140 |
| 0.1423 | 0.0506 | 0.2666 | - | 0.0190 | 4.3720 |
| 0.1132 | -0.0243 | 0.0975 | + | 0.2320 | 3.1080 |
| 0.0790 | -0.2010 | 0.4277 | - | 1.8210 | 4.7900 |
| 0.0552 | 0.0095 | 0.2318 | - | 2.8310 | 6.7940. |
| 0.0279 | -0.1990 | 0.2609 | - | 0.2150 | 4.6460 |
| -0.0071 | 0.1617 | 0.2606 | + | 1.0390 | 3.0050 |
| -0.0285 | 0.0754 | 0.2432 | + | 2.4160 | 7.4850 |
| -0.0543 | 0.0333 | 0.2306 | + | 0.7127 | 3.1110 |
| -0.0899 | -0.1640 | 0. 1268 | + | 0.0077 | 3.5850 |
| -0.1220 | -0.1411 | 0.1716 | - | 0.0002 | 2.9510 |
| -0.1579 | -0.1750 | 0.3326 | + | 0.0900 | 3.7170 |
| -0.1877 | -0.1072 | 0.1365 | - | 0.3039 | 3.1840 |
| -0.2223 | -0.2651 | 0.1879 | - | 0.0471 | 3.2160 |
| -0.2490 | -0.3538 | 0.3099 | - | 1.7060 | 5.7410 |
| -0.2874 | -0.1961 | 0.2835 | - | 0.2598 | 4.2830 |
| -0.3227 | -0.3070 | 0.2228 | + | 0.0734 | 2.3500 |
| -0.3629 | -0.2994 | 0.1781 | + | 0.5728 | 5,2210 |
| -0.3915 | -0.2314 | 0.0801 | - | 0.2300 | 2.8580 |
| -0.4234 | -0.5021 | 0.2557 | - | 0.0311 | 2.6790 |
| -0.4650 | -0.4722 | 0.2024 | - | 0.0028 | 2.5710 |
| -0.4971 | -0.4724 | 0.3465 | - | 0.1554 | 3.7380 |
| -0.5264 | -0.6.109 | 0.1963 | - | 0.6406 | 3.6440 |
| -0.5714 | -0.5529 | 0.2270 | - | 0.9516 | 4.7050 |
| -0.6238 | -0.4368 | 0.2361 | - | 0.246 ? | 3.0990 |
| -0.6873 | -0.6020 | 0.3793 | - | 0.6198 | 4.5360 |


| M Mbustry 10 | （ご心㇒い） |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CLASS MIL－WT | MEAN | VARIANCE | SIGN | SKENUESS | KURTOSIS |
| $-0.7320$ | －0．6689 | 0.2565 | ＂ | 2.1890 | 5.6330 |
| －0．7910 | －0．8461 | 0.3660 | － | 0.0251 | 3.0760 |
| －0．8481 | $-0.8370$ | 0.5165 | ＋ | 0.0570 | 3.0920 |
| －0．9331 | －0．7999 | 0.3777 | － | 0.9749 | 5.3590 |
| －1．0210 | －0．8043 | 0.2742 | － | 0.0773 | 3.1920 |
| －1．1060 | －0．8763 | 0.2862 | ＋ | 0.1002 | 3.8620 |
| －1．2210 | －0．8379 | 0.3273 | － | 1.6340 | 6.2960 |
| －1．4160 | －0．9402 | 0.6648 | － | 0.3282 | 2.6880 |
| －1．5970 | $-1.1810$ | 0.5970 | ＋ | 1.3760 | 4.3680 |
| －2．1860 | $-1.1530$ | 0.4838 | － | 0.1253 | 2.5250 |


| CLASS HIDET | MEAN | VARIAMCE | SIGN | SKEANESS | KURTOSI |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6800 | 1.0990 | 0.3926 | - | 0.0327 | 2.1460 |
| 1. 2130 | 1.0110 | 0.1819 | - | 0.0136 | 2.3340 |
| 0.9603 | 0.9371 | 0.2216 | - | 0.2181 | 2.5159 |
| 0.7620 | 0.4811 | 0.1769 | * | 0.0592 | 2.3910 |
| 0.5890 | 0.4391 | 0.3050 | - | 0.0008 | 3.2650 |
| 0.4531 | 0.4620 | 0.2431 | - | 0.0000 | 3.2820 |
| $0.3 \pm 75$ | 0.2052 | 0.2314 | - | 0.2657 | 3,3830 |
| 0.1984 | 0.2749 | 0.2984 | - | 0.1820 | 3.8040 |
| 0.0936 | 0.0185 | 0.1 .446 | - | 0.0820 | 2.4820 |
| -0.0014 | -0.0476 | 0.1864 | $\cdots$ | 1.0210 | 3.7690 |
| -0.0947 | -0.0725 | 0.1440 | - | 0.1810 | 3.0730 |
| -0.1.843 | -0.0.2293 | 0.2287 | * | 0.8782 | 4.2540 |
| -0.2639 | -0.2460 | 0.2702 | - | 0.0002 | 3.6410 |
| -0.3440 | -0.3656 | 0.8592 | + | 0.0104 | 3.8170 |
| -0.4744 | -0.2839 | 0.2567 | - | 2.0660 | 9.2540 |
| -0.6199 | -0.5929 | 0.2448 | - | 0.2674 | 2.5130 |
| -0.8267 | -0.5906 | 0.4002 | + | 0.0130 | 2.8680 |
| -1.0420 | -1.0200 | 0.2865 | - | 0.7194 | 3.5900 |
| -1.5360 | $-1.2170$ | 0.4470 | - | 0.2804 | 2.7760 |


| CLASS M1L-FT | MEAN | variance | $\operatorname{sign}$ | SKEWESS | Kiftosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.6680 | 1.7080 | 0.9337 | - | 1.3620 | 4.0230 |
| 2.2090 | 1.4910 | 0.6837 | - | 0.0198 | 2.0620 |
| 1.6630 | 1.3980 | 0.3653 | - | 0.0384 | 4.4820 |
| 1.5400 | 1.0320 | 0.4959 | - | 0.3604 | 3.8750 |
| J. 5130 | 0.9924 | 0.6379 | " | 1.9490 | 5.1790 |
| 1.4200 | 1.1230 | 0.5197 | - | 0.0077 | 3.3010 |
| 1.3260 | 1.0780 | 0.5634 | - | 0.6683 | 4.4250 |
| 1.2470 | 1.1310 | 0.3535 | - | 0.0739 | 4.1310 |
| 1. 1910 | 0.8903 | 0.6467 | " | 0.0753 | 3.5860 |
| 1.0980 | 0.9340 | 0.4121 | + | 0.0502 | 4. 9700 |
| 1.0280 | 1.0130 | 0.3051 | + | 0.14 .47 | 3.5020 |
| $0.969 \%$ | 0.8474 | 0.2674 | * | 0.1901 | 3.1320 |
| 0.9229 | 0.6236 | 0.3530 | + | 0.8532 | 4.61 .60 |
| 0.3830 | 0.7601 | 0.2748 | - | 0.0962 | c. 1770 |
| 0.3439 | 0.8419 | 0.3969 | + | 1.1700 | 4.5230 |
| 0.6007 | 0.7172 | 0.1750 | 4 | 0.0006 | 2.5290 |
| 0.7552 | 0.5624 | 0.5301 | - | 2.9490 | 8.7350 |
| 0.7116 | 0.5101 | 0.1836 | + | 0.5555 | 3.2390 |
| 0.6686 | 0.51 .51 | 0.5474 | + | 0.0102 | 4.1520 |
| 0.6291 | 0.5697 | 0.2180 | - | 0.2878 | 4.6010 |
| 0.5911 | 0.4066 | 0.3227 | - | 0.1754 | 3.9020 |
| 0.5552 | 0.5620 | 0.5095 | + | 0.0798 | 4.7570 |
| $0.5 \% 13$ | 0.4936 | 0.1293 | - | 0.0010 | 2.9570 |
| 0.4977 | 0.3165 | 0.3933 | - | 4.1590 | 10.4300 |
| 0.4711 | 0.4074 | 0.1610 | - | 0.2998 | 3.0910 |
| 0.4498 | 0.4280 | 0.2009 | + | 1.6090 | 6.2490 |
| 0.4215 | 0.3428 | 0.0839 | - | 1.0790 | 5.3510 |
| 0.3931 | 0.3718 | 0.1890 | + | 0.0063 | 2.1680 |
| 0.3690 | 0.2063 | 0.2444 | + | 0.0247 | 5.0530 |
| 0.3462 | 0.3926 | 0.0967 | + | 0.1778 | 4.0300 |
| 0.3241 | 0.2410 | 0.2062 | - | 0.3006 | 5.0040 |
| 0.3025 | 0.2504 | 0.2207 | - | 3.0540 | 6.8510 |
| 0.2831 | 0.2545 | 0.2184 | + | 1. 0.0555 | 4.3750 |
| 0.2601 | -0.0052 | 0.2285 | - | 0.0866 | 2.8000 |
| 0.2388 | 0.1872 | 0.2257 | - | 1.2960 | 7.2110 |
| 0.2191 | 0.1283 | 0.2308 | - | 0.0763 | 3.4110 |
| 0.1030 | 0.0429 | 0.0798 | * | 0.1213 | 3.3140 |
| 0.177 ? | 0.1030 | 0.1684 | - | 0.0422 | 4.2920 |
| 0.1521 | 0.0843 | 0.2005 | - | 0.0115 | 6.0070 |
| 0.1 .233 | 0.1137 | 0.1388 | - | 0.3598 | 4.6850 |
| 0.0982 | 0.0704 | 0.1212 | + | 2.2930 | 7.7670 |
| 0.0762 | 0.0071 | 0.1696 | - | 0.0078 | 3.1270 |
| 0.0554 | -0.0999 | 0.3291 | + | 0.2985 | 5.0180 |
| 0.0301 | -0.1571 | 0.2413 | - | 0.4751 | 4.2870 |
| 0.0019 | 0.0793 | 0.1837 | + | 0.3814 | 3.2090 |
| -0.0194 | -0.08884 | 0.1768 | + | 0.1896 | 3,4060 |
| -0.0453 | 0.0545 | 0.2147 | + | 8.1330 | 12.9400 |
| -0.0717 | -0.1434 | 0.3097 | - | 2. 2140 | 5.1490 |
| -0.0968 | -0.2382 | 0.4584 | - | 4.8 ¢90 | 9.0950 |
| -0.1225 | -0.2758 | 0. 4192 | - | 2.6750 | 9.5550 |
| -0.1415 | -0.0743 | 0.1973 | * | 0.2781 | 3.0600 |
| -0.1584 | -0.0262 | 0.2437 | + | 0.6690 | 3.6840 |


| CLASS HID-FT | MEAN | VARIANCE | SIGN | Staxnts | Muntos: |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.1634 | -0.3069 | 0.1966 | - | 0.0097 | 2.16 |
| -0.2073 | $-0.3399$ | 0.4022 | - | 1.6500 | 5.4310 |
| -0.225s | $=0.3404$ | 0.3867 | - | 1.6390 | 6.9150 |
| -0.2481 | $-0.3537$ | 0.2336 | * | 0.9244 | 4.2900 |
| -0.2728 | -0.2389 | ก. 3258 | + | 0.7334 | 4. 6 bs? |
| -0.2920 | $-0.2104$ | 0.2638 | + | 3.6440 | 9.2620 |
| -0.3162 | $-0.5051$ | 0.4158 | - | 4.5990 | 6.1430 |
| $-0.3356$ | $-0.4038$ | 0.1930 | " | 0.0976 | 3.6310 |
| -0,3593 | -0.3685 | 0.1796 | - | 3.4010 | 7.8100 |
| -0.3828 | -0.3639 | 0.2329 | $+$ | 4.1250 | 10.1500 |
| -0.4079 | -0.3556 | 0.1075 | $+$ | 0.0677 | 3, 55 50 |
| -0.4332 | -0.3621 | 0.2104 | t | 0.71 .49 | 5.0060 |
| -0.4595 | -0.5877 | 0.3357 | - | 0.0448 | 3.7190 |
| -0.4925 | -0.5711 | 0.1522 | ** | 0.3339 | 4.4950 |
| -0.5065 | -0.5022 | 0.1331 | $+$ | 0.2588 | 3.4430 |
| -0.5372 | -0.671.4 | 0.1 .414 | - | 0.0105 | 2. 8270 |
| -0.5748 | -0.4728 | 0.3693 | - | 0.2845 | 9.7950 |
| -0.0103 | $-0.4085$ | 0.1887 | + | 2.5470 | 7.0150 |
| $-0.6387$ | $-0.6315$ | 0.3975 | - | 3.5300 | 7.3090 |
| -0.6677 | $-0.6577$ | 0.1340 | " | 1.0900 | 4.8780 |
| -0.7007 | $-0.6630$ | 0.2243 | " | 0.2070 | 3.8750 |
| $-0.7314$ | -0.6208 | 0.3158 | $\cdots$ | 2.8030 | 8.9760 |
| -0.7671 | $-0.5712$ | 0.2885 | - | 0.1458 | 5.1530 |
| -0.8111 | -0.5994 | 0.4421 | + | 10.0000 | 15.7300 |
| -0.8613 | -0.4141 | 0.3153 | + | 1. 0310 | 3.6760 |
| -0.9085 | -0.9319 | 0.2442 | $\bullet$ | 0.0767 | 2.2940 |
| -0.9570 | -0.7646 | 0.4264 | $+$ | 0.2637 | 3.7570 |
| -1.0160 | -0.7165 | 0.242 .9 | $+$ | 0.1073 | 3.4650 |
| -1.0780 | $-0.9770$ | 0.2913 | $+$ | 0.4926 | - 2.5870 |
| -1.1590 | $-1.0030$ | 0.3798 | $\cdots$ | 0.1205 | 2.8190 |
| -1.2230 | $-0.8340$ | 0.6800 | + | 0.3402 | 3.7900 |
| -1.3380 | $-1.1300$ | 0.3624 | $\pm$ | 0.4151 | 5.2120 |
| -1.4820 | -1.1430 | 0.4255 | * | 0.7667 | 3.6850 |
| -1.6970 | -1.2540 | 0.6503 | + | 0.0024 | 6.7430 |
| $-2.3200$ | -0.9662 | 0.6058 | + | 1.2810 | 7.0190 |


| CLASS M:D-ET | MEAV | VARIANCE | Sign | SkEMNESS | Kuktosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.3900 | 1.7850 | 0.4575 | " | 1.5430 | 5.5360 |
| 1.9670 | 1.6410 | 0.3713 | - | 1.1000 | 4.1530 |
| 1.7500 | 1.3830 | 0.2672 | - | 0.1599 | 2.8250 |
| 1.6050 | 1.3390 | 0.2223 | - | 0.2505 | 3.0910 |
| 1.4940 | 1.4109 | 0.3854 | - | 0.1919 | 3.2700 |
| 1.3670 | 1.2510 | 0.1472 | - | 0.4750 | 4.4500 |
| 1.2770 | 1.1880 | 0.2942 | + | 0.0565 | 2.1670 |
| 1.2010 | 1.0880 | 0.3269 | - | 0.1595 | 3.6850 |
| 1.1250 | 0.8970 | 0.2457 | - | 0.1301 | 2.9430 |
| 1.10330 | 0.8784 | 0.2052 | - | 0.0648 | 3.1170 |
| 0.9664 | 0.909 ? | 0.1712 | - | 0.321 .4 | 2.6690 |
| 0.8840 | 0.7484 | 0.3785 | - | 0.2353 | 3.1360 |
| 0.6195 | 0.6046 | 0.2727 | - | 0.5765 | 2.8380 |
| 0.7717 | 0.6902 | 0.4204 | - | 1.6470 | 6.7420 |
| 0.7102 | 0.5901 | 0.1863 | - | 0.3151 | 2.8350 |
| 0.6522 | 0.6107 | 0.1041 | $\stackrel{+}{+}$ | 0.0233 | 3.4750 |
| 0.6003 | 0.3617 | 0.1350 | - | 0.7211 | 3.7250 |
| 0.5476 | 0.4584 | 0.1736 | - | 0.3894 | 3.6050 |
| 0.4905 | 0.4542 | 0.5142 | - | 5.8600 | 10.5800 |
| 0.4447 | 0.4592 | 0.2389 | + | 0.0047 | 2.1280 |
| 0.1001 | 0.3666 | 0.1474 | + | 0.0164 | 3.4560 |
| 0.3508 | 0.3199 | 0.1458 | - | 0.0022 | 2.7610 |
| 0.3122 | 0.1510 | 0.3280 | - | 0.3134 | 5.7300 |
| 0.2629 | 0.2300 | 0.1080 | " | 0.0310 | 2,3210 |
| 0.2238 | 0.2832 | 0.1238 | + | 0,0708 | 3.0250 |
| 0.1 .963 | 0.1193 | 0.5218 | + | 0.4749 | 6.7850 |
| 0.1611 | 0.1132 | 0.0893 | + | 0.0160 | 2.7600 |
| 0.1360 | 0.0412 | 0.1328 | - | 0.5349 | 6.5570 |
| 0.1025 | 0.0764 | 0.1607 | + | 0.0952 | 2.9080 |
| 0.0712 | 0.0097 | 0.2089 | + | 0.1068 | 3.4350 |
| 0.0313 | 0.0190 | 0.0617 | * | 0.1338 | 2.3600 |
| -0.0069 | 0.0311 | 0.0987 | + | 7.1460 | 13.4200 |
| -0.0447 | -0.1425 | 0.1262 | * | 0.4726 | 3.8720 |
| -0.0810 | -0.1288 | 0.1555 | + | 0.3757 | 2.8930 |
| -0.1189 | 0.0246 | 0.2747 | * | 1.3840 | 6.4290 |
| -0.1551 | -0.1026 | 0.0363 | + | 0.0058 | 3.8760 |
| -0.1860 | -0.1580 | 0.1913 | + | 0.1320 | 3.2250 |
| -0.2187 | -0.2597 | 0.0867 | + | 0.3410 | 4.3510 |
| -0.2525 | -0.2790 | 0.0763 | $+$ | 0.1604 | 3.0560 |
| -0.2848 | -0.1550 | 0.1290 | + | 0.0949 | 4.0500 |
| -0.3270 | -0.3987 | 0.0927 | + | 0.2145 | 2.7900 |
| -0.3625 | -0.4002 | 0.1713 | - | 2.1520 | 7.4140 |
| -0.3987 | -0.4419 | 0.2331 | - | 0.22 .46 | 5.1240 |
| -0.4435 | -0.3770 | 0.0751 | - | 0.0204 | 2.4300 |
| -0.4813 | -0.3529 | 0.1026 | + | 0.1169 | 2.4410 |
| -0.5091 | -0.6034 | 0.1666 | - | 0.7875 | 4.5590 |
| -0.5473 | -0.0.6301 | 0.1710 | - | 0.2191 | 3.1110 |
| -0. 0.5676 | -0.5221 | 0.3206 | $\stackrel{ }{ }$ | 4.9670 | 12.8700 |
| -0.630n | -0.5913 | 0.1427 | + | 0.0259 | 3.0220 |
| -0.6698 | -0.6611 | 0.2273 | * | 0.3001 | 3.7500 |
| -0.7209 | -0.6959 | 0.1018 | + | 0.0131 | 4.2980 |
| -0.771.8 | -0.7371. | 0.0810 | + | 0.3550 | 3.7190 |

1 WHOTRY j9 (GOHTD)

| CLASS MID-PT | HEAN | VARIANCE | SIGN | SKEAMESS | Kuptosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $-0.3351$ | $-0.8343$ | 0.2169 | + | 0.8373 | 7.1850 |
| -0.8856 | -0.7640 | 0.2418 | + | 2.5780 | 6.9930 |
| -0.9519 | $-0.8454$ | 0.1984 | + | 0.0255 | 3.3830 |
| -1.0190 | $-1.0450$ | 0.3069 | - | 1.3760 | 5.0510 |
| -1.1060 | $-1.0210$ | 0.1090 | $+$ | 1.1990 | 3.7650 |
| $-1.1950$ | -1.1160 | 0.1568 | - | 0.1269 | 2.5110 |
| -1.3340 | -1.1840 | 0.3989 | + | 2.8890 | 8.8330 |
| $-1.5300$ | $-1.4980$ | 0.4120 | + | 0.5071 | 4.7850 |
| $-1.9550$ | $-1.3600$ | 0.6073 | + | 1.1770 | 4.4390 |

1 UDUSTRY 20

| CLASS MIT－PT | MEAN | VARIANCE | S16N | SKEWHESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.2570 | 1.7330 | 0.5146 | $\cdots$ | 0.1644 | 2．7430 |
| 1.7050 | 1.5630 | 0． 2469 | ＋ | 0.2685 | 2.9470 |
| 1.3830 | 1.1560 | 0.3593 | － | 0.0028 | 2.2390 |
| 1.0490 | 0.6447 | 0.5591 | \％ | 1.7820 | 5.3060 |
| 0.8498 | 0.7176 | 0.1946 | ＋ | 0.1484 | 3.0400 |
| 0.6959 | 0.5300 | 0.2300 | ＋ | 0．0．250 | 2.8370 |
| 0.5596 | 0.5662 | 0.4519 | ＋ | 1.0480 | 4.4590 |
| 0.4561 | 0.3734 | 0.4446 | ＋ | 0.0000 | 3.3220 |
| 0.3680 | 0.1035 | 0.1682 | ＋ | 0.1590 | 2.6960 |
| 0.2438 | 0.2127 | 0.2041 | － | 0.1073 | 2.0820 |
| 0.1584 | 0.0304 | 0.1775 | ＊ | 0.0065 | 1．9980 |
| 0．0735 | $=0.0397$ | 0.2397 | ＊ | 2.0270 | 6.5720 |
| －0．0020 | 0.0074 | 0.103 B | － | 0.0102 | 2.0660 |
| －0．0517 | －0．1．105 | 0.1897 | ＊ | 0.2262 | 6.2400 |
| －0．1513 | $-0.1290$ | 0.1828 | ＋ | 0.1105 | 6.7020 |
| －0．2094 | $-0.1571$ | 0.3766 | ＋ | 5.6220 | 10.3700 |
| －0．2729 | －0．3193 | 0.1070 | － | 2.9670 | 8.00 .30 |
| －0．3419 | －0．2524 | 0.0976 | ＋ | 0.8193 | 3.4000 |
| －0．4047 | －0．2866 | 0.1105 | ＋ | 0.0356 | 3.2700 |
| －0．465？ | －0．3850 | 0.2650 | ＋ | 0.0029 | 4.5140 |
| －0．5346 | －0．4509 | 0.1321 | ＋ | 0.6673 | 3.5100 |
| －0．0004 | －0．6660 | 0.1234 | ＋ | 0.0110 | 2.7230 |
| －0．6652 | －0．6417 | 0.1807 | － | 0.0377 | 2.7820 |
| －0．7589 | －0．6671 | 0.1158 | ＋ | 0.6217 | 3.8440 |
| －0．8468 | －0．7484 | 0.1362 | ＊ | 2.0700 | $5.9500$ |
| －0．9663 | $-0.8923$ | 0.2959 | ＋ | 0.1620 | 3.8530 |
| －1．0910 | $-0.8677$ | $0.235 ?$ | ＋ | 1.7670 | 4.3740 |
| －1．2700 | －1．0930 | 0.2068 | － | $0.0267$ | $4.4180$ |
| －1．7570 | $-1.0670$ | 0.4271 | $\cdots$ | 0.4592 | 4．2590． |


| ， | －\％ 6 an |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| －${ }^{\text {a }}$ | －4，$=124$ | 6 有49 | A，${ }^{2}$ ， | $4+78$ |
|  | $\cdots \mathrm{x}$ | H．tht | traty | \％，${ }^{3}$ |
| $\cdots 8$ | －1．2． 28 | 15.68 | \％ 8 \％ | 515 |
| $3-50$ | An， 9 ¢ | C． $\mathrm{C}^{5} 8$ |  | － 450 |
| me $0 \cdot \mathrm{c}$ | 4） y 768 | ＋ 8082 | 4．3， 30 | 6． E |
|  | $\cdots$ \％${ }^{\text {a }}$ | W， 654 | स⿵冂卄 | 78 |
|  | \％．．． |  | －tas | 6．4\％ |


| CLASS MIDRFT | MEAN | variance | SIGN | SkEMMESS | kuptosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.6420 | 1.9370 | 0.4320 | - | 0.4280 | 3.0140 |
| 2.2490 | 2.0820 | 0.2553 | - | 0.6347 | 3.5340 |
| 2.0130 | 1,5060 | 0.5816 | - | 1.2310 | 3.6980 |
| 1.8530 | 1.6220 | 0.7738 | " | 0.1633 | 2.0810 |
| 1.5380 | 1.4370 | 0.5031 | - | 3.0480 | 8.4710 |
| 1.4360 | 1.0380 | 0.6475 | - | 0,8847 | 4.0430 |
| 1.3100 | 0.9354 | 0.51 .83 | + | 0.0687 | 4.1430 |
| 1.2010 | 1.0740 | 0.1904 | - | 0.2881 | 2.9440 |
| 1.0820 | 1.0350 | 0.3987 | - | 0.0024 | 3.8990 |
| 0.9972 | 0.9379 | 0.3165 | + | 0.2853 | 3.0240 |
| 0.9125 | 0.9042 | 0.2980 | + | 0.4014 | 3.8470 |
| 0.8306 | 0.7690 | 0.1789 | - | 0.01 .7 | 2.6020 |
| 0.7580 | 0.6064 | 0.3253 | + | 0.0010 | 3.6040 |
| 0.71971 | 0.4363 | 0.4213 | - | 0.3955 | 5.4480 |
| 0.6457 | 0.5485 | 0.1324 | - | 0.0138 | 3.5920 |
| 0.5823 | 0.4413 | 0.131 .6 | + | 0.0076 | 4.1140 |
| 0.5205 | 0.4117 | 0.2862 | - | 0.0164 | 2.6040 |
| 0.4635 | 0.3537 | 0.3833 | - | 3.9390 | 9.1730 |
| 0.4165 | 0.2947 | 0.0783 | - | 0.0346 | 4.1920 |
| 0.3817 | 0.2117 | 0.1122 | - | 0.0590 | 2.92010 |
| 0.3557 | 0.3531 | 0.2922 | + | 0.0193 | 5.1970 |
| 0.3189 | 0.3110 | 0.2188 | + | 0.0589 | 3.3430 |
| 0.2856 | 0.2391 | 0.1487 | " | 0.2015 | 3.1390 |
| 0.2589 | 0.1668 | 0.1860 | + | 0.0000 | 2.4420 |
| 0.2321 | 0.2334 | 0.1477 | + | 0.5994 | 3.7380 |
| 0.2091 | 0.1887 | 0.0647 | - | 0.2846 | 3.5250 |
| 0.1843 | 0.1493 | 0.2803 | - | 0.0098 | 5.2300 |
| 0.1594 | 0.0415 | 0.2278 | + | 0.0061. | 3.5050 |
| 0.1325 | 0.0014 | 0.0908 | - | 0.7560 | 4.1160 |
| 0.1012 | -0.0441 | 0.1420 | " | 1.0670 | 3.881 .0 |
| 0.0717 | 0.0832 | 0.1092 | - | 0.3500 | 3.1040 |
| 0.0449 | -0.0449 | 0.1910 | - | 0.0019 | 2.6700 |
| 0.0184 | -0.0152 | 0.1392 | + | 0.0065 | 2.1420 |
| -0.0102 | -0.1553 | 0.2402 | - | 0.1241 | 3.1130 |
| -0.0345 | -0.0232 | 0.1022 | - | 0.0010 | 3.5240 |
| -0.0610 | -0.1282 | 0.1055 | + | 0.0889 | 3.4960 |
| -0.0916 | -0.0659 | 0.0871 | - | 0.0568 | 2.3650 |
| -0.1254 | -0.1303 | 0.0571 | - | 0.3924 | 3.0160 |
| -0.1563 | -0.1229 | 0.1581 | + | 5.0020 | 11.0900 |
| -0.1805 | -0.0962 | 0.1 .835 | + | 0.3582 | 5.2740 |
| -0.2048 | -0.2087 | 0.1419 | + | 0.3530 | 4.7650 |
| -0.2288 | -0.2995 | 0.1050 | - | 0.0090 | 3.5500. |
| -0.2543 | -0.2798 | 0.1120 | - | 0.0077 | 5.0220 |
| -0.2769 | -0.3550 | 0.1053 | - | 9.2960 | 1.5.3000 |
| -0.2932 | -0.2708 | 0.1052 | + | 2.3900 | 6.9550 |
| -0.3132 | -0.3421 | 0.0624 | - | 0.4130 | 3.7160 |
| -0.3336 | -0.2999 | 0.1058 | + | 0.1846 | 3.5390 |
| -0.3509 | -0.2578 | 0.0974 | + | 0.5181 | 3.0630 |
| -0.372.4 | -0.3016 | 0.1015 | - | 0.1982 | 3.7010 |
| -0.3550 | -1).3561 | 0.0543 | - | 1.2050 | 6.1610 |
| -0.4126 | -0.4022 | 0.0931 | - | 0.0125 | 2.9300 |
| -0.4302 | -0.4993 | 0.1525 | - | 0.0642 | 3.3920 |



| CLASS HTDPT | MEAM | VAR)ANCE | SIGN | SkEMNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.4550 | $-0.4272$ | 0.0898 | + | 2.4160 | 7.8120 |
| -0.4776 | -0.4547 | 0.0811 | - | 5.7290 | 11.1200 |
| -0.4980 | -0.4259 | 0.2823 | + | 3.9210 | 11.1500 |
| -0.0173 | -0.5110 | 0.1870 | - | 2.4830 | 7.5160 |
| -0.5368 | -0.5072 | 0.153 .3 | - | 0.5130 | 5.6300 |
| -0.3005 | -0.6830 | 0.1565 | - | $0.1 \pm 10$ | 4.6090 |
| -0.534\% | -0.5438 | 0.0772 | + | 0.8037 | 4.7900 |
| -0.5073 | -0.6053 | 0.1388 | + | 0.1522 | 4.7700 |
| -0.0.319 | -0.4914 | 0.2106 | $+$ | 7.6630 | 12.3900 |
| -0.6562 | $-0.7312$ | 0.2313 | - | 0.7295 | 3.5490 |
| -0.6894 | $-0.7124$ | 0.1372 | - | 1.8590 | 5.7490 |
| $-0.7147$ | -0.6652 | 0,0593 | + | 0.0219 | 3.0640 |
| -0.7457 | -0.7399 | 0.1259 | + | 0.2919 | 3.4800 |
| -0.7957 | -0.7420 | 0.0976 | + | 2.2430 | 6.7640 |
| -0.5434 | -0.8456 | 0.1508 | - | 0.6137 | 4.3780 |
| -0.0986 | -0.7160 | 0.1667 | - | 0.0178 | 3.0000 |
| -0.9697 | -0.0203 | 0.2344 | " | 3.6980 | 8.6470 |
| $-1.0450$ | -0.9232 | 0.2727 | + | 1.2500 | 6.4100 |
| $-1.1270$ | -0.9907 | 0.2335 | + | 0.1913 | 3.1010 |
| -1.2350 | -0.9449 | 0.4836 | + | 2.3060 | 7.5640 |
| -1.3750 | -1.1690 | 0.4175 | + | 0.5174 | 3.4620 |
| -1.5690 | -1.5420 | 0.6567 | + | 0.0989 | 2.7070 |
| -2.1080 | -1.4030 | 0.3952 | + | 0.0032 | 2.5230 |

## APPFNDTXC

TRANSTTTON MATRICES - STIRCROUPS

See Chapter V Section I for definitions and explanations and Section 3 for discussion of these data.

INDUSTRY 1 SUBGROUP 2

| CLASS MID-PT | MEAN | VARIANCE | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0180 | 1.6800 | 0.3121 | - | 0.0126 | 2.3070 |
| 1.4740 | 1.2470 | 0.2288 | - | 0.0000 | 2.2750 |
| 1.2380 | 1.0480 | 0.2674 | - | 0.0003 | 2.7990 |
| 1.0320 | 0.4577 | 0.3601 | - | 0.0078 | 3.3000 |
| 0.8698 | 0.7961 | 0.2018 | - | 0.2915 | 2.9770 |
| 0.7416 | 0.5817 | 0.2562 | - | 0.4413 | 3.8930 |
| 0.6240 | 0.6067 | 0.2172 | + | 0.0264 | 2.9460 |
| 0.5294 | 0.5220 | 0.3242 | - | 0.4543 | 3.4400 |
| 0.4492 | 0.4508 | 0.3063 | + | 0.0695 | 3.0350 |
| 0.3523 | 0.3034 | 0.2019 | - | 0.0233 | 2.7450 |
| 0.2539 | 0.2345 | 0.1420 | + | 0.1468 | 2.4910 |
| 0.1684 | 0.1554 | 0.1620 | + | 0.5917 | 4.3710 |
| 0.0567 | -0.0774 | 0.1973 | - | 0.1402 | 3.5190 |
| 0.0268 | 0.0207 | 0.2739 | + | 0.0003 | 3.0020 |
| -0.0602 | 0.0545 | 0.1123 | - | 0.3435 | 3.6940 |
| -0.1582 | -0.2014 | 0.1694 | + | 0.0002 | 2.5060 |
| -0.2484 | -0.2702 | 0.2118 | + | 0.0252 | 2.5880 |
| -0.3186 | -0.2906 | 0.1692 | + | 1.6140 | 5.7790 |
| -0.4140 | -0.4078 | 0.3267 | + | 0.4955 | 4.51 .10 |
| -0.5249 | -0.3413 | 0.1879 | - | 0.0596 | 3.0900 |
| -0.6441 | -0.6004 | 0.1752 | + | 0.0028 | 4.4180 |
| -0.7541 | -0.5653 | 0.1643 | + | 0.8203 | 4.0100 |
| -0.8667 | -0.8717 | 0.3234 | - | 0.4098 | 4.6370 |
| -1.0150 | -0.9001 | 0.2836 | + | 0.7346 | 4.5270 |
| -1.2040 | -1.0920 | 0.2609 | + | 0.6286 | 5.1820 |
| -1.8480 | -1.6370 | 0.4722 | + | 0.0960 | 3.6130 |


| CLASS MIIMFT | MEAN | VARIANCE | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 1.7520 | 1.3850 | 0.3514 | - | 0.0260 | 3.1940 |
| 1.1400 | 1.2480 | 0.2684 | - | 0.1926 | 3.6730 |
| 0.8264 | 0.6662 | 0.1500 | - | 0.6598 | 3.7280 |
| 0.5544 | 0.3554 | 0.1563 | + | 0.2819 | 2.8440 |
| 0.2916 | 0.1680 | 0.2668 | + | 0.4651 | 4.1730 |
| 0.1273 | -0.0003 | 0.3802 | - | 0.3110 | 2.4110 |
| -0.0389 | -0.1204 | 0.1793 | + | 0.4427 | 3.4660 |
| -0.2596 | -0.3550 | 0.2483 | - | 0.3555 | 2.5580 |
| -0.5500 | -0.6803 | 0.3155 | - | 0.0227 | 2.1590 |
| -0.9067 | -0.7841 | 0.5567 | + | 0.0008 | 2.1120 |
| -1.6790 | -1.1150 | 0.5322 | - | 0.0163 | 2.3390 |


| CLASS MID-PT | MEAN | VARIANCE | SIGN | SKENNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | ---: |
| 2.0190 | 1.0810 | 0.8442 | - | 0.0299 | 2.1360 |
| 1.2530 | 1.1090 | 0.3879 | + | 0.0050 | 3.3430 |
| 0.9577 | 0.7916 | 0.5267 | + | 0.0015 | 3.6970 |
| 0.7120 | 0.5072 | 0.2758 | + | 0.0535 | 2.1560 |
| 0.4765 | 0.3930 | 0.5949 | - | 0.0391 | 2.3340 |
| 0.3208 | 0.2387 | 0.4096 | - | 0.2425 | 2.8420 |
| 0.1042 | 0.1143 | 0.2522 | + | 2.0270 | 5.2550 |
| 0.0904 | 0.1155 | 0.2321 | - | 0.4509 | 2.6650 |
| -0.0508 | -0.0771 | 0.2876 | + | 0.1314 | 2.4540 |
| -0.1793 | -0.0767 | 0.7507 | + | 3.6710 | 6.0670 |
| -0.2632 | -0.2887 | 0.3655 | - | 0.1870 | 2.7350 |
| -0.3766 | -0.4604 | 0.2314 |  | 1.4130 | 4.1730 |
| -0.4976 | -0.3350 | 0.2926 | - | 0.4744 | 4.0950 |
| -0.6704 | -0.3592 | 0.7236 | + | 1.3220 | 4.7710 |
| -0.8490 | -0.7677 | 0.3252 | + | 0.1119 | 2.6510 |
| -1.1200 | -0.8693 | 0.3717 | + | 0.7981 | 3.0680 |
| -1.5450 | -1.0070 | 0.4682 | + | 0.0560 | 3.4050 |


| CLASS MIDMPT | MEAN | VARIANCE | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8450 | 1.4240 | 0.4630 | - | 0.0000 | 3.1240 |
| 1.2340 | 1.0420 | 0.4290 | + | 0.8527 | 3.7690 |
| 0.8834 | 0.8244 | 0.4597 | - | 0.1232 | 2.8460 |
| 0.6319 | 0.4256 | 0.4005 | - | 0.2940 | 2.8900 |
| 0.3661 | 0.1362 | 0.7189 | - | 1.3330 | 5.0780 |
| 0.2014 | 0.2006 | 0.4529 | - | 0.0006 | 2.6630 |
| 0.0367 | -0.1355 | 0.2905 | - | 0.0126 | 1.9110 |
| -0.1434 | -0.1880 | 0.4755 | + | 0.0028 | 3.4900 |
| -0.2836 | -0.2418 | 0.3095 | - | 0.0716 | 3.9110 |
| -0.3838 | -0.2268 | 0.2148 | - | 0.5838 | 2.9210 |
| -0.5179 | -0.6012 | 0.1857 | + | 0.5839 | 4.6110 |
| -0.6792 | -0.4026 | 0.2016 | + | 0.1102 | 3.8870 |
| -0.8180 | -0.6062 | 0.3647 | - | 0.0317 | 3.6330 |
| -1.0240 | -0.8427 | 0.4511 | - | 0.3359 | 2.41 .10 |
| -1.5990 | -1.0550 | 0.3918 | + | 0.6827 | 5.9350 |


| CLASS MID-FT | MEAN | VARIANCE | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | ---: |
| 1.7360 | 1.5340 | 0.4932 | + | 0.0141 | 2.4230 |
| 0.9700 | 0.9372 | 0.1691 | + | 0.0968 | 2.7800 |
| 0.7135 | 0.3564 | 0.3980 | - | 1.0620 | 4.1790 |
| 0.4368 | 0.4610 | 0.4605 | + | 0.2136 | 3.7820 |
| 0.2028 | 0.1061 | 0.2331 | - | 0.3632 | 3.0440 |
| -0.0599 | 0.0908 | 0.2669 | - | 0.4959 | 3.3080 |
| -0.4155 | -0.5568 | 0.1 .911 | + | 0.01 .87 | 2.0660 |
| -0.7213 | -0.6514 | 0.2664 | - | 0.3326 | 2.8980 |
| -0.9773 | -0.8917 | 0.1136 | - | 1.2860 | 4.9120 |
| -1.4000 | -1.0530 | 0.4307 | - | 0.9148 | 4.8110 |

CLASS MDOTT
MEAN
2.3320
1.7630
1.5980
1.4550
1.3360
1.2500
1.1870
1.1070
1.1460
0.9769
0.9125
0.8600
0.8034
0.7471
0.6962
0.6569
0.6104
0.5709
0.5364
0.5064
0.4786
0.4536
0.4217
0.3071
0.3715
0.3509
0.3237
0.2931
0.2635
0.2345
0.2027
0.1735
0.1446
0.11 .55
0.0902
0.0614
0.0342
0.0119
$-0.0180$
$-0.0477$
$-0.0776$
$-0.1052$
-0.134 ?
$-0.1615$
$-0.1872$
$-10.21 .40$
$-0.2376$
$-0.2620$
$-0.2936$
$-0.3309$
$-0.3631$
$-0.3985$
1.6930
1.2720
1.2300
1.2610
1.0260
0.9572
1.0240
0.9122
0.8723
0.8026
0.7464
0.6393
0.4165
0.6714
0.5871
0.6046
0.5705
0.4868
0.5946
0.4698
0.2801
0.2349
0.2126
0.1868
0.4500
0.3585
0.1925
0.2829
0.3758
0.1083
0.0751
0.1853
0.1168
0.0389
$-0.1117$
0.0626
$-0.0823$
$-0.0316$
$-0.1093$
$-0.2715$
$-0.1298$
$-0.1175$
$-0.2045$
$-0.3031$
$-0.2604$
$-0.4339$
$-0.1950$
$-0.3213$
$-0.4450$

VARIANCE SIGN SKEWNESS
KURTOSIS

| 0.4937 | " | 1.0000 | 5.1160 |
| :---: | :---: | :---: | :---: |
| 0.3579 | - | 0.1676 | 2.0920 |
| 0.5356 | + | 0.0664 | 3.7860 |
| 0.3333 | * | 0.1383 | 2.6300 |
| 0.3001 | - | 1.3040 | 5.9870 |
| 0.3150 | - | 0.1160 | 3.4340 |
| 0.2212 | + | 0.491 .1 | 3.9450 |
| 0.3113 | + | 0.0002 | 3.7010 |
| 0.4390 | + | 0.0865 | 3.4450 |
| 0.5340 | - | 0.0561 | 2.5860 |
| 0.2001 | " | 4.4670 | 9.3130 |
| 0.2290 | " | 0.2914 | 3.3370 |
| 0.2901 | $\cdots$ | 0.0098 | 3.7600 |
| 0.2454 | $\rightarrow$ | 4.6850 | 10.3400 |
| 0.3920 | + | 0.4566 | 3.2410 |
| 0.2529 | + | 0.0364 | 2.1860 |
| 0.2725 | + | 0.3990 | 2.9020 |
| 0.1395 | - | 0.5541 | 4.0520 |
| 0.1102 | + | 0.0747 | 3.0230 |
| 0.1814 | - | 0.5228 | 4.6270 |
| 0.4582 | - | 5.1610 | 10.6600 |
| 0.2679 | " | 0.9770 | 4.6390 |
| 0.1277 | - | 0.0110 | 3.0630 |
| 0.4863 | - | 7.2630 | 12.35.10 |
| 0.2589 | - | 3.3090 | 6.9190 |
| 0.3658 | + | 3.4960 | 8.5210 |
| 0.1091 | + | 0.0001 | 3.0080 |
| 0.1818 | - | 0.4130 | 4.1280 |
| 0.2026 | + | 0.1807 | 5.7780 |
| 0.1243 | + | 2.4000 | 6.2620 |
| 0.2823 | + | 2.4240 | 5.3210 |
| 0.1588 | - | 0.7907 | 4.8920 |
| 0.2467 | + | 0.11 .41 | 5.1230 |
| 0.2204 | - | 0.0418 | 2.9450 |
| 0.1863 | - | 0.9944 | 5.3800 |
| 0.1715 | + | 0.6103 | 3.3210 |
| 0.2569 | - | 4.1560 | 7.7370 |
| 0.1695 | - | 0.0284 | 4.7310 |
| 0.1733 | - | 1. 5330 | 3.9450 |
| 0.2545 | - | 1.81 .30 | 7.1840 |
| 0.1933 | " | 0.9024 | 4.0410 |
| 0.2639 | - | 0.2375 | 3.8580 |
| 0.1878 | - | 0.7138 | 4.0150 |
| 0.1755 | + | 0.3630 | 2.7970 |
| 0.1418 | - | 0.2953 | 2.5300 |
| 0.1045 | + | 0. .087 | 2.6320 |
| 0.1286 | - | $0 .+242$ | 3.2830 |
| 0.1609 | * | 0.6464 | 3.3000 |
| 0.2211 | - | 3.3460 | 7.6000 |
| 0.1080 | + | 0.0348 | 2.0900 |
| 0.1118 | - | 0.0433 | 2.0150 |
| 0.3893 | - | 1.8510 | 7.7150 |

INDUGTRY 4 SUBGROUP 5 (CONTD)

| CLASS :TO-PT | MEAN | VARIANCE | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $-0.4333$ | $-0.5344$ | 0.3021 | $\cdots$ | 2.2990 | 6.4390 |
| $-0.4653$ | -0.4324 | 0.1 .629 | - | 0.0157 | 3.1540 |
| -0.4946 | $-7.5613$ | 0.3037 | - | 1.4510 | 4.8890 |
| -0.5255 | -0.5489 | 0.1352 | - | 0.1569 | 4.7040 |
| -0.5980 | $-0.4221$ | 0.1559 | - | 0.0422 | 2.9860 |
| -0.6020 | -0.5981 | 0.1165 | + | 0.3747 | 3.7100 |
| -0.6549 | -0.5949 | 0.1636 | - | 3.1870 | 8.7970 |
| -0.6986 | -0.5500 | 0.1099 | - | 0.5084 | 3.3510 |
| -0.7212 | $-0.7110$ | 0.1383 | - | 0.0198 | 5.9310 |
| $-0.7553$ | -0.6970 | 0.2384 | + | 0.5180 | 3.6390 |
| -0.8001 | -0.6432 | 0.3107 | + | 1.1770 | 5.6920 |
| -0.0.8413 | -0.7371 | 0.3716 | * | 1.6260 | 7.5280 |
| -0.8350 | -0.9378 | 0.2511 | - | 1.9430 | 5.7860 |
| $-0.9372$ | -0.7664 | 0.1836 | + | 2.5900 | 8.3770 |
| -0.9801 | -0.7477 | 0.2954 | - | 0.0443 | 4.1760 |
| -1.0510 | -0.8881 | 0.4117 | $\cdots$ | 0.4459 | 5.3390 |
| -1.1190 | $-1.2200$ | 0.4608 | - | 0.4195 | 2.7410 |
| -1. 2060 | -0.9467 | 0.1833 | - | 0.0775 | 2.3330 |
| -1.3320 | -1.1.040 | 0.5096 | - | 0.1575 | 2.5700 |
| -1. 1.190 | $-1.2930$ | 0.3892 | - | 0.0690 | 3.4960 |
| $-2.1680$ | $-1.4680$ | 0.3733 | - | 0.3130 | 2.5540 |


| CLASS MIDMT | MEAN | VARIANCE | SIGN | SKEHNESS | KURTOS1S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.3980 | 0.9917 | 0.5957 | + | 0.0577 | 2.7750 |
| 0.7521 | 0.5820 | 0.1400 | * | 0.5017 | 2.5080 |
| 0. .4798 | 0.3119 | 0.1377 | + | 1.6910 | 4.4090 |
| 0.3527 | 0.2332 | 0.3654 | $\cdots$ | 3.2120 | 7.2650 |
| 0.2577 | 0.1 .344 | 0.0744 | - | 0.1762 | 2.3150 |
| 0.1420 | -0.0126 | 0.2052 | - | 0.0978 | 2,8670 |
| 0.0487 | 0.0435 | 0.1717 | - | 1.8430 | 4.5570 |
| -0.1134 | -0.2398 | 0.4122 | - | 0.5397 | 3.4160 |
| -0.2302 | -0.3216 | 0.4915 | - | 5.7360 | 9.6030 |
| $-0.3407$ | -0.2295 | 0.1580 | - | 0.0957 | 2.4750 |
| -0. 0.5508 | -0.4636 | 0.3258 | - | 0.0704 | 2.5150 |
| -0.7945 | $-0.7057$ | 0.1597 | + | 0.0589 | 1.7970 |
| $-1.3540$ | $-0.7929$ | 0.5288 | - | 0.0206 | 3.2600 |


| CLASS MID-PT | MEAN | VARIANCE | S1GN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0410 | 1.5520 | 0.7205 | * | 4.5610 | 10.8300 |
| 1.4630 | 1.1790 | 0.2778 | " | 0.6441 | 3,4000 |
| 1. 181.0 | 1.1150 | 0.2919 | \% | 0.3449 | 3.8450 |
| 0.9352 | 0.6872 | 0.3304 | + | 0.3190 | 3.2650 |
| 0.7056 | 0.5682 | 0.2684 | + | 0.1138 | 3.0250 |
| 0.5410 | 0.4318 | 0.3977 | $\cdots$ | 0.3635 | 4.3550 |
| 0.3929 | 0.1682 | 0.2626 | - | 1.8360 | 4.8740 |
| 0.2444 | 0.0689 | 0.2753 | - | 0.0312 | 3.5500 |
| 0.1340 | -0.1548 | 0.5865 | - | 3.3410 | 6.9820 |
| 0.0079 | -0.0124 | 0.2400 | - | 0.1368 | 4.2900 |
| -0.1146 | -0.0858 | 0.3388 | - | 0.0430 | 4.4700 |
| -0.2203 | $-0.2363$ | 0.3302 | + | 0.5056 | 3.5510 |
| -0.2995 | -0.4368 | 0.1646 | - | 0.6587 | 5.3030 |
| -0.3914 | -0.2903 | 0.2555 | * | 0.1367 | 4.0530 |
| -0.4825 | $\sim 0.7005$ | 0.3254 | $\pm$ | 1.6110 | 3.6860 |
| -0.5950 | -0.4075 | 0.3238 | - | 0.0465 | 3,6190 |
| -0.7521 | -0.8267 | 0.3283 | $\cdots$ | 0.1721 | 3.1270 |
| -0.9156 | -0.7840 | 0.3962 | - | 0.0161 | 3.0780 |
| -1.0980 | -0.9129 | 0.3553 | * | 0.1045 | 5.3460 |
| -1, 6600 | -0.7491 | 0.5250 | + | 1.0240 | 4.9020 |

INDUSTRY 6 SUBgROUP 4

| CLASS HIDEPT | MEAN | variance | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8950 | 1.3350 | 1.3680 | - | 2.0620 | 4.5920 |
| 1.2390 | 1.0230 | 0.3231 | + | 0.1058 | 2.7780 |
| 0.9348 | 0.7431 | 0.2372 | + | 0.2018 | 2.6620 |
| 0.7292 | 0.5514 | 0.2957 | - | 1.6630 | 4.8170 |
| 0.6056 | 0.4094 | 0.4328 | + | 0.1952 | 4.2140 |
| 0.4749 | 0.2301 | 0.3607 | - | 0.3520 | 2.9200 |
| 0.3562 | 0.2386 | 0.1720 | + | 0.0815 | 3.2540 |
| 0.2021 | 0.1882 | 0.2282 | - | 0.0225 | 3.2220 |
| 0.0819 | -0.0760 | 0.26 .13 | - | 1.1350 | 3.6830 |
| -0.0052 | -0.0161 | 0.1911 | - | 0.7488 | 3.6360 |
| -0.1255 | -0.0371 | 0.3837 | - | 0.7208 | 3.0900 |
| -0.2333 | -0.2432 | 0.3024 | - | 3.3120 | 7.5720 |
| -0.3883 | -0.2044 | 0.3400 | - | 1.3270 | 5.0660 |
| -0.6012 | - 0.0 .5013 | 0.2696 | - | 0.1348 | 4.1400 |
| -0.8491 | -0.7358 | 0.3842 | - | 0.1820 | 3.1460 |
| -1.1640 | -1.0010 | 0.5549 | + | 0.0873 | 3.1760 |
| -2.0430 | -1.3600 | 0.9229 | + | 0.8895 | 4.5270 |




| 1.7230 | 0.3431 | － |
| :---: | :---: | :---: |
| 1.4270 | 0.3164 | － |
| 1.2540 | 0.3734 | － |
| 1.0240 | 0.2726 | － |
| 0.9161 | 0.2555 | － |
| 0.671 .3 | 0.1 .406 | ＋ |
| 0.6323 | 0.2824 | － |
| $0.608 ?$ | 0.2599 | ＋ |
| 0.6070 | 0.1952 | － |
| 0.3806 | 0.1293 | － |
| 0.5932 | 0.3 .337 | － |
| 0.2432 | 0.1901 | － |
| 0.3429 | 0.1058 | － |
| 0． 3711 | 0.1100 | － |
| 0.3110 | 0.1105 | － |
| 0.0350 | $0.107 ?$ | － |
| 0.3012 | 0.1016 | ＋ |
| 0.0677 | 0.1140 | ＋ |
| 11.2383 | 0.2041 | ＋ |
| 0.1 .199 | 0.1384 | － |
| －0．0491 | 0.1111 | － |
| －0．0337 | 0.1306 | ＂ |
| －0．1078 | 0.2106 | － |
| －0．0848 | 0.0844 | ＋ |
| $-0.0213$ | 0.0906 | ＋ |
| －0．141．4 | 0.1668 | － |
| $-0.1552$ | 0.0882 | ＋ |
| $-0.1327$ | 0.1193 | － |
| －0．1321 | 0.0747 | － |
| $-0.1063$ | 0.0620 | ＋ |
| －0．2184 | 0.1393 | － |
| － 0.1 .035 | 0.1604 | ＋ |
| －0．2033 | 0.1542 | ＋ |
| －0．2570 | 0.0925 | － |
| －0．3016 | 0.1880 | － |
| －0．3560 | 0.1034 | － |
| －0．3734 | 0.1658 | ＋ |
| －0．3690 | 0.1531 | ＋ |
| －0．4389 | 0.0943 | － |
| －0．3514 | 0.2138 | ＋ |
| －0．6913 | 0.5137 | － |
| －0．57月6 | 0.1897 | － |
| －0．4169 | 0.2363 | ＋ |
| －0．0001 | 0.3096 | － |
| －0．590．4 | 0.1676 | － |
| －0．6135 | 0.2539 | ＋ |
| －0．7185 | 0.3455 | － |
| －0．9336 | 0.2 .727 | － |
| －0．8567 | 0.2157 | － |
| 5290 | 0.8854 |  |

0.0779
1.0250
2.1120
0.3261
0.3508
0.0448
0.6745
0.0589
0.2898
0.5031
2.5480
0.1497
0.2035
0.1448
0.6286
0.1000
0.0121
0.0026
4.2220

1． 2240
0.5859
4.6610
0.6795
0.3206
0.3939
0.1003
0.0149

0． 2058
0.2918
5.8470
0.7669
0.1655
2.859
0.0194
0.4101
0.2376
0.6316
0.1619
0.1652
1.2300
1.2320
0.3200
0.0006
0.0085
0.0128
0.1256
0.6874
0.2594
0.0593
2.4410

2． 86.30
6.8420
3.3090
3.6110
2.2610
4.7890
3.1910
4.0250
3.8400
8.5700
2.3060
2.3720
3.2300
2.8700
1.8780
2.4330

2． 2330
$7.9110^{\circ}$
3.2080
5.0720
4.9500
7.5860
3.1760
3.4090
3.3700
2.8640
3.9240
3.8900
4.6700
13.0400
4.2390
3.4180
8.1630
6.6050
2.7060
4.4480
4.5970
2.8880

4．8650
4.3430
6.2000
3.2230
2.7600
3.3550
2.1360
4.6620
2.6410
4.1650
2.6380

## InDUSTAY 7 SUBgROUP 2

| CLASS FIDMPT | MEAN | VARIANCE | SIGN | SKEWNESS | KIRTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.9950 | 1.5590 | 0.1967 | - | 0.0191 | 2.5760 |
| 1.4800 | 1.1000 | 0.1714 | + | 0.1491 | 2.6050 |
| 1.3020 | 1.0880 | 0.2774 | - | 0.2974 | 4.0490 |
| 1.1320 | 0.9823 | 0.1679 | - | 0.2853 | 2.4090 |
| 0.9604 | 0.8910 | 0.4476 | - | 0.0236 | 2.5550 |
| 0.8553 | 0.5947 | 0.2240 | - | 0.1979 | 3.2060 |
| 0.7770 | 0.6499 | 0.1802 | + | 0.1708 | 3.5070 |
| 0.6790 | 0.5957 | 0.1428 | + | 0.2024 | 2.2300 |
| 0.5863 | 0.4821 | 0.2299 | - | 0.8961 | 3.0810 |
| 0.4772 | 0.4946 | 0.2184 | - | 0.0409 | 3.6550 |
| 0.3546 | 0.2473 | 0.1584 | + | 1.5860 | 4.6860 |
| 0.2525 | 0.4593 | 0.1185 | - | 0.0229 | 2.5470 |
| 0.1618 | 0.1691 | 0.1729 | + | 1.5280 | 4.4130 |
| 0.0772 | 0.0922 | 0.1678 | + | 2.7920 | 6.3470 |
| -0.0252 | -0.1449 | 0.2516 | - | 0.2187 | 3.5910 |
| -0.1151 | -0.0724 | 0.1870 | + | 0.7236 | 5.1740 |
| -0.2341 | -0.0247 | 0.2807 | + | 0.1338 | 2.7550 |
| -0.3678 | -0.0333 | 0.1791 | + | 0.1609 | 2.6110 |
| -0.5563 | -0.3500 | 0.2193 | + | 0.1316 | 3.1690 |
| -1.1530 | -0.8143 | 0.4253 | + | 0.1011 | 2.8880 |

I NDUSTRY 8 SUGGROUP 1

GLASS MIM-PT MEAN
VARIANCE SIGN
SKENNESS
KIRTOSIS
1.7150
1.1150
0.7407
0.5399
0.3305
0.1648
0.0090
-0.1392
-0.2941
-0.5003
-0.6828
-1.3600
1.0990
0.6383
0.5458
0.4268
0.3540
0.2466
0.0035
-0.2240
-0.3322
-0.0567
-0.4610
-0.6422

| 0.7712 |  | 0.7968 |
| :--- | :--- | :--- |
| 0.3730 |  | 0.7696 |
| 0.5572 |  | 0.1 |
| 0.3537 |  | 0.6127 |
| 0.3078 | + | 0.0266 |
| 0.1 .969 | + | 0.1978 |
| 0.4381 |  | 0.0266 |
| 0.8533 |  | 2.2140 |
| 0.3989 |  | 0.1060 |
| 0.4469 | + | 0.4181 |
| 0.2687 |  | 0.0578 |
| 0.7148 | + | 0.0523 |
| 0. | 0.0453 |  |

2.7820
2.5890
3.8040
2.6690
2.9860
2.1710
2.1 .710
4.2840
3.6090
2.8720
3.5110
2.9990
2.9450

CLASS MID-FT MEAN
1.7700
1.3210
1.10530
0.3234
0.7199
0.5973
0.4891
0.3632
0.2739
0.2560
0.0466
-0.0414
-0.1407
-0.2533
-0.3725
-0.4650
-0.5939
-0.7463
-0.8557
-1.1050
-1.6840

MEAN
1.3430
0.9015
0.4953
0.7717
0.4434
0.3436
0.4708
0.0764
0.2484
$-0.1935$
0.0907
0.1000
$-0.2054$
$-0.2077$
$-0.5685$
$-0.4202$
$-0.4179$
$-0.4627$
$-0.4949$
$-0.6964$
$-0.9653$

VARIANCE SIGN
SKEWNESS
KIRTOSIS

| 0.4383 | - | 0.91 .03 | 3.9650 |
| :--- | :--- | :--- | :--- |
| 0.5173 | - | 0.3749 | 3.1760 |
| 0.5740 | - | 0.4212 | 2.7560 |
| 0.3779 | - | 0.1227 | 3.4520 |
| 0.6250 | - | 0.8003 | 3.5800 |
| 0.3654 | - | 0.8753 | 3.5040 |
| 0.3317 | + | 0.0116 | 2.3140 |
| 0.6707 | - | 1.4060 | 4.1520 |
| 0.4293 | - | 0.2528 | 3.0060 |
| 0.4020 | + | 0.0019 | 1.9890 |
| 0.4305 | + | 0.8415 | 4.5750 |
| 0.2965 | + | 0.2564 | 3.3500 |
| 0.5565 | - | 0.0081 | 3.7040 |
| 0.6375 | + | 0.0661 | 2.9770 |
| 0.5414 | + | 0.1100 | 2.4840 |
| 0.4115 | - | 0.2411 | 2.1390 |
| 0.8100 | + | 0.0008 | 3.6730 |
| 0.5654 | - | 0.7221 | 3.3570 |
| 0.1424 | + | 0.8727 | 3.3600 |
| 0.4205 | - | 0.3369 | 3.0190 |
| 0.5912 | - | 0.0273 | 2.4860 |


| $6 L 153: 1+1$ | 位A的 | VABIAINCE | SIGN | SKEN中忥S | KURTOSTs |
| :---: | :---: | :---: | :---: | :---: | :---: |
| O．06110 | 1．2760 | 0．7311 | $\cdots$ | 2，3310 | 6.8410 |
| 1．1／1 | 1．1．400 | 0.3454 | ＋ | 0.0172 | 3.5570 |
| 1． | 19.9716 | 11.4302 | － | 0.3118 | 3.7280 |
| 0.95515 | 0.0361 | 0.2393 | ＋ | 0.0707 | 2.3440 |
| 10.71993 | 0.7307 | リ． 2922 | － | 0.1083 | 3．5860 |
| $0 \cdot 010$ | 11.5413 | 0.2529 | － | 0.1050 | 3.4510 |
| 0.2130 | 0.3910 | 0． 2370 | － | 1.9310 | 6.1020 |
| 0.4151 | 0．5423 | 0.24 Ha | $\cdots$ | 2.4310 | 7.3500 |
| 0． 6153 | 0．2．376 | 0.35015 | － | 0.7052 | 4.9470 |
| （1）． 26.17 | 0．1943 | 11．3358 | － | 0.13 .14 | 4.9070 |
| 0.1732 | －0，36713 | 0.0962 | － | 0.0429 | 1.9530 |
| 0.1793 | 11.0914 | （1） 66 ？ | － | 0.0451 | 4.1920 |
| 11．1392 | － 11.2702 | 0.5275 | － | 4.7220 | 7.7430 |
| －0． 0.01 | $\cdots 0.1633$ | 0.2314 | $\cdots$ | 1.0930 | 4.1200 |
| －0．1549 | $-0.3544$ | 0.1820 | － | 0.0880 | 2.3330 |
| －0． a S沙9 | － $0.39 \% 0$ | 0.4960 | － | 0.0693 | 4.0250 |
| －0．31．07 | －0．4573 | 0.5214 | － | 1.6930 | 5.2160 |
| －0．4：119 | $-0.4020$ | 0.1530 | － | 0.0670 | 3.1210 |
| －0．4927 | －11．4023 | 0.2016 | ＋ | 0.0405 | 3.6800 |
| －0．5097 | －11．32．43 | 0.2117 | ＋ | 0.0000 | 2.6690 |
| － 10.7100 | －0．6025 | 0.4938 | － | 2.0150 | 4.8420 |
| －0．865\％ | －11．5332 | 0.2209 | － | 0.0115 | 2.3260 |
| －1．1500 | －0．7391 | 0.3384 | $\cdots$ | 0.0907 | 3.5270 |
| －1． 1.530 | $-0.9268$ | 1.2400 | ＋ | 0.5923 | 4.0510 |

iWDUSTR 11 Subgroup 2

| CLASS HIDMT | MEAN | vaplance | SIGN | SKEWAESS | Kurtosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6050 | 1.4180 | 0.2905 | + | 0.6204 | 3.2530 |
| 1.0520 | 0.9724 | 0.1777 | - | 0.0324 | 4.3200 |
| 0.0105 | 0.6223 | 0.2310 | - | 1.1110 | 4.0160 |
| 0.6288 | 0.5444 | 0.2986 | - | 0.0898 | 2.4800 |
| 0.4119 | 0.3440 | 0.1138 | - | 0.1870 | 2.5360 |
| 0.2567 | 0.1734 | 0.1502 | + | 0.0002 | 2,2660 |
| 0.0425 | -0.2704 | 0.2745 | - | 1.0970 | 3.4490 |
| -0.1024 | -0.0799 | 0.2290 | + | 0.0017 | 3.0860 |
| -0.2407 | -0.3143 | 0.2112 | - | 0.5911 | 3.9770 |
| -0.3507 | -0.3224 | 0.3638 | + | 0.5874 | 3.4990 |
| -0.5152 | -0.6373 | 0.1687 | * | 0.1251 | 2.4850 |
| -0.7550 | -0.621.5 | 0.3180 | - | 0.3006 | 3.2060 |
| -1.4720 | -1.1450 | 0.4998 | - | 0.0151 | 2.5420 |

IMDUSTMY 12 Surgroup 2

| CLASS MID-PT | MEAN | VARIAPCE | 516 N | SKEANESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8570 | 1.5980 | 0.3016 | - | 1.4730 | 4.4710 |
| 1.0520 | 0.9863 | 0.3047 | + | 0.4656 | 4.4660 |
| 0.7795 | 0.5278 | 0.3250 | - | 5.0980 | 9.06\% |
| 0.5533 | 0.4558 | 0.4209 | - | 3.5620 | 7.4590 |
| 0.3468 | 0.4789 | 0.1461 | - | 0.0017 | 2.2500 |
| 0.1561 | 0.0512 | 0.1010 | - | 1.1010 | 4.7230 |
| 0.0538 | 0.0549 | 0.1025 | \% | 0.3685 | 3.8850 |
| -0.0717 | -0.0993 | 0.0949 | + | 0.4504 | 2.7650 |
| -0.2236 | -0.0915 | 0.1178 | - | 0.0606 | 3.0650 |
| -0.3665 | -0.3829 | 0.0806 | * | 0.1589 | 2.1720 |
| -0.4739 | -0.4080 | 0.1546 | + | 3.1200 | 7.0230 |
| -0.6114 | -0.5842 | 0.0954 | + | 1.2200 | 4.7010 |
| -0.7616 | -0.7773 | 0.1733 | - | 0.6407 | 4.2820 |
| -0. 0.9635 | $=1.0620$ | 0.1038 | - | 0.0140 | 2.1940 |
| -1.5620 | -1.3160 | 0.5740 | + | 0.0109 | 3,3450 |


| moustry l2 | SuFGodep |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CLASS H11-11 | HEAN | VARIANCE | SIGN | SKEHUESS | KURTOSIS |
| 1.5870 | 1.2530 | 0.3640 | + | 0.0479 | 3.7290 |
| 1.0150 | 0.8318 | 0.1901 | + | 0.0621 | 2.3670 |
| 0.7595 | 0.4677 | 0.3541 | - | 0.0444 | 3.7630 |
| 0.5550 | 0.4761 | 0.1630 | - | 0.3326 | 3.3310 |
| 0.412 .3 | 0.5247 | 0.2204 | $\uparrow$ | 1.0090 | 4.3500 |
| 0.2050 | 0.1169 | 0.3532 | $\cdots$ | 2.4420 | 5.9950 |
| 0.1377 | 0.1528 | 0.21 .48 | + | 0.1052 | 2.3190 |
| $-0.0180$ | $-0.0161$ | 0.3361 | - | 3.01 .60 | 7.2360 |
| -0.2350 | -0.1461 | 0.1456 | $\cdots$ | 0.1630 | 2.1520 |
| -0.4544 | -0.3637 | 0.1801 | - | 0.01 .50 | 3.2940 |
| -0.7822 | -0.6424 | 0.2523 | - | 0.0004 | 3.0010 |
| -1.4600 | -1.3070 | 0.4024 | - | 0.0726 | 3.2710 |


| CLASS MID-FI | MEAN | VARIANCE | SIGN | SKEnMESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. 9930 | 1.3990 | 0.6278 | - | 1.5650 | 4.2270 |
| 1.4600 | 2.1360 | 0.1721 | T | 0.5191 | 2.5610 |
| 1.11.60 | 0.9362 | 0.5326 | - | 0.2702 | 3. 5620 |
| 0.8652 | 0.7233 | 0.5961 | " | 3.9620 | 7.9580 |
| 0.6247 | 0.3886 | 0.6694 | * | 0.5074 | 2.8910 |
| 0.4216 | 0.4895 | 0.3208 | + | 0.9543 | 4.5470 |
| 0.2734 | 0.3147 | 0.341 .4 | + | 1.2750 | 4.3090 |
| 0.1541 | $-0.0153$ | 0.0310 | - | 0.3216 | 4.4230 |
| 0.0291 | 0.1026 | 0.2660 | * | 1.6770 | 6.4530 |
| $-0.1058$ | -0.0961 | 0.1583 | + | 0.0864 | 2.3030 |
| -0.2096 | -0.3025 | 0.3955 | \% | 2.5490 | 6.4100 |
| $-0.3164$ | -0.4904 | 0.1628 | - | 0.4799 | 2.6800 |
| -0.4398 | $-0.3936$ | 0.0989 | + | 0.0001 | 3.1320 |
| -0.5490 | -0.4974 | 0.2313 | \% | 0.1262 | 3.7340 |
| -0.6778 | -0.5394 | 0.0440 | + | 0.0004 | 2.5190 |
| -0.8156 | $-0.4529$ | 0.3774 | + | 0.0188 | 2.7470 |
| -1.0620 | -0.8368 | 0.4562 | + | 1.6320 | 5.6220 |
| $-1.8290$ | -1.1480 | 0.7418 | - | 0.0017 | 2.4940 |


| CLASS MID-FT | MEAN | variance | SIGN | Sikndess | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.2110 | 1.8170 | 0.4222 | - | 8.0810 | 15.0900 |
| 1.7520 | 1.4890 | 0.1819 | - | 0.0380 | 3.9250 |
| 1.5660 | 1.3140 | 0.1894 | * | 0.5200 | 4.9380 |
| 1.3690 | 1.2640 | 0.1792 | - | 0.0721 | 2.2050 |
| 1. 21.60 | 1.0180 | 0.4092 | $\square$ | 2.7790 | 5.9130 |
| 1.0710 | 1. 01005 | 0.3141 | - | 6.3890 | 12.7400 |
| 0.9643 | 0.9615 | 0.0705 | + | 0.1927 | 4.2300 |
| 0.3747 | 0.7406 | 0.1326 | + | 0.3713 | 2.4910 |
| 0.8149 | 0.6259 | 0.5572 | - | 6.8200 | 12.9100 |
| 0.7461 | 0.5713 | 0.2616 | - | 1.9060 | 5.1950 |
| 0.6702 | 0.6761 | 0.1366 | + | 0.3938 | 3.5500 |
| 0.6039 | 0.5378 | 0.2121 | + | 0.0064 | 3.6880 |
| 0.5572 | 0.4942 | 0.2063 | - | 0.3124 | 5.4840 |
| 0.4887 | 0.4740 | 0.1745 | * | 2.8690 | 7.6380 |
| 0.4399 | 0.3216 | 0.1398 | $\cdots$ | 2.6370 | 5.9330 |
| 0.4007 | 0.3329 | 0.1432 | - | 1.7410 | 4.4900 |
| 0.3569 | 0.51 .87 | 0.2344 | + | 4.8800 | 9.5650 |
| 0.2973 | 0.2409 | 0.0476 | + | 0.2594 | 2.9990 |
| 0.2555 | 0.2284 | 0.0895 | + | 0.0001 | 1.9740 |
| 0.2225 | 0.1390 | 0.3475 | - | 10.9000 | 16.7800 |
| 0.1829 | 0.2147 | 0.0700 | + | 0.152 ? | 3.2070 |
| 0.1434 | 0.0910 | $0,1,401$ | * | 2.7720 | 7,7630 |
| 0.1186 | 0.0754 | 0.1141 | + | 1.0630 | 6.5050 |
| 0.0820 | 0.0462 | 0.0659 | - | 0.0570 | 2.6490 |
| 0.0427 | 0.0005 | 0.2312 | + | 2.6310 | 8.2960 |
| 0.0081 | -0.0225 | 0.1427 | - | 0.1227 | 3.3170 |
| -0.0238 | -0.0869 | 0.2433 | - | 4.5700 | 10.9400 |
| -0.0805 | -0.0360 | 0.0865 | - | 3.5590 | 9.8500 |
| -0.0961 | -0.1029 | 0.2839 | - | 3.4100 | 7.2080 |
| -0.1294 | -0.1377 | 0.0874 | - | 0.7069 | 3.4870 |
| $-0.1667$ | -0.1170 | 0.0841 | + | 0.2073 | 3.7300 |
| -0.2093 | -0.1838 | 0.3253 | $\div$ | 5.5340 | 12.6400 |
| -0.2383 | -0.2998 | 0.15154 | - | 6.0490 | 11.6700 |
| -0.2810 | -0.2943 | 0.1092 | * | 0.4002 | 3.3170 |
| -0.3?12 | -0.4561 | 0.0951 | - | 0.0120 | 2.9970 |
| -0.3724 | -0.4007 | 0.2549 | + | 0.0402 | 6.2490 |
| -0.4223 | -0.4046 | 0.0781 | - | 0.6118 | 3.2500 |
| -0.4669 | -0.4234 | 0.0626 | + | 0.7886 | 4.7750 |
| -0.5158 | -0.5154 | 0.21 .64 | - | 4.2870 | 9.8580 |
| -0.5641 | -0.5668 | 0.0648 | + | 0.0002 | 2.7170 |
| -0.6132 | -1. 0.6368 | 0.1154 | + | 0.0046 | 4.2120 |
| -0.5629 | -0.6793 | 0.1802 | - | 3.2900 | 9.5510 |
| -0.7253 | -0.6639 | 0.1103 | - | 0.3101 | 3.3410 |
| -0.7741. | -0.7677 | 0.1180 | - | 0.2524 | 3.6020 |
| -0.8477 | -0.6699 | 0.2183 | + | 3.0080 | 6.0550 |
| -0., 9164 | -0.8097 | 0.1183 | - | 0.0228 | 2.6130 |
| -1.0040 | -0.8826 | 0.0991 | + | 1.c000 | 4.6720 |
| -1.1120 | $-1.1020$ | 0.0981 | - | 0.0501 | 2.5280 |
| -1.2130 | -1.1360 | 0.2401 | + | 0.1438 | 3.7010 |
| -1.2090 | -1.1560 | 0.0703 | + | 0.4873 | 2.8180 |
| -1.4350 | -1.2170 | 0.1561 | + | 0.0405 | 2.8880 |
| -1.9900 | -1.5610 | 0.2692 | + | 0.1197 | 2.3830 |


| CLASS MIDMFT | MEAN | VAPIANCE | SlGN | SKEWMESS | KUPTOS1S |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.9770 | 1.5280 | 0.6687 | - | 0.1317 | 2.3150 |
| 1.3340 | 1.1550 | 0.2587 | - | 1.9930 | 3.9660 |
| 1.0480 | 0.7409 | 0.4478 | - | 1.7250 | 5.3620 |
| 0.8654 | 0.6406 | 0.3174 | - | 0.1547 | 3.4100 |
| 0.7095 | 0.4911 | 0.3153 | + | 1.0500 | 6.4240 |
| 0.6212 | 0.3994 | 0.3044 | $\cdots$ | 0.3638 | 4.9600 |
| 0.5159 | 0.2301 | 0.5602 | - | 1.1530 | 4.0450 |
| 0.4277 | 0.1731 | 0.4135 | - | 0.6 .312 | 3.1830 |
| 0.3527 | 0.3693 | 0.2891 | + | 0.0875 | 5.5920 |
| 0.2645 | 0.31 .69 | 0.1633 | + | 0.0160 | 2.2440 |
| 0.1596 | 0.1840 | 0.1509 | - | 0.1495 | 6.0200 |
| - 0.0528 | -0.0039 | 0.4481 | - | 0.0933 | 4. 6490 |
| -0.0501 | -0.0735 | 0.2467 | - | 0.0471 | 2.5560 |
| -0.1513 | -0.2495 | 0.3659 | " | 0.0991 | 2. 4570 |
| -0.2786 | -0.281.3 | 0.1645 | + | 0.2259 | 3.2550 |
| -0.4027 | -0.3853 | 0.5038 | a | 0.0011 | 3.1690 |
| -0.5412 | -0.4564 | 0.4589 | - | 0.0010 | 3.0750 |
| -0.6014 | $-0.5763$ | 0.5733 | - | 0.1268 | 2.9870 |
| -0.0487 | $-0.7047$ | 0.4966 | - | 0.3369 | 5.4620 |
| -1.0730 | -0.7782 | 0.6402 | + | 0.0325 | 1.8820 |
| $-1.4120$ | -0.9811 | 0.9383 | + | 1.0780 | 5.3400 |
| $-2.0020$ | -1.2490 | 0.6072 | + | 0.2915 | 2.8830 |

IMDUSTRY 15 SUBGROUP ?

| CLASS MIDWPT | MEAN |  | VARIANCE | SlGN | SKENNESS | KURTOS15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0630 | 1.4310 |  | 1.0580 | " | 4.7190 | 8.7570 |
| 1.1600 | 0.9660 | 1 | 0.1349 | - | 0.7993 | 3.3600 |
| 0.7331 | 0.7447 |  | 0.1266 | + | 0.4036 | 2.1310 |
| 0.6167 | 0.5646 |  | 0.1293 | \% | 0.0060 | 2.8930 |
| 0.4481 | $0.167 ?$ |  | 0.1839 | - | 5.1820 | 9.0280 |
| 0.2871 | 0.1201 |  | 0.3750 | - | 6.3960 | 10.6000 |
| 0.1399 | 0.0897 |  | 0.0672 | - | 0.0003 | 3,0340 |
| 0.0011 | 0.0596 |  | 0.1123 | ar | 0.0704 | 2.9700 |
| $-0.1004$ | -0.2033 |  | 0.0815 | - | 0.8459 | 4.7340 |
| -0.2649 | $-0.1574$ |  | 0.1886 | + | 1.4970 | 4.1390 |
| -0.4054 | -0.4358 |  | 0.1144 | $+$ | 0.0054 | 3.3350 |
| -0.5323 | -0.4680 |  | 0.3239 | + | 3.6220 | 8.6500 |
| -0.0.6965 | -0.5401 |  | 0.1067 | + | 0.0004 | 2.1090 |
| -0.8713 | -0.7663 |  | 0.2107 | + | 0.0000 | 2.8770 |
| -1.0940 | -0.9830 |  | 0.1312 | * | 0.4236 | 3.7500 |
| -1.6580 | $-1.1030$ |  | 0.3874 | - | 0.0108 | 3.2890 |

JNDUGIRY IS SUPGRUAP 3

| CLASS MITMT | MEAN | VARIANCE | SIGN | SKEWNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.9990 | 1.4060 | 0.4803 | + | 0.1730 | 2.3430 |
| 1.3130 | 0.8434 | 0.2618 | + | 0.1227 | 2.2110 |
| 1.0290 | 1.0740 | 0.3136 | + | 0.5233 | 3.0200 |
| 0.8341 | 0.8726 | 0.2737 | + | 0.1763 | 1.9420 |
| 0.6972 | 0.6294 | 0.0871 | $\cdots$ | 0.7612 | 4.3780 |
| 0.5941 | 0.4320 | 0.2084 | + | 0.0006 | 2.6980 |
| 0.5057 | 0.4761 | 0.0724 | - | 0.2654 | 2.8800 |
| 0.4502 | 0.5259 | 0.201 .3 | - | 0.71 .74 | 4.7490 |
| 0.3794 | 0.3440 | 0.1047 | * | 0.1974 | 3.6890 |
| 0.3174 | 0.3989 | 0.1257 | + | 0.0012 | 3.3510 |
| 0.2509 | 0.1208 | 0.1356 | $\cdots$ | 0.2625 | 2,2630 |
| 0.1580 | 0.0335 | 0.1332 | - | 0.2630 | 2.7670 |
| 0.0696 | -0.0064 | 0.0672 | - | 0.4923 | 4.1740 |
| -0.0030 | - 0.1 .48 S | 0.1872 | + | 0.0549 | 2.9120 |
| -0.0.071 | - 0.2470 | 0.4706 | - | 2.6260 | 6.3520 |
| -0.1.665 | $-0.2717$ | 0.1368 | " | 0.0780 | 2.2800 |
| -0.2564 | -0.1.376 | 0.1752 | + | 0.7282 | 7.8760 |
| -0. 0.356 | -0.1528 | 0.1813 | - | 0.0618 | 5.0640 |
| -0.4377 | -0.4293 | 0.2746 | $\cdots$ | 3.1740 | 7.3550 |
| -0.5618 | -0.4830 | 0.1063 | + | 0.2209 | 2.3680 |
| $-0.7121$ | $-0.7300$ | 0.3689 | + | 0.0193 | 2.0030 |
| -0.8823 | -0.9628 | 0.3397 | - | 0.1120 | 3.6740 |
| -1.1270 | -0.8996 | 0.2684 | + | 0.0026 | 2.2730 |
| -1.6680 | -1.0780 | 0.7359 | + | 0.2930 | 4.7400 |


| TMDUSTRY 16 | SUBGROUH | 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CLASS MIDMPT | MEAN | VARIAMCE | SidN | SKENVESS | KuRTosis |
| 1. 7660 | 1.5000 | 0.4652 | - | 0.0526 | 2,6700 |
| 1. 0.030 | 0.81 .37 | 0.3688 | + | 0.7224 | 3.5010 |
| 0.7471 | 0.3990 | 0.5495 | + | 1.8490 | 4.1720 |
| 0.5178 | 0.5380 | 0.1619 | + | 0.0317 | 2.1000 |
| 0.3816 | 0.2463 | 0.1085 | - | 0.3599 | 2.6610 |
| 0.2669 | 0.4438 | 0.3271 | 4 | 0.9422 | 3.3300 |
| 0.1688 | 0.0822 | 0.0991 | + | 0.2589 | 2.3380 |
| 0.0533 | 0.0023 | 0.3312 | - | 0.0424 | 2.9370 |
| -0.0585 | -0.2496 | 0.3876 | * | 1.7390 | 4.9380 |
| -0.1764 | -0.2354 | 0.2327 | - | 0.0662 | 2.7420 |
| -0.3200 | -0.2410 | 0.0653 | + | 0.1243 | 2.1790 |
| -0.4636 | -0.3711 | 0.1245 | = | 0.3621 | 2.9750 |
| -0.6973 | -0.6946 | 0.4256 | - | 0.0542 | 2.0660 |
| -1.0530 | $-0.9093$ | 0.4669 | - | 1.3040 | 4.4870 |
| -1. 6.660 | $-1.1920$ | 0.6178 | + | 0.9075 | 4.0030 |


| CLASS ALD-PT | MEAN | VARIANCE | SIGN | SKEANESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.7800 | 1.1900 | 0.7074 | - | 4.8990 | 9.4420 |
| 1.0950 | 0.9405 | 0.2792 | + | 1.5910 | 4.3370 |
| 0.9066 | 0.4523 | 0.721 .4 | $\cdots$ | 4.5860 | 9.3390 |
| 0.7557 | 0.0937 | 0.2249 | - | 0.1236 | 2.8230 |
| 0.5723 | 0.3961 | $0 . \pm 082$ | + | 0.7206 | 4.0100 |
| 0.4437 | 0.3722 | 0.2114 | + | 0.0110 | 2.3710 |
| $0.344 \%$ | 0.1024 | 0.2474 | " | 1.0090 | 3.1770 |
| 0.2279 | 0.3260 | 0.1486 | $\cdots$ | 0.0466 | 2.0260 |
| 0.1146 | -0.0855 | 0.2537 | + | 0.4652 | 3.9150 |
| 0.0042 | -0.0671 | 0.41 .82 | - | 1.7090 | 5.8750 |
| -0.0708 | $-0.1740$ | 0.2144 | + | 0.2949 | 3.6990 |
| $-0.1643$ | -0.1444 | 0.2168 | + | 0.1908 | 2.7240 |
| -0.2536 | -0.2263 | 0.1171 | - | 0.1061 | 2.5700 |
| -0.3457 | -0.2791 | 0.3727 | " | 0.1134 | 2.0910 |
| -0.4630 | -0.4554 | 0.1846 | * | 0.1560 | 3.0790 |
| -0.6074 | -0.4852 | 0.3638 | $\cdots$ | 0.0156 | 3.0700 |
| -0.8640 | $-0.5155$ | 0.4919 | + | 0.9385 | 7.3130 |
| -1.4220 | $-0.8757$ | 0.441 .7 | - | 0.8846 | 3.0930 |


| CLASS MJD-PT | MEAN | VARIANCE | SION | SKEVAESS | KURTOS 15 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.6950 | 1.0790 | 0.3354 | + | 0.1532 | 2.9850 |
| 1. 1960 | 1.0040 | 0.2401 | - | 0.0482 | 2.3800 |
| 0.9888 | 0.7255 | 0. 21.63 | + | 0.0322 | 2.3000 |
| 0.8585 | 0.7924 | 0.2524 | - | 0.0575 | 2.2540 |
| 0.7222 | 0.6233 | 0.1675 | + | 0.0637 | 2.7030 |
| 0.6014 | 0.4653 | 0.3091 | $\cdots$ | 0.4855 | 2.6110 |
| 0.4468 | 0.3513 | 0.3316 | - | 0.7370 | 3.3170 |
| 0.3193 | 0.3224 | 0.31 .39 | + | 2.7300 | 6.7240 |
| 0.1940 | 0.2021 | 0.1710 | + | 0.0132 | 2.2590 |
| 0.0607 | $-0.0933$ | 0.1227 | - | 0.1421 | 3.1100 |
| -0.0700 | $-0.0737$ | 0.2684 | + | 0.0112 | 2.6560 |
| -0. 0.2061 | $=0.161 .4$ | 0.2735 | + | 0.0327 | 2.3640 |
| -0.3634 | -0.2666 | 0.2475 | - | 1.0210 | 3.8690 |
| -0.4431 | $-0.4972$ | 0.2266 | + | 0.0010 | 2.2170 |
| -0. 0.5322 | -0.4496 | 0.1634 | - | 0.0073 | 3.6820 |
| -0.7066 | -0.8760 | 0.3960 | - | 1.6960 | 3.9230 |
| -0.9577 | -0.6812 | 0.2542 | + | 0.4887 | 3.6110 |
| -1.6660 | $-1.2290$ | 0.6499 | - | 0.0000 | 2.3920 |

IODUSTMY 16 SJBGROUP 4

| CLASS $\because$ CDHFT | MEAN | VARIANCE | SICN | SKENMESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8680 | 1.3040 | 0.7325 | - | 0.9332 | 5.3640 |
| 1.1820 | 0.7681 | 0.1317 | + | 0.3894 | 2.1930 |
| 0.6717 | 0.6076 | 0.9364 | - | 0.2148 | 2.4920 |
| 0.6601 | 0.4573 | 0.5166 | - | 0.3168 | 2.4600 |
| 0.4551 | 0.3462 | 0.2004 | + | 0.0382 | 2.7840 |
| 0.3005 | 0.3327 | 0.4518 | $+$ | 0.0172 | 3.0210 |
| 0.1418 | 0.0242 | 0.2539 | + | 0.0738 | 2.2880 |
| 0.0076 | $-0.1343$ | 0.3304 | - | 0.1463 | 1.8370 |
| -0.1017 | $-0.3117$ | 0.3357 | - | 0.6699 | 3.7750 |
| -0.3103 | -0.2648 | $0.242 \%$ | + | 0.0001 | 2.5960 |
| -0.4497 | -0.2521 | 0.3036 | - | 0.0005 | 1.7500 |
| -0.5715 | -0.4794 | 0.3618 | - | 0.4862 | 3.5140 |
| -0.8043 | -0.4485 | 0.4137 | - | 0.3139 | 2.2550 |
| -1.0700 | $=0.7108$ | 0.7927 | + | 0.2057 | 2.1940 |
| -1.5070 | $-0.7652$ | 0.5364 | - | 0.0043 | 2.0240 |


| y whlorRy 16 | 51900090 | 5 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CLASGMID-PT | MEAM | VARIANCE | 516 M | SKEWNESS | KURTOS1S |
| 1.8390 | 1.5470 | 0. 3256 | + | 0.0035 | 2.7540 |
| 1.1600 | 0.8682 | 0.3255 | + | 0.0486 | 3.1800 |
| 0.8375 | 0.8210 | 0.2377 | + | 0.3077 | 5.7890 |
| 0.6776 | 0.6147 | 0.2253 | $+$ | 0.0000 | 2.3520 |
| 0.5044 | 0.4991 | 0.2150 | - | 0.0571 | 3.8600 |
| 0.3177 | 0.22017 | 0.1962 | * | 0.0000 | 2.8580 |
| 0.1134 | -0.0071 | 0.2656 | + | 0.0519 | 3.5870 |
| -0.01.38 | -0.1019 | 0.11 .41 | - | 0.5528 | 5.1820 |
| -0.1437 | -0.2219 | 0.1255 | + | 0.2506 | 3.3060 |
| -0.2519 | -0.2911 | 0.1320 | + | 0.1808 | 3.2540 |
| -0.3657 | -0.5096 | 0.1242 | - | 3.8790 | 9.6710 |
| -0. 0.4734 | -0.4143 | 0.2464 | 4 | 0.2105 | 3.5280 |
| -0.5662 | -0.6189 | 0.1273 | m | 0.9449 | 4.5590 |
| -0.6857 | -0.6293 | 0.2081 | - | 1.1430 | 4.3020 |
| -0.8731 | -0.8802 | 0.2494 | + | 0.0101 | 2.9600 |
| -1.4510 | $-1.0220$ | 0.3571 | - | 0.3476 | 2.6770 |


| CLASS - 11-1-T | HEAN | VARIANCE | SIGN | Suthress | Kukiosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. $1.3 \%$ | 1.4090 | 0.4374 | - | 10.9025 | 6.4170 |
| 1.3\% | $10.845 \%$ | 0.2760 | + | 0.0112 | 2.4150 |
| 0.96 | 0.91166 | 0.1495 | $+$ | 0.0023 | $3 \cdot 1640$ |
| 0.768 | 0.5392 | 0.1331 | + | 0.3436 | 3.4600 |
| $0 \cdot 3403$ | 0.3017 | 0.2350 | - | 0.0094 | 2.67 , 0 |
| 0. ${ }^{1}$ |  | 0.2100 | * | 0.6075 | 4.6610 |
| 0. | 11.3045 | 0.1724 | $+$ | 0.0632 | 3.0590 |
| 0.103 | 0.1515 | 0.1290 | - | 1.6870 | 5.9900 |
| 0.1) ind | -0.11755 | $0.19 \% 1$ | - | 1.9340 | 7.1620 |
| - $0 \cdot 0 \cdot 0 \cdot 6$ | 0.0110 | 0.1991 | + | 1. 6990 | 5.5110 |
| -0.1. 0.5 | - 0.1431 | 0.2936 | $+$ | 0.1648 | 3.3710 |
| -0. 020 | - $0 \cdot$ - ${ }^{\text {areo }}$ | 0.1040 | - | 0.063 .3 | 2.9500 |
| -0.6. 0 | -0.5713 | 0.2365 | - | 0.0565 | 2.5240 |
| - 0.4 mot | - 0.6369 | 0.3027 | - | 2.6540 | 6.3460 |
| - 0. Sast | - 0.41977 | 0.0921 | - | 0.0335 | 2.3550 |
| -0. 0 ¢ $\%$ a | $-11.0753$ | 0.1 .726 | - | 0.0784 | 2.5140 |
| -1.4290 | -1.0650 | 0.7450 | + | 5.0450 | 12.6410 |


| CLASS MIDMPT | MEAN | variance | SIGN | SKEWMESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.1770 | 1.5210 | 0.6489 | - | 0.5157 | 3.1130 |
| 1.3440 | 1.2350 | 0.3059 | + | 2.0490 | 5.0750 |
| 1.0820 | 0.9867 | 0.1484 | + | 0.0211 | 3.9250 |
| 0.8744 | 0.5244 | 0.5170 | + | 0.0685 | 3.5340 |
| 0.7363 | 0.5457 | 0.216 ? | + | 3.1680 | 7.1130 |
| 0.6361 | 0.4281 | 0.1571 | - | 0.1539 | 2.5480 |
| 0.5500 | 0.4046 | 0.2396 | + | 1.5090 | 6.1000 |
| 0.4613 | 0.3950 | 0.1389 | - | 0.2553 | 3.0050 |
| 0.3626 | 0.3562 | 0.1078 | - | 0.2000 | 3.4620 |
| 0.2875 | 0.2815 | 0.2296 | - | 0.6239 | 3.3600 |
| $0.185 \%$ | 0.1841 | 0.1136 | - | 0.0012 | 3.4830 |
| 0.1248 | 0.1696 | 0.0640 | + | 0.0004 | 2.0270 |
| 0.0477 | 0.0649 | 0.2582 | - | 0.0049 | 3.7270 |
| -0.0402 | -0.0696 | 0.2545 | + | 0.0875 | 3.8260 |
| -3.1089 | -0.1514 | 0.0354 | + | 0.4216 | 2.1740 |
| -0.2100 | -0.3529 | 0.1131 | - | 0.0059 | 2.0130 |
| -0.3211 | -0.3251 | 0.0762 | + | 0.1044 | 2.4620 |
| -0.4150 | -0.41.99 | 0.1338 | - | 2.7620 | 8.0900 |
| -0.4723 | -0.3202 | 0.1236 | + | 0.1372 | 2.8950 |
| -0.5440 | -0.5417 | 0.1116 | - | 0.0665 | 2.3070 |
| -0.0546 | -0.4721 | 0.2861 | + | 0.9561 | 3.1300 |
| -0.7606 | -0.7250 | 0.3684 | + | 0.0056 | 3.9510 |
| -0.8725 | -0.8650 | 0.1230 | - | 0.5522 | 2,5750 |
| -1.0440 | -0.8992 | 0.2271 | + | 0.2448 | 2.0950 |
| -1.2450 | -1.2380 | 0.3884 | - | 0.7475 | 2.8840 |
| -1.8730 | -1.3250 | 0.2229 | - | 1.6870 | 5.7140 |

CLASS MID-FT MEAN

| 1.7200 | 1.2120 |
| :--- | ---: |
| 1.3110 | 0.9242 |
| 1.1360 | 0.7960 |
| 0.9265 | 0.5738 |
| 0.7223 | 0.5499 |
| 0.6033 | 0.6510 |
| 0.4689 | 0.4764 |
| 0.3702 | 0.3264 |
| 0.2917 | 0.1627 |
| 0.2385 | 0.1774 |
| 0.1738 | 0.1928 |
| 0.1125 | 0.1320 |
| 0.1444 | -0.0184 |
| -0.0057 | -0.2423 |
| -0.0669 | -0.0516 |
| -0.1477 | -0.2375 |
| -0.2192 | -0.3294 |
| -0.3129 | -0.4390 |
| -0.4376 | -0.4052 |
| -0.5517 | -0.5441 |
| -0.6618 | -0.3825 |
| -0.7902 | -0.6552 |
| -0.9254 | -0.7360 |
| -1.1700 | -0.9397 |
| -1.8520 | -1.2380 |

1.7200
1.3.10
0.9242
0.7960
0.5730
0. 6510
0.4764
0.1627
0.1774
0.1320
-0.0184
-0.2423
-0.0516
$-1) .2375$
-0.3294
-0.4300
-0.4052
$-0.5441$
$-0.6552$
$-0.9397$
$-1.2380$

VARIAMCE SIGN
$0.1637+$
0.5107 +
$0.8219 \quad-$
$0.3699+$
$0.2326-$
$\begin{array}{ll}0.1553 \\ 0.3821\end{array}+$
$\begin{array}{ll}0.3821 \\ 0.16453\end{array}-$
$0.1391-$
0.134
$0.4230 \rightarrow$
0.1926 -
$0.2333+$
$\begin{array}{ll}0.3068 & - \\ 0.2834\end{array}$
$0.3409 \quad 0.0211$
$0.0963+0.0302$
$0.2641 \rightarrow 0.4143$
$0.3334+1.0400$
$0.1721+0.0182$
$0.2201-1.5200$
$0.3131 \quad 0.0208$
0.3041

KURTOSIS
1.9860
2.10440
3.5010
11.2400
2.2890
2.0540
2.3060
5. 1200
2.3300
3.8340
4.4460
2.8870
5.7300
3.5720
4.7590
4.3960
4.8100
2.9170
2.4800
2.1340
5.5790
2.2000
4.9720
2.3850
2.4290

| CLASS MID-PT | MEAN | VARIANCE | SIGN | SKEWNESS | Kurtosis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.2970 | 1.6240 | 0.5433 | - | 0.0392 | 2.3680 |
| 1.6830 | 0.7211 | 0.6117 | - | 0.1784 | 2.7560 |
| 1.4430 | 0.0076 | 0.7603 | - | 0.2704 | 3.8310 |
| 1.2890 | 1.0580 | 0.5394 | - | 0.1055 | 3.7840 |
| 1.1360 | 0.8056 | 0.3986 | - | 0.9398 | 4.2110 |
| 1.0210 | 0.8688 | (1.2890 | + | 0.0239 | 2.1470 |
| 0.9291 | 0.7649 | 0.6274 | - | 0.0000 | 5.0090 |
| 0.3357 | 1.0689 | 0.3076 | + | 0.2166 | 2.5660 |
| 0.7720 | 0.8063 | 0.2104 | - | 0.4189 | 3.2700 |
| 0.7000 | 0.4282 | 0.2545 | + | 0.0253 | 3.2990 |
| 0.6532 | 0.4484 | 0.2318 | - | $1.1800^{\circ}$ | 4.81.30 |
| 0.5038 | 0.5381 | 0.1620 | + | 0.1128 | 3.6450 |
| 0.5193 | 0.5373 | 0.1563 | " | 0.0094 | 2.6910 |
| 0.4557 | 0.4490 | 0.2222 | + | 0.5208 | 4.5860 |
| 0.4092 | 0.3167 | 0.2157 | + | 0.0333 | 2.8070 |
| 0.3662 | 0.231 .6 | 0.1776 | $\cdots$ | 0.0001 | 3.7650 |
| 0.3171 | 0.2685 | 0.4118 | - | 0.0344 | 4.3250 |
| 0.2601 | 0.1134 | 0.1198 | + | 0.2394 | 3.3580 |
| 0.2092 | 0.1612 | 0.3136 | + | 0.1374 | 4.6360 |
| 0.1735 | 0.1856 | 0.2496 | + | 2.8800 | 8.0580 |
| 0.1393 | 0.1036 | 0.1151 | + | 0.0011 | 2.1390 |
| 0.0716 | -0.1252 | 0.2165 | $\cdots$ | 0.4791 | 4.5660 |
| 0.0251 | 0.0772 | 0.1559 | - | 0.0462 | 3.1210 |
| -0.0162 | -0. 0.754 | 0.4714 | - | 0.8361 | 5.1130 |
| -0.0.0487 | -0.2387 | 0.4577 | $\cdots$ | 1.7980 | 6.7460 |
| -0.0850 | -0.1334 | 0.1627 | - | 0.6069 | 3.6850 |
| -0.1300 | -0.2762 | 0.4403 | - | 1.5580 | 9.3360 |
| -0.1739 | -0.2953 | 0.6476 | - | 0.2638 | 4.7120 |
| -0.2132 | -0.0972 | 0.2080 | * | 0.2483 | 3.5270 |
| -0.2663 | -0.5041 | 0.5200 | - | 0.5796 | 3.3050 |
| -0.3045 | -0.2602 | 0.1497 | + | 0.0134 | 1.9770 |
| -0.3375 | -0.3985 | 0.1912 | - | 0.0190 | 3.2170 |
| -0.3708 | -0.4972 | 0.2829 | - | 7.6270 | 12.5300 |
| -0.4170 | -0.2633 | 0.1843 | + | 2.8280 | 6.4040 |
| -0.4601 | $-0.6950$ | 0.3209 | + | 0.0006 | 4.5180 |
| -0.5j.46 | -0.3362 | 0.3974 | - | 0.0057 | 6.5510 |
| -0.5665 | -0.4404 | 0.2589 | + | 1.7170 | 5.7080 |
| -0.6205 | -0.5706 | 0.3853 | + | 0.7133 | 4.6570 |
| -0.6796 | -0.4070 | 0.2270 | + | 0.7526 | 3.3780 |
| -0.7415 | -0.5044 | 0.3121 | * | 1.4840 | 5.9810 |
| -0.8044 | -0.7247 | 0.3040 | - | 2.1070 | 5.3880 |
| -0.3663 | -0.7782 | 0.2708 | - | 0.0094 | 3.7870 |
| -0.9482 | -0.7977 | 0.2331 | + | 0.3792 | 5.1370 |
| -1.0550 | -0.8971 | 0.4383 | + | 1.7510 | 5.2280 |
| -1.1800 | $-1.021 .0$ | 0.4980 | - | 0.0210 | 3.7250 |
| -1.4070 | -1,1960 | 0.5025 | - | 0.2056 | 2.0920 |
| -2.0260 | -0.9532 | 0.6645 | + | 3.8660 | 9.1730 |

IWHatey 19 shpsponp

| CLASS MID-PT | MEAN | VARIANCE | SIGN | SKEMNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0990 | 1.6260 | 0.6714 | - | 0.1888 | 2.5410 |
| 1.2150 | 0.9665 | 0.4213 | + | 1.6960 | 4.8240 |
| 0.9313 | 0.7713 | 0.0596 | - | 0.0268 | 2.5230 |
| 0.7222 | 0.5599 | 0.1529 | - | 0.0443 | 2.3790 |
| 0.5472 | 0.4870 | 0.2763 | + | 0.3553 | 3.5000 |
| 0.4105 | 0.4453 | 0.2127 | + | 0.9435 | 2.6080 |
| 0.2881 | 0.1639 | 0.1317 | + | 0.0114 | 1.8950 |
| 0.1791 | 0.1587 | 0.1980 | - | 0.0311 | 2.9330 |
| 0.0667 | -0.0760 | 0.1863 | - | 0.3723 | 3.1740 |
| -0.0004 | 0.0091 | 0.1233 | - | 0.0349 | 2.0090 |
| -0.1103 | -0.0556 | 0.3293 | + | 2.7140 | 6.9660 |
| -0.2119 | -0.1664 | 0.0885 | + | 0.0544 | 2.3830 |
| -0.3320 | -0.2765 | 0.0995 | + | 0.7403 | 3.8580 |
| -0.4528 | -0.4039 | 0.1729 | + | 0.7183 | 4.3450 |
| -0.5729 | -0.5607 | 0.1308 | - | 0.0633 | 2.5570 |
| -0.6938 | -0.8827 | 0.1088 | - | 0.0907 | 3.5570 |
| -0.0442 | -0.7074 | 0.1285 | - | 0.8154 | 2.9350 |
| -1.1830 | -1.01200 | 0.2172 | + | 0.0002 | 2.0370 |
| -1.7020 | -1.6060 | 0.2083 | - | 0.1464 | 2.8670 |


| CLASS MIM-PT | MEAN | VARIANCE | SIGN | SKEMNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.1420 | 1.9500 | 0.4235 | 4 | 0.0003 | 1.9810 |
| 1.2400 | 1.01910 | 0.1867 | - | 0.3286 | 3.5390 |
| 0.7930 | 0.6050 | 0.1364 | + | 1.8170 | 5.4200 |
| 0.5380 | 0.5019 | 0.0379 | + | 0.2041 | 2.6220 |
| 0.3433 | 0.1943 | 0.0750 | - | 0.0779 | 3.3060 |
| 0.2461 | 0.1293 | 0.0664 | - | 0.4795 | 3.2700 |
| 0.1204 | 0.0837 | 0.0943 | - | 0.2006 | 4.6490 |
| 0.0264 | -0.0263 | 0.0547 | $\cdots$ | 0.0233 | 3.0710 |
| -0.0.593 | -0.1052 | 0.0441 | + | 0.2322 | 3.7340 |
| -0.1363 | -0.1049 | 0.0537 | " | 0.3275 | 2.4120 |
| -0.2304 | -0.1942 | 0.0535 | + | 2.0780 | 6.4630 |
| -0.3474 | -0.3640 | 0.0459 | - | 0.0033 | 2.5840 |
| -0.0.4972 | -0.5191 | 0.0628 | - | 0.5266 | 3.2910 |
| -0.6520 | -0.7131 | 0.0578 | + | 0.0609 | 2.8670 |
| -0.3360 | -0.9835 | 0.3032 | " | 3.0220 | 6.2490 |
| -1.5540 | -1.3730 | 0.6809 | - | 0.0665 | 2.3130 |


| CLASS HIUAPT | HEAN | VARIANCE | SIGN | SKENHESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.1320 | 1.6460 | 0.5311 | - | 1.9920 | 5.8100 |
| 1.7280 | 1.3490 | 0.3488 | " | 0.4751 | 3.2810 |
| 1.4880 | 1.1610 | 0.3258 | + | 0.1241 | 3.4230 |
| 1.2960 | 1.2220 | 0.2627 | - | 2.0240 | 7.0930 |
| 1.1550 | 0.8948 | 0.221 .3 | - | 0.8329 | 4.5530 |
| 1.0610 | 0.8621 | 0.3635 | - | 0.1147 | 2.3130 |
| 0.9775 | 0.8193 | 0.2492 | - | 0.0569 | 2.7800 |
| 0.8917 | 0.7030 | 0.2300 | - | 0. 0.9095 | 3.5490 |
| 0.2209 | 0.6038 | 0.3220 | - | 0.2022 | 2.5640 |
| 0.7144 | 0.6210 | 0.3925 | - | 0.4352 | 3.3900 |
| 0.6111 | 0.6043 | 0.3477 | - | 1.6800 | 5.7740 |
| 0.5341 | 0.4864 | 0.3351 | - | 2.5840 | 8.7510 |
| 0.4659 | 0.4507 | 0.0969 | + | 0.0781 | 4.5440 |
| 0.3826 | 0.4013 | 0.1633 | - | 0.1714 | 2.5190 |
| 0.3049 | 0.2982 | 0.1261 | - | 0.2823 | 3.3670 |
| 0.2493 | 0.0612 | 0.4429 | - | 2.3250 | 6.5520 |
| 0.1721 | 0.1818 | 0.3623 | - | 0.6463 | 4.8760 |
| 0.1104 | 0.2412 | 0.3568 | + | 4.0390 | 7.9430 |
| 0.0454 | -0.0783 | 0.2034 | - | 0.0120 | 2.8360 |
| -0.0272 | 0.0213 | 0.1095 | + | 0.3725 | 3.2750 |
| -0.0714 | -0.0756 | 0.2035 | - | 1.1650 | 5.1280 |
| -0.1271 | -0.1.1558 | 0.2120 | + | 0.0726 | 2.8090 |
| -0.1883 | -0.1977 | 0.1251 | + | 0.0013 | 2.1350 |
| -0.2.634 | -0.2343 | 0.2562 | + | 2.7800 | 7.5430 |
| -0.3330 | -0.3377 | 0.1726 | + | 0.5757 | 5.6170 |
| -0.4189 | -0.4212 | 0.3667 | + | 0.0458 | 3.2990 |
| -0.4944 | -0.5061 | 0.1214 | + | 0.0136 | 3.0010 |
| -0.5686 | -0.5419 | 0.1836 | - | 0.2537 | 2.8190 |
| -0.6277 | -0.4668 | 0.4013 | + | 1.3190 | 6.8400 |
| -0.6935 | -0.6468 | 0.1251 | + | 0.0697 | 2.8070 |
| -0.7659 | -0.6551 | 0.1360 | + | 0.0018 | 2.3870 |
| -0.8336 | -0.8735 | 0.3828 | - | 0.0124 | 3.3340 |
| -0.9369 | -0.8596 | 0.2091. | - | 0.0206 | 5.1190 |
| -1.0410 | -1.0220 | 0.2710 | - | 0.0018 | 3.0770 |
| -1.2100 | -1.0480 | 0.1688 | - | 0.0053 | 2.9900 |
| -1.4390 | $-1.2260$ | 0.6136 | + | 3.6750 | 7.1740 |
| -1.9340 | -1.3050 | 0.6276 | * | 0.5243 | 3.4160 |

IMDUSTRY 20 SURGROUp

| $C L A S S ~ M I D-P T$ | $M E A B$ |
| :---: | :---: |
| 1.03880 | 1.4940 |
| 1.3080 | 1.0670 |
| 0.8220 | 0.5752 |
| 0.5497 | 0.5080 |
| 0.3316 | 0.0963 |
| 0.1211 | 0.1789 |
| -0.1065 | -0.0095 |
| -0.2391 | -0.1372 |
| -0.4377 | -0.2624 |
| -0.6393 | -0.7042 |
| -0.8005 | -0.7261 |
| -1.0710 | -0.9621 |
| -1.6110 | -1.2290 |

VAriance sign skenness kurtosis
$0.4266 \quad 1.1970 \quad 3.8590$
$1.3080 \quad 1.0670$
0.2317 -
0.2200 -
$0.3330 \quad+$
0.1642
0.5944 $0.3116 \quad+\quad 0.2324$
2.8160
0.8220
0.5752
0.5497
0.5080
0.1789
$\begin{array}{ll}0.3116 & \\ 0.0951\end{array}$
0.3675
3.4590
0.3316
0.1211
$-0.2391$
$-0.4377$
$-0.6383$
$-0.8005$
$-1.6110$
-1.2290

14DJSTRY 20 SUGGROUP

| CLASSMID-HT | MEAN | VAFIANCE | SIGN | SKEANESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.3050 | 1.3710 | 0.9794 | - | 1.8920 | 4.4410 |
| 1.3000 | 1.2720 | 0.3988 | - | 0.0091 | 2.2990 |
| 0.9666 | 0.6789 | 0.3109 | - | 1.3160 | 4.8950 |
| 0.7201 | 0.7546 | 0.1352 | - | 0.0713 | 2.6110 |
| 0.5183 | 0.2989 | 0.1814 | + | 0.2375 | 2.2810 |
| 0.3906 | 0.0991 | 0.2275 | - | 1.5620 | 4.4770 |
| 0.2556 | 0.0673 | 0.1517 | - | 0.0613 | 2.2200 |
| 0.1484 | 0.1056 | 0.0879 | + | 0.0240 | 2.0700 |
| 0.0421 | -0.0633 | 0.1028 | - | 0.3130 | 4.2120 |
| -0.0360 | -0.0674 | 0.0542 | - | 0.4432 | 2.8630 |
| -0.0866 | 0.0842 | 0.0779 | + | 1.8700 | 4.7800 |
| -0.1530 | -0.1207 | 0.0626 | + | 0.2647 | 3.1780 |
| -0.2116 | -0.1155 | 0.1353 | - | 0.0006 | 2.2250 |
| -0.2796 | -0.3092 | 0.0518 | + | 0.0042 | 2.4250 |
| -0.3579 | -0.3267 | 0.0548 | + | 0.2566 | 2.6670 |
| -0.4403 | -0.4531 | 0.1264 | + | 0.1190 | 2.5250 |
| -0.4996 | -0.5762 | 0.2298 | - | 0.9463 | 3.5400 |
| -0.5668 | -0.6729 | 0.1640 | - | 0.9774 | 3.5860 |
| -0.6537 | -0.6818 | 0.0952 | + | 0.9563 | 3.5710 |
| -0.7815 | -0.5715 | 0.1288 | + | 1.4470 | 4.2840 |
| -0.9002 | -0.7676 | 0.1755 | + | 0.3252 | 4.0210 |
| -1.0110 | -0.8477 | 0.1538 | + | 0.2906 | 3.0860 |
| -1.3900 | -0.9794 | 0.1367 | + | 0.0003 | 2.5700 |

maustry 21 subgroup 2

| CLASS.MDDMI | MEAV | Vaiclance | SIGN | SKEHNESS | KURTOSIS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.2210 | 1.6730 | 1.0190 | - | 1.6090 | 4.2600 |
| 1. 3560 | 1.2460 | 0.4175 | - | 0.4271 | 3.3400 |
| 0.9231 | 0.91 .57 | 0.1444 | + | 0.1325 | 2.6480 |
| 0.7310 | 0.6966 | 0.3235 | + | 0.4678 | 4.7270 |
| 0.360 .5 | 0.5805 | 0.1138 | - | 0.0928 | 2.6150 |
| 0.4204 | 0.00350 | 0.0723 | $+$ | 0.0909 | 2.1150 |
| 0.3272 | $0.295 ?$ | 0.1902 | + | 0.0020 | 3.1640 |
| 0.2437 | 0.0441 | 0.1475 | - | 0.9999 | 3,8010 |
| 0.1530 | 0.0644 | 0.2478 | - | 0.0334 | 2.8860 |
| 0.0570 | 0.0225 | 0.2083 | - | 0.1019 | 2.5350 |
| -0.0279 | 0.034 J | 0.1093 | + | 0.1099 | 2.4410 |
| -0.1.1.75 | $-0.1133$ | 0.1044 | + | 0.0194 | 2.6000 |
| -0.2002 | $-0.3523$ | 0.1783 | - | 1.3390 | 4.7300 |
| -0.2966 | - -0.253 j | 0.2775 | - | 3.0340 | $8.24<0$ |
| -0.3657 | -0.2466 | 0.2544 | + | 2.4800 | 7,4550 |
| -0.4457 | -0.4801 | 0.1496 | - | 0.7462 | 3.5920 |
| -0.5429 | -0.51.88 | 0.1377 | - | 0.4661 | 3.4830 |
| -0.6165 | -0.6469 | 0.1230 | - | 3.3830 | 7.2590 |
| -0.7222 | -0.6927 | 0.1 .959 | - | 0.3254 | 3.4760 |
| -0.9118 | -0.781.9 | 0.2914 | - | 0.0011 | 2.8620 |
| $-1.1870$ | -1.2560 | 0.3595 | + | 0.0323 | 3.6310 |
| -1.7890 | -1.2570 | 0.5590 | $+$ | 0.0025 | 3.1950 |


| CLASS 4 H-T | nate ${ }^{\text {a }}$ | VATMAGE | $516 N$ | SikEMJESS | KURTOS 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. 3538 | 1.4450 | 0.2368 | - | 0.9568 | 2.8520 |
| 1.2100 | $1.02 \mathrm{S0}$ | 0.6002 | - | 2.7270 | 5.8450 |
| 7.30.4 | a. 8050 | 0.1032 | + | 0.0778 | 2.1340 |
| 0.4605 | 0.4055 | 0.4541 | - | 0.1052 | 6.0110 |
| 0.2854 | 7. 2.295 | [.2392 | + | 0.0044 | 1.7460 |
| -6. 0.82 | -0.1844 | 0.1308 | + | 0.15940 | 2.2620 |
| -7.1915 | -0.2471 | 0.1604 | - | 2.0780 | 5.5040 |
| -9.3-26 | -0.3697 | 0.1155 | - | 1.3860 | 5.7940 |
| -0.50.34 | -0.3365 | 0.0575 | + | 0.0474 | 2.6750 |
| - 0.6724 | -0.5340 | 0.1488 | - | 0.0002 | 4.1560 |
| -0.58te | -0.7663 | 0.2279 | - | 0.0148 | 2.4370 |
| $-1.5510$ | -1.3010 | 0.6400 | + | 2.6100 | 6.7200 |


| Howstor 21 | Subcroup |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Class anm | MEAT | VARIANCE | SIGN | SkEntuss | KURTOSİ |
| 1.8000 | 1.3570 | 0.3891 | - | 0.3048 | 2.4500 |
| 1.1354 | 0.8599 | 0.5865 | - | 2.0330 | 5.4770 |
| 0.6301 | 0.5621 | 0.4310 | - | 0.1273 | 3.4290 |
| 0.3215 | 0.3349 | 0.6934 | + | 0.0042 | 3.2610 |
| 0.2569 | 0.4052 | 0.3977 | + | 0.0860 | 2.8360 |
| 0.0547 | 0.2754 | 0.5953 | + | 0.1191 | 3.5590 |
| -0.1653 | -0.2435 | 0.3592 | + | 0.0028 | 2.5350 |
| -0.3504 | -0.5812 | 0.2448 | - | 0.1136 | 2.7960 |
| -0.5027 | -0.4913 | 0.2616 | - | 0.0059 | 2.6150 |
| -0.0.453 | -0.7513 | 0.3989 | + | 0.4175 | 2.7260 |
| -1.1150 | -0.8543 | 0.3061 | + | 0.0401 | 2.7110 |
| -1.5170 | -0.9281 | 0.2567 | + | 0.3175 | 2.4910 |



| muvstry 21 | Surgroup | 6 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CLASS MIT-PT | MEAN | VARIAMCE | SIGN | SKEVUESS | KURTOSIS |
| 1.9410 | 1.4850 | 0.3604 | - | 0.5388 | 3.7750 |
| 1.4200 | 1.1120 | 0.3713 | " | 0.3730 | 2.7990 |
| 1.1100 | 1.2410 | 0.3895 | + | 0.4537 | 4.0040 |
| 0.8542 | 0.8273 | 0.3356 | $+$ | 0.0053 | 4.0550 |
| 0.6700 | 0.6273 | 0.1546 | + | 1.3640 | 4.0110 |
| 0.4836 | 0.4453 | 0.1172 | + | 0.2518 | 3.2240 |
| 0.2950 | 0.2685 | 0.0955 | - | 0.0220 | 2.3470 |
| 0.1230 | 0.1124 | 0. 2899 | + | 0.3119 | 3.5960 |
| -0.0416 | -0.1962 | 0.3103 | - | 0.7023 | 4.0520 |
| -0.2092 | -0.1774 | 0.1655 | + | 0.1392 | 3.3630 |
| -0.3628 | -0,3491 | 0.3757 | + | 1.1710 | 4.1140 |
| -0.3605 | -0.5701 | 0.1445 | + | 0.2123 | 3.6540 |
| $-0.6926$ | -0.5047 | 0.0766 | + | 0.0508 | 2.6760 |
| -0.0870 | -0.9368 | 0.0727 | $+$ | 0.0156 | 3,3830 |
| -1.0050 | -0.8278 | 0.3521 | + | 1.7110 | 5.0270 |
| $-1.1820$ | -1.0330. | 0.0784 | + | 0.0141 | 3.9540 |
| -1.61.60 | -1.4980 | 0.1777 | + | 1.0350 | 3.5030 |

$A P P E N D T X D$

THE FTTTFD EOTTATTONS

The results of fitting the nower function forms (see Chapter VI Section I) are given for both industries and subgroups:

See Chapter VIT for discussion of these results.

























[^25]See Chapter VTII Sections $I$ and 2 for discussion of these results.


| Linear |
| :--- |
| Cubic $\left(1 C D^{-}\right)$ |


Negative Range
合


Linear
Cubic（LCD ${ }^{+}$）

 on
0
0 $\stackrel{\circ}{\circ}$ $\stackrel{\circ}{\circ}$ $\stackrel{\text { a }}{\infty}$ N 동 $\stackrel{+}{\circ}$ に．
$\stackrel{\circ}{0}$
$\stackrel{0}{\circ}$ $\stackrel{-1}{\circ}$ 웅 $\stackrel{n}{\infty}$
$\dot{\circ}$
$\dot{\circ}$

Industry Level Decay Measures

＋
0.792
0.760
0.871
0.710
0.851
0.630

0.706 $\stackrel{\text { n }}{\substack{0 \\ \vdots \\ 0 \\ ~}}$ 등 $\begin{array}{ccc}0 & 0 \\ 0 & N \\ 0 & 0 & 0 \\ 0 & 0\end{array}$ N 운 \begin{tabular}{c}
N <br>
\hline <br>
0 <br>
0

 $\stackrel{\infty}{\circ}$ 

$\infty$ <br>
$\infty$ <br>
0 <br>
0 <br>
\hline
\end{tabular} Industry

No H + に $6 \rightarrow \infty$ の ๙ ～ Æ － 거N


Industry No.
 $\underset{\sim}{n} \underset{\sim}{\infty} \underset{\sim}{\infty} \underset{\sim}{\infty}$ 98 N

| Subgroup |
| :---: |
| No |

$$
\sim i n \rightarrow H H H H H m H \sim m m m \text { N }
$$

$$
5
$$

| Industry No. | Subgroup No. | Linear ( $1 \mathrm{D}^{+}$) | Linear Cubic ( $1 \mathrm{CD} \mathrm{D}^{\mathrm{f}}$ ) |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 0.843 | 0.841 |
|  | 3 | 0.972 | 1.100 |
| 4 | 1 | 0.812 | 0.838 |
|  | 2 | 0.759 | 0.763 |
|  | 4 | 0.888 | 0.921 |
|  | 5 | 0.845 | 0.847 |
| 5 | 3 | 0.800 | 0.916 |
|  | 4 | 0.858 | 0.862 |
| 6 | 4 | 0.802 | 0.843 |
| 7 | 1 | 0.808 | 0.809 |
|  | 2 | 0.737 | 0.737 |
| 8 | 1 | 0.636 | 0.503 |
| 9 | 1 | 0.665 | 0.645 |
| 11 | 1 | 0.800 | 0.792 |
|  | 2 | 0.907 | 0.999 |
| 12 | 2 | 0.940 | 1.025 |
|  | 4 | 0.794 | 0.763 |
|  | 6 | 0.807 | 0.799 |
| 13 | 1 | 0.897 | 0.901 |
| 15 | 1 | 0.760 | 0.758 |
|  | 2 | 0.850 | 0.881 |
|  | 3 | 0.890 | 0.816 |
| 16 | 1 | 0.823 | 0.766 |
|  | 2 | 0.750 | 0.762 |
|  | 3 | 0.859 | 0.822 |
|  | 4 | 0.690 | 0.721 |
|  | 5 | 0.914 | 0.823 |
| 17 | 1 | 0.802 | 0.740 |
| 18 | 1 | 0.880 | 0.906 |
|  | 2 | 0.780 | 0.739 |
|  | 3 | 0.787 | 0.781 |
| 19 | 1 | 0.887 | 0.870 |
|  | 2 | 0.980 | 0.967 |
|  | 3 | 0.860 | 0.864 |
| 20 | 2 | 0.840 | 1.055 |
|  | 3 | 0.868 | 0.966 |
| 21 | 2 | 0.958 | 0.964 |
|  | 3 | 0.871 | 0.852 |
|  | 4 | 0.812 | 0.725 |
|  | 5 | 0.741 | 0.672 |
|  | 6 | 0.910 | 0.894 |

$A P P F_{1} N D I X N$


Gawver's 1-firm enncent, ration Ratio
Four-firm fimnlnument, Conneentrotinn ratins for rinimum tist Heading industries for 1958 are şiven hy Gayer in his artiote "Concentration in Rritish Manufoncturines Tndustry", Nxened ioronnmin Papora vol 231971,

 this stury. Dofinitinng of tho suhgroung in torme: rf myts aro givan in

 maximum and minimum ration, thoir avoras, has hoon losed. This afforte Tnduatiry 4 subgroun 5, 7 amberoun 1 and $1 \underline{3}$ suhgrouns $?$ and 3. :More $a$ subgenim invalues some non-manufacturins, antivitiss, these have heen dispogarded in calculating the enneontration ratin. rases of this are: Tndustry 1 suhgroun ?, Tndustry 6 , Tniluctry 17 suhermin $?$ and Tndustery 13 anherning 1. Tn a mumber of sages, the suhgraun dofinjtinns involuo
 whorever a enmmonent, of it, is oallod for: Tnduatry a suhermons a and 5 , Tndustry 7 suberoun 1 and insustry 16 aubgroun 5.

Finally, Fawrer does not, sive a fiserme for Sonstruntion hilt, using the methnd he descrihes in nngendix TTT of his artinle and data from the 1958 Cenalv of fronuction Pt, 133, Ta.hา 4 , maximum ant minimum values were calcmlated.

## Whittinerton's Noneentration Ration

The values are riven in "rho Predintion or Profitahility" riohle 3" . The ratin is defined as "the ration of tho sum of the not. :ssets of large onmpanies ton the sum of the not asceta nf all runtor momnanios in the relevant indmatery". (inly enmbanjes which montimod from 7951 ton lakn aro
included and 'large' is defined as having net assets of greater than \&4 million in 1954. It is only available at the industry level.

## Variance of Logarithm of Size, Variance of Size and Average size

These three measures all used net assets as the measure of size and all are calculated for quoted companies only. The logarithms in the first are Napierian.

## Industry Level Measures of Structure

| Industry No. | Sawyer's 4-firm Concentration Ratio | Whittington's Concentration Ratio | Variance of Logarithm of Size | Variance of size | Average Size |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 29.0 | 66.8 | 1.55 | $1.64 \times 10^{7}$ | 1546 |
| 4 | 25.4 | 61.4 | 1.48 | $2.63 \times 10^{7}$ | 2043 |
| 5 | 42.6 | 86.2 | 2.20 | $1.01 \times 10^{8}$ | 4148 |
| 6 | 46.7 | 81.2 | 2.67 | $9.46 \times 10^{7}$ | 4238 |
| 7 | 19.5 | 60.9 | 1.33 | $2.84 \times 10^{7}$ | 1491 |
| 8 | 16.8 | 68.8 | 0.74 | $3.04 \times 10^{6}$ | 1003 |
| 9 | 10.6 | 37.1 | 0.84 | $8.42 \times 10^{6}$ | 1424 |
| 11 | 14.3 | 39.9 | 1.28 | $5.96 \times 10^{6}$ | 1150 |
| 12 | 34.3 | 84.6 | 2.08 | $3.75 \times 10^{7}$ | 2553 |
| 13 | 25.1 | 75.2 | 1.39 | $8.59 \times 10^{7}$ | 3490 |
| 15 | 20.3 | 70.9 | 2.02 | $3.86 \times 10^{7}$ | 2502 |
| 16 | 20.9 | 56.0 | 1.12 | $3.54 \times 10^{7}$ | 1267 |
| 17 | 5.6 | 26.5 | 1.55 | $3.20 \times 10^{6}$ | 1158 |
| 18 | * | 42.5 | 1.16 | $8.61 \times 10^{6}$ | 1330 |
| 19 | * | 76.2 | 1.94 | $4.37 \times 10^{7}$ | 2305 |
| 20 | * | 72.5 | 2.22 | $3.06 \times 10^{7}$ | 1340 |
| 21 | * | 62.3 | 1.95 | $1.17 \times 10^{7}$ | 1329 |


| Industry No. | Subgroup No. | Sawyer's 4-firm Concentration Ratio | Variance of Logarithms of Size | $\begin{gathered} \text { Variance of } \\ \text { Size } \end{gathered}$ | Average Size |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & 2 \\ & 3 \end{aligned}$ | $\begin{aligned} & 21.5 \\ & 14.5 \end{aligned}$ | $\begin{aligned} & 1.56 \\ & 0.75 \end{aligned}$ | $\begin{aligned} & 3.85 \times 10^{6} \\ & 5.78 \times 10^{5} \end{aligned}$ | $\begin{array}{r} 1191 \\ 881 \end{array}$ |
| 4 | $\begin{aligned} & 1 \\ & 2 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{aligned} & 34.2 \\ & 17.5 \\ & 20.5 \\ & 22.6 \end{aligned}$ | $\begin{aligned} & 1.48 \\ & 0.93 \\ & 0.58 \\ & 1.43 \end{aligned}$ | $\begin{aligned} & 1.95 \times 10^{7} \\ & 4.50 \times 10^{5} \\ & 4.16 \times 10^{5} \\ & 2.70 \times 10^{7} \end{aligned}$ | $\begin{array}{r} 3010 \\ 1061 \\ 779 \\ 2123 \end{array}$ |
| 5 | $\begin{aligned} & 3 \\ & 4 \end{aligned}$ | $\begin{aligned} & 40.1 \\ & 38.2 \end{aligned}$ |  | $\begin{aligned} & 1.64 \times 10^{7} \\ & 1.72 \times 10^{7} \end{aligned}$ | $\begin{aligned} & 3431 \\ & 2057 \end{aligned}$ |
| 6 | 4 | 44.4 | 1.25 | $3.79 \times 10^{6}$ | 1357 |
| 7 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 18.7 \\ & 24.3 \end{aligned}$ | $\begin{aligned} & 1.30 \\ & 1.32 \end{aligned}$ | $\begin{aligned} & 3.13 \times 10^{7} \\ & 6.19 \times 10^{6} \end{aligned}$ | $\begin{aligned} & 1442 \\ & 1682 \end{aligned}$ |
| 8 | 1 | 24.0 | 0.68 | $2.61 \times 10^{6}$ | 915 |
| 9 | 1 | 10.6 | 0.81 | $5.75 \times 10^{6}$ | 1389 |
| 11 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 14.3 | 1.15 1.25 | $4.83 \times 10^{7}$ $3.82 \times 10^{6}$ | $\begin{array}{r} 953 \\ 1651 \end{array}$ |
| 12 | $\begin{aligned} & 2 \\ & 4 \\ & 6 \end{aligned}$ | $\begin{aligned} & 28.6 \\ & 38.9 \\ & 31.9 \end{aligned}$ | $\begin{aligned} & 1.61 \\ & 1.73 \\ & 2.19 \end{aligned}$ | $\begin{aligned} & 8.88 \times 10^{6} \\ & 5.79 \times 107 \\ & 1.74 \times 10^{7} \end{aligned}$ | $\begin{aligned} & 1482 \\ & 3152 \\ & 2083 \end{aligned}$ |
| 13 | 1 | 19.1 | 1.29 | $2.18 \times 10^{7}$ | 3146 |
| 15 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | 27.4 29.3 7.8 | $\begin{aligned} & 2.13 \\ & 1.48 \\ & 1.43 \end{aligned}$ | $\begin{aligned} & 5.21 \times 10^{7} \\ & 2.83 \times 10^{7} \\ & 1.06 \times 107 \end{aligned}$ | $\begin{array}{r} 2907 \\ 3973 \\ 903 \end{array}$ |
| 16 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{array}{r} 36.1 \\ 7.6 \\ 16.7 \\ 24.8 \\ 23.4 \end{array}$ | $\begin{aligned} & 2.30 \\ & 0.67 \\ & 0.69 \\ & 0.57 \\ & 1.20 \end{aligned}$ | $1.89 \times 10^{8}$ $2.82 \times 10^{5}$ $4.36 \times 10^{5}$ $2.76 \times 10$ $2.77 \times 10^{2}$ | $\begin{array}{r} 4008 \\ 602 \\ 568 \\ 603 \\ 1030 \end{array}$ |
| 17 | 1 | 5.6 | 1.62 | $1.89 \times 10^{6}$ | 1125 |
| 18 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | * | $\begin{aligned} & 1.02 \\ & 1.29 \\ & 1.14 \end{aligned}$ | $\begin{aligned} & 5.83 \times 10^{6} \\ & 1.59 \times 10^{7} \\ & 4.23 \times 10^{6} \end{aligned}$ | $\begin{aligned} & 1310 \\ & 1255 \\ & 1362 \end{aligned}$ |
| 19 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | * | $\begin{aligned} & 1.51 \\ & 2.70 \\ & 1.69 \end{aligned}$ | $\begin{aligned} & 2.13 \times 10^{7} \\ & 1.06 \times 107 \\ & 2.58 \times 107 \end{aligned}$ | $\begin{aligned} & 1881 \\ & 5544 \\ & 1521 \end{aligned}$ |
| 20 | $\begin{aligned} & 2 \\ & 3 \end{aligned}$ | * | $\begin{aligned} & 1.18 \\ & 1.98 \end{aligned}$ | $\begin{aligned} & 1.89 \times 10^{5} \\ & 4.96 \times 10^{6} \end{aligned}$ | $\begin{aligned} & 317 \\ & 732 \end{aligned}$ |
| 21 | 2 3 4 5 6 | $*$ $*$ $*$ $*$ $*$ | $\begin{aligned} & 1.26 \\ & 1.18 \\ & 2.49 \\ & 1.83 \\ & 1.84 \end{aligned}$ | $\begin{aligned} & 7.11 \times 10^{5} \\ & 3.91 \times 100^{5} \\ & 3.64 \times 10^{7} \\ & 2.87 \times 100^{2} \\ & 2.74 \times 10^{2} \end{aligned}$ | $\begin{array}{r} 608 \\ 536 \\ 2197 \\ 2841 \\ 388 \end{array}$ |

This is calculated as the averace of the ratee of return of companies for each of the years 1948-1960.

Trowth Rate of Net Assets
Taken from. Whit, ington's "Prediction of Profitability" Table 2.4. Tt, is the compound annual rate calculated after ad juatmont,s have heon made for asset pevaluations and chanses of accounting date. These figures are based on continuine comnanjes only.

Measures of the Variability of Tncuatrer ivorase Rate of Retium
For each induatry, the annual average rates of return were salmilated. Then each industry series was regressed on a linear fime trend The coefficient of time in that enuation is referred to as "the Trend".

## Industry Level Measures of Performance

| Industry <br> No. | Average <br> Rate of Return <br> on Net Assets | Growth <br> Ret Assets | Trend in the <br> Average Rate <br> of Return | Standard Deviation <br> of the Average <br> Rate of Return | Standard Deviation <br> of Errors about <br> the Trend in |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (\%) |  |  |  |  |  |


| Industry No. | Subgroup No. | Average Rate of Return on Net Assets <br> (\%) | Trend in Average Rate of Return <br> (\% pts. p.a.) | Standard <br> Deviation of Average Rate of Return (\% pts.) | Standard Deviation of Errors about the Trend of Average Rate of Return |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 15.5 | 0.32 | 1.6 | 0.8 |
|  | 3 | 15.7 | -0.56 | 3.0 | 2.6 |
| 4 | 1 | 16.3 | -0.41 | 1.9 | 1.1 |
|  | 2 | 22.0 | -0.92 | 4.3 | 2.7 |
|  | 4 | 17.1 | -0.87 | 4.0 | 3.1 |
|  | 5 | 19.9 | -0.60 | 2.3 | 0.4 |
| 5 | 3 | 19.0 | -0.23 | 2.1 | 2.0 |
|  | 4 | 20.3 | -0.03 | 1.6 | 1.3 |
| 6 | 4 | 20.5 | -0.68 | 2.9 | 1.5 |
| 7 | 1 | 20.7 | -0.53 | 2.8 | 1.1 |
|  | 2 | 15.1 | -0.06 | 1.6 | 1.3 |
| 8 | 1 | 13.7 | -2.10 | 9.5 | 6.4 |
| 9 | 1 | 16.5 | -1. 50 | 6.1 | 2.5 |
| 11 | 1 | 14.3 | -0.59 | 4.6 | 3.0 |
|  | 2 | 15.5 | -0.34 | 2.8 | 2.6 |
| 12 | 2 | 17.9 | -0.70 | 2.8 | 1.3 |
|  | 4 | 19.9 | -1. 20 | 6.3 | 4.9 |
|  | 6 | 15.7 | 0.05 | 1.0 | 0.8 |
| 13 | 1 | 11.2 | 0.25 | 1.0 | 0.2 |
| 15 | 1 | 18.3 | -0.54 | 5.8 | 4.1 |
|  | 2 | 17.0 | 0.19 | 1.6 | 1.3 |
|  | 3 | 13.9 | -0.32 | 2.3 | 1.5 |
| 16 | 1 | 15.8 | -0.48 | 3.5 | 2.8 |
|  | 2 | 12.3 | -0.05 | 2.9 | 2.4 |
|  | 3 | 13.8 | -0.05 | 3.4 | 2.9 |
|  | 4 | 11.7 | -0.16 | 6.8 | 3.6 |
|  | 5 | 16.5 | -0.08 | 2.4 | 2.1 |
| 17 | 1 | 17.2 | -0.13 | 1.8 | 1.5 |
| 18 | 1 | 14.5 | -0.15 | 0.9 | 0.5 |
|  | 2 | 14.2 | -0.10 | 1.7 | 1.2 |
|  | 3 | 13.6 | -0.66 | 3.5 | 1.4 |
| 19 | 1 | 14.6 | 0.20 | 1.3 | 0.9 |
|  | 2 | 13.6 | 0.20 | 1.4 | 1.1 |
|  | 3 | 18.0 | -0.20 | 1.5 | 0.7 |
| 20 | 2 | 15.5 | 0.05 | 2.4 | 2.4 |
|  | 3 | 11.7 | 0.03 | 1.5 | 1.1 |
| 21 | 2 | 11.4 | 0.46 | 1.8 | 0.7 |
|  | 3 | 15.4 | -0.42 | 2.2 | 1.7 |
|  | 4 | 13.1 | 0.07 | 0.6 | 0.6 |
|  | 5 | 11.3 | 0.32 | 1.8 | 1.4 |
|  | 6 | 12.9 | 0.21 | 1.1 | 0.6 |


[^0]:    1) References to this literature are given in Chapter II.
[^1]:    3) Arrow K J, "The Firm in General Equilibrium Theory" in Marris R and Woods A (Eds) "The Corporate Economy", Macmillan London 1972, p 68 where he states that in classical theory the role of the firm was "that of overcoming disequilibria."
[^2]:    5) See "Company Income and Finance 1949-1953", NIESR 1956, Appendix $A$, where it is shown that an average of $87 \%$ of the employees of quoted companies classified to a particular SIC order worked in establishments of that order. It is thus not a serious simplification to regard firms as operating in a single industry.
[^3]:    6) Downie J, "The Competitive Process", Duckworth 1958, similarly concludes that births are unimportant, p 101
[^4]:    9) This range of competence is likely to include more than just technological factors; different marketing skills are clearly of relevance. It might be useful to regard both firms and markets as embedded in a space whose dimensions are measures of such relevant factors as technology and marketing. The subset examined by any firm might then be defined as an area of that space centred upon the point at which the firm is located.
[^5]:    10) Bain J S, "Rarrjers to New Competition", Cambridge Mass. Harvard University Press 1956: "Entry requires both the arrival of a new legal entity in the industy and an addition to industry capacity in use."
[^6]:    22) Andrews P W S, "Industrial Analysis in Economics" in Andrews P W S and T Wilson (eds) "Oxford Studies in the Price Mechanism", 00 P 1951 p 169, "The market will be in equilibrium as long as - any loss of capacity due to businesses being driven from production is made up by extensions to existing capacity or by the entry of new capacity."
[^7]:    27) op cit $p 113$ "... makes it possible to virtually ignore the fact that an expanding firm needs to pursue an active selling policy in order to win new customers."
    28) op cit p 33
[^8]:    29) He calls these "dependent firms." and argues that "independent firms" (i.e. new firms) cannot be significant influences because of the constraint upon new capital.
    30) op cit p 103
    31) op cit p 86
[^9]:    3) This is a steady state solution for that particular transition matrix. Tt has not been proved here that this will be reached whatever the starting distribution, but this must, be so for a onmnletely ergodic process.
[^10]:    7) Feller W, "An Introduction to Probability Theory and its Applications" Vol I 3rd edition, John Wiley \& Co, New York 1967, p 375 refers to A Kolmogorov for the theory of chains with infinitely many states, his work being briefly reported in a German language paper and fully developed only in Russian.
    8) Feller op. cit. p 115
[^11]:    9) The demonstration of this analogy is obviously non-rigorous, hopefully it does have a heuristic value. It is done in the spirit expressed by Feller op. cit. p 444 when, having stated that much of Markov process theory is beyond the scope of the book, he says: "However many problems connected with such processes can be treated by quite elementary methods provided it is taken for granted that the processes actually exist. We shall now proceed in this manner."
[^12]:    9) Stigler G J, "Capital \& Rates of Return in Mamufacturing Industry", NBEK 1963, Ch 3 deals with inter-industry equilibration and its effects on rates of return. See alan Chapter IX sertion $?$ of this study.
[^13]:    * indicates - for skewness: significantly different from zero at the $10 \%$ level
    - for kurtosis: significantly different from 3 at the $5 \%$ level

[^14]:    7) See Pretorius S J, "Skew Bivariate Frequency Surfaces", Biometrika Vol 22 1930-31 for a study of the type that would be appropriate to the characteristics of the data so far uncovered.
    8) Kendall M G \& Stuart A, "Advanced Theory of Statistics" Vol I, Charles Griffin \& Co Ltd London, 3rd edition 1969 p 173.
[^15]:    is the leptokurtosis. Both these being enhanced when only "signjficant" values of these two statistics are taken.

    The general conclusion is that none of the characteristics of the transition matrices found for industries stemmed from their being aggregates of heterogeneous subgroups. Or at least reducing the degree of aggregation and increasing the homogeneity has not removed any of these characteristics.

[^16]:    i?) Three industries ( $8,9,20$ ) show mo relntionshin in either rance, inductries 16 and 17 none in the nocitive rance and 6 in the negative ranco.

[^17]:    1) It may be argued that the unavoirable omission of the death of firms will also lead to a bias in the lowest class.
[^18]:    5) An F-test could be applied to the pair of coefficients of $r^{2}$ and $r^{3}$, or we might use the corrected $R^{2}$ to guide equation choice. The final selection would not be affected. It is helpful, though, to keep the t-statistics of the individual parameters in view in this particular exercise, as the precision of parameters is important when
[^19]:    1) Aspin A A, "Pables for Use in Comnarisons Whose Acouracy Involves Two Variances Separately Fistimated", Biometrika 1949 nn 290-293
    2) Aspin A A, op cit p 290
[^20]:    6) See Section 4.5
[^21]:    The conclusion of this section is that what evidence we have found supports the idea that the less competitive the industry the slower the rate of decay of profitability, but the evidence is by no means conclusive.

[^22]:    5) Whittington $r$, on cit np 91-97
[^23]:    21) Whittington on cit pp 97-707 consiriers various enmhinations of profitabilities in previous years as indenendent variables in his linear decay function.
[^24]:    Summary statistics of the distributions of rates of return on net assets are provided for each industry for each of the years 1948-1960.

[^25]:    The rates of decay are given for each range and each equation form at both industry and subgroun level.

