THE DEVELOPMENT OF THE CHILD'S UNDERSTANDING OF FULLNESS AS A RATIO

A thesis submitted in fulfilment of
the regulations governing the Degree of
Doctor of Philosophy of the University of Stirling

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December, 1970.

ACKNOWLEDGEMENTS

First of all I wish to thank Professor P. McEwen for permitting me to continue my research as a part-time student in the Psychology Department of the University of Stirling.

I am also grateful to the principals, teachers and pupils of the schools where I carried out the study and to the adult subjects from the University of Stirling.

Finally, I wish to acknowledge the continued guidance and assistance of Mr. Michael F. Moore who supervised the thesis work in all its stages.

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SUMMARY

One of the central problems in the understanding of the child's cognitive development is his acquisition and subsequent modification of concepts. The present study is concerned with the mathematical concept of ratio, and more specifically with the development of the child's understanding of fullness as a ratio.

The idea of ratio might be said to involve a quantitative relation between two or more features that remains invariant regardless of the nature of the component features. Such a quantitative relation is typified by the idea of a physical proportion, and it is this which will be investigated: with proportion as embodied in the question, "Which of two containers is fuller (or emptier)?"

"Fullness" is a very interesting concept to investigate, for it involves in its very definition a ratio or proportion between the volume of a container and the volume of the substance contained. Thus in the case of proportion, looked at formally, there is an especially great difficulty for the child: in order to make a correct judgment of fullness or emptiness, the child must resist the temptation to follow perceptual appearances, and instead must use a symbolic operation somewhat like computing a ratio. He must estimate the volume of the container that is filled and then relate this volume to the total volume of the container.

This is quite a difficult task for the child to master, but it is plain that in some primitive form the child can deal with 'full' and 'empty' long before he understands ratios. The child's first use of the terms is limited and he does not grasp what is meant by the comparative forms 'fuller' and 'emptier'. However, at some stage the child goes beyond this partial and restricted idea to a more complete understanding of fullness as a ratio.

At the Harvard Centre for Cognitive Studies, Jerome S. Bruner (1964, 1966) has recently been concerned with this very aspect of the child's cognitive development. He investigated the concept of fullness by requiring subjects aged five, six, seven, nine and eleven to judge which of two glasses was fuller and which emptier and presented eleven pairs of glasses in all differing with respect to the height of the glasses, the diameter of the glasses, the height of the water, the volume of the water, the volume of unfilled space in each glass, and finally the proportion full and proportion empty. On the basis of his results he claims that it is not until age eleven that the child can resist perceptual cues and judge fullness correctly.

There were, however, several shortcomings in both Bruner's design and his procedure which cast doubt upon these findings. It was therefore decided to replicate this experiment modifying and elaborating where considered necessary. (Hanlon, 1967)

The results of this experiment showed that the eleven year old child, contrary to Bruner's conclusions, did not fully understand the concept of ratio through the idea of fullness so the age range studied was subsequently extended to 15 years (Moore and McWhirter, in press). However, it soon became clear that even the older children were not responding correctly, and it seemed that the concept was causing difficulty through misunderstanding rather than inability to perform the necessary operations. The children judged according to their interpretation of the concept and in so doing gave sporadic glimpses that if the problem were communicated to them more precisely they could respond correctly.

A further extension was therefore undertaken in which an attempt was made to communicate the problem more exactly to a representative sample of children in the same age range and a comparison was also made between the performance of adult subjects in the original task and in the new experiment where aid was given. The original subjects served as a control group. The results were considered both within the context of the development of the child's understanding of fullness as a ratio and also in relation to the findings of Bruner and Kenney.

PART 1

REVIEW OF THE RELEVANT AND RELATED LITERATURE

CHAPTER I

INTRODUCTION

Webster's Dictionary defines mathematics as the "science treating of the exact relations existing between quantities or magnitudes and operations". Mathematical concepts thus include those of quantity, space and number as a system, and like all conceptual development may be investigated in two ways - how concepts are acquired, and what concepts are known at different age levels. These are the overlapping categories of process and product.

Studies of the understanding and use of mathematical concepts have included the following: inventories of the number knowledge of children at various age levels; experiments on the learning of quantitative concepts; longitudinal studies of the mathematical ability of individual children; studies of the mathematical vocabulary of children; experiments on the teaching of number to very young, or older but mentally deficient, children; and studies which attempt by various procedures to accelerate the acquisition of mathematical concepts both in adults and in children. Although these studies may differ in details of procedures, tests used, populations sampled, and the nature of the sampling, the greater number of them do have at least one feature in common. They represent an attempt to determine the competences of subjects at differing ages in the area of mathematics. Furthermore, as Martin (1951) rightly points out, most of the studies tend to give "maximal" performance rather than "typical" performance. Because the majority of them involve structured interviews, their findings may be more forced than those that would have been obtained using less structured situations; as indeed, children's language in the classroom differs from that in the playground.

According to Goodenough (1930), however, "it seems to be true that .... the situations which tend to call forth the maximum display of a given form of behaviour for the majority of the subjects may be regarded as optimum for securing a reliable series of observations on such behaviour". As long as the present, everyday, typical behaviour of children is appreciably reflective of their capacities, any distinction between what they can do and what they do do has little meaning. Unfortunately, this argument was not supported by Martin's (1951) study on quantitative expression in young children. Martin, however, does concede that the absence of any relationship may have been due to the inadequacy of the sample of behaviour he chose to study, with respect both to content and to the dimension of time involved.

Studies of the type enumerated above do not give directly information regarding the developmental process by which children become adept in the area of mathematics. Rather they give only the resultants of that process as it has operated over a period of time determined by the age of the child studied. This does not mean, however, that the data obtained from such investigations do not permit inferences to be made concerning the developmental process. It is in effect, experiments of this type which reveal the nature of the growth of the mathematical concept of ratio or proportion and how it may be modified by experience and situational factors.

CHAPTER II

PROPORTION

A. ARITHMETICAL PROPORTION

The relation between two similar magnitudes in respect of quantity, determined by the number of times one contains the other, either integrally or fractionally, may be defined as ratio, and the equivalence of two ratios is proportionality.

Lunzer and Pumfrey (1966) use the simple example of converting a set of measurements from yards to metric units. Any pair of measurements in yards yields a ratio, and this same ratio will appear between the corresponding metric measures. By the same token, the ratio of the first length in yards and the corresponding measurement in metres will also be that between the other pair of corresponding measures. For we know that if

$$a/b = c/d \text{ then } ad = bc \text{ and } a/c = b/d.$$

Simple conversions of this type are instances of direct proportionality and such problems can be solved by the discovery of just one ratio: there is no need for the child to think of both equivalences. For example, if he knows that two inches is equivalent to five centimetres then he can easily discover the metric equivalent of a foot and the only ratio with which he need be concerned is 5:2 or 5/2. This ratio can be carried over to twelve inches, and the result expresses the equivalence in ratios

$$2:5 = 12:30$$

The child is not at all concerned to know that

$$2:12 = 5:30$$

Proportional association appears to function very early in the child but first of all in an unquantified sense. It is only later that the quantified form appears.

Winch (1913), for example, found that seven year old children who had never attempted or been given work on proportions, could deal with proportion sums unquantitatively especially when only proceeding to or from unity, could give correct numerical answers to incomplete proportion sums and could be taught to give the numerical solution of complete proportion sums by the method of unity.

In a study (Ginsburg H. & Rapoport A., 1967) of children's estimates of proportions of elements similar results were found. Here six and eleven year old subjects succeeded in estimating proportions of black and white marbles fairly accurately (with a slight tendency to overstimate low proportions and underestimate high ones) without using numerical operations. The experimenters suggest that early in the sequence the children made rough estimates of the relative frequencies in an "intuitive way" (e.g., "there are slightly more white ones than black") and then, as new elements were presented, appropriately modified the original estimate. The proportions first occurred in an unquantified form and then became quantified.

Ginsburg himself (1967) corroborates this evidence and concludes on the basis of another study that the young child is a much better "intuitive statistician" than one would expect and is, indeed, surprisingly adept at dealing with a variety of statistical problems: by the age of four he is generally accurate at estimating simultaneously presented proportions; by the age of six his estimates of successively presented proportions are generally accurate and by the age of twelve he can derive from a sample of data reasonable inferences concerning population proportions.

Piaget's (Piaget and Inhelder, 1948, 1951) work on the quantification of probabilities reveals a similar pattern. Initially the child is unable to apply any systematic or probabilistic strategy to this kind of problem. During middle childhood the child begins to quantify the probabilities but predicts solely on the basis of the absolute number of quantities involved; he seems incapable of reasoning in terms of the proportions. Piaget believes that proportions involving, as they do, relations established between other relations and thus operations performed on operations, require a formal-operational structure for their mastery. And, indeed, the protocols he cites suggest that few of his subjects under ten or eleven years of age can systematically solve these quantification of probability problems.

From eleven years onwards the relations between the relevant and all possible instances are established for all cases and the child understands the probabilities even when the favourable and all possible cases both differ. The problem is solved by the calculation of fractions or at the beginning of this, the third stage, by successive analyses of the different intuitive operations and their logical relations.

"C'est ainsi que .... la loi des grandes nombres suppose l'emploi des proportions et que les proportions elles - mêmes, étant des rapports de rapports, requièrement psychologiquement l'intervention du formel .... mais surtout, l'achèvement des notions probabilistes implique le recours aux opérations combinatoires (combinaisons, permutations et arrangements)." (Piaget and Inhelder 1951, page 172).

Lovell (1968), too, believes that the schema of proportion requires formal thought. Using a Piagetian-type approach, he gave subjects aged nine to fifteen years twenty tasks involving both proportion and ratio or arithmetic series or geometric series that could be solved by multiplication or division by a whole number: he demonstrated that proportion depends on some central intellective ability which underlies performance on all tasks involving proportion but that specific abilities also contribute to the ability to use proportionality in particular tasks, although tasks involving ratio will depend less on the central-intellective ability than those involving proportion. Even at fifteen years of age the number of responses at or near the state of formal thought in about one third of his tasks was still less than 50 per cent.

B. PHYSICAL PROPORTION

Apart from this arithmetical sense the notion of a ratio may be viewed purely as a relational concept; that is, it depends upon a relationship between perceptible features in the world but cannot itself be indicated ostensibly. As such, it is typified by physical proportion, and has been studied by Piaget and other experimenters interested in the development of the concepts of time, movement and velocity, space, weight, quantity and volume. Physical proportion has also been investigated in such studies as those dealing with the equilibrium balance, the hauling of a weight on an inclined plane, the projection of shadows, centrifugal force and compensations.

(i) Time, movement and velocity

The concepts of time, movement and velocity are closely related both logically and in terms of their psychological development. In a series of experiments on these constructs, Piaget (Piaget, 1946) investigated children's ability to estimate the velocities of successive movements, their skill at conserving uniform velocity and their understanding of uniformly accelerated movement. These studies entailed the quantification of velocities and velocity differences either by establishing simple proportions among times and distances or by actually making estimates of the arithmetic value of the distance-over-time ratio and revealed that developmental progress continues into early adolescence.

Proportion at this stage becomes extensive rather than intensive and mathematical rather than qualitative. It is understandably a late acquisition as compared to simple simultaneous comparisons because the child must compare not only the part to the whole but also the part in comparison to the remainder: "L'opération constitutive de la proportion n'est pas autre chose que cette relation entre deux parties". (Piaget 1946)

(ii) Experiments on Logical Thinking

(a) Equilibrium in the Balance

Although all Piaget's work has something to do with logic, some of his specific studies (Inhelder & Piaget 1958) in this area pursue the question of proportionality. In one study the problem of proportion was introduced through the use of a simple balance-type weighing instrument. When two unequal weights W and W^1 are balanced at unequal distances from the axis L and L^1, the amounts of work WH and WH^1 needed to move them to heights H and H^1 corresponding to these distances are equal. Thus there is an inverse proportion:

$$W/W^1 = L^1/L = H^1/H$$

Finding the law presupposes the construction of the proportion $W/W^1 = L^1/L$ and explaining it requires an understanding of the proportion $W/W^1 = H^1/H$. Again it is discovered that it is not until ten or eleven years of age that the child understands the proportions involved.

Similar results were also found by Lunzer and Pumfrey (1966) for children six to fifteen years of age in a study using both the pantograph and the balance, but pupils who solved the latter problem using proportional reasoning in this study did so quite mechanically: they multiplied and divided because this procedure gave them the correct solution. Both Lovell (1961) and Jackson (1963) agree with this finding and suggest that only bright pupils could interpret the situation correctly even at the age of fifteen; although against this it should be noted that the Lunzer and Pumfrey study was confined to average pupils.

Piaget and Inhelder introduce six logical formulae to express symbolically how proportions are actually discovered. They claim that before introducing numbers as measurements for weight and distance the subject usually begins by assuming

$p.\bar{q} = R\ (\bar{p}.q)$ (that is, increasing the weight and reducing the distance on one of the arms is the same as reducing the weight and increasing the distance on the other arm).

This is none other than the proportion $\frac{p}{q} = R\frac{\bar{p}}{q}$ thus $(p.\bar{q}) = R(\bar{p}.q)$, which then implies $\frac{p}{q} = \frac{\bar{q}}{p} = (pu\bar{q} = qu\bar{q})$ and $\frac{p.q}{\bar{p}.\bar{q}} = \frac{puq}{\bar{p}u\bar{q}}$ thus $\frac{1x}{Rx} = \frac{Cx}{Nx}$ (where $x = p.q$) and leads to the metrical proportion $\frac{p}{q} = R\,\frac{\bar{p}}{q}$ which corresponds to

$\frac{nw}{n1} = \frac{w:n}{1:n}$: for example, $\frac{2 \text{ x } 4}{2 \text{ x } 8} = \frac{4:2}{8:2}$

Piaget and Inhelder take as their problem the establishment of how the subjects construct the first two proportions, and explain it in terms of the I(identity)N(inversion) R(reciprocity)C(correlative) group above. In the first stage the child fails to distinguish between his action and the external process and this is followed by integration of intuitions in the direction of the compensation of weights. In substage 2a concrete operations are performed on the weight and distance but there is no systematic co-ordination between them; but in substage 2b there follows an inverse correspondence of the weights and distances. The law is finally discovered and explained during stage 3; this is true for all Piaget's work on proportions and not only in the balance experiments.

(b) Hauling a weight on an inclined plane

In this study there are three variables: the weight the truck carries, the counterweight suspended by a cable fastened to the wagon and the inclination of the track. Here again it is during the third stage and especially in the final part of this stage that the subject discovers the proportionality of the heights and weights, that is, as soon as he considers the height rather than the angle.

The forms of the equilibrium schema (INRC) group and the proportionality schema are the same here as in the balance-scale problem. In both cases, after the same preoperational representations of stage 1 and the same initial operations at substage 2a, the inverse correspondence is discovered at substage 2b. The operational schema of equilibrium is established only when the INRC group comes into play at the level of formal or propositional operations, and in both instances this leads to the schema for proportions and compensations in their general form.

(c) The Projection of Shadows

In this study Piaget and his co-workers turn from physical to geometric proportion. The proportions involved here denote relationships between distances and diameters in a physical phenomenon which can be explained in terms of simple projective geometry.

Rings of varying diameter are placed between a light source and a screen. The law is extremely simple: the size of their shadows is directly proportional to the diameters and inversely proportional to the distance between them and the light source. The subject is asked specifically to find two shadows which cover each other exactly using two unequal rings. To do so he need only place the larger one further from the light in proportion to its size and there will be compensation between distance and diameter.

Once again Piaget's subjects pass through three stages and eventually go beyond additive differences to the true multiplicative relationships or compensations. In so doing they distinguish and co-ordinate transformations by inversion, which cancel the modification in question, and transformations by reciprocity which compensate it without cancelling it. Again the subjects learn unconsciously to make effective use of the group of four transformations, the INRC group (identity, negation, reciprocal, correlative).

Lunzer (1965) has suggested that proportionality is an intrinsically higher order method of ordering experience, since it involves matching two relations: it is a second-order relation, being a relation between relations. This is in essence, a modification of Piaget's contention above, that proportionality and reciprocity are both expressions of the INRC group.

(d) Centrifugal force and Compensations

In this study two possibilities are open to the subject: he can construct metrical proportions, or he can isolate the factors that determine the equilibrium in terms of the "all other things being equal" method. Piaget is here aiming to discover whether, psychologically, proportions carry with them the idea of compensation or whether it is the other way around.

Three metal balls of different weights are placed on a disc at three different distances from its centre. The disc is rotated faster and faster until the balls roll off because of centrifugal force. The problem is to predict in what order they will leave their initial positions and why. Obviously the law of centrifugal force is a complex one, but the child need isolate only factors m and r, that is, he need understand only that the ball is displaced sooner in direct proportion to its weight and later in inverse proportion to the distance from the centre. The problem of compensation arises, but the three weights are calculated in such a way as to compensate exactly for the three distances.

As usual, the child passes through stage 1 with preoperational interpretations (less than seven years), to stage 2a with concrete operations and partial correspondences and the beginning of concrete compensation. Proportions however necessitate formal thought and it is not until stage 3 that there appears the spontaneous isolation of variables and compensation by proportionality.

In all cases the structure of proportions requires an element of compensation, and the ease with which proportionality can be isolated depends on whether direct or inverse proportions are involved and the facility with which factors can be compared from the standpoint of their units of measurement. The subject in all the above cases begins with compensations and works through to proportions, although the former does not always imply the latter.

The compensation schema is more directly accessible than the proportionality schema, and for Piaget, this is due to three factors. First, compensation is based directly on qualitatively logical relationships whereas proportions acquire an experimentally verifiable structure only when they are quantified. Consequently, there is a kind of logical anticipation of proportions before they are put into metrical form. Second, the compensation may be additive or multiplicative as is the case for logical but not metrical proportions. This accounts for the initial tendency of the child to look for proportionality in the equality of additive diffferences. Finally, compensation derives directly from the idea of reciprocity which is at the core of reversibility. Moreover, reciprocity cannot be combined with inversion except through mediation of the INRC group which for the Geneva group of experimenters is the basis of all proportions.

In an experiment using Cuisenaire Rods, Lunzer and Pumfrey (1966, Pumfrey 1968) at Manchester University also found that below the age of fourteen, even in a grammar school, few children spontaneously chose the multiplicative method rather than the additive, when reasoning in terms of proportionality.

(iii) Space

Proportionality in dealing with spatial problems is much clearer than in non-geometric forms. Long before he can think about 'similar' figures the child can directly perceive whether figures having different dimensions possess similar relationships. Therefore, the origin of the idea of proportions must be sought in the actual perception of figures. So far as this goes, the problem has been narrowed down considerably by Gestalt theory, not only by showing the part played in every perception by patterns and 'good configuration' but also by demonstrating that one criterion of 'good configuration' is precisely the possibility of recognizing structures as identical despite variations in absolute size.

Piaget and Inhelder (1948) explore the development of the child's perception of proportions with particular reference to triangles, rectangles and open figures. Using two different techniques, asking children to describe and draw pairs of similar or dissimilar inscribed triangles and requiring them to sort cardboard triangles into groups, they permitted the similarities to be recognized either through parallelism of the sides or through equality of the angles.

With both methods it proved impossible to carry out useful experiments with children younger than four or five years (stage 1). The child discovers the parallelism of the sides (method 1) and the equality of the angles (method 2) at the very same age (seven and a half years on the average), and he learns to ignore the actual lengths of the sides. With each method the discovery of qualitative similarities appears to take place prior to any understanding of the relative proportions of the dimensions themselves.

Using the technique of perceptual comparison and pictorial construction involving the enlargement of a standard the above results were further reinforced for rectangles. The child's judgements progress from a global comparison at four to five years, resulting in an over-estimation of length, through an intuitive transposition of dimensional relations in perceptual comparisons but not in drawings at seven to eight years, to the operational generalization of proportions at approximately twelve years.

This correspondence between the spatial and numerical concept of proportionality is well known mathematically and it is all the more interesting to find that it occurs at the same level of development in two contexts - similarities and proportions - which might at first sight seem to be different.

Proportionality in the case of open figures was examined in a control experiment with the enlargement of the open figure by drawings and perceptual comparisons. The development here was seen to be comparable with that found for perceptual and graphic comparison of the rectangle, minus the complications due to the child envisaging the global shape, and plus others due to the presence of a third distance to be compared along with the other two.

In summary then, the formation of mathematical proportions - as the equality of two ratios - raises a psychological problem only because it does not take place during the concrete-operational stage. The child at this level can already construct fractions or numerical ratios and from the qualitative standpoint, beginning with the concrete level, we see evidence of what Spearman (1923) has called the "eduction of correlates" in which the subject formulates the links in a double entry table in such a way as to forecast proportions like "Rome is to Italy as Paris is to France". This is why Piaget and Inhelder wonder why eight to eleven year-olds are not able to discover the equality of two ratios which form a proportion and why the discovery is not made before the formal level. Their experiments in the most diverse areas demonstrate repeatedly that proportions are not acquired before substage 3a. Since academic timing is not the cause, the reason for its late comprehension must lie in the actual structure of the operations available to the child at different levels.

Why, we may also ask, when a child does try to establish a relation between relations, does he prefer to add and subtract rather than multiply or divide? Lunzer (1965) suggests that part of the reason may lie in the inadequacy of generally practised modes of introducing multiplication which fails to show that it is inversely related to division. Pumfrey (1968), also at Manchester, confirms the suggestion, from the results obtained in a series of experiments using the balance, structural arithmetic material and the pantograph, with five to fifteen year-olds, that an introduction to the understanding of proportion can profitably be made before the secondary stage of education, provided that the content and structure of the materials used are related to the strategies available to the child. In this respect, the findings are in agreement with Winch (1913).

(iv) Quantity

For Piaget (1947, 1952, Piaget and Inhelder 1960, Piaget and Szeminska 1939), the development of quantity concepts is equivalent to the formation of the concept of conservation of quantity, and conservation involves proportionality. As he says in the opening page of his book on number (1952): "Every notion, whether it be scientific or merely a matter of common sense presupposes a set of principles of conservation, either explicit or implicit. Conservation is a necessary condition of all rational activity ... This being so, arithmetical thought is no exception to this rule. A set or collection is conceivable only if it remains unchanged irrespective of changes occurring in the relationship between the elements ... A continuous quantity such as a length or a volume can be used in reasoning only if it is a permanent whole, irrespective of the possible arrangement of its parts. In a word, whether it be a matter of continuous or discontinuous quantities, or quantitative relations perceived in a sensible universe or of sets and numbers conceived by thought, whether it be a matter of the child's earliest contacts with number or the most refined axiomatisations of any intuitive system, in each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding. From the psychological point of view the need for conservation appears then to be a functional 'a priori' of thought."

The method of inquiry varies with the type of quantity notion investigated, matter, weight or volume. The basic technique is a simple one. In order to test for conservation of matter and weight, the experimenter gives the subject a ball of clay and asks him to make another exactly like it, "just as big and just as heavy'. After the child has done this, the experimenter returns one of the balls as a standard for comparison, and changes the appearance of the other by stretching it into a 'sausage', flattening it into a cake, or cutting it into several pieces. The experimenter then questions the child as to whether the amount of clay or the weight have changed or remained invariant as a result of the transformation. A scale balance is used to determine the weight, and a glass container with water is used to determine the volume. The child is shown that each ball of clay, when immersed in the water, causes the water level to rise to the same height. The experimenter then alters one of the balls and asks if it will still make the water rise to the same height.

Piaget (Piaget and Inhelder, 1962) also measures conservation of volume with a task involving water poured into different shaped vessels. The subject is asked to judge whether two quantities of water (lemonade), previously judged equal remain equal when transferred to containers of different shapes or when the contents of one of these is transferred to several smaller containers.

The principal findings of these studies are as follows. First, each type of quantity concept (matter, weight and volume) shows about the same developmental trend: firstly, perceptual factors all important, resulting in no conservation; secondly, an empirically founded, "on and off" sort of conservation where the child tentatively hypothesizes conservation for some transformations but denies it for others; and thirdly, a logically certain, almost axiomatic assertion of conservation in the case of all transformations for the type of quantity concept in question. The other major finding is that, despite this apparent similarity among tasks, the conservation of matter, weight and volume are not achieved of a piece. For Piaget's subjects, conservation of matter seems to become common at eight to ten years of age, of weight at ten to twelve, and of volume only at twelve years and after. Of one hundred and eighty children aged four to ten years, fifty-five showed no conservation of any kind, sixty-seven showed conservation of matter alone, thirty-eight of matter and weight but not volume, and only twenty of all three.

The increasing difficulty of these tasks for the child is not surprising when we consider that weight and volume are each concepts which are much more 'abstract' in nature than matter or sheer bulk. Weight cannot be observed directly and perceptually, it must be inferred. Similarly volume must be calculated. Piaget however interprets his findings in a more complex and detailed fashion. He conjectures that there are certain schemas concerning the physical characteristics of objects whose acquisition at least facilitates the formation of the quantity conservations. There is first the general capacity to 'multiply relations', or recognize that changes in one direction may be compensated for these occurring in the opposite direction. The second schema which Piaget proposes is closely related to the first and is called 'atomism'. Conservation becomes more probable if the child can conceive of the clay as a whole composed of tiny parts or units which simply change their location vis-a-vis one another when the whole undergoes a transformation of shape. However, in judging weight, the child may yet believe that the weight of each unit varies with the location in the whole. Similarly with respect to volume, the child may believe that each tiny unit of clay varies in the amount of space it occupies, compressing or decompressing, altering its density, as a function of its position in space following transformation of the whole.

These are not simply ad-hoc explanations for Piaget. He investigates the atomism schema in the following way. Working with one hundred children four to twelve years of age, Piaget showed two identical glasses containing equal quantities of water, and established their equivalence of weight on a scale balance. The experimenter then put two or three pieces of sugar in one of the glasses and marked the height to which the water rose. The child was then questioned as to what he thought would remain invariant as the sugar slowly changed state.

In the first developmental stage, children appear to think that the sugar becomes completely annihilated as an existence when it dissolves, but at the same time believe that the sugar taste will somehow be conserved. Many however, feel that even this will disappear within a day or two.

Stage 2 is a complex one, comprising various transitional phases. Sugar-as-existent does indeed remain invariant, and still exists as very tiny, 'atomistic' particles. However, the tiny grains by their very diminutiveness are not endowed with either weight or volume. These invariances are achieved later: first weight, then volume.

There are several experiments which deal with the development of conceptions of density and compression-decompression. In one, the experimenter heats a piece of popcorn until it pops, and in another the child is shown several objects of different density. Piaget draws two major conclusions from the results of these and other experiments.

These concepts begin by generally being confused and undifferentiated and only gradually emerge from this undifferentiated totality as separate, stabilized quantity concepts. Thus, in the beginning there really is no concept of amount of matter, or weight or volume, distinct and separate from each of the others. A little later amount of matter differentiates from this conglomerative concept to become a rational affair for which conservation can be predicted, for which subquantities always sum to the same total quantity, and so forth. However, weight and volume are at this point still undifferentiated and, a posteriori, are not yet separably rational concepts which can submit to reversible operations. Still later, these two also articulate from one another, and each in its turn goes on to become a genuinely quantitative construct. The popcorn experiment illustrates quite clearly the earlier stages of this differentiation process. The younger subjects immediately assume that the piece of corn weighs more after it has popped because it is "bigger". Weight and volume are, for these children, a kind of global "bigness" and are apparently not seen as distinct and different properties which can vary independently, although usually correlated in nature. It is only after the child recognizes the logical independence of these properties that any idea of a genuine quantification of either becomes possible.

The second major conclusion is more specific: a genuine grasp of the concept of volume and of its relation to weight requires the development of a schema of substance density and related concepts concerning compression and decompression of matter. Piaget's protocols and data suggest that about the time the child becomes capable of managing volume problems he also conceives of matter as composed of numerous tiny parts or elements with empty spaces in between. Substances can vary as to how tightly these elements are compressed. Objects which are heavy for their size are thus composed of tightly packed elements; lighter ones are more loosely packed, with lots of spaces. A further study, reported in the book on geometry, (Piaget, Inhelder and Szeminska, 1960) deals more specifically with the conservation and measurement of volume. The child is shown a block which is quite solid and is told that the block is an old house built on a little island. It seems that this house is threatened, and so the inhabitants decide to build another in its place. The new house is to have exactly as much room as the old, although it is being built on another island, which differs from the first in size or shape or in both. The problem consists in reproducing the volume of the first block while altering its form to comply with the base which is given. Furthermore, the equal volume must be built out of little wooden cubes each of which is 1 $cm.^3$, while the original is a solid block. Conservation of volume was tested in two ways. In one form of the test the thirty-six cubes were arranged to make different "houses" and the child was asked if they had the same or different amount of "room" in them. The other form was the displacement-of-water one used in the original conservation of volume study.

For the measurement part of the study, the developmental sequence was roughly as follows. The youngest children compare houses on a single dimension only, frequently the height, and if asked to copy the original house directly, they tend to do so by enclosing it with unit blocks, rather than by making a replica alongside the model. At the next stage, the child begins to bring logical multiplication to bear on the problem. That is, he knows that a smaller base necessitates a higher structure but how much higher it should be he has no procedure for determining. Eventually, in middle childhood and early adolescence, the child manages to solve the problem by arithmetically multiplying dimensions.

This study on the conservation of volume uncovered one interesting new fact about its development. There appears to be a stage in middle childhood (average age around nine years) when, in Piaget's phraseology the child conserves "interior volume" but does not yet conserve "occupied volume". He recognizes that the number of cubes always remains the same and therefore asserts that the amount of room made is also the same from house to house. However, he paradoxically refuses to conclude from this that when finding the volume of the house by displacement the water level in the bowl will also stay the same whatever the arrangement of the submerged cubes comprising the house. In Piaget's interpretation, the child of this stage does have a beginning conception of volume; however, the volume he conceives of pertains to the object alone, with no implication for the volume of its surrounding medium.

Lunzer (1960 b) repeated this experiment and concludes that there is little reason to doubt that Piaget's main lines of development are correct in broad outline. However, he found no evidence that conservation of volume should be regarded as limited by the priority of topological over Euclidean conceptions of geometry.

Piaget's work in this field has been replicated in a series of experiments by Elkind (1961 a,b,c, 1962, Flavell 1966) on subjects of different ages. He found that the age 'décalage' between the conservation of matter, weight and volume, may be considerably greater than Piaget had thought. About 75% of Elkind's subjects had acquired conservation of matter and weight around the seven to nine year period, but the 75% level for conservation of volume does not seem to occur until about age fifteen.

Evidence in broad support of Piaget is found in the work of Lovell and Ogilvie (1960, 1961) on testing junior school children (age seven to ten years) for conservation of global quantity and interior volume, and complementary or displaced volume. These experimenters offer the hypothesis - very much in accord with Piaget's views on the task-and-area-specific nature of concrete as opposed to formal-operational thought - that the child can at first apply a concept of conservation of global quantity only in particular, selected situations; with additional maturation and experience, the concept becomes a stable and consistent instrument of cognition, applicable to a broad range of concrete instances.

A further replication study involving cross-cultural comparisons was carried out by Hyde (1959). Aden subjects were given several quantity tasks in addition to the number ones and unlike Elkind, this experimenter found no strong evidence for the global quantity-weight-volume 'décalage'. Careful scrutiny of these studies does not suggest any immediate explanation for the discrepancy in results but data from the Hyde study regarding the conservation task in which sugar is dissolved in water largely confirmed Piaget's qualitative observations. There is also some tentative information regarding the difficulty level of the various quantity tasks. For example, conservation of global quantity in the plasticine-balls task appears to be more difficult to achieve than in the water-in-vessels setting, but easier than in the sugar-in-water test.

In yet another study Piaget's conclusions with young children were again confirmed but it was found that the subjects may be influence by the form in which the problem is presented (Carpenter, 1955). Lunzer (1956, 1960a, 1965), however, extended the typical situation and asked his subjects aged six to twelve years to pour the same amount of liquid into two different shaped containers. The child had at his disposal one pint bottle filled with water, one baby's feeding bottle graduated in fluid ounces and inccs., and one funnel. It was to be expected that there will be some gap between the appreciation of the constancy of a liquid through changes in its shape, as measured in the original experiment of Piaget, and the insightful planned application of that knowledge required in this experiment. This was in fact found to be the case, and Lunzer tentatively concludes that: "The absence of the notion of the constancy of a liquid in a subject does not necessarily imply complete failure to co-ordinate the two dimensions involved but that, the co-ordination is not quantitatively adequate."

Françoise Frank (reported in Bruner 1964, 1966) did firstly classic conservation tests to determine which children (of a group aged four to seven) exhibited conservation and which did not. She then went on to other procedures among which was the following. Two standard beakers are partly filled so that the child judges them to contain equal amounts of water. A wider beaker of the same height is introduced and the three beakers are now, except for their tops, hidden by a screen. The experimenter pours from a standard beaker into the wider beaker. The child, without seeing the water, is asked which has more to drink, or do they have the same amount. In comparison with an unscreened pretest there is a striking increase in correct equality judgements. With the screen present, most children justify their correct judgement by noting that "It's the same water" or "You only poured it".

Now the screen is removed. All the four year-olds change their minds. The perceptual display overwhelms them and they decide that the wider beaker has less water. Virtually all the five year-olds stick to their judgement, after invoking the difference between appearance and reality. All of the six and seven year-olds retain correct judgement. Some minutes later, Frank then does a post-test on the children using a tall thin beaker along with the standard ones, and no screen. The four year-olds are unaffected by their prior experience; none of them is able to grasp the idea of invariant quantity in the new task. The older children show marked improvement in performance. Finally, control groups doing just a pretest and a post-test show no significant learning.

A related experiment by Patricia Nair (reported in Bruner, 1964) in 1963 explores the arguments children use when they solve a conservation task correctly and when they do not. Her forty subjects were all five year-olds. She transferred water from one rectangular clear plastic tank to another that was both longer and wider than the first. She had a toy duck swimming in the first container, and when the water was poured into the new container she told the child that "The duck was taking his water with him".

Three kinds of arguments were set forth by the children to support their judgements. One is perceptual- having to do with the height, width, or apparent "bigness" of the water. A second type has to do with action: The duck took the water along, or the water was only poured. A third one, "transformational" argument, invokes the reversibility principle: if you poured the water back into the first container, it would look the same again. (Not one of the children in this experiment used the compensation argument - that though the water was lower it was correspondingly wider and was, therefore, the same amount of water, although this type of reasoning by compensation is said by Piaget and Inhelder (1962) to be the basis of conservation).

Bruner does not report how many children judged correctly, but of those who thought the water was not equal in amount after pouring, fifteen per cent used non-perceptual arguments to justify their judgement. Of those who recognized the equality of the water, two-thirds used non-perceptual arguments. It is plain that if a child is to succeed in the conservation task, he must have some internalized formula that shields him from the overpowering appearance of the visual displays much as in in the Frank experiment. The explanations of the children who lacked conservation suggest how strongly oriented they were to the visual appearance of the displays they had to deal with. Thus it may be concluded from all these studies that the conservation of volume is not worked out conceptually before the beginnings of the final level at eleven to twelve years. Applied to quantity conceptions, Piaget's theory holds that the child is ready for attaining the abstract volume conception at ages of eleven to twelve; he is ready in the maturational sense because the conceptualization of volume requires only concrete operations (internalized actions) which are present in most children by age of seven (Elkind 1961 b,c, Piaget 1952, Piaget and Inhelder 1962) and he is also ready in the experiential sense that he has had sufficient object contacts to form abstract conceptions of mass and weight which are the structural prerequisites for the attainment of the volume conception (Piaget 1954, Smedslund 1961a).

The relatively late acquisition of the volume concept may be due to several factors: in part, one would guess that only very few could arrive by themselves at a valid method of calculating volume in the absence of geometrical teaching which takes place at twelve or thirteen. On the other hand, that equal volumes must displace equal quantities of water if totally immersed, or that water poured from one container to another must remain invariant, seem to be ideas which children might arrive at spontaneously. But the attainment of such an idea must involve: (i) an implicit recognition of the equivalence of volume in the two solids; (ii) an implicit recognition of the conservation of volume in the liquid itself - despite its physical displacements and transformation; and (iii) an appreciation of the relationship between the volume of the body immersed in a liquid and the volume of liquid displaced. Hence, one cannot be surprised to find that this understanding is considerably delayed.

Furthermore, in contrast to the simple forms of conservation which the child masters by simple additive compensations, the conservation of volume throughout changes of form presupposes the ability to handle complex proportions (Inhelder and Piaget, 1958).

In the first stage of attainment of the conservation of volume, the child compares the volumes only in a perceptual way, and cannot employ "la partition en unités égales ou la décomposition en dimensions proportionées" (Piaget and Szeminska, 1939). The child fails to understand that the increased height is compensated by the decreased width and that the two values are inversely proportional.

In the second stage of intermediate responses, the child is beginning to establish proportionality "proprement dite" and not merely qualitative correlation.

In the third and final stage, conservation is so widespread that it seems an 'a priori' and analytic deduction independent of observation of relationships and experience, but it is not so; it is a co-ordination of relations, including both logical multiplication of relationships and of the mathematical composition of parts and of proportions. Piaget concludes that "ce n'est pas la découverte de la conservation qui entraîne la possibilité de multiplier les relations, mais bien l'inverse ...

Nous prétendons - et c'est là toute notre hypothèse - qu'à un moment donné le sujet comprend que les differences se compensent .... et c'est ainsi, que débute la quantification extensive, parce qu'alors deux rapports qualitatifs hétérogènes .... sont conçues comme égaux tout en conservant leur signification de difference asymetrique. Ainsi naît donc la proportion, par combinaison de l'égalité avec la relation asymétrique." (Piaget and Szeminska 1939, page 62).

It is noteworthy that the child does not come to control the notions of multiplicative compensations and conservation of volume until the age when he discovers proportions in other areas. However, he does make the discovery without metrical calculation and without realizing that the numerical calculation of the compensations that he conceptualizes qualitatively implies the use of proportions. The indication is (Piaget and Inhelder, 1962) that when an operational schema is organized the subject discovers its various consequences simultaneously, even without bringing together the various aspects of the schema, and this applies equally to multiplicative compensations and proportions.

This idea of proportion or ratio as embedded in the task of judging volume is examined in the experiment by Halford (1968) where six to eight year old children in conservation and non-conservation groups were tested for their ability to predict whether the contents of comparison containers differing in height and diameter of glass were equal or unequal to that of the original container. For the experimental subjects, a standard container equal to the comparisons in height or breadth, but differing from the original in both, was filled from the original in the subject's view. The results show that the conserving subjects were superior in recognizing equality and unequality even without the use of the standard and made superior use of the knowledge that the standard and original, though of different dimensions, contained the same quantity.

The proportion notion is pursued further in the study carried out at the Harvard Centre for Cognitive Studies by Bruner and Kenney (1964, 1966). The above studies on conservation of volume by Piaget, his co-workers, and also by other experimenters validating Piaget's results, were concerned only with changes in water level as liquid was poured from one container to another of different dimensions; they did not involve directly the concept of fullness.

Bruner and Kenney, however, went beyond the concept of volume itself and asked their subjects to judge which of two containers was fuller and which was emptier. Volume in the earlier studies was clearly visible to the child; the physical proportion or ratio involved in this study is not. It depends upon a relationship between the perceptible features of the containers, but the relationship itself cannot be defined ostensively. Rather, it requires the child to carry out a symbolic operation somewhat like computing a ratio.

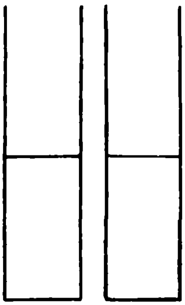
The basic problem posed for the children is, on the surface, a simple one: which of two vessels in a pair is fuller and then which one is emptier. The experiment itself consisted of paired comparisons of glasses, filled to varying degrees with ordinary water. In all, one hundred and sixty children served as subjects, forty in each of the age groups five, six and seven, and twenty each in the age groups nine and eleven. Originally the study (Bruner 1964) encompassed only the three younger age groups, but when it became quite clear that by age five, a definite idea of "proportion", albeit an incorrect one, is already present and that by age seven the children still had not grasped the idea in its proper mathematical sense, the two older groups were included in the study.

Bruner presented his subjects with eleven pairs of glasses in fixed order (see Figure 1). The task begins with a very easy judgement involving two identical glasses filled to the same proportion. From there on it becomes more complicated by virtue of there being pairs

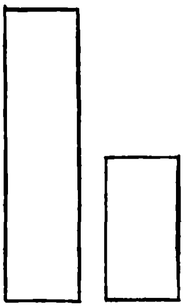
Figure 1

The eleven displays of glasses used in the
experiment on fullness by Bruner and Kenney (1964, 1966)

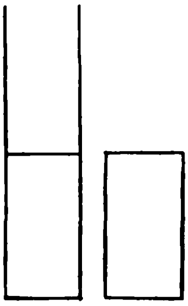
Scale ¼ inch to 1 inch



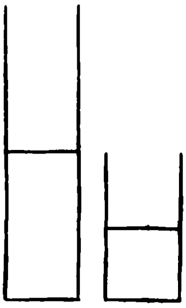
Display 1
Type IV



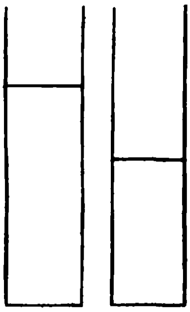
Display 2
Type II



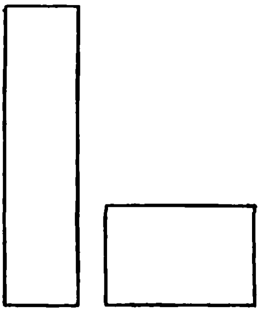
Display 3
Type III



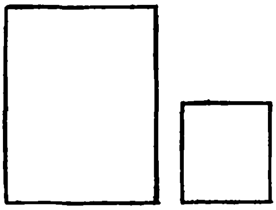
Display 4
Type I



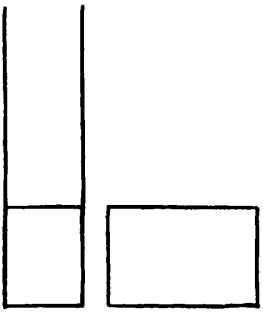
Display 5
Type IV



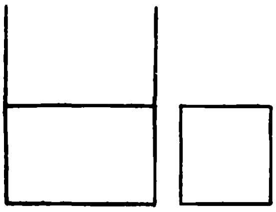
Display 6
Type II



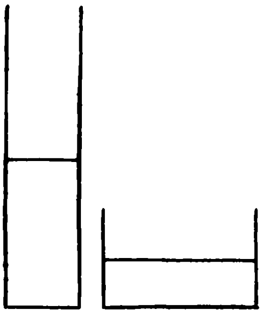
Display 7
Type II



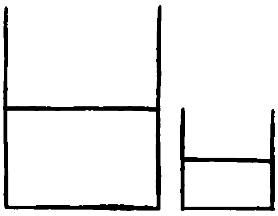
Display 8
Type III



Display 9
Type III



Display 10
Type I



Display 11
Type I

that are made up of glasses of unequal volume and pairs that are equally or unequally full. Any two pairs of glasses can differ or be the same in the following respects:

the proportion they are filled
the proportion they are empty
their water levels
the height of the glasses
the diameter of the glasses
the volume of water they contain
the volume of empty space they contain.

Aside from the first two of these measures, which bear a complementary relation to each other, Bruner states that all the other measures are independently variable of each other as well as independent of the first two indices. This is not exactly true, since the first two ratio measures and also the last two volume measures depend entirely on the water levels, the height of the glasses and also the diameter of the glasses. It is only these latter three measures which can vary independently. However, Bruner used this feature of independent variability in constructing the displays, selecting among all combinations of features.

His display pairs were of four types. In type I (Displays 4, 10 and 11), glasses of unequal volume are partly filled but to an equal proportion. Type II displays (numbers 2, 6 and 7 are the examples) have unequal volume but both glasses in the pair are filled to the top. Type III displays (numbers 3, 8 and 9) again contain pairs of unequal volume in which one glass is fractionally full, the other completely full. Finally, type IV consists of a pair of identical glasses in which the only factor that differs is water level (display 5).

In the above pairs the perceptual features can be arranged in such a way as to throw the perceptual factors either for or against a correct ratio judgement. Pair 11, for example, is one in which everything perceptual goes against a judgement of equally full: one of the glasses is taller, has a higher water level, is wider and has, therefore, also a greater volume of water than the other.

Bruner and Kenney attempted "linguistic activation" with the three youngest age groups, in order to test whether alerting the children to the specific perceptual properties of the displays would alter their judgements of proportion. No significant improvement was found, and the experimenters concluded that it "avails little to activate children to separate cues when their difficulty is not so much with the cues as such but with their combination".

Although they do not give percentages of subjects correct in each age group, the experimenters do consider the bases for judgements of fuller and emptier. It was found that the younger children equate "fuller" with the glass having the larger volume of water, and "emptier" with the smaller volume. For the seven and nine year-olds, on the other hand, "empty" means more volume of empty space, and it is only the nines and the elevens who attempt or successfully succeed in formulating the ratio idea in terms of a relation between quantities.

Because the older children have developed some appreciation of a complementary relation of filled and empty space, they are led into logical contradiction; for indeed, with this rule they may say, given a large glass and a small glass, both half full, the glass that is fuller is also emptier. The younger children with their more global approach are no more correct but are not led into any contradiction.

In analysing for the effect of perceptual factors on the child's judgement of fullness, Bruner and Kenney subdivide the displays into those in which all cues (namely, water level, volume of water, and "reaching to the top") favour the correct judgement (1 and 5), those where most do (6 and 8), those where they are balanced (2 and 10); those where most cues mislead (3, 7 and 9), and those where all mislead (presumably 4 and 11). They hypothesized that the number of correct judgements would conform to this order, and such turned out to be the case. Furthermore, the older the child the less perceptual factors dominated judgement in this way.

Finally, Bruner and Kenney analysed the number of children who named several perceptible attributes as a basis for their judgements. It was found that older children tend to be somewhat more pluralistic about the criteria they use than the younger subjects, who refer to some single attribute. At eleven, they are able to co-ordinate the crucial relationships and state these economically as well as being able to consider more attributes. The proportion of children giving multiple bases for their judgements of fullness quintuples between age five and age nine, and triples for their judgements of emptiness. "The eleven year-old," however, "knowing the answer, need not give multiple reasons. He merely remarks on the common fraction". The eleven year-old child has ordered his perceptual world so that he is able to apply concepts of relationship that are not dependent upon simple ostensive definition. In short, according to Bruner and Kenney, he has acquired the concept of ratio or proportion through the idea of fullness.

CHAPTER III

FACTORS INVOLVED IN PERFORMANCE IN CONCEPT FORMATION EXPERIMENTS

Piaget lays great stress on the spontaneous development of cognitive abilities as the child matures and maintains that experimental factors can become effective only to the extent that they build on the child's previously developed structures of thought, as through the activation of the reasoning process prior, but logically related, to the one to be developed.

Maturation is undoubtedly of great importance but, on the other hand, there are other factors which are influential in the child's performance in concept formation experiments and in his acquisition of concepts like proportion: these include language, instructions, relevant and irrelevant information, feedback and reinforcement, perceptual and practical experience and training.

(i) Language

Since the successful performance on Piagetian-type tasks appears to be influenced by the ease of communication between the experimenter and the subject, one might reasonably expect children who attend school to understand and use language more effectively than those who have been deprived of schooling and thus perform at a higher level on conservation tasks involving verbal communication. Yet, ask Mermelstein and Shulman (1967), "do we in this manner confound the attainment of a particular cognitive structure with the ability to manifest it in verbal communication?"

As Piaget maintains that perceptions precede language developmentally, we might expect that children's performances on non-verbal tasks will be categorised at a higher developmental level than their performances on tasks measuring ostensibly the same cognitive structure but requiring verbal communication. Also, we might expect greater differences between schooled and unschooled children on verbal tasks.

Utilizing a nonverbal measure of conservation of quantity as well as the standard verbal tasks, Mermelstein and Shulman compared performances of six and nine year old Negro children from two different communities, one of which had been without public schools for four years, on five tasks involving the conservation of continuous quantities (one nonverbal and two verbal tasks), and the conservation of discontinuous quantities and its relation to one-to-one correspondence (two verbal tasks). Differences between verbal and nonverbal tasks were found to be highly significant, and no significant differences were found generally that were attributable to the effects of non-schooling except within one questioning condition, where the perception of the conservation of the discontinuous substances were connected with the needs of the subjects.

Although opposed to Dodwell's (1960, 1961) suggestion that socio-economic status and length of schooling do make a difference and Rothenberg's (1969) finding that lower-class children are more likely to be inaccurately assessed in terms of conservation status than middle-class children due to their poorer verbal ability especially as many studies require justification of performance by subjects, this finding is in accord with Price-Williams' (1962) results for African Bush children and Goodnow and Bethon's (1966) study in Hong Kong where again no difference in performance on conservation-type tasks between children who had had formal schooling and those who had not was found.

Mermelstein and Shulman suggest that the syncretic nature of egocentric thought may help to explain these results: if six and nine year old children emphasize the events implied by the questions rather than the relationships of the questions to the events, this may account for the similarity in performances of the non-schooled and schooled nine year olds. It is also conceivable, they state, that specific training in language does not ensure analytic understanding of the questions used prior to nine years of age.

There is also evidence in this study that the absence or presence of language itself significantly affects performance, since there was a lower mean age of acquisition for the nonverbal continuous task than the verbal discontinuous task. Variations in question phrasing appeared here to influence significantly performance on certain number tasks and the hypothesis of no difference in performance is supported for the continuous but not the discontinuous tasks. These factors throw doubt both on Piaget's results, which do not take into account the language variable, and on his techniques of assessment.

Recent studies have measured conservation attainment through standard question formats because they provide a more comparable situation for all Ss than does Piaget's more flexible, clinical method of questioning. Within the limits of standard formats the major difficulties are in the vocabulary level and structure of the questions. Some investiators (Shantz and Sigel in Rothenberg 1969; Wallace, 1966) have indicated that with the most common standard questions a major problem arises in the interpretation of the Ss' responses: among the Ss who fail to conserve, it has not been possible to know whether this failure was due to inability to understand the language of the question, the concept of conservation, or both. Recent studies (Donaldson and Balfour, 1968, Griffiths, Shantz and Sigel, 1967, Pratoomraj and Johnston, 1966) have dealt with the first aspect of the measurement problem, the language level. These studies have pointed to the importance of assessing the S's comprehension of the key words (i.e., "same", "more") used in the conservation questions or training the S to criterion in his understanding of these words prior to testing his level of conservation attainment. Pratoomraj and Johnston (1966), for example, found with children aged four to seven years that the kind of question they used ("is it the same? more? less? different?") had very little effect. Donaldson (Donaldson and Balfour 1968) likewise found that three and four year-old children did not differentiate between "less" and "more": of the fifteen subjects studied only one gave consistently correct responses to "less" whereas the others showed a marked tendency to make the same response irrespective of which word was used in the instruction. Information gained from this type of prior assessment or training would then enable one to determine more clearly the reasons for any S's failure to conserve.

The second aspect of the language problem, that is, the structure of the questions, has been considered to a lesser extent. Rothenberg (1969) has shown in her study on number conservation, among four and five year-old children that important differences occurred in the number of conservers identified, depending on the question or questions asked as well as on the number of transformations presented. The problem is seen in the format of the commonly posed conservation question, "Does this row (or side) have more, or does this row have more, or do they both have the same number (amount, etc.)?" (Wallach and Sprott, 1964; Zimiles, 1966). Questions such as this, which contains a number of separate parts, are particularly difficult for young children to remember because they are long and complex. Moreover, it seems unlikely that the Ss can reliably answer the three-section question with a single response. Hood (1962) has noticed, for example, that with this three-part question children tended to repeat the last thing they were asked. This suggests that the extent to which children appear to be conserving may depend on the order of the parts of the question. Two-part questions (Fleischmann, Gilmore and Ginsburg, 1966) tend to present similar problems. There is one further common type of question (Dodwell, 1960; Elkind, 1961b; Wallach, Wall and Anderson, 1967; Wohlwill and Lowe, 1962), which deals only with a single event. While this type of question has the advantage of presenting a single phrase short enough to be remembered, it does have the disadvantage of tending to favour either a conserving response or a nonconserving response by virtue of the emphasis in the question on "same" or "more". This single-question method can result in either many more or fewer conservers than might be obtained through more complete questioning of the child. Dodwell (1960), for example, when using two presentations with equal numbers in arrays that differed only in the nature of the materials, found a much greater percentage conserving when the Ss were asked if there were the same number in the two arrays than when they were asked which array had more.

Some investigators (Griffiths et al., 1967; Hood, 1962; Mermelstein and Shulman, 1967) have suggested that further changes in the conservation question format are necessary for one to be fairly sure of the real character of the child's thinking. Hood (1962) has suggested that this can best be accomplished by presenting standard single-event questions in several forms for each problem.

Gruen (1966) has noted the varying criteria used in different studies to define conservation. Primarily, the difference is whether or not the Ss are asked to justify or explain their response to the conservation question. This criterion difference has tended to result in more conservers being selected when justification of the answer was not requested. Since most of the previous research has used either an excessively long question or a short but somewhat biased question to determine conservation status, it is understandable why these differences in the number of conservers have been found depending on whether justification was included. Changes in the nature of the conservation questions in the direction of providing a larger and possibly more reliable sample of responses to each problem may tend to decrease the discrepancy in the results found using these two definitions of conservation and still avoid the problem of false positives, noted by Shantz and Sigel, that has occurred when justification for determing conservation was not required.

Thus attempts to determine the effect of language on concept formation through the means of requiring the subject to verbalize during the experiment have produced conflicting results. Wolff (1967) claims that verbalization like pretraining has a positive effect on concept formation while mediational ability is not related to performance on this type of task, but Rommetveit and Kvale (1965) produce results which are not so clear cut.

Working with complex geometrical patterns to signal good and bad luck at a wheel of fortune, these experimenters identify a "pre-functional" stage of concept formation, where it was found that a detrimental effect occurs if the subjects are first required to verbalize the difference, after sorting verbal descriptions of "good" and "bad" patterns. Furthermore, subjects who anticipated being rehearsed about the difference between good and bad wheels of fortune patterns had a successively poorer performance at sorting verbal descriptions of the differences, as their ability to sort actual stimulus patterns into appropriate conceptual categories improved.

Carolyn Stern (1967) also investigated the importance of verbalization in children's ability to learn problem solving strategies. Using treatment conditions including multiple and single hypotheses testing with and without speaking, she found that children taught to use knowledge of results to test one concept at a time scored significantly higher on post-tests than did those taught to test several hypotheses at once. Also, a request that the subjects explain the basis of problem solution provided no reliable difference in the efficiency with which such problems involving simple materials are solved.

Confirmation that verbalization may produce a detrimental effect on concept formation comes from another study where it is hypothesized that concepts are formed without verbalization, and unsuccessful attempts to put concepts into words can destroy or distort the concepts themselves (Phelan 1965). Investigating concept formation in ninety college students it was found that when correct verbalization followed correct concept utilization facilitation in the application of the concept to new situations was observed. But spurious rules can be developed from inadequate verbalization and these tend to get substituted for the incorrect but efficiently functioning concept and so lead to maladaptive problem solving behaviour. Consequently incompatibilities between the formal concept and the incorrect verbalization can result in their mutual negation leaving open no conceptual mode of approach and necessitating a new start in problem solving. Similarly, the experiment of Rasmussen and Archer (1961) which was based on Whorf's (1956) hypothesis that language determines the categories into which we place objects and provides the labels by which discrimination among them occurs, also pinpointed the danger of language training. Rasmussen and Archer (1961) compared the effects of two types of familiarization with randomly generated forms, language pretraining, which entailed learning a verbal lable, and aesthetic pretraining, which entailed making aesthetic judgements. Surprisingly, the aesthetic pretraining facilitated performance in a subsequent concept identification task. The same forms were employed in both tasks, and the facilitation under aesthetic pretraining occurred when the shape of the form was the relevant dimension. To explain these results, Rasmussen and Archer (1961) inferred that subjects making aesthetic judgements must attend to various dimensions of the stimulus forms in contrast to the others who probably responded to more limited aspects of the shape. In other words, those who learned the verbal label may have merely learned the minimum basis for discriminating between the shapes, while those who made aesthetic judgements may have noticed a multiplicity of aspects.

The above study was carried out with adult subjects, but Amster and Marascuilo (1965) have studied the effect on ten year-old children of instructional set on pretraining performance and subsequent learning of the mathematical concepts of set-union and set-intersection. Verbal pretraining was found to facilitate learning during pretraining, compared with aesthetic pretraining. However, in the subsequent concept learning task, for which the same materials were employed, the subjects who had aesthetic pretraining learned the concepts more readily that those who had rote-learning instructions during pretraining. This difference due to pretraining was only statistically reliable among relatively low socio-economic status subjects in the condition that received a small variety of instances of the concept. The variety of instances presented during the concept learning task did not significantly affect concept formation _per se_, but generalization to new instances was significantly greater when the concept had been learned originally from a small rather than a large variety of instances.

Basing their work on a similar premise Kofsky (1967) and Prehm (1966) have investigated the effect of verbal pretraining on concept learning in culturally disadvantaged children. The former experimenter found that training disadvantaged children in labelling and discriminating component stimulus attributes resulted in greater attention to these attributes in inductive concept formation but in no greater success in solving concept tasks. Prehm, on the other hand, found no difference in performance between subjects classed as high and low risk groups of becoming mentally retarded at some future time but his results do suggest that verbal pretraining involving verbal label, attention and control had a significantly positive effect on performance. Stern (1965) also found that young children give a superior performance in concept identification tasks when they are allowed to rehearse the concept rather than its instance labels and verbal training has also been seen to facilitate transfer (Ervin, 1960), if several necessary conditions are present.

Wohlwill (1960a, b) has also produced results to show performance on his number tasks were not a mere reflection of the verbal labels involved but of a true concept. The same conclusion appears to be indicated by Martin's (1951) finding that children's spontaneous use of quantitative vocabulary in the description of pictures failed to correlate significantly with a more conceptual test of number ability.

Support for the notion that verbal labels may direct attention toward or away from criterial dimensions comes from work on relevant and irrelevant verbalization in discrimination and reversal learning. Milgram and Noce (1968), found, with children aged seven years and mental retardates of comparable mental age, that verbalization of the relevant class of cues facilitated performance, that verbalization of one relevant and one or more irrelevant dimensions is also facilitative but to a lesser degree, and finally that verbalization of irrelevant dimensions only interferred.

Language is seen to be unimportant for concept formation comparing deaf and hearing children. Furth (1961), for example, found that in the performance of seven to twelve year old deaf children on three conceptual tasks, the lack of a capacity to verbalize did not, in itself, interfere with a general capacity for abstract conceptualization. Similarly, Kates (1962) discovered that deaf and hearing adolescents proceed in concept learning tasks by the use of similar strategies.

One important factor which influences the ability of children to verbalize the principle involved in problem solving is the abstractness of the material (Long 1939, Long and Welch 1941, 1942). Also, the subjects who were able to solve the simpler problems without a hint were seen to be more able to solve the more difficult second hierarchy test.

Further work on the ability to generalize a principle involved in solving problems or forming concepts and how it is affected by the experimenter giving help or hints is found in the work of Maier (1931) and in the configurational theories of thinking by Duncker (1945) and Wertheimer (1945). Maier's work on reasoning in humans indicates that when suggestions of "helps" were necessary to solve such problems as tying two cords together which the subject could not reach, the very "help" (E casually swinging one cord, or giving the subject a pair of pliers) which brought about the solution was not consciously experienced except in cases in which the solution appeared in steps.

In his configurational theory of thinking Duncker (1945 and Miller 1951) speaks of a pattern in which the component parts of the problem are arranged, the reorganizing principles or directions imposed upon the components and the eventual restructuring of the components in a form that coincides with the form of the desired solution.

In giving his problem "Why are all numbers of the form abc abc divisible by 13?", Duncker compares seven learning situations, one where the subjects are not given any aid and six where the clue given to aid solution becomes more and more specific, and varies between 'the numbers are divisible by 1001' to 'look for a more fundamental common character from which the divisibility by 13 becomes evident'. The structural property becomes clearer when the subject considers the number 1001, that is abc thousands and abc units. Duncker interprets his findings by concluding that the material given is restructured according to the demands of the situation, that is, a solution always arises "out of demands made by what is required on what is given". This means that a solution which involves a recentering of the given material can be facilitated if the new concept is somehow brought into prominence, either by an explicit hint or by suitable variation of the material.

(ii) Concept formation as a function of instructions

Further evidence to substantiate the critical role of language in concept learning is seen in the way relatively small changes in experimental instructions yield highly significant changes.

Instructions may vary according to the amount of guidance, the specificity of information presented, the amount of information presented, the method of presentation and according to purpose. The first three dimensions are relative and cannot be described precisely except in connection with a specific experiment. The method of presentation may be audio, visual or audio-visual, while the instructions may be formulated to achieve various purposes: (i) to acquaint subjects with the specific stimulus material or the more general task characteristics; (ii) to acquaint subjects with the specific response or the more general performance desired; (iii) to present the subjects with information of a procedural type such as a strategy, or a method, to apply to the solution of a task; (iv) to provide subject with information of a substantive type, such as an advance organizer or a principle, to employ in performing the task; (v) to provide a set related to the recall or use of information or abilities, and (vi) to manipulate the level of motivation of the subject.

Purposes (i) and (ii) deal with clarification of the nature of the task and assume some degree of unfamiliarity of the task by the subject. Purposes (iii) and (iv) involve the presentation of additional information, usually designed to facilitate performances of the task by the subject. Purposes (v) and (vi) involve an attempt to directly manipulate the thought processes or perceptions of the subject.

In 1932 the effect of verbal instructions upon the formation of concepts was investigated by Ewert and Lambert (1932). In their study of the influence of the set following minimal, standard and maximal instructions on the ability to transfer discs on circles in a certain way with the least number of moves, it was found that increasing instructions increased the variability of the scores but decreased the average number of trials, the excess moves, and the learning time. A similar result was obtained by Stern and Keislar (1966) with children eight and nine years of age. The latter experimenters discovered that a group which had been taught the strategy of testing one hypothesis at a time was significantly superior to those children who were given an equal amount of practice with no special training as well as to those given task familiarization only. The children taught to test several hypotheses at one time were not found to differ reliably from these groups.

Intelligence and age, however, may play a part in determining the effect of instructions. Superior intelligence has been seen (Osler and Weiss 1962) to give subjects aged six, ten and fourteen years an advantage in the problem finding phase of a concept learning task but not in the actual problem solution. Under non-specific instructions the subjects of superior intelligence produced more effective concept formation. Under explicit instructions, however, the average intelligence group improved while those of the superior intellectual ability remained unchanged. Bittner and Shinedling (1968) have also shown in their methodoligical investigation of the conservation of substance that younger first grade subjects are influenced by both instructional and task variations but as the children grow older (third grade) the importance lessens and several studies do show that conservation is positively correlated with intelligence and verbal ability (Dodwell 1960, Elkind 1961, Goldschmid 1967, Hood 1962, Keasey and Charles 1967).

Furthermore, Feigenbaum (1963) working with the conservation of discontinuous quantities has revealed how the child's grasp of the conservation concept tends to vary with his IQ and also with the nature of the concrete experimental operations; task parameters such as number of beads used and also container size differential were seen to be important.

The solution of problems and its relation to instructions has also been found to depend on whether the subject is encouraged towards analytic or non-analytic problem solving. Archer, Bourne and Brown (1955) in carrying out just such a comparison found that analytical instructions did not significantly improve overall performance but reduced variability and facilitated performance on the more complex tasks.

Although Braley (1963) could find no difference in the efficiency of concept formation between instructions oriented towards concept attainment procedures and those oriented towards problem solving, instructions which convey information about a principle have been seen by others to facilitate initial learning and transfer, yet as Beilin et al (1966) found in their study on water level representation with six to seven year-old children, verbal instructions of the principle involved were inferior to perceptual confirmation training. In Fowler's study in 1931 where he tried to teach three artificial concepts by different methods a combination of deductive presentation with explanation was found to be more effective than either working from particulars to generalizations or vice versa.

The importance of cognitive control of human behaviour has been described by Miller, Calanter and Pribram (1960) who stated that all human behaviour is controlled by a plan which in turn is comprised of strategies and tactics. Strategy refers to the control of the global units of behaviour and tactics to the specific units. Bruner et al (1956) reported the first extensive research with strategies in concept formation and identified four strategies. Apparently the same person may use any of the four strategies depending on situational conditions. Not only a strategy but a principle for securing information efficiently can be described and provision of this information to subjects markedly facilitates concept learning over a short period of time. In one experiment, for example, (Hilgard, Irvine and Whipple, 1953) one group of subjects learned to perform card tricks by memorizing a sequence while a second group learned a procedure for determining the sequence. Initial learning was faster for the memory group, and there were no differences in retention of the first trick. However, significantly more subjects in the understanding group performed the second trick on the retention test and also demonstrated more transfer to new tasks where simple transposition was not an effective aid to solution.

This finding has been corroborated by Forgus and Schwartz (1957), Haslerhud and Meyers (1958) and also by Sassenrath (1959) who found that learning to learn a principle during training facilitated learning to learn a reversal principle during the transfer period.

The facilitating effects of the knowledge of a principle have also been demonstrated by Judd (1908) in his investigation of the relation of special training to general intelligence, by Craig (1956) in his experiment on directed versus independent discovery of established relations and by Hendrickson and Schroeder (1941) in their study on the transfer of training in learning to hit a submerged target, and by Klausmeier and Meinke (1968) in their manipulation of instructions which vary according to purpose and amount of information. In this last study one set of instructions outlined a conservative focusing strategy, another described the structure of the material and a third set presented only mimimal information about the task. Concept formation was more efficient for subjects (aged between nineteen and thirty years) receiving the principle; also the rank order of the effects of instruction from most to least efficient was strategy instructions, structure instructions and minimum instructions.

Another factor seen to influence the learning and retention of concepts is the effect of set. Reed (1946) has found it to affect college students' ability to learn words and nonsense syllables while Callantine (Callantine and Warren 1955) showed that concept formation under single problem conditions was superior to that involving multiple problems. Byers (1963) corroborates this in his study on strategies and learning set and Rosenbaun and Arenson (1967) in their experiment on einstellung by observation on the "water-jar" problem revealed that observation of a direct solution did not facilitate problem solving but that of the indirect solution did hinder this activity.

In conclusion, it may be stated that very minor and subtle variations in experimental instructions may yield highly significant results. Due consideration should therefore be paid to every aspect of the way in which the problem is presented to the subjects, to the manner in which the subject is required to perform and to how he is required to communicate his responses back to the experimenter.

(iii) The effect on concept formation proficiency of relevant and irrelevant information

Between the ages of ten and thirteen years children have been seen consistently (Hagen 1967, Hagen and Sabo 1967, Crane and Ross 1967, Maccoby and Hagen 1965, Siegel and Stevenson 1966) to show marked improvement in the ability to select what is relevant from what is irrelevant in various experimental situations. The results of these studies have been interpreted to indicate that young children's relatively inefficient performance in learning and perceptual tasks is partly due to their inability to focus attention on the relevant and irrelevant aspects of the task, but a further study (Druker and Hagen 1969) discovered that the developmental change responsible for selective information processing did not involve improved visual discrimination; older subjects in this experiment were characterized by more efficient encoding and rehearsal strategies.

Attempts to show that concept formation proficiency is highly dependent upon the procedures used to present the information to subjects rather than, as has usually been contended, upon the strategies used to transform information into conclusions about the concept have been made by Denny and Gamblin (1965), who considered memory load and the form of information input. They gave subjects information in optimized or non-optimized form, presented when needed for use or prior to usage. The optimized form of presentation and no memory load conditions were associated with more proficient concept formation when both these conditions obtained: 73% of subjects showed errorless concept formation supporting the contention that proficiency is most importantly dependent upon input factors.

Generally speaking, performance deteriorates as the amount of irrelevant information increases. Wallach, Wall and Anderson (1967), for example, induced six and seven year-old children into conserving a number of objects by a procedure involving experience with reversibility and feel that their success may have been due to leading the subjects to stop using misleading perceptual cues. Phyllis Katz (1968) also, in her study with lower class children on a perceptually oriented concept formation task with the stimulus dimensions verbalized in advance and positive exemplaries of the concept continuously visible to the children, found that while increasing the amount of irrelevant stimulus information elicited more errors in all age groups it did not interact significantly with the developmental level. Response latencies of the older and brightest subjects increased with the number of irrelevant cues, whereas those of the less intelligent children did not, indicating possible developmental differences in information processing.

Similarly, Huttenlocher (1962) found that boys required to select correct blocks of a set with relevant dimensions of form and brightness performed less well when allowed to manipulate the blocks to construct instances for the test and it was surmised that planning of the manipulative responses distracted the subjects from the task of remembering and interpreting the material previously presented.

Similarly, Petre (1966) examined the effect of amount of pretraining with identical and dissimilar stimuli on concept learning. He used stimulus equivalence training designed to facilitate performance on a subsequent matching-to-sample concept task in which correct matching was on the basis of size and linearity. Half the experimental groups (D.R.) received training based on stimuli identical with the concept stimuli and the others (G.F.) were trained on stimuli which were physically dissimilar but also either curvilinear or rectilinear. It is interesting that the control group did not differ from the combined experimental groups in the mean number of errors on the concept problem but the D.R.

groups were superior to the G.F. groups.

The assimilation and utilization of information in concept learning under varying conditions of information presentation has also been examined by Sechrest and Wallace (1962). Their task concerned the attainment of a single two attribute concept by means of free selection from an array completely visible to the subject, and the four conditions of information transmission involved the initial positive instance alone and also accompanied by a list of the possible hypotheses remaining or eliminated and by exposure of the array. The groups did not differ in the number of instances required for solution or in the relevance of the first verbalized hypotheses. Inefficient performance was also not attributable to failure of information assimilation but to the fact that the subjects did not utilize all the available information. The third and fourth conditions also resulted in a significantly greater number of redundant hypotheses.

Concept learning and the degree of synchrony of relevant and irrelevant stimulus components occuring during the learning period has been investigated by Detambel and Stolurow (1956) who conclude that they are negatively related.

The effects of sequential manipulations of relevant and irrelevant stimulus dimensions or concept learning have been investigated by several experimenters. Kurtz and Hovland (1956), using geometrical designs as the concept material verified the hypothesis that "learning would proceed more rapidly under a condition in which the instances of a given concept were presented one after another without interpolation of instances of other concepts as compared with a condition in which instances of several concepts were presented in an intermixed order". Gengerelli (1927) also investigated the mutual interference in the evolution of concepts and saw that a common element contained in a series of symbols was learned less readily when the same symbols had previously been learned in the formation of other concepts than when they had not entered into other concepts. Newman (1956) and Anderson and Guthrie (1966), among others, corroborated this finding. Utilizing a paired associate method with line azimuth and the position of dots as the concept determinants, Newman found that the closer the grouping of representations of the same concept, the greater the liklihood that the responses to concept-relevant features would be temporally contiguous.

Anderson and Guthrie used an alternating series in their study, in which one set of relevant cues appears at odd numbered trials and another set at even numbered trials, contrasting with a constant series in which the same relevant cues appear at every learning trial. Their main finding was the interaction in both training and test trials between type of series and the number of stimulus dimensions changing from trial to trial. For the alternating series, the group where one stinulus dimension changed between each pair of adjacent training trials showed fewer errors than the group in which three dimensions changed. The reverse held for the constant series.

(iv) Feedback and reinforcement

Not only do the instructions given to the subjects, and the amount and type of information available influence performance in concept learning experiments, but feedback and reinforcement including knowledge of results have also been seen to play an important role.

Feedback may possess motivational as well as informational potential, and can be interpreted in terms of three intervening variables: response availability, the need for centainty and the need for confirmation (Wallace 1964).

In traditional concept learning (of didactic concepts) knowledge of results provides the information necessary for the selection of relevant attributes and the construction of assignment rules. In schematic concept learning (concepts defined by objects in the environment) the necessary information is extracted from the unfamiliar stimuli themselves with no external feedback.

Brown, Walker and Evans (1968) have investigated schematic concept formation as a function of constraint redundancy and knowledge of results. Their study involved a modified oddity task requiring the assignment of stimuli to categories corresponding to three different schema families and revealed that a high level of schematic redundancy (70%) facilitated schematic ooncept formation and that a lower level (40%) prevented its occurrence, but it was also seen that didactic concept learning did occur at this level if knowledge of results was provided.

Similar results showing the effectiveness of feedback have also been found by Beilin (1966) in invariant area conceptualization and was seen to be superior to translocation and iteration procedures.

Reinforcement and praise also are influential. Hatano (1966), for example, required subjects to judge greater than, equal to, or less than of two stimuli described by one of two values of four dimensions according to inclusion or exclusion of these stimuli to a given concept. The task proved difficult for children even when possible alternatives of criteria were explained. However, in the course of trials without reinforcement which were given after reinforced practice, about one third of the elementary school pupils changed their performances in the direction of increasing consistency. In another study (Overbeck and Schwartz 1970) on the training of weight conservation reinforcement was also seen to facilitate formation of the concept whereas active participation (as opposed to passive observation) by the subject did not.

In another experiment (Juzak 1955) praise and reproof used as false norms produced no significant difference in the generalization of learning concepts, although it was seen that when reproof was given in addition to praise the subject revealed a greater effort to perform correctly.

Various other experimenters (Høvland and Weiss 1953, Smoke 1932, 1933, Braley 1963) have studied the effects of negative, non-reinforced instances in concept learning tasks with varying results. In Braley's study involving exclusion solutions, task performance was seen to be unrelated to conditions of reinforcement as well as to instruction and to type of concept. It was, therefore, hypothesized that failure to use negative information was related to fundamental information handling strategies, such that limited memory storage induces a trial and error search based on positive identification of the reinforced instances.

Negative instances may, however, be rather effective in themselves in the learning of concepts. Smoke equated the amount of information so that differences in learning could be attributed to differences in difficulty of assimilating information concerning what it "is not". All negative instances were found to be inferior to all positive but his results disprove the generalization that concepts cannot be learned from negative instances, since under appropriate conditions more than half the subjects were able to arrive at the correct concept exclusively on the basis of negative instances.

(v) The effect of training on concept learning studies

A further and important influence on concept learning proficiency is the effect of experience through training. The type of training studied has varied and includes techniques such as nonverbal reinforcement, verbal orientation-reinforcement, verbal rule instruction, language activation, multiple classification, equilibration and cognitive conflict.

Training effects on Piagetian concepts

Although most of Piaget's theory and research revolves around the characteristics of separate states he also tackles the transition mechanisms leading from one stage to the next.

The mechanism of transition which Piaget proposes is an equilibration process. This process, continuously operating in all exchanges between the growing subject and his environment, is the propellant for change and transition. This continuous process of equilibration gives rise to successive, essentially discontinuous equilibrium states, that is, organized systems of actions (sensory-motor, perceptual, concrete-operational) whose attributes as systems are describable in equilibrium terms. As Flavell (1966) states "Piaget's analysis of transitions and states maps these two components, respectively, into a homogeneous and continuous equilibration process (the formative process) which gives rise to heterogeneous and discontinuous equilibrium states (the processes formed)".

The role of experience through training on the developmental patterns thus formed is not clear. Although the Geneva group considers experience to have a significant place in cognitive development Piaget seems lukewarm to the idea that experience alone may significantly alter logical thought development.

In support of this position numerous studies utilizing a variety of training techniques, as well as diverse populations, have been unsuccessful in accelerating the operational level of Piaget's stages or in facilitating the acquisition of conservation. Slater (1958), for example, has suggested that if the attainment of Piaget's stages can be accelerated and is not solely maturational, it would seem likely that children from different types of environments might attain the "operational" levels at different rates. Testing fifty infant children for conservation of number, one-to-one correspondence, seriation, cardination, additive composition, and relations of equivalence, Slater found that environment does play a part, although not always to the advantage of the children from the "better" environment. The subjects from the 'good' homes produced more operational results and were vastly superior when language was necessary. The 'poorer' children, however, produced better results when their interests were involved in the experimental situation. Motivation was seen to be very important in that the 'poorer' children performed better when they were required to share sweets or counters or the like.

Another study investigating the influence of the environment in fostering the development of 'operational structures' is the cross-cultural study by Hyde (1959). This investigator found that non-European children in Aden tend to acquire a basic numerical understanding considerably later than Europeans. The writer points out that this latter group was especially privileged. By the same token, the non-European groups were underprivileged and culturally deprived. There is therefore clear evidence of the influence of cultural factors on the development of operational reasoning.

It may be, however, that Hyde discounts too readily the effect of schooling on performance. Although many of her non-European subjects did not commence schooling until the age of 7 or 8, correlations of length of schooling with scores on Piaget's tests were not significant for even the European sub-group, although, at any rate at the eight-year level, all of these figures are positive and several are large.

The cogent and well designed study of antecedent processes in the area of number by Wohlwill and Lowe (1962) also produced negative results. In this study, training was directly towards conservation of number only, and the training procedures themselves were highly specifiable and of narrow compass. Seventy-two kindergarten children took part in the study, which was in the form of a non-verbal matching-from-sample type learning experiment, preceded and followed by verbal questions to measure the child's understanding of the conservation principle. There were four conditions of training, involving respectively the role of reinforced practice on conservation, of dissociation of biasing perceptual cues, and of inferential mechanisms based on the recognition of the effects of addition and subtraction of elements and a control group was also included.

The subjects as a whole showed an over-all increase in non-verbal conservation responses from a pre- to a post-test, within the limited context of the learning task, but there were, however, no significant differences attributable to the conditions of training. Transfer of conservation learning to the verbal post-test was negligible under all conditions, indicating that whatever learning may have taken place was of a rather restricted type, representing perhaps more the formation of an empirical rule than the understanding of a general principle.

Further support for Piaget's position comes from Beilin and Franklin (1962) in their study into the effects of age and training on logical operations in area and length measurement. It was found that length measurement is achieved prior to area measurement contrary to Piaget's description. For the first and third grade children the achievement of measurement is affected by the test itself and the third grade subjects were influenced further by instruction in measurement concepts. However, no first grade subject achieved operational area measurement even with training and instruction.

Support for Piaget's position that the young child is incapable of conservation is also forthcoming. In a series of five experiments Fleishman et al (1966) showed that failure of the four or five year-old child to conserve continuous or discontinuous quantities cannot be attributed to a simple non-repetition response to set. Their results also indicate that the incidence of conservation cannot be increased by methods which employ emphasis on (i) quantification, (ii) continuity of a transformation or (iii) continuity of transformation combined with reduction of visual cues. They also found that the feedback method, judged by stringent criteria, was also ineffective, although when more liberal standards were applied, a medicum of success was achieved.

Thus it would seem that Piaget's assertions are substantiated, and that it is indeed impossible to alter the sequence or bring about too rapid a change. After reviewing these attempts to train conservation, Flavell (1966) indicates, "Almost all the training methods reported impress one as sound and reasonable and well suited to the educative job at hand. And yet, most of them have had remarkably little success in producing cognitive change".

However, these conclusions are not unchallenged. Other investigators, utilizing a variety of techniques have induced different kinds of conservations and have succeeded in altering the sequence of Piaget's stages.

One technique, propounded principally by Jan Smedslund (1961 a, b, c, d, e, 1962 a, b), found to be successful in inducing conservation is that involving cognitive conflict. This investigator eventually reached the decision that induction of conflict coupled with external reinforcement was the key to successful training in conservation.

This theory involves a set of competing responses in a given situation which induces a conflict which reorganizes the intellectual structures and ensures conservation. This view proceeds along the lines of Piaget's equilibrium model and is generally consistent with it. Piaget's discussion on strategies to decrease egocentric thought in the child provides an illustration of the similarity of the cognitive conflict position and Piaget's adaptation position. In particular, the confrontation of different children's viewpoints generates a cognitive conflict and facilitates the acknowledgement of another point of view. Gruen (1966) points to the similarity of Piaget's and Smedslund's theoretical framework when he states that "Smedslund has used criteria more appropriate for assessing conservation as Piaget defines it than do criteria by Frank" (Bruner 1964). But the two positions differ in respect to the origin of the cognitive conflict and perhaps this distinction highlights Piaget's negative posture on early training.

Tentative support for Smedslund is found in Gruen's (1965) study where five year-old children were given pre- and post-tests on conservation of number, and also conservation of length and substance. Half the subjects were given pretraining on verbal discrimination of length and number, and half were not. One third of the subjects in each of the pretraining groups were given direct training on number conservation, one third were exposed to situations designed to produce "internal cognitive conflict" and one third received no training at all on number conservation. It was found that neither confronting the child repeatedly with the invariance of numerical values in face of irrelevant perceptual changes, nor devising situations to induce internal conflict, is particularly effective in inducing number conservation. Over half of the subjects who received one of those two types of training did not make even one conserving response on the post-test of number conservation. But the direction of the results, although not statistically significant, tends to support the equilibrium-through-internal-conflict hypothesis. There was also a significant difference between the conflict and verbal training groups combined and the control groups which suggests that number conservation can be acquired without any kind of direct external feedback if one can induce appropriate cognitive conflict in the child.

Another study which artificially induced conservation of substance in a situation where the subject's expectation of an event was reversed was that by Brison (1966). Here the five to six year-old child had to integrate the elements of the conservation situation to obtain a desired reward. Half the experimental subjects showed evidence of acquiring conservation and the concept transferred to substances not used in experimental training.

These results are corroborated for the conservation of length and number by different experimenters. In Murray's (1968) study on length the nonconservers trained by the reversibility and cognitive conflict procedure did significantly better than the untrained nonconservers on the Muller-Lyer task and on a transfer task with the same sticks distorted by the Oppel inverted T illusion. In Wallach and Sprott's (1964) experiment on number, first grade children were induced into conservation through experience with the reversibility of rearrangements which were regarded as implying changes in number. Such recognition of reversibility may account for the normal development of such conservations as those involving number and perhaps also amount.

This successful training procedure assumes to affect the cognitive structure, emphasizes internal rather than environmental variables and claims success only with children who are in possession of the proper cognitive structure but have not yet reached equilibration. Kingsley and Hall (1967), however, offer an alternative hypothesis for the training failures which emphasizes experiential rather than internal variables. Their contention is that most of the training attempts which failed have ignored the large amount of background knowledge necessary and thus time needed to train children for conservation mastery. They tested this position through the use of training based on Gagné's learning set procedure.

As a result of dissatisfaction with the application of standard learning theory principles, Gagné (Gagné and Paradese 1961, Gagné 1962, 1963, 1965) has developed an approach which analyses the material to be taught into a hierarchy of subtasks. Attainment of these provides a successful route to the mastery of a criterion task. In contrast to Piaget's stage theory, Gagné's analysis of learning sets depends much more heavily on experience and ignores the role of maturationally related unfolding internal structures in determining acquisition sequence.

"It is of importance to note that there is no intention of describing a 'developmental sequence' relating the learning of an individual to his stage of growth or chronological age. Simple kinds of learning .... occur throughout the learning of an individual, no matter how old he may be ... At the same time it is inevitable that some correlation between the sequence of simple-complex learning and the sequence of earlier-later events will be present. The individual learns simpler things first, than more and more complex things; while all this is happening, he is also growing older" (Gagné, 1965, p.175).

This would suggest that not only could different types of conservation be taught quite rapidly but that they could also be mastered out of sequence as long as they do not require identical knowledge.

In their study to test the efficacy of this approach, Kingsley and Hall used Gagné's learning set analysis as a basis for teaching five and six year-old children length and weight conservation. As a result of their demonstration and their explanation strategy with continual eliciting of subjects responses, they obtained highly significant training effects. In addition, experimental groups improved significantly more than control groups in substance conservation although this may have been due to the similarity of weight and substance tasks. Although only three of the appropriate seventeen subjects in the trained group resisted extinction, all fifteen of a "natural conservers" control group also extinguished.

Also convinced that reasoning and problem solving capabilities are best conceived to consist of repertoires of behaviour, modifiable in accordance with principles of learning, instead of inaccessible structures and processes, and that basic changes in level of intellectual functioning are therefore not matters of "internal equilibration" but rather of learning as the result of "external reinforcement" is Anderson (1965). This experimenter demonstrated that children can acquire and transfer a rather advanced complex problem solving skill when taught by the procedure of varying each factor in succession while holding all other factors constant. This is, of course, a classical strategy of experimental science and has been argued favourably by Bruner et al (1956). The skill is believed to be advanced and is thought by Piaget to appear naturally only at fourteen to sixteen years of age (Inhelder and Piaget 1958, p.335), but it is of value as it may be a "tactic" in a more sophisticated strategy.

Corroborative results also come from another study which also shows that the acquisition of compound concepts may be facilitated by previous training with simple concepts. Kendler and Vineberg (1954) required subjects to learn two simple concepts and then a test concept which involved combinations of simple concepts. The rate of learning the test concept was directly related to the number of simple concepts appropriate to the test concept which had been learned.

As previously stated in the section concerning the effect of instructions on subjects' performance, concept learning proficiency may be facilitated by training on the principle or verbal rule involved. Beilin (1965) has firmly adopted this successful training procedure which provides the child with a statement of a rule to be applied to the problem in each instance of an unsuccessful trial response or a conservation task.

Although induction of conservation in this technique is based on the understanding of a specific rule and this is not possible in Piagetian reasoning, Beilin has shown it to be effective. In his study with kindergarten children on length, number and area conservation he used various training procedures - nonverbal reinforcement, verbal orientation reinforcement, "equilibration" and verbal rule instruction methods. The results indicate training to be effective in facilitating conservation performance but principally with provision of a verbal statement of the problem-solving principle. No conservation learning transferred to the area task and correct verbalization of the conservation principle, both before and after training, was less predictive of correct performance in conservation tasks than the reverse.

Beilin's position is reinforced by further findings (Smith 1968), which suggest that verbal rule instruction improved the performance of both nonconservers and transitional conservers of weight, whereas modification of Smedslund's addition-subtraction method produced no effect on conservation of either group. Neither group, however, proved to be resistant to the countersuggestion involved in a demonstration of apparent nonconservation.

Also Lister (1969, 1970) using a flexible teaching procedure including set experiences and explanations has shown that the concepts of weight and volume conservation can be taught to educationally subnormal children. Her nonconserving subjects (aged nine to fifteen with IQs between 46 and 75) succeeded in recognizing, generalizing and giving reasons for the conservation and post-testing as late as five months afterwards, revealing that the concept was durable as well as generalized.

A further technique believed to facilitate the acquisition of conservation involves multiple classification. Sigel (Sigel, Roeper and Hooper, 1966) assumes that one bases the learning of complex structures on simpler structures and claims that acquisition of conservation of substance follows the acquisition of simpler structures such as multiple labelling, multiple classification, multiple relations, and reversibility. These investigators argue that conservation is possible only when the child is capable of these three mental operations and their study shows that training on the three operations led to an increase in conserving responses.

A belief which postulates that focusing on the linguistic aspects of a situation and so decreasing the strength of perceptual cues will aid conservation is held by Jerome S. Bruner, (Bruner 1964, Bruner et al 1966). In their study on fullness, Bruner and Kenny hypothesized that activating the linguistic aspects of the problem would minimize the effect of misleading perceptual cues and so facilitate acquisition of the concept of fullness. However, alerting their five, six and seven year-old subjects to the specific perceptual properties of the displays did not effect a significant improvement and the experimenters conclude that it "avails little to activate children to separate cues when their difficulty is not so much with the cues as such but with their combination".

One body of opinion which is not surprised at Bruner's failure to induce conservation and which also seriously doubts the techniques of Beilin, Sigel and Smedslund, is that of Mermelstein and his associates (Mermelstein and Shulman 1967, Mermelstein and Meyer 1969). These researchers firmly believe in the Piagetian theory and point out that whereas Smedslund's theoretical position is generally consistent with Piaget's, Sigel, Bruner and Beilin present arguments for inducing conservation which are either inconsistent or only partially consistent with Piaget's viewpoint.

According to Mermelstein, Sigel's procedures are indeed necessary conditions for conservation of substance but they are not sufficient conditions. Sigel's training assumes that once the prerequisites for conservation have been met, the concept of conservation necessarily has been attained. Sigel further assumes that because the material is presented in a prescribed manner, the child will assimilate the material in the corresponding order. Such a claim is not consistent with Piagetian theory; for, if the child constructs his reality, it does not necessarily follow that he will assimilate the material in the same order presented.

They also cast doubt on the verbal rule procedure that Beilin presents, as this "training procedure appears to violate Piagetian theory". They state that the use of a rule in a training technique appears inconsistent with Piaget and go on "To begin with, the egocentric nature of the child's thought seriously hampers his ability to adopt another point of view. Second, the syncretic nature of the child's mental structure with its behavioural manifestations of juxtaposition prevents him from accurately perceiving the rule because in juxtaposition the child does not take into account the necessary connections of the rule." In a related study, Mermelstein and Shulman (1967) indicate that children under nine years of age perceive only the gist of questions or the events of the questions. Since questions and rules are both statements implying necessary connections, it follows from this that two statements which stress "amount", for example, but in different ways, will be perceived as similar by the child. Thus the child will not truly understand a "specific" rule in that a specific rule implies a particular connection to an event. The child, on the other hand, acts as if all specific rules had the same connection to the event.

Also, Bruner's contentions on language training also appear to be inconsistent with Piagetian theory in that the resulting mental structure is a function of the language activated whereas Piaget believes that the mental structure precedes language development. Furthermore, even though the Bruner, the Beilin and the Sigel techniques employ more verbal content than Smedslund's, the contention that the advantaged child should do better on these training procedures than the disadvantaged child is, according to Mermelstein, ill-founded.

Although advantaged children manifest language facility superior to disadvantaged children, Mermelstein seriously questions the impact of the difference at this stage of development. In other words, all children tend to rely more on concrete-manipulative experiences rather than linguistic experiences. And, further because of the syncretic characteristic of child thought, it is questionable whether the child's greater language facility can be appropriately harnessed to assist in the acquisition of the concept. For according to Piaget, without the concept the language facility cannot be appropriately directed. Consequently, it is contended that the particular population is of no consequence in an examination of the acquisition of the concept of conservation.

In summary then, because the concept of conservation derives from Piagetian theory and because the rationals for the Smedslund, Sigel, Beilin and the Bruner training are judged by these authors to be not completely consistent with Piaget, they hypothesized that these training procedures would not be effective with a diverse population. Tests three weeks, two and a half months and five months after the last training session indicate that the "Piagetian" concept of conservation (involving generalizability and durability) was not induced by any of the training techniques on any of their different populations of nursery and kindergarten subjects. They found that, with time, improvement in conservation occurs, a result which is consistent with the contention that acquisition of conservation is a function of factors other than training.

Wohwill (in Bresson et al 1966), in his consideration of the role of experience in cognitive development, emphasizes that it is important to consider what actually is learned after any particular training. Similarly, Mermelstein does concede, however, that because the other experimenters employ different criteria it is possible that the success which they report relates not to the concept of conservation of substance as Piaget sees it but rather to some other concept or some deformation of the concept. Adding credence to such a contention, Piaget points to the different conceptions of conservation that he and Bruner have. Further, Almy et al (1966, p.41) and Gruen (1966) stated that the selected training techniques depend on whatever theory about the nature of human learning the experimenter holds. Directly testing such an interpretation, Gruen (1966) demonstrated that the different criteria and the different procedures employed by Bruner and Smedslund led to a different classification of of conserving responses. Although Smedslund's different criteria offer one explanation of results, a difference between the cognitive conflict position and Piagetian theory which Smedslund has reported may provide an alternative explanation for the lack of success of his training.

Smedslund (1962b) suggests that the organism-object conflict which he espoused is not of sufficient moment to cause a modification of the intellectual structures. He suggests that the organism-object relation is too neutral for the child and does not or cannot create the conflict in the child. In other words, he suggests a confrontation of different points of view among children rather than a confrontation of different viewpoints between the child and object as a necessary condition for modification of intellectual structures.

Sigel (personal communication to Mermelstein, 1968) also, in his latest research, is modifying his position on acquisition of conservation. Rather than suggesting that multiple labelling, classification, seriation, and reversibility training in that order facilitate conservation, he now suggests that classification and discrimination learning may be among the several variables for conservation. Supporting Sigel's most recent position, Meyer and Mermelstein (1968) have demonstrated that the "sequence" within Sigel's training technique does not play a role in the acquistion of conservation, and they have suggested that if training is successful classification may play a role.

CHAPTER IV

SUMMARY OF THE LITERATURE

In reviewing the relevant and related literature an examination was made of how the concept of proportion, in the arithmetical and more especially the physical sense, grows gradually in the child's cognitive development. A demonstration of how the formation of just such a concept is influenced by a variety of factors both verbal and perceptual was also made.

Physical proportion is typified by problems involving time, movement and velocity, the equilibrium in the balance, hauling a weight on an inclined plane, the projection of shadows, centrifugal force and compensation, space and quantity.

As the research on quantity concepts is most pertinent to the present study it was investigated most fully and great attention paid to the work on conservation. Extensive research by Piaget et al on the conservation of volume indicates that the child does not learn to conserve volume until about eleven or twelve. It is only then that children discover the true notion of volume; it is then that they find out for themselves, and often independently of what they are taught in school (a) how to multiply the length, height and breadth to ascertain the volume of a cuboid, and (b) recognize that the space which it takes up in the water is invariant whatever its transformations, and that the volume which it displaces is likewise invariant. It is only then that they learn to cope with the proportions involved.

It is also about this age that Bruner et al state that the child appreciates the idea of fullness, but the findings of this work is questioned and doubts are cast upon the experimental design and method. An understanding of fullness demands more than the recognition of the relationship of one volume of liquid to another; it requires the ability to relate the volume of liquid contained in a vessel to the total volume of the vessel. Since this ratio is more abstract or symbolic, it would be expected that the child who had achieved the notion of volume would not necessarily be able to judge with equal confidence which of two containers was fuller, or which was emptier. The concept of volume as a multiplicative function of linear dimensions is a comparatively late acquisition, as also is the understanding of the equivalence of quantities of water displaced by equal but dissimilar volumes, but it should be expected that the concept of fullness would be achieved even later by the child.

Although the ability to conserve and also the appreciation of fullness appear to develop spontaneously in the child's cognitive development, situational factors such as language, instructions, information and reinforcement, and also experience and training do influence the formation of such concepts. The issue is not clear, however, and contradictory results abound with some investigators succeeding in accelerating concept formation and others failing to alter the developmental sequences postulated by the Geneva group.

The study which follows investigates the proportions and ratios involved in judging fullness and also demonstrates the importance of the factors enumerated above in the performance of this task.

PART 2

THE INVESTIGATION

CHAPTER V

METHOD OF FIRST EXPERIMENT

In the first part of the study two hundred and twenty children, one hundred and ten boys and one hundred and ten girls, aged five to fifteen years (twenty in each yearly age group), from six Northern Ireland schools (three primary, two grammar and one secondary intermediate) and also twenty adults served as subjects. The schools - Finaghy Primary School, Seymour Hill Primary School (Dunmurry), Tonagh Primary School (Lisburn), Larkfield Secondary School (Dunmurry), Annadale Grammar School (Belfast) and Friends' School (Lisburn) - were chosen in different areas so that combined they formed a fairly representative sample. On the other hand, the classes within each grade and the children within each class were chosen at random by the experimenter. The twenty adult subjects were a selection of students, laboratory technicians and secretaries from the University of Stirling, Scotland.

Only the five to eleven year-olds from the three primary schools were tested in the first instance but when it became evident that even the eleven year-old children did not fully understand the relationships involved, the older subjects were subsequently included in the study.

Half the children in each age group to eleven years (five boys and five girls) were asked "fuller" and the rest "emptier". The verbatim questions were as follows. E: "Is one glass fuller/emptier than the other?" If S replied "Yes", E asked "Which one?" and then "Why?" If S replied "No" E asked "Why not?" The older subjects were all asked 'fuller'.

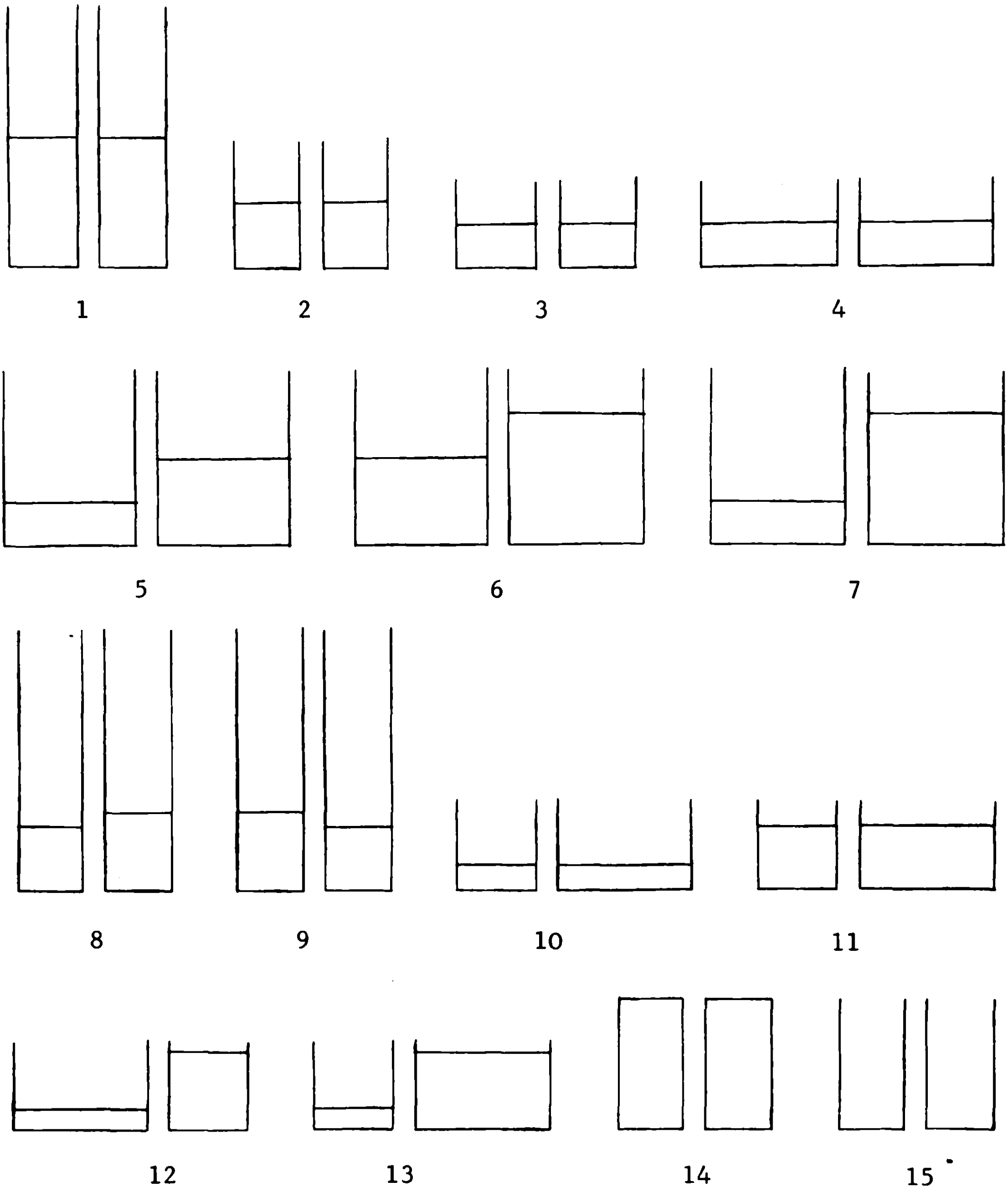
Forty-five pairs of glasses, filled to varying degrees with blue-tinted water were presented in three separate displays each containing fifteen pairs. The dimensions of the glasees are shown in Table 1 while Figures 2, 3 and 4 show diagramatically the forty-five pairs as presented in the three displays. Within the displays the combinations of glasses and water levels were arranged in such a way as to facilitate the changing

Figures 2, 3 and 4

The 45 pairs used in the experiment
as presented in the three displays

DISPLAY 1

Scale ¼ inch to 1 inch



DISPLAY 3

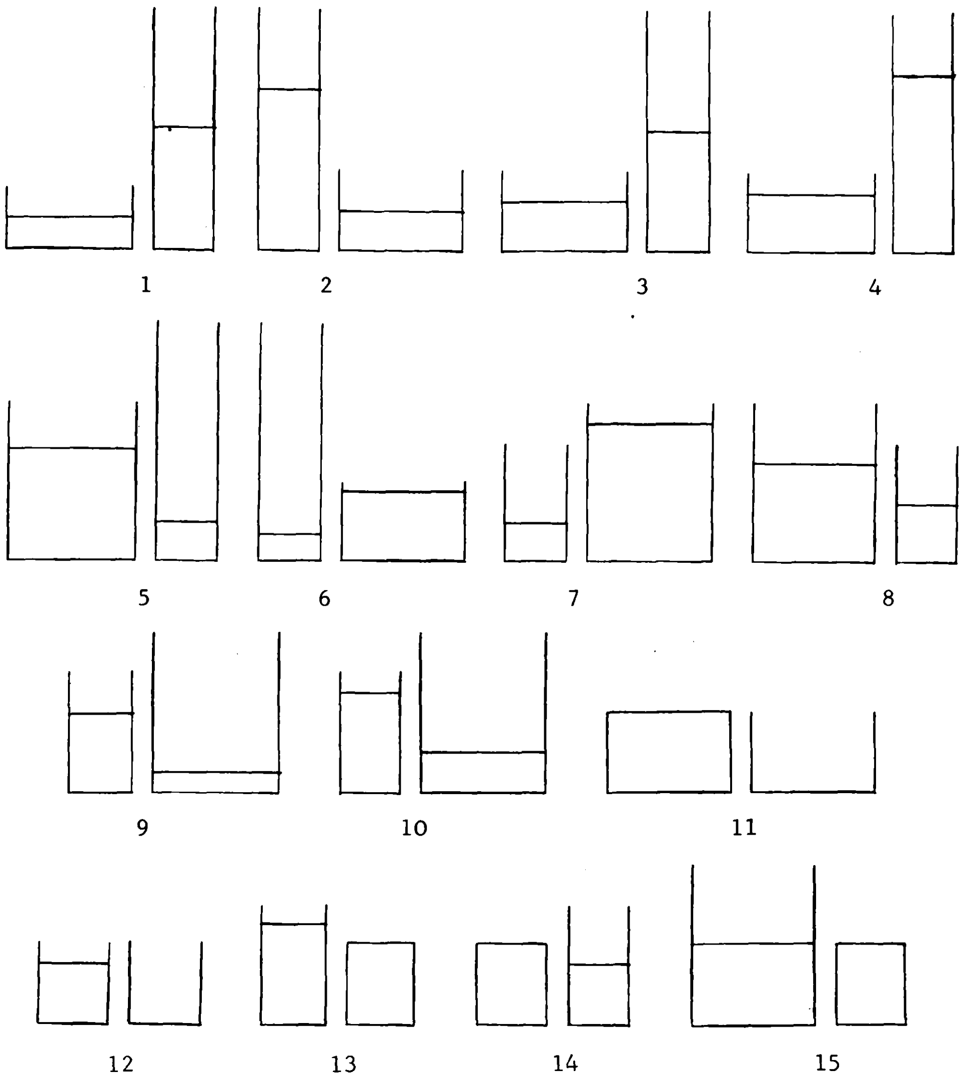


Figure 5

The minimum number of pairs necessary to represent all the possible combinations of features

Scale 1/6 inch to 1 inch

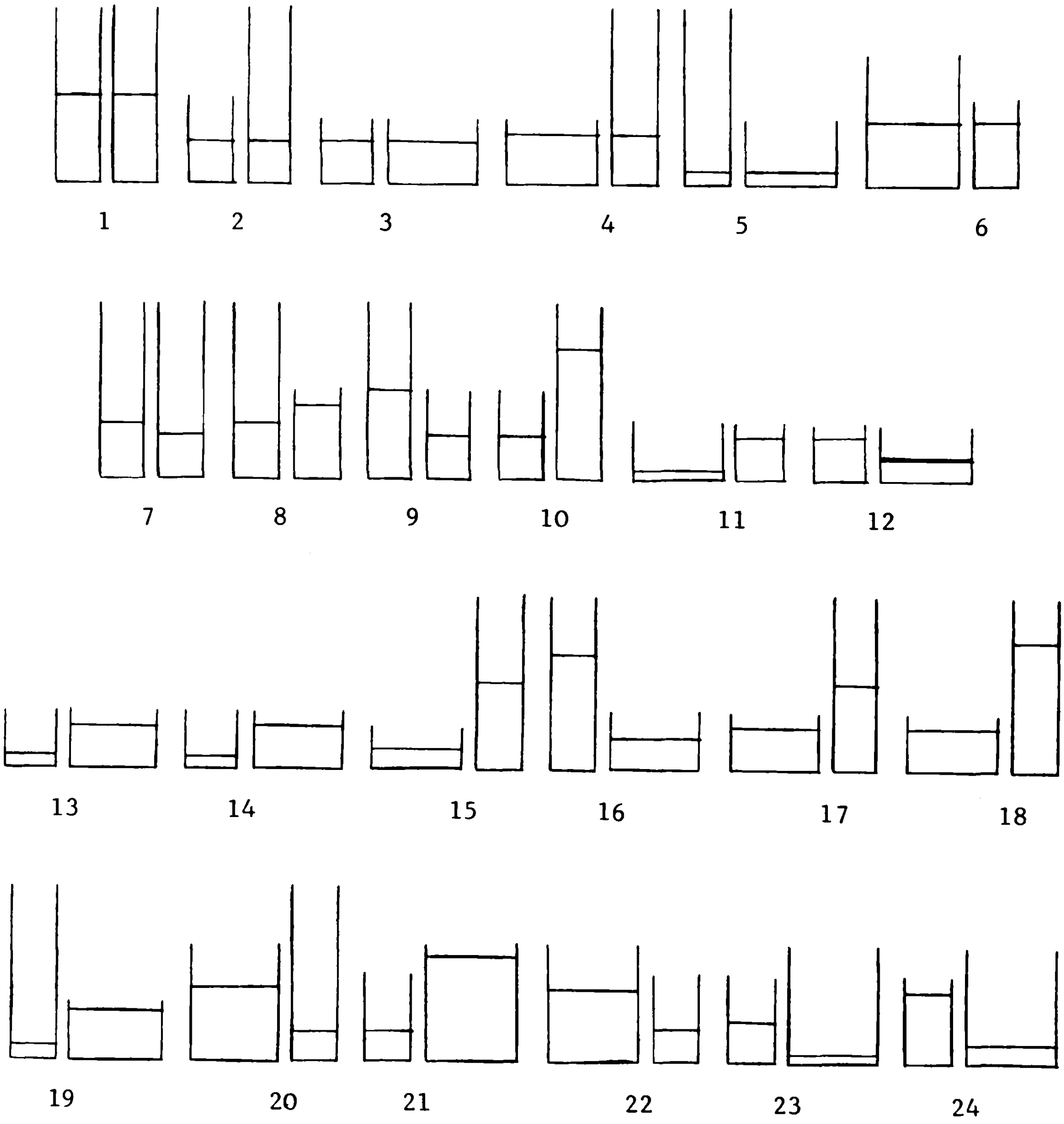


TABLE 1

NUMBER AND DIMENSIONS OF GLASSES USED IN THE DISPLAYS

Number of glasses	Height in inches	Diameter in inches
6	6	1½
6	3	1½
6	4	3
6	2	3
6	2	1¾
1	1½	3

of the displays.

In each pair the water level (W.L.), the height of the glass (H.G.), the diameter of the glass (D.G.), are all purely perceptual and can very independently but the volume full (V.F.) and the volume of unfilled or empty space (V.E.), and also the proportion full (P.F.) and the proportion empty (P.E.), all require some computational thought; they are also complementary to each other and dependent on how W.L., H.G. and D.G. are combined.

All possible combinations of features were presented and as Table 2 and Figure 5, show, the minimum number of pairs is twenty-four.

In Type 1 (pair 1 minimum 24) the glasses are identical and the water levels are the same. Thus the volume of water and the volume of space in the two glasses are the same. In short, the glasses are partly filled to an equal proportion.

Type II (pair 2) consists of pairs of glasses of the same diameter but different heights. Since the water levels are equal, the volumes full are also the same. The taller glass has a greater volume empty and is therefore not as full as the shorter glass.

Type III (pair 3) pairs comprise glasses of different diameter but the same height. The water levels are the same which means that both the volume full and the volume empty differ in the two glasses, but they are equally full.

TABLE 2

ALL POSSIBLE COMBINATIONS OF FEATURES, EXCLUDING PAIRS WITH EITHER ONE OR BOTH GLASSES FULL OR EMPTY

Types	Water Level	Height of glass	Diameter of glass	Volume full	Volume empty	Minimum number of different pairs
I	S	S	S	S	S	1
II	S	D	S	S	D	1
III	S	S	D	D	D	1
IV	S	D	D	D	S/D	3
V	D	S	S	D	D	1
VI	D	D	S	D	S/D	3
VII	D	S	D	S/D	D/S	4
VIII	D	D	D	S/D	S/D	10
						Total 24

Key S: Same D: Different

Type IV represents pairs of glasses of different dimensions with the same height of water. Three possibilities exist here: (1) both the volumes full and empty differ with the greater volume of liquid in the shorter glass (pair 4). (2) The volumes full differ while the volumes empty are equal (pair 5). (3) Volume full and volume empty again both differ but in this case the greater volume of water is in the taller glass (pair 6). Since the water levels are the same, the shorter glass in all three instances is the fuller.

Type V is composed of identical glasses with different water levels (pair 7). Both the volumes full and empty differ, and the glass with the higher water level and greater volume of water is the fuller.

In Type VI we have examples of pairs of glasses of different heights but same diameters. In this type, the water levels differ. As in Type IV, three possibilities exist: (1) Both the volumes differ, and the higher water occurs in the shorter glass (pair 8). It therefore has a lesser volume of space, and a greater volume of water and is fuller. (2) Here again the volumes differ but now the higher water level and greater volumes of water and space occur in the taller glass, which can thus be filled to a greater, less, or equal proportion. The glasses in this pair were made equally full (pair 9). (3) The volume of water is different but the volume of space is the same (pair 10). The taller glass with the higher water level and greater volume of water is fuller.

In Type VII, the glasses in each pair are the same height with different diameters. The height of the water also differs. There are four different instances of this combination of features: (1) The volume full is the same in the two glasses while the volume empty differs (pair 11). The higher water occurs in the narrow glass which is fuller. The volume of unfilled space is greater in the wider glass. (2) Volumes full and empty differ and the higher water occurs in the narrow glass which is therefore fuller (pair 12). In this case both volume full and volume empty are greater in the wider glass. (3) The volume of water can differ while the volume of space remains equal (pair 13). The higher water level and greater volume of liquid occur in the wide glass which is filled to a greater proportion. (4) Volumes full and empty both differ with the higher water level and greater volume full in the wider glass which is fuller. The greater volume of space occurs in the narrow glass. (pair 14)

In Type VIII, all three features differ within the pairs. There are ten different instances when water levels, heights and diameters all differ: (1) Volume full and volume empty the same, so that the glasses are equally full (pair 15). The higher water level occurs in the tall glass. (2) Volume full the same and volume empty different. In this pair (pair 16), the volume empty is greater in the short wide glass, while the tall narrow glass has a higher water level and is fuller. (3) Volume full different and volume empty the same (pair 17). The higher water level occurs in the tall thin glass, while the volume full is greater in the short fat glass which is filled to a greater proportion. (4) Volumes full and empty both different. The higher water level is in the tall thin glass which can be filled to a greater, less or equal proportion to the short fat glass.

Here (pair 18) the glasses were made equally full, with the volumes full and empty both greater in the short wide glass. (5) Volumes of liquid and space both differ (pair 19). Volume empty is greater in the tall narrow glass, while the volume full is more in the short wide glass which is fuller. (6) Volume full different and volume empty the same (pair 20). Water level higher and volume full greater in the tall wide glass which is filled to a greater proportion. (7) Volume full different and volume empty the same (pair 21). Water level is higher and volume full more in the tall wide glass which is fuller. (8) Volumes full and empty both differ. Water height and volumes full and empty are all greater in the taller wide glass, which could be filled to a greater, equal or less proportion than the smaller narrow glass. The taller glass was made the fuller (pair 22). (9) Volume full the same and volume empty different (pair 23). The volume of empty space is here greater in the tall wide glass. The shorter, narrow glass has a higher water level and is fuller. Finally in this last type (10), volumes of water and space both differ (pair 24). They are greater in the tall, wide glass, but the short narrow glass has the higher water level and is fuller.

Table 3 shows the eight different types of pairs as found in the three displays and Table 4 where the minimum twenty-four pairs were located in the displays.

TABLE 3

THE 8 DIFFERENT TYPES OF PAIRS AS PRESENTED IN THE 3 DISPLAYS

Types	Display Number	Pair Numbers
I	1	1 to 4
II	2	1 and 2
III	1	10 and 11
IV	2	3 to 5
V	1	5 to 9
VI	2	6 to 8
VII	1 and 2	12 and 13, 9 and 10
VIII	3	1 to 10

TABLE 4

THE MINIMUM 24 PAIRS AS LOCATED IN THE 3 DISPLAYS OF THE FIRST STUDY

DISPLAY 1		DISPLAY II		DISPLAY III	
Pair No.	Min. 24 No.	Pair No.	Min. 24 No.	Pair No.	Min. 24 No.
1	1	1	2	1	15
9	7	3	6	2	16
11	3	4	5	3	17
12	12	5	4	4	18
13	14	6	8	5	20
		7	9	6	19
		8	15	7	21
		9	12	8	22
		10	11	9	23
				10	24

In addition to using glasses which were partially full, pairs were included to test the effects when one or both glasses in a pair are completely full or empty: all the eight types of pairs were represented in this respect except types II, VI, and VII.

By presenting in display 1 more than one pair of a particular combination of features, where thought desirable and where the number of glasses permitted, it was possible to consider whether the results were due to sporadic chance factors or to faulty knowledge (Grossnickle F.E. and Synder J.H. 1939); it was also possible to test to see if varying experimental parameters such as the size of container (pairs 1 to 4 in display 1), height of water (pairs 10 and 11), position reversal (pairs 8 and 9) and ordering effects (pairs 5 to 7) produced any effects.

The basic and underlying variations inherent in the task are thus tested at the outset of the experiment. The child is introduced to the task via relatively easy judgments; all the glasses within each pair in display 1 are of the same height, and indeed except for pairs 10 to 13 the glasses in each pair are of identical size and shape.

The fifteen pairs in each display were arranged and numbered on a table in the same way for each subject, and all subjects judged the displays in the same order (1, 2 and then 3) but the order of judging the pairs in each display was determined randomly, for each subject.

Furthermore, the position of each individual member in all pairs was determined randomly, except where undesirable, i.e., in the pairs testing ordering effects, position reversal and changing the height of the water.

The adult subjects were given only the twenty-four minimum pairs and asked only to judge "fuller", for reasons explained later.

CHAPTER VI

METHOD OF DEMONSTRATION EXPERIMENT

In investigating where the ability of the subjects to judge what is involved in deciding which of a pair of glasses is fuller breaks down, it might be said that they either do not understand what to do or, on the other hand, that they cannot do what is required, and if they cannot do this, then what can they do?

In order to determine if in fact the original task, for one reason or another, was misunderstood, it was decided to give a short demonstration with explanations and then administer the original task, giving only the twenty-four minimum pairs and asking only "fuller".

General Procedure

Two hundred and twenty children aged five to fifteen years of age (twenty in each age group) and twenty adults served as subjects. Again the children were chosen randomly and in this study half the secondary pupils came from Grosvenor High School (Belfast), and the rest from Larkfield Secondary Intermediate School (Dunmurry). (The Primary School pupils were chosen from Brown Street P.S. and Glenwood P.S., both in Belfast, and the twenty adult subjects were a selection of students, laboratory technicians and secretaries from the University of Stirling, Scotland.) The demonstration was the same for all subjects and was as follows with E. speaking as follows.

E: "We are going to have a look at what we mean when we say that one glass is fuller than another. First of all, here is a completely full glass; the water reaches right to the top of the glass; there is no empty space.

Next we have an empty glass; there is no water in it at all; it is all space.

Now, here are some glasses which are only partly full; they have some water and some empty space. This glass is half full; it has the same amount of water and empty space. This glass is more than half full;

it has more water than space. This glass is less than half full; it has less water than empty space.

This glass is also less than half full (with reason) and this one is more than half full (reason given). If we put these two glasses together (E demonstrating) to form a pair, this glass (pointing) would be fuller than that one. Wouldn't it?"

E continued in this way with the remaining glasses, giving the proportion full in terms of "half full", "less than half full" or "more than half full" and then explaining with reference to the V.F. and V.E. E moved the second glass of each pair in turn to form the pair and then moved it back to its original position in the display.

The demonstration took the form of Piaget's clinical approach to the extent that if S disagreed with E the explanation was repeated or even rephrased in order to try to convince S. The S was involved in the demonstration by being required in a casual way to agree (or disagree) with E.

Since the child in judging fullness must learn to resist the perceptual cues, water level, height of glass and diameter of glass, and consider the necessary relationships, it might be hypothesised that until he is capable of doing so he will tend to be led by these visible cues. The pairs of glasses to be used for the demonstration were therefore chosen with respect to the influence of perceptual cues on the child's judgment: the three perceptual cues can be arranged in each pair either to bias the child in favour of or to militate against a correct solution. Within the eight types of combinations of features the number of cues which ~~can~~ vary within each pair, that is, ~~be the same or~~ differ in the two glasses of a pair, is constrained to different extents: in type I, none differs, in types II, III and V, one differs, in types IV, VI and VII two can differ, and in type VIII all three factors differ within each pair.

Tables 5 and 6 show how the twenty-four pairs can be analyzed for

TABLE 5

ARRANGEMENT OF PERCEPTUAL CUES

No. of free factors	No. of factors favouring correct judgment	No. of factors favouring wrong judgment	Type No.	Display No.	Pair Nos.
3	3	0	VIII	3	7 and 8
3	3	0	VIII	2	13
2	2	0	VI	2	8
2	2	0	VII	2	10 and 13
2	2	0	VII	3	15
1	1	0	V	1	5 to 9
1	1	0	V	3	11 and 12
3	2	1	VIII	3	2, 5 and 6
3	2	1	VIII	3	14
2	1	1	IV	2	4 and 5
2	1	1	VI	2	6
2	1	1	VII	1	9 and 12
3	1	2	VIII	3	3, 9 and 10
3	1	2	VIII	2	14
3	1	2	VIII	3	13
1	0	1	II	2	1 and 2
2	0	2	IV	2	3
3	0	3	VIII	2	15

TABLE 6

ARRANGEMENT OF PERCEPTUAL CUES IN PAIRS WHICH DEMAND EQUALITY OF JUDGMENT

No. of free factors	No. of factors favouring one glass	No. of factors favouring other glass	Type No.	Display No.	Pair Nos.
0	0	0	I	1	1 to 4
0	0	0	I	1	14 and 15
1	1	0	III	1	10 and 11
1	1	0	III	2	11
2	2	0	VI	2	7
3	2	1	VIII	3	1 and 4
3	3	0	VIII	2	12

their arrangement of perceptual cues and provides the basis for the selection of the demonstration pairs: as seen in Figure 6 one pair was chosen at random from each of the six different categories: (1) pairs where all 'free' factors favour the correct judgment (2) pairs where 2 out of 3 cues favour the correct solution, (3) pairs where 2 factors are free to differ and are balanced, (4) pairs in which the 3 factors differ and only 1 bias the child in favour of the correct choice, (5) pairs where all free factors militate against the correct solution, and finally, (6) pairs which demand equality of judgment.

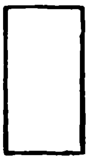
In order that none of the demonstration pairs would be repeated in the testing later, any pair where V.F. and V.E. both differed within the pair was duplicated.

The first pair was representative of type V with V.F. and V.E. both different. This is a pair where all free perceptual factors favour the correct judgment. The second pair is a duplicate of one of the ten type VIII pairs (display 3 pair 5). This is a case where 2 out of 3 perceptual cues favour the correct answer. The third pair is a duplicate of a type IV pair (display 2 pair 4) and represents an instance where 2 perceptual cues are free to differ and are balanced. Pair 4 is another duplicate of a type VIII pair (display 3 pair 9) and demonstrates the case when the 3 perceptual factors differ and only 1 favours the correct judgement. Pair 5 is a duplicate of a type IV pair (display 2 pair 3) and is an example where all free factors militate against the correct solution. Pair 6 is one of the type I pairs and demonstrates equality of judgment.

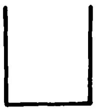
Following the demonstration subject then moved to the testing pairs. In order not to lengthen the procedure more than necessary, it was decided to test only the twenty-four minimum pairs rather than the forty-five administered earlier, and to compare results for these pairs only. The position of the glasses in each pair was the same as in the original study.

Figure 6

The glasses used for the demonstration
(pair numbers refer to minimum 24 pairs)



full



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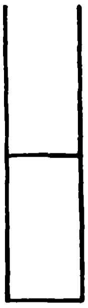

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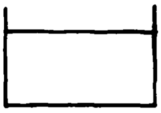
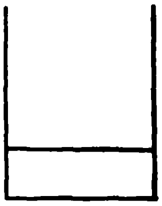
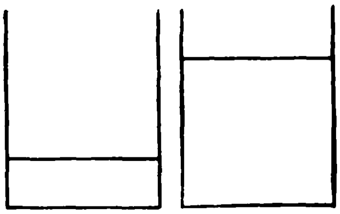


more than
½ full



less than
½ full



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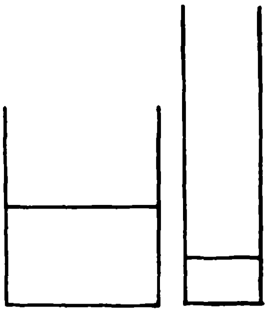

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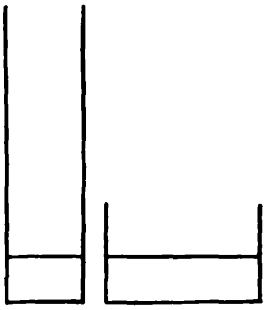

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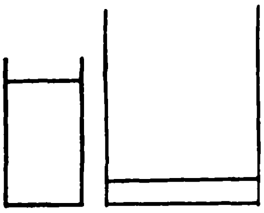

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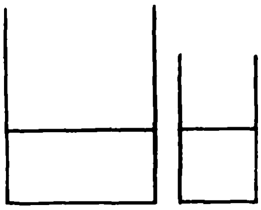

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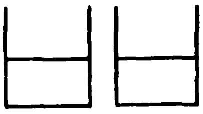


No. 1 duplicated

In order to prevent a serial effect, the order of the twenty-four pairs on the table was arranged in a random fashion and kept constant for all subjects and the order of judging the pairs, was also randomized for each individual subject.

Figure 5 shows the pairs and their arrangement in the display was as follows: 9, 1, 2, 5, 24, 8, 17, 19, 11, 10, 18, 7, 23, 20, 15, 12, 21, 16, 14, 3, 22, 4, 13, 6.

CHAPTER VII

RESULTS OF THE FIRST STUDY

(a) Comparison between judgements of 'fuller' and 'emptier'(for subjects aged five to fifteen years)

The number of subjects correct for each pair in the two independent groups was calculated (Tables 7, 8, 9 and 10) and submitted to the Fisher Exact Probability Test (2 tail). The observed differences were no more than one would expect by chance: only three cells were significantly different at the 5 per cent level of rejection and this is marked by an asterisk in the tables, five others differed at the 10 per cent level of significance, and one at α = .025.

Close analysis of both the number of subjects correct and the reasons given reveals that the child thinks generally in terms of fullness when required to judge which of a pair of glasses if fuller or emptier. Indeed, no child at all under eleven years old mentioned the words 'space', 'air' or 'emptiness' and of the thirteen aged twelve to fifteen years who did, six were judging fullness. Since the two samples carry out the tasks in essentially the same way their results were combined and considered as one group for the purpose of analysis. Also the adult subjects and those participating in the second 'demonstration' experiment were asked to judge only 'fuller'.

(b) Preliminary Tests

The total number of children correct at each age level on each pair is shown in Tables 11 and 12, while Table 13 shows the results for the basic twenty-four pairs for all the subjects including the adults.

Examination of the pairs testing for variations in experimental parameters reveals that no difficulties arise from them and that the errors in judgments observed were not due to these variations.

TABLE 7

NUMBER OF CORRECT RESPONSES FOR SUBJECTS ASKED FULLER

Type No.	Display No.	Pair No.	Age in Years										
			5	6	7	8	9	10	11	12	13	14	15
I	1	1	4	7	9	10	9	8	10	10	10	7	9
		2	5	9	9	10	10	8	9	9	7	6	9
		3	4	8	8	10	9	8	9	10	7 .05	6*	9
		4	4	7	8	9	10	8	9	6	5	6	8
II	2	1	4	0	2	1	0	1	1	0	0	1	0
		2	3	1	3	1	2	1	1	0	0	2	0
III	1	10	3	5	5	5	3	2	1	3	3	1	5
		11	1	4	3	3	3	1	1	3	2	2	0
IV	2	3	2	0	1	2	0	0	0	1	0	0	0
		4	3	2	8	5	9	8	9	6	6	7	9
		5	6	4	8	6	8	9	9	7	6	10	9
V	1	5	10	10	10	10	10	10	10	10	10	10	10
		6	10	10	10	10	10	10	10	10	10	10	10
		7	10	10	9	10	10	10	10	10	10	10	10
		8	10	9	10	10	10	10	10	9	10	9	8
		9	8	9	8	10	10	10	9	10	10	10	10
VI	2	6	10	10	10	10	10	10	10	10	10	10	9
		7	0	0	0	0	1	0	0	0	0	1	0
		8	10	10	10	10	10	10	10	10	10	10	10
VII	1	12	10	9	6	7	9	8	6	4	5	5	4
		13	10	10	9	10	10	10	10	10	10	10	10
	2	9	10	8	6	5	2	2	1	3	4	1	2
		10	10	10	10	10	10	10	10	10	10	10	10
VIII	3	1	0	0	1	1	2	3	3	3	3	2	6
		2	10	10	9	7	5	7	6	7 .05	9*	9	5
		3	0	2	4	3	7	6	8	8	5	8	7
		4	0	0	0	1	1	2	3	1	1	3	4
		5	10	10	10	10	10	10	10	10	10	10	10
		6	10	10	10	10	10	10	10	10	10	10	10
		7	10	10	10	10	10	10	10	10	10	10	10
		8	9	10	10	10	10	10	10	10	10	9	9
		9	10	10	10	7	8	9	8	6	7	10	6
		10	10	9	8	6	4	7	2	4 .025	7*	5	5

TABLE 8

NUMBER OF CORRECT RESPONSES FOR SUBJECTS ASKED EMPTIER

			Age in Years										
Type No.	Display No.	Pair No.	5	6	7	8	9	10	11	12	13	14	15
I	1	1	7	7	9	7	8	10	9	9	9	7	8
		2	4	9	10	9	10	10	10	9	6	7*	8
		3	6	8	10	8	10	10	10	8	9	10	9
		4	5	9	10	9	10	8	10	8	6	7	8
II	2	1	1	0	0	0	2	1	0	0	0	4	4
		2	4	0	1	0	0	1	1	2	0	3	2
III	1	10	3	5	6	7	6	6	1	2	0	5	0
		11	4	5	5	2	7	4	0	2	1	4	1
IV	2	3	2	2	0	1	1	2	0	3	0	1	2
		4	5	2	5	6	4	7	9	6	9	7	8
		5	4	3	5	5	3	6	10	10	10	9	9
V	1	5	9	10	9	10	10	10	10	10	10	10	10
		6	9	9	9	10	10	10	10	10	10	10	10
		7	9	10	9	10	10	10	10	10	10	10	10
		8	8	7	9	10	9	8	10	10	10	10	9
		9	7	6	9	9	9	9	10	9	9	10	9
VI	2	6	8	9	9	10	9	10	10	10	10	10	10
		7	2	1	1	0	0	0	0	1	0	1	0
		8	8	9	9	10	10	10	10	8	10	10	10
VII	1	12	9	9	7	7	10	9	5	3	4	6	5
		13	9	10	10	10	10	10	10	10	10	10	10
	2	9	9	8	6	5	6	6	0	4	1	4	4
		10	9	9	9	10	10	9	10	10	10	9	10
VIII	3	1	1	0	2	2	1	0	8	4	4*	4	3
		2	9	10	7	7	8	7	2	8	4	6	8
		3	0	0	3	4	3	4	9	8	8	8	9
		4	1	1	4	2	2	1	7	1	4	1	5
		5	9	9	9	10	10	10	10	10	10	10	10
		6	9	9	9	10	10	10	10	10	10	10	10
		7	9	9	9	10	10	10	10	9	10	10	10
		8	9	8	9	10	10	10	10	8	10	8	7
		9	9	9	9	9	9	8	8	6	9*	9	7
		10	9	9	6	8	8	7	0	4	1	4	5

TABLE 9

NUMBER OF CORRECT RESPONSES FOR SUBJECTS ASKED FULLER ON PAIRS WITH ONE OR BOTH GLASSES EITHER FULL OR EMPTY

	Display No.	Pair No.	Age in Years										
			5	6	7	8	9	10	11	12	13	14	15
Both full	1	14	7	10	10	10	10	10	10	10	9	9	10
	2	11	6	7	3	5	2	3	1	3	4	1	2
		12	5	4	2	1	2	3	1	2	1	1	2
Both empty	1	15	9	9	10	10	10	10	10	10	10	10	10
One full	2	13	10	10	10	10	10	10	10	10	10	10	10
		14	10	8	6	5	4	7	4	3	6	5	4
		15	7	1*	3	1	2	2	0	2	1	2	2
		13	4	2	2	4	1	5	6	2	3	3	3
		14	9	9	10	10	10	10	10	9	10	10	10
		15	3	3	1	2	0	0	0	1	0	3	2
One empty	3	12	8	10	10	10	10	10	10	10	10	10	10
One full and one empty	3	11	8	9	10	10	10	10	10	10	10	10	10

TABLE 10

NUMBER OF CORRECT RESPONSES FOR SUBJECTS ASKED EMPTIER ON PAIRS WITH ONE OR BOTH GLASSES FULL OR EMPTY

	Display No.	Pair No.	Age in Years										
			5	6	7	8	9	10	11	12	13	14	15
Both full	1	14	9	10	10	10	10	10	10	8	9	10	10
	2	11	9	7	6	5	7	5	0	6	0	3	4
		12	8	5	4	3	4	4	0	5	0	3	4
Both empty	1	15	10	10	10	10	10	10	10	10	10	10	10
One full	2	13	7	9	9	10	10	10	10	10	10	10	10
		14	7	9	6	5	7	7	2	7	2	5	4
		15	4	7*	2	2	3	4	0	4	0	4	5
		13	6	7	4	6	2	4	2	4	3	5	5
		14	7	9	8	10	10	10	10	10	9	10	10
		15	2	5	1	4	0	1	0	4	0	3	4
One empty	3	12	9	8	10	10	10	10	10	10	10	10	10
One full and one empty	3	11	9	8	10	10	10	10	10	10	10	10	10

TABLE 11

TOTAL NUMBER OF CORRECT RESPONSES (N = 20 IN EACH AGE GROUP)

Type No.	Display No.	Pair No.	Age in years										
			5	6	7	8	9	10	11	12	13	14	15
I	1	1	11	14	18	17	17	18	19	19	19	14	17
		2	9	18	19	19	20	18	19	18	13	13	17
		3	10	16	18	18	19	18	19	18	16	16	18
		4	9	16	18	18	20	16	19	14	11	13	16
II	2	1	5	0	2	1	2	2	1	0	0	5	4
		2	7	1	4	1	2	2	2	2	0	5	2
III	1	10	6	10	11	12	9	8	2	5	3	6	5
		11	5	11	8	5	10	5	1	5	3	6	1
IV	2	3	4	2	1	3	1	2	0	4	0	1	2
		4	8	4	13	11	13	15	18	12	15	14	17
		5	10	7	13	11	11	15	19	17	16	19	18
V	1	5	19	20	19	20	20	20	20	20	20	20	20
		6	19	19	19	20	20	20	20	20	20	20	20
		7	19	20	18	20	20	20	20	20	20	20	20
		8	18	16	19	20	19	18	20	19	20	19	17
		9	15	15	17	19	19	19	19	19	19	20	19
VI	2	6	18	19	19	20	19	20	20	20	20	20	19
		7	2	1	1	0	1	0	0	1	0	2	0
		8	18	19	19	20	20	20	20	18	20	20	20
VII	1	12	19	18	13	14	19	17	11	7	9	11	9
		13	19	20	19	20	20	20	20	20	20	20	20
	2	9	19	16	12	10	8	8	1	7	5	5	6
		10	19	19	19	20	20	19	20	20	20	20	20
VIII	3	1	1	0	3	3	3	3	11	7	7	6	9
		2	19	20	16	14	13	14	8	15	13	15	13
		3	0	2	7	7	10	10	17	16	13	16	16
		4	1	1	4	3	3	3	10	2	5	4	9
		5	19	19	19	20	20	20	20	20	20	20	20
		6	19	19	19	20	20	20	20	20	20	20	20
		7	19	19	19	20	20	20	20	19	20	20	20
		8	18	18	19	20	20	20	20	18	20	17	16
		9	19	19	19	16	17	17	16	12	16	19	13
		10	19	18	14	14	12	14	2	8	8	9	10

TABLE 12

TOTAL NUMBER OF CORRECT RESPONSES ON PAIRS TESTING THE EFFECTS OF FULL AND EMPTY (N = 20)

			Age in Years										
	Display No.	Pair No.	5	6	7	8	9	10	11	12	13	14	15
Both full	1	14	16	20	20	20	20	20	20	18	18	19	20
	2	11	15	14	9	10	9	8	0	9	4	4	6
	2	12	13	9	6	4	6	7	1	7	1	4	6
Both empty	1	15	19	19	20	20	20	20	20	20	20	20	20
Both full	2	13	17	19	19	20	20	20	20	20	20	20	20
	2	14	17	17	10	10	11	14	6	10	8	10	8
	2	15	11	8	5	3	4	5	0	6	1	6	7
	3	13	10	9	6	10	3	9	8	6	6	8	8
	3	14	16	18	18	20	20	20	20	19	19	20	20
	3	15	5	8	2	6	0	1	0	5	0	6	6
One empty	3	12	17	18	20	20	20	20	20	20	20	20	20
One full and one empty	3	11	17	17	20	20	20	20	20	20	20	20	20

TABLE 13

NUMBER OF SUBJECTS AT EACH AGE LEVEL IN THE FIRST STUDY WHO JUDGED EACH OF THE 24 MINIMUM PAIRS CORRECTLY (N = 20)

	Age in Years										
Pair No.	5	6	7	8	9	10	11	12	13	14	15	Adults
1	11	14	18	17	18	18	19	19	19	14	17	20
2	5	0	2	1	6	2	1	0	0	5	4	12
3	5	11	8	5	8	5	1	7	3	6	1	10
4	10	7	13	11	16	15	19	18	16	19	18	19
5	8	4	13	11	18	15	18	12	15	14	17	18
6	4	2	1	3	3	2	0	4	0	1	2	12
7	15	15	17	19	19	19	19	13	12	20	19	20
8	18	19	19	20	20	20	20	20	20	20	19	17
9	2	1	1	0	2	0	0	1	0	2	0	11
10	18	19	19	20	20	20	20	18	20	20	20	17
11	19	18	13	14	20	17	11	8	8	11	9	12
12	19	16	12	10	15	8	1	7	5	5	6	11
13	19	19	19	20	19	19	20	20	20	20	20	20
14	19	20	19	20	20	20	20	20	20	20	20	20
15	1	0	3	3	2	3	11	7	7	6	9	14
16	19	20	16	14	20	14	8	15	13	15	13	14
17	0	2	7	7	8	10	17	16	13	16	16	16
18	1	1	4	3	1	3	10	2	5	4	9	11
19	19	19	19	20	20	20	20	20	20	20	20	20
20	19	19	19	20	20	20	20	20	20	20	20	18
21	19	19	19	20	20	20	20	19	20	20	20	19
22	18	18	19	20	20	20	20	18	20	17	16	18
23	19	19	19	16	20	17	16	13	16	19	13	14
24	19	18	14	14	19	14	2	11	8	9	10	12

(c) Comparison between the judgments of the Grammar School and the Secondary Intermediate pupils

The number of secondary school age children correct for each pair was calculated (Tables 14 and 15) and submitted to the Fisher Exact Probability Test (2 tail). Those cells which differed significantly at the 0.05 level of rejection are marked in the tables with an asterisk but it is clear that the observed differences are no more than one would expect by chance.

The Mann Witney U test (2 tail) was also carried out on the total number of pairs correct for each subject and revealed that the two populations do not differ at any of the four age levels at the 0.05 level of rejection.

Table 16 and figure 7 also give the average number of pairs correct for the two populations. Again it is seen that no obvious difference occurs except that the grammar school subjects at year 12 seem somewhat superior but the situation is reversed at year 15.

(d) The influence of perceptual cues on judgments of fullness

Since the child, in judging fullness, must learn to resist the perceptual cues, water level, height of glass and diameter of glass, and consider the necessary relationships, it might be hypothesized that until he is capable of doing so he will tend to follow visible cues.

The three perceptual cues can be arranged in each pair either to bias the child in favour of a correct judgment or to militate against a correct solution. Within the eight types of combinations of features the number of factors which ~~can~~ vary within each pair, that is, differ ~~or be the same~~ in the two glasses of a pair, is constrained to different extents: in type I none differs , in types II , III and V one differs , in types IV, VI and VII two ~~can~~ differ, and in type VIII all three factors differ within each pair.

TABLE 14

NUMBER OF CORRECT RESPONSES FOR THE TWO GROUPS OF CHILDREN AT SECONDARY SCHOOL

Type No.	Display No.	Pair No.	Intermediate				Grammar			
			12	13	14	15	12	13	14	15
I	1	1	9	9	8	9	10	10	6	8
		2	10	7	7	10	8	6	6	7
		3	10	*10	8	10	8	*6	8	8
		4	7	*8	7	9	7	*3	6	7
II	2	1	0	0	2	3	0	0	3	1
		2	0	0	4	0	2	0	1	2
III	1	10	3	4	3	3	2	1	3	2
		11	1	2	2	1	4	1	4	0
IV	2	3	0	0	0	1	4	0	1	1
		4	6	8	8	10	6	7	6	7
		5	9	8	10	10	8	8	9	8
V	1	5	10	10	10	10	10	10	10	10
		6	10	10	10	10	10	10	10	10
		7	10	10	10	10	10	10	10	10
		8	9	10	10	9	10	10	9	8
		9	10	9	10	10	9	10	10	9
VI	2	6	10	10	10	9	10	10	10	10
		7	0	0	2	0	1	0	0	0
		8	10	10	10	10	8	10	10	10
VII	1	12	5	7*	5	7*	2	*2	6	*2
		13	10	10	10	10	10	10	10	10
	2	9	2	2	2	3	5	3	3	3
		10	10	10	10	10	10	10	9	10
VIII	3	1	4	1*	4	5	3	*6	2	4
		2	7	6	8	7	8	7	7	6
		3	7	6	9	8	9	7	7	8
		4	1	2	2	4	1	3	2	5
		5	10	10	10	10	10	10	10	10
		6	10	10	10	10	10	10	10	10
		7	10	10	10	10	9	10	10	10
		8	10	10	9	9	8	10	8	7
		9	2	7	9	6	10	9	10	7
		10	2	4	4	5	6	4	5	5

TABLE 15

TOTAL NUMBER OF CORRECT RESPONSES ON PAIRS WITH ONE OR BOTH GLASSES FULL OR EMPTY

	Display No.	Pair No.	Intermediate				Grammar			
			12	13	14	15	12	13	14	15
Both full	1	14	9	10	9	10	9	8	10	10
	2	11	2	2	2	3	7	2	2	3
		12	2	0	2	3	5	1	2	3
Both empty	1	15	10	10	10	10	10	10	10	10
One full	2	13	10	10	10	10	10	10	10	10
		14	4	3	4	4	6	5	6	4
		15	2	0	2	3	4	1	4	4
	3	13	2	2	3	4	4	4	5	4
		14	9	9	10	10	10	10	10	10
		15	1	0	1	3	4	0	5	3
One empty	3	12	10	10	10	10	10	10	10	10
One full or one empty	3	11	10	10	10	10	10	10	10	10

TABLE 16

AVERAGE NUMBER OF PAIRS CORRECT FOR GRAMMAR AND SECONDARY INTERMEDIATE SCHOOL SUBJECTS

	12	13	14	15
Intermediate School	28.5	28.6	30.6	31.8
Grammar School	31.7	28.4	30.5	29.6

Figure 7

The average number of pairs correct for the Grammar and Secondary Intermediate School subjects

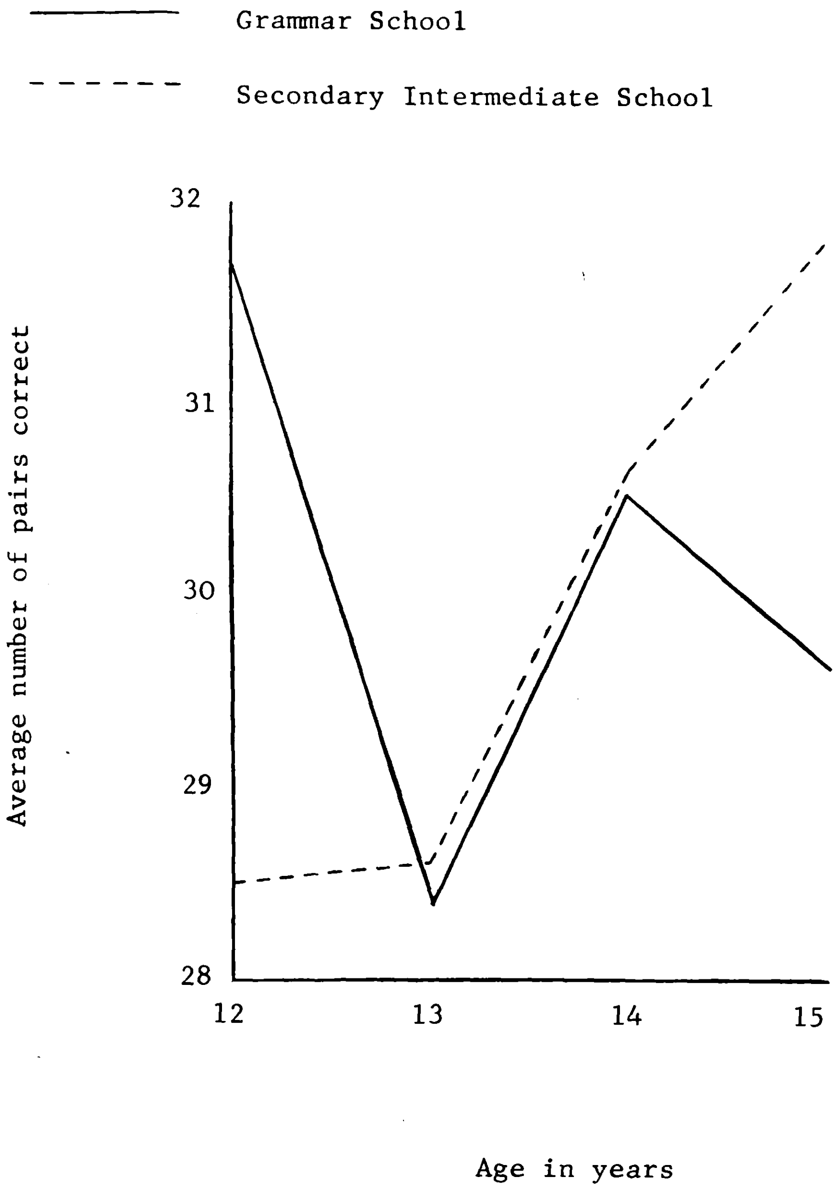

Grammar School
Secondary Intermediate School
Average number of pairs correct
32
31
30
29
28
12
13
14
15
Age in years

Table 17 shows the different trends in results observed while tables 5 and 6 earlier show how the forty-five pairs can be analysed for their arrangement of perceptual cues.

The Spearman Rank Correlation Co-efficient between the subjects' judgments and the effect of perceptual cues was determined for each age level and is shown in Table 18. All the correlation between the subjects' judgements and the influence of perceptual indices are positive and significantly greater than zero at $\alpha = 0.05$ (indeed p is less than 0.01 in each case except that of the five year olds with the pairs testing equality).

It might have been expected that the influence of perceptual indices would decrease with age. However, this trend did not occur and the older children are still heavily dependent upon "how the glasses look". Closer examination of the results corroborates this observation.

1. The pairs in which all perceptual features are free to vary and do so in such a way that they favour the correct response, proved to be very easy indeed for all the subjects. There appears to be no difference between the pairs where D.G. was the same, and where H.G. and W.L. were in favour of the correct glass, and those where H.G. was the same and D.G. and W.L. were in favour. All seem to be judged with equal facility. There was one notable exception, however: pair 15 display 3, in which H.G. and D.G. favoured the incorrect answer but where W.L. was equal, proved extremely difficult, even for the older children. The fuller glass was full to the brim but had a smaller volume of water. Some of the subjects who erred said that the glasses were equally full, while others judged that the taller glass with the wider diameter and greater volume of liquid was the fuller.

2. When two out of three perceptual indices favour the correct judgment, it seems that the task is easy so long as W.L. and D.G. are the two which favour, thus making the fuller glass contain the greater volume of water. Type VIII pairs 5 and 6 (in display3) and pair 14 display 3 all represent pairs of this type and all are judged correctly by most subjects. On the other hand, when W.L.

TABLE 17

TRENDS OBSERVED IN RESULTS

	Type No.	Display No.	Pair Nos.
Majority of subjects judged correctly	V	1	5 to 7
	VI	2	6 and 8
	VII	2	10 and 13
	VIII	3	5 to 8
		1	14 and 15
		2	13
		3	11, 12 and 14
Number of subjects correct increases with age	I	1	1 to 4
	IV	2	4 and 5
	V	1	8 and 9
	VIII	3	1, 3 and 4
Number of subjects correct decreases with age	VII	1	9 and 12
	VIII	3	2, 9 and 10
		2	11, 12, 14 and 15
Majority of subjects judged wrongly	II	2	1 and 2
	III	1	10 and 11
	IV	2	3
	VI	2	7
		3	13 and 15

TABLE 18

SPEARMAN RANK CORRELATION COEFFICIENTS BETWEEN THE SUBJECTS' JUDGMENTS AND THE INFLUENCE OF PERCEPTUAL CUES

Age in years	5	6	7	8	9	10	11	12	13	14	15
Pairs testing equality of judgement	.523	.889	.897	.895	.910	.876	.786	.808	.785	.842	.712
Remaining 32 pairs	.855	.565	.682	.726	.737	.686	.707	.655	.841	.657	.688

and H.G. are for the correct response and D.G. is against the task is not so easy for the child, and the number of subjects correct decreases with age. The glass which is fuller has the same V.F. and a smaller V.E. than the other glass, as represented by type VIII pair 2 (display 3)

3. When two factors differ and one is for, the other against the correct solution difficulty of judgment seems to depend on the particular factors involved. In type VI pair 6 (display 2), for example, where D.G. is the same, and W.L. is for and H.G. against and where the fuller glass thus has the greatest volume of water, the children find no difficulty in responding correctly. However, when, as in type IV pairs 4 and 5 (display 2), W.L. is the same and H.G. is for and D.G. against the younger subjects tend to err in their judgments. They reason that since the W.Ls are equal the glasses are full to the same proportion. The older children, however, realize that the wider glass contains more liquid and so they say it is fuller.

When H.G. is the same and W.L. is for and D.G. is against the correct judgment a different patterning of results emerges. In type VII pairs 12 and 9 (display 1) the number of subjects correct decreases markedly with age. In pair 12 both glasses contain the same amount of water and in pair 9 the fuller glass has a smaller V.F. Fewer older children judge pair 9 correctly because they argue that it is fuller because it contains more liquid. In both pairs the younger children judge correctly by simply responding as the perceptual indices dictate: W.L. is more important to the younger child than D.G.

4. When only one of the three factors suggests the correct judgment the task, as is expected, proves relatively difficult. Differences in responding as age increases once again depends on which factors bias for or against the correct member. When W.L. and H.G. militate against the right choice, and D.G. biases in favour of it, as in type VIII pair 3 (display 3), the younger subjects tend to choose the taller glass with the higher W.L. and so he judges incorrectly. The older child estimates the proportions full sufficiently

accurately to judge correctly but his final choice depends on the volume rather than the fractions calculated.

When W.L. is for and H.G. and D.G. are against the correct judgment, as represented by pair 14 display 2, and type VIII pairs 9 and 10 (display 3), the younger child arrives at the correct solution while the older child does not. This is to be expected in view of earlier pairs where it was seen that W.L. is important to the younger subjects while D.G. seems to be most influential in determining the judgments of the older children. In pair 9, the glasses contain the same amount of water, but in pairs 10 and 14 the fuller glass holds the smaller V.F. The latter, as should now be obvious, induces more incorrect responses in the older subjects than the former.

Finally, when H.G. is 'for' but W.L. and D.G. are 'against', the task proves very difficult for all the children. This is as would be expected since the younger subjects appear to be influenced by W.L. to a large extent while the older subjects pay greater attention to D.G. and in pair 13 display 3, where this arrangement of features occurs, it is clear that the fuller glass contains less liquid.

5. When all the 'free' factors, one, two or three, militate against the correct judgment the task is difficult for all the children. In type II pairs 1 and 2 (display 2) W.L. and D.G. are the same in each pair and H.G. is against the correct choice. Almost all the children recognise the equality of volume and so conclude that the glasses are full or empty to the same proportion.

In pair 3 type IV (display 2) two indices differ and both bias against the correct judgment: W.L. is the same but H.G. and D.G. are against and the fuller glass has less liquid. As in earlier instances the younger children tend to agree on equality as determined by the W.Ls while the older subjects state that the glass with the wider diameter and larger volume full is filled to a greater proportion. Both groups, of course, judge erroneously.

When three factors differ within a pair (Example: Pair 15 display 2) and all militate against the correct judgment the majority of all the children tend to choose the wrong glass as fuller.

6. The pairs requiring equality of judgment on the part of the subjects can also be ordered with respect to the influence of water level, height of glass and diameter of glass.

(a) When all three factors are the same in each member of a pair as in type 1 pairs (display 1, pairs 1-4) and also pairs 14 and 15 display 1, the children find little trouble in judging equality. Where some children erred it was generally not due to inability to recognize equality but rather due to the meticulous care and caution exercised in comparing the water levels, and perhaps concluding that there "was a tiny dribble" or "a few drops more" in one of the glasses. It is very difficult to ensure that the heights of water are exactly level. Equality on both the part of the experimenter and the subjects is, to a certain extent, a subjective thing. It is important to point out in this respect that once again the majority of subjects who judged equality, did not point out specifically that the glasses were filled to an equal proportion but were satisfied simply that the volumes were equal. (The results of type II pairs confirm this observation.)

(b) When only one perceptual indice differs within a pair, most of the children tend to choose whichever glass looks fuller. In type III, pairs 10 and 11 (display 1), for example, most of the children who erred chose the wider glass with the greater volume of water. Here again, though, scrupulous care in examining the water levels was evident and a few subjects concluded that the narrower glass contained a "tiny drop" or a "wee bit" more, or that it was a "wee bit higher" and so choose it as fuller.

(c) When two factors differ and both favour the same glass as fuller the child finds it extremely difficult to see that the glasses are equally full. In type VI pair 7 (display 2) (display 2), for example, even all the older children choose the taller glass of same diameter with the higher water level and greater volume as fuller.

(d) When three factors differ and two favour one glass and the third the second glass, it is very difficult for the child to see that both glasses are equally full. In type VIII pairs 1 and 4 (display 3), H.G. and W.L. tend to suggest that the tall slim glass is fuller while D.G. favours the short wide glass. In both these pairs there is a sudden rise at age eleven in the number of subjects who judge correctly. Only the oldest children, as suggested already, can take into consideration the compensatory relationships involved. However, once again they do so in order to equate volume rather than proportions full or empty.

It is interesting to note that more children reason that the short wide glass is the fuller in pair 4 than in pair 1, since the greater volume of liquid does occur in this glass in pair 4, while the volume full (empty) in pairs 1 and 4 are equal. Both pairs, one representing the ratio of one half, and the other three quarters are very difficult indeed for the children.

(e) When all three factors differ within a pair and all suggest that one particular glass is fuller than the other, it is difficult for the children to judge equality. In pair 12 display 2, for example, where both glasses are full to the brim, most of the subjects choose the tall, wide glass with the higher water and greater volume as the fuller. In fact, all those who erred chose this glass as being filled to a greater proportion, and fewer older children judged correctly than younger. However, it could be said, without much doubt, that if there had been a pair where all three factors favoured one glass as fuller, but where equality of judgment was required, and where the glasses were only partially filled, fewer children in all the age groups would have judged correctly.

The importance of perceptual factors on the child's judgment of which of two vessels is fuller or emptier can be summarized as follows. The height of the containers <u>per se</u> appears to influence the judgments very little: the children in all the age groups pay greatest attention to the dimensions of the 'volume full', that is, diameter of the glass and height of water or water level. Water level appears to be the most important perceptual determinant, especially for the younger children, and when it is equal the diameter of the container is taken into account. This finding is in accord with Lumsden's study (Lumsden and Poteat, 1968) which revealed a disproportionate relevance of the vertical dimension in the concept of "bigger" for young children. However, more important than either of these features individually is how they combine to produce the volumes of liquid contained. Most of the two hundred and twenty children appear to be influenced by the amount of liquid the glasses contain, and believe that the fuller glass is the one which holds the greater volume of water.

An interesting observation emerges as the children grow older; convinced that they are required to judge which container holds more water, the older subjects learn to take account of the compensatory relationships involved when W.L. and D.G. both differ within a pair, and the diameter of the glasses seems to become more important in their estimations than even the water levels. The younger children can co-ordinate several dimensions and respond to the volumes full in global terms, e.g. "it has more", "it is bigger" or "it is poured in biggest"; only one five year old, and four six year olds in any one pair, qualified their judgments further by pointing out a single feature of the pairs. As the children mature and their experience widens, they become capable of reasoning verbally and in a more complicated fashion. However, even at age fifteen, the child still seems to believe that fuller means "more liquid". The belief in this is so firm that, as we have seen, the older children judged some pairs wrongly which most of the younger subjects, simply by yielding to perceptual cues, judged correctly.

A rather surprising effect of this is produced when the average number of pairs correct out of forty-five is caluclated for each age group (Table 19 and figure 8). The number of pairs where the number of

Figure 8

The average number of pairs correct for each age level

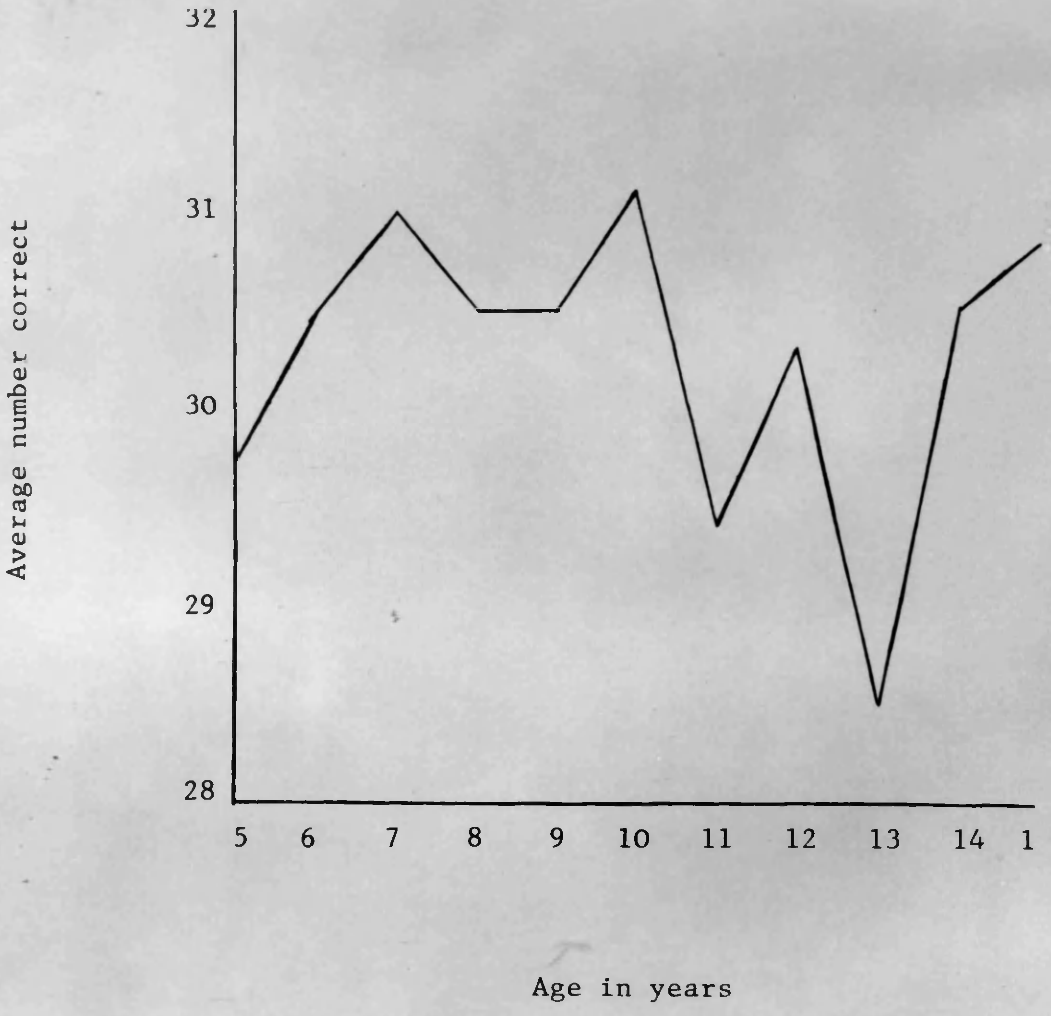

32
31
30
29
28
Average number correct
5
6
7
8
9
10
11
12
13
14
1
Age in years

TABLE 19

AVERAGE NUMBER OF PAIRS CORRECT FOR EACH AGE GROUP

Age	5	6	7	8	9	10	11	12	13	14	15
No. of pairs	29.7	30.5	31.0	30.5	30.5	31.1	29.4	30.3	28.5	30.5	30.7

subjects correct decreases balances out with those where the number increases to produce the above table, where at first glance it would appear that the child makes no progress at all, as he grows older, in his ability to judge fullness.

The older subjects appear well equipped to estimate the physical proportions involved but they do not realize that they must take into account the ratios existing between the volume of liquid contained and the total volume of the container; that is they do not fully understand the concept of ratio through the idea of fullness.

(e) The Adult Subjects

Having tested five to fifteen year olds on all possible combinations of features with duplications and having reached the above conclusion it was decided to carry out a further study and also include adults, but this time using only the minimum twenty-four pairs and asking only 'fuller'.

For the purpose of comparison the data obtained for the children had to be reorganized and the results for the surplus twenty-one pairs excluded.

Tables 13, 20 and 21 give the results for the minimum twenty-four pairs for all the subjects including adults who participated in the non-demonstration study. As can be seen from Tables 13 and 19 there is an improvement in performance between the fifteen year olds and the adults although the judgments are still not perfect but Table 21 demonstrates that the performance remains closely dependant upon perceptual cues.

TABLE 20

AVERAGE NUMBER CORRECT OUT OF THE MINIMUM 24 PAIRS FOR EACH AGE GROUP IN THE ORIGINAL EXPERIMENT

Ages	5	6	7	8	9	10	11	12	13	14	15	Adults
Average	15.3	15	15.65	15.4	15.9	16.05	15.65	15.35	15.4	16.05	16	18.7

TABLE 21

SPEARMAN RANK CORRELATION COEFFICIENT (1 TAIL) BETWEEN PERCEPTUAL CUES AND THE JUDGMENT OF FULLNESS FOR THE 24 ESSENTIAL PAIRS IN THE FIRST STUDY ($\alpha = .05$)

Ages	5 Equality Pairs	19 Remaining Pairs
5	0.975*	0.3928*
6	0.825	0.6755*
7	0.6	0.7115*
8	0.675	0.822*
9	0.675	0.874*
10	0.675	0.7975*
11	0.2	0.8275*
12	0.7952	0.6636*
13	0.2	0.6810*
14	0.7952	0.6033*
15	0.1539	0.5508*
Adults	0.225	0.6955*

CHAPTER VIII

DISCUSSION OF THE ORIGINAL STUDY

The present experiment compared with that by Bruner and Kenney

The most crucial and deliberate differences between the two studies, apart from the extension of the present study to include twelve, thirteen, fourteen, fifteen year olds, and later adult subjects, are briefly as follows. Bruner and Kenney presented only eleven pairs of glasses chosen "with malice of aforethought" but apparently without evidence to substantiate the choice of pairs; only on the basis of results obtained from a thorough investigation presenting all possible combinations of features could one be justified in selecting certain pairs.

In order to differentiate between sporadic, chance errors and persistent ones due to faulty knowledge, more than one example of each type should be included in the study. Also, the possible effect of varying experimental parameters, such as size of container, height of water, position reversal and transitivity, should be examined. Since the experiment assumes that the child has mastered these elementary abilities we, unlike Bruner and Kenney, in our first display included pairs to test these skills in order that we might be confident that no difficulties arise from them.

Bruner and Kenney do not appear to have taken adequate precautions against possible contamination of results by extraneous factors; while we determined the position of each individual pair member by chance, (except when in some of the preliminary tests it was undesirable to do so) all the tall glasses in Bruner and Kenney's pairs are positioned on the left. Furthermore, their study also runs the risk of a serial effect by administering the pairs in the same sequence to all subjects. Although in the present study the fifteen pairs in each display are arranged and numbered on a table in the same way for each subject, and though all subjects judged the displays in the order 1, 2 and 3, the order of judging the pairs within each display was randomized for each subject.

The choice of proportions in the original study may also be questioned; most of the partially filled glasses appear to be exactly half-full. A study to determine how the general concept of ratio is acquired should include more than one proportion, especially when that proportion is an 'easy' one; it is only when the child has mastered more than one example that he can be said to have learned the general principle. The use of coloured water rather than ordinary water should render the glasses more attractive to the children and should guarantee that they see the water levels more clearly.

A further criticism of the previous study concerns the questions asked of the subjects. Bruner and Kenney used questions involving a number of separate parts (a sequence of questions about fullness/emptiness) and as stated earlier this type of questioning is particularly difficult for young children to remember because of its length and complexity. Also research cited earlier (Hood, 1962, Fleishmon et al, 1966) indicates that the extent to which children appear to be performing correctly may depend on the order of the parts of the question. We simplified the wording in the present experiment to deal with only a single event. This has the advantage of presenting a single phrase short enought to be remembered, and although if we extend Rothenberg's (1969) argument, it may have the disadvantage of tending to favour a positive response by virtue of the emphasis in the question of a comparative rather than an equality judgment we feel it to be an improvement. It is also tentatively suggested that the failure to achieve linguistic activation, one aspect of the study not repeated in the first experiment, may have been due to alerting the children's attention to the different features after, rather than before, they judged for fuller or emptier.

The issue, however, becomes more critical when we consider that Bruner and Kenney distinguish between the reasoning involved in judgments of fuller and that determining judgements of emptier from results obtained by the same subjects on the same pairs. Objective judgments on fullness and emptiness can be obtained only from two independent samples, one of which is asked 'fuller' and the other its complement 'emptier'.

In view of the above differences it is perhaps not surprising that the present findings conflict rather sharply with Bruner and Kenney's results. The major reasons for the discrepancy appear to be the following. By presenting only a limited number of pairs, rather than all possible combinations of features, these investigators seem to have omitted some of the most crucial pairs: out of nineteen pairs which present difficulty to the older subjects in this study they had only four, and of the eight pairs where the number of subjects correct decreases with age they had only one.

Bruner and Kenney's observations on the difference between judgments of fuller and emptier - the latter being determined by volume empty rather than volume full - are also negated in this investigation; the original doubts regarding their procedure on this point thus seem to be justified. Also, support for the present findings may be found in the study on children's language comprehension cited earlier by Donaldson and Balfour (1968). These investigators showed that young children tend to regard "less" as undifferentiated from "more", and that "more" was dominant as the interpretation given to the undifferentiated pair of words.

It is not possible to compare the two sets of results in detail, as Bruner and Kenney do not give the number or percentage of subjects correct (or incorrect) for each individual pair, but consider the percentage results only for their different types. As Table 19 shows, it is usually meaningless to consider averages in this context. Furthermore, they do not report whether any statistical tests were used to analyse their data.

The analysis of perceptual influences in the previous study is also inconsistent: in their experimental method Bruner and Kenney consider that height of glass, diameter of glass, water level and volume full are the operative perceptual factors but in discussing the results they observe that water level, volume of water and 'reaching to the top' are the predominant influences. They then proceed to analyse their findings in terms of the latter set of factors which they have just extracted from the results as being most influential, and not surprisingly the analysis supports their hypothesis.

Furthermore, Bruner and Kenney do not consider equal volume full and equal volume empty when the proportions full and empty are unequal, factors which proved very revealing in the present study. They also err in considering independent variability: they state that the height of the glass, the water level, volume full and volume empty, "are independently variable of each other" as well as of proportion full and proportion empty. Independent variability of course applied only to the three perceptual indices, height of glass, diameter of glass and water level; the other features depend entirely, as Table 2 shows, on how these cues are combined.

Bruner and Kenney concede that their experiment "is not designed in such a way as to pick up the interaction among factors" so they "must leave the details unexplained". However, we have illuminated the details and their findings are clearly contradicted; the child, even at fifteen years of age, still thinks in terms of the volume contained rather than the proportion full. Indeed, the task is not easy even for adults who are still influenced by how the glasses look. There is clear evidence that the older child can cope with fractions and estimate the proportions or ratios existing between two volumes of liquid in containers, but even the adult finds it difficult to remain sufficiently detached from perceptual cues to also take into account the total volume of the containers.

Viewed against the literature cited earlier our results are as we might expect: in order to judge ratio through fullness the child must go beyond the volume concept, and this, as Piaget et al have shown, is achieved only at the beginning of formal operations at about eleven years, Bruner's ceiling age level. Our failure to communicate to the older subjects exactly what the concept of fullness involves perhaps suggests that we are investigating a concept which is misused in everyday life by adults and children alike.

CHAPTER IX

RESULTS OF THE DEMONSTRATION STUDY

(a) Comparison between the results of the Grammar School and the Secondary School subjects

The number of subjects correct for each pair in the two independent groups was calculated and is shown in Table 22. Since the demonstration study expects the subjects to benefit from a learning situation which requires, in addition to attention to the situation, a comprehension of the reasoning involved both with respect to the operative visual factors and the ratio-like computations carried out, as well as a willingness to accept the decisions made, it might be postulated that the grammar school pupils selected on the basis of higher I.Q.'s would perform better than the secondary school pupils. A number of one tail non-parametric tests was therefore carried out. The Fisher Exact Probability Test revealed that at the 0.05 level of rejection, fourteen of the total ninety-six cells were significant: three at the twelve year level, none for thirteen year olds, three for the fourteen year olds, and eight at the fifteen year old level. These cells are marked in the table with an asterisk.

The Mann Witney U Test was also carried out on the total number of pairs correct for each subject. This test shows that with $\alpha = .05$ (1 tail) all the subjects except the twelve year group differed significantly. Indeed it is interesting to note that as age increases so do the differences: the twelve year olds do not differ significantly at $\alpha = .05$, the thirteen year olds differ significantly at $\alpha = .05$, the fourteen year olds at $\alpha = .025$ and the fifteen year olds at $\alpha = .001$.

These results contrast sharply with the earlier finding that the two different school populations did not differ significantly in the first study and is further seen clearly in Table 22 and figure 9 for the average number of pairs correct for the two groups. It therefore appears that the grammar school children do improve and perform better following the

Figure 9

The average number of pairs correct for the minimum 24 pairs in both experiments for the Secondary Intermediate and the Grammar School pupils

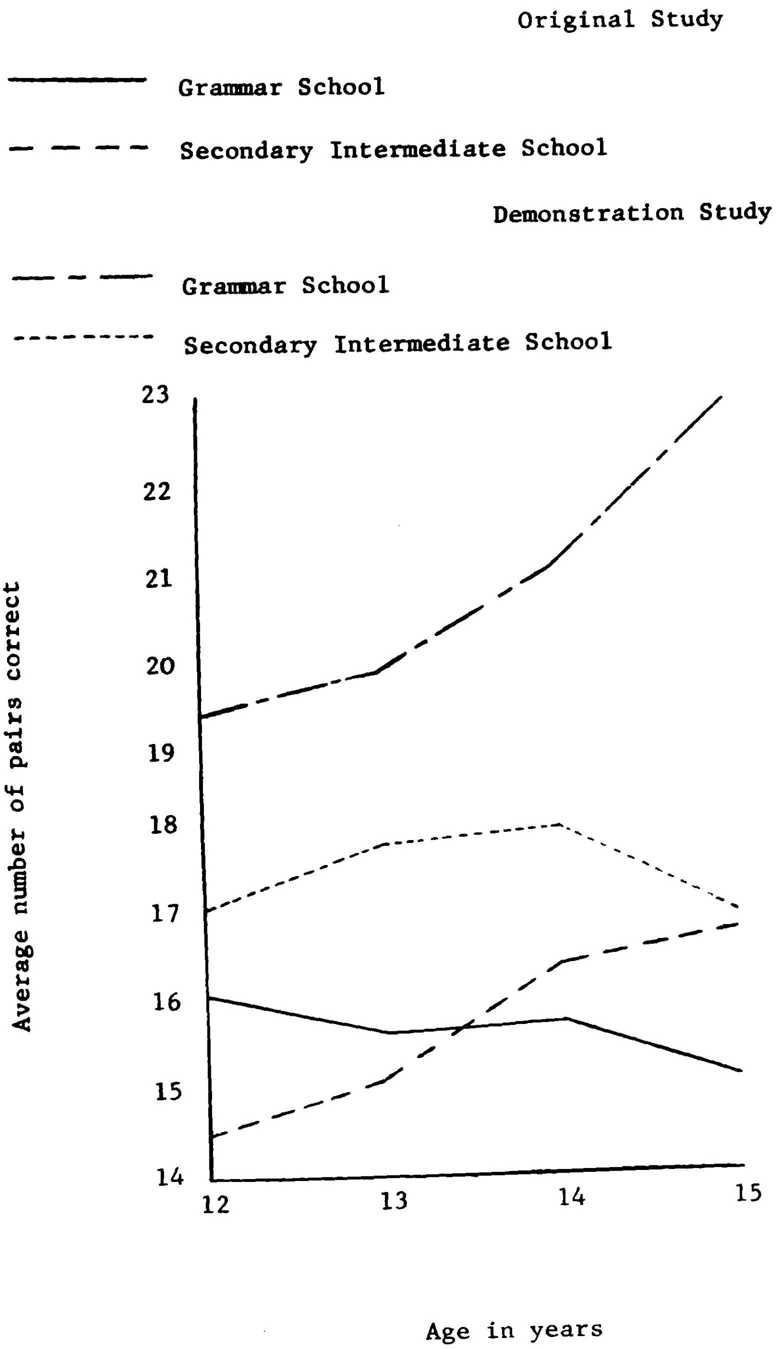

Original Study
Grammar School
Secondary Intermediate School
Demonstration Study
Grammar School
Secondary Intermediate School
23
22
21
20
19
18
17
16
15
14
Average number of pairs correct
12
13
14
15
Age in years

TABLE 22

NUMBER OF CORRECT RESPONSES FOR THE DEMONSTRATION STUDY OF THE TWO GROUPS OF CHILDREN AT SECONDARY SCHOOL

Pair No.	Secondary Intermediate School				Grammar School			
	12	13	14	15	12	13	14	15
1	10	10	10	9	9	10	10	9
2	2*	3	4	1*	8*	6	8	9*
3	2	6	6	3*	6	8	7	10*
4	10	7	8	10	8	8	10	10
5	6	4	6	6	7	5	10	9
6	1*	4	2*	3*	6*	6	7*	9*
7	8	7	10	10	9	9	7	10
8	10	10	10	9	9	8	10	10
9	2	3	3	2*	5	7	7	8*
10	9	10	10	9	9	8	10	10
11	7	9	10	8	9	9	9	10
12	3*	8	8	4*	9*	10	9	10*
13	10	10	10	10	10	10	9	10
14	10	10	10	10	10	10	9	10
15	8	4	2*	4*	5	7	8*	9*
16	7	8	7	8	8	8	7	10
17	8	5	4	6*	7	9	8	10*
18	5	3	2*	4	3	6	7*	6
19	10	10	10	10	10	10	10	10
20	10	10	10	10	10	10	10	10
21	10	10	10	10	10	9	10	10
22	9	9	10	9	7	8	9	10
23	9	10	10	9	10	9	10	10
24	5	8	8	6*	9	9	9	10*

TABLE 23

AVERAGE NUMBER OF PAIRS CORRECT FOR THE 24 MINIMUM PAIRS FOR THE TWO GROUPS OF SECONDARY SCHOOL AGE CHILDREN

Type of School	Secondary Intermediate				Grammar			
Age	12	13	14	15	12	13	14	15
Original study	14.5	15.1	16.4	16.8	16.1	15.7	15.8	15.1
Demonstration study	17.1	17.8	18.0	17.0	19.3	19.9	21.0	22.9

demonstration than the secondary intermediate pupils, that is, they seem more able to benefit from the learning situation.

As we are dealing with an overall representative sample the results of both groups of demonstration subjects in the twelve to fifteen year range were combined and treated as one sample for the rest of the analysis.

(b) Comparison between original subjects and demonstration subjects

Since it was hypothesized that the subjects who participated in the demonstration would be more efficient in judging which of a pair of glasses is fuller than those who did not, a 1 tailed Mann-Witney U Test was carried out on the total number of pairs judged correctly (Tables 13 and 24) for each subject. With p less than 0.05 all the age groups are significantly different. Indeed the seven, nine, ten and fourteen year olds are significantly different with p less than 0.05 and the six, eight, eleven, twelve, thirteen and fifteen year olds and the adult subjects with p less than 0.001.

This observation is corroborated by Table 25 and Figure 10 which show that the average number of pairs correct for each age level increases with age and is also higher in each case for the demonstration group even at the lower edge of the age range. The Secondary Intermediate School pupils alone in the four age groups from twelve to fifteen years also show higher averages than the original non-demonstration subjects.

Thus it may be concluded that when the subjects are shown what the judgment of fuller involves, they do perform better than when simply required to judge which of a pair of glasses if fuller, without having any indication of what exactly it is that they are being asked to do; it is just as much a question of learning what the task requires as learning how to perform it.

Closer scrutiny of the nature of the demonstration reveals that there are two major influences operating to bring about the improvement in performances: the effect of perceptual cues and the influence of E's verbal instructions.

TABLE 24

NUMBER OF SUBJECTS IN EACH AGE GROUP IN THE DEMONSTRATION STUDY WHO JUDGED EACH PAIR CORRECTLY (TOTAL NUMBER OF SUBJECTS IN EACH AGE GROUP = 20)

	Age of subjects										
Pair No.	5	6	7	8	9	10	11	12	13	14	15	Adults
1	10	11	14	18	18	18	19	19	20	20	18	20
2	10	10	5	6	3	5	3	10	9	12	10	17
3	2	7	4	8	6	7	10	8	14	13	13	17
4	16	17	14	16	15	16	16	18	15	18	20	19
5	12	11	13	18	17	15	13	13	9	16	15	15
6	0	2	2	3	4	5	3	7	10	9	12	17
7	16	18	19	19	17	20	19	17	16	17	20	20
8	20	20	18	20	19	17	20	19	18	20	19	20
9	0	0	1	2	2	2	3	7	10	10	10	17
10	20	20	19	20	20	19	20	18	18	20	19	19
11	20	19	20	20	19	18	19	16	18	19	18	20
12	18	18	16	15	12	9	15	12	18	17	14	18
13	20	20	19	19	19	20	20	20	20	19	20	18
14	20	20	20	20	20	20	20	20	20	19	20	20
15	0	0	2	2	2	5	7	13	11	10	13	17
16	20	20	19	20	17	17	18	15	16	14	18	19
17	2	0	5	8	10	11	13	15	14	12	16	18
18	0	0	0	1	3	3	6	8	9	9	10	15
19	20	20	18	20	19	20	20	20	20	20	20	20
20	20	20	19	20	20	20	20	20	20	20	20	19
21	20	20	18	20	20	20	20	20	19	20	20	19
22	19	20	20	20	19	19	18	16	17	19	19	19
23	20	20	20	20	17	19	20	19	19	20	19	20
24	20	20	20	19	18	16	15	14	17	17	16	19

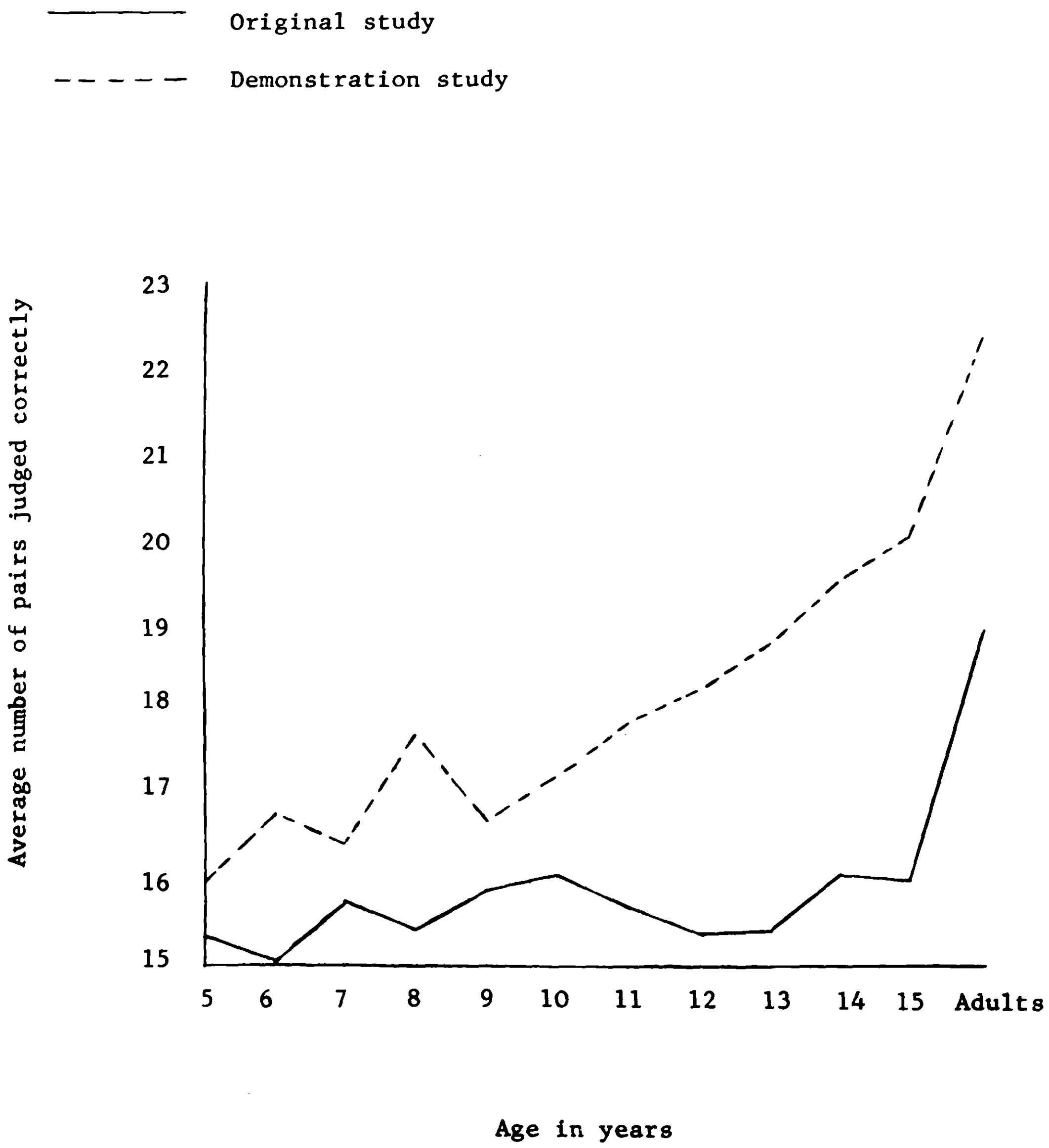
TABLE 25

AVERAGE NUMBER OF PAIRS CORRECT FOR BOTH STUDIES

Ages	5	6	7	8	9	10	11	12	13	14	15	Adults
Without demonstration	15.3	15.0	15.65	15.4	15.9	16.05	15.65	15.35	15.4	16.05	16.0	18.7
With demonstration	16.35	16.75	16.4	17.7	16.8	17.2	17.8	18.2	18.75	19.5	20.0	22.1

Figure 10

The average number of pairs judged correctly for the minimum 24 pairs in both experiments


Original study
Demonstration study
Average number of pairs judged correctly
23
22
21
20
19
18
17
16
15
5
6
7
8
9
10
11
12
13
14
15
Adults
Age in years

(c) The Influence of Perceptual Cues on the Judgment of Fuller

Since the original experiment demonstrated clearly how susceptible the child is to perceptual cues in choosing which of a pair of glasses is fuller, the demonstration pairs were selected, as previously stated in the method, with respect to the arrangement of the three perceptual factors H.G., D.G. and W.L. in each pair, which together determine the volume full and volume empty and also the proportion full and proportion empty of each container. How effective therefore was the demonstration in overcoming this powerful influence?

The Spearman Rank Correlation Coefficient (1 tail) between the subjects' judgments and how the pairs may be analyzed for the arrangement of perceptual cues was determined for each age level and is shown in Tables 26 and 27. All the correlations in both studies are positive and those marked with an asterisk are significant at the 0.05 level of rejection. Indeed, for the nineteen pairs requiring comparative judgments all the correlations are significant at $\alpha = .01$ except for the five year old subjects in both studies and the seven year old demonstration subjects; the correlation for the adult subjects in the demonstration study is the only one for the nineteen comparative pairs not significant at $\alpha = .05$. Also of interest is the observation that in the five equality pairs the correlations for the demonstration group of subjects exceed those for the non-demonstration subjects except for the five and twelve year olds where the original correlations are greater than the demonstration ones, and the nine year olds where the correlations are equal. On the other hand, in the remaining nineteen pairs the demonstration correlations are all less than those obtained in the original study excluding the five and fifteen year olds where they are greater.

Several points are worthy of consideration in an analysis of these correlations. The three visual indices are operative in producing the volumes full in each pair and if a glass looks as if it contains more water

TABLE 26

SPEARMAN RANK CORRELATION COEFFICIENT (1 TAIL) BETWEEN PERCEPTUAL CUES AND THE JUDGMENT OF FULLNESS IN THE STUDY FOR THE 24 PAIRS, (α = .05) DEMONSTRATION

Ages	5 Equality Pairs	19 Remaining Pairs
5	0.9*	0.4842*
6	0.9*	0.6151*
7	0.9*	0.4407*
8	0.975*	0.5511*
9	0.675	0.7415*
10	0.7	0.7885*
11	0.7	0.6545*
12	0.3691	0.6437*
13	0.9*	0.5750*
14	0.975*	0.5381*
15	0.7379	0.7*
Adults	0.9*	0.3008

TABLE 27

SPEARMAN RANK CORRELATION COEFFICIENT (1 TAIL TEST) BETWEEN PERCEPTUAL CUES AND SUBJECTS' JUDGMENTS FOR BOTH DEMONSTRATION (D) AND ORIGINAL (O) STUDIES ($\alpha = .05$)

Age		5 Equality Pairs	19 Remaining Pairs
5 years	D	0.9*	0.4842*
	O	0.975*	0.3928*
6	D	0.9*	0.6151*
	O	0.825	0.6755*
7	D	0.9*	0.4407*
	O	0.6	0.7115*
8	D	0.975*	0.5511*
	O	0.675	0.822*
9	D	0.675	0.7415*
	O	0.675	0.874*
10	D	0.7	0.7885*
	O	0.675	0.7975*
11	D	0.7	0.6545*
	O	0.2	0.8275*
12	D	0.3691	0.6437*
	O	0.7952	0.6636*
13	D	0.9*	0.5750*
	O	0.2	0.6810*
14	D	0.975*	0.5381*
	O	0.7952	0.6033*
15	D	0.7379	0.7000*
	O	0.1539	0.5508*
Adults	D	0.9*	0.3008
	O	0.225	0.6955*

D: demonstration study
O: original study

Those pairs with an asterisk are significant at the 0.05 level of rejection.

than it may be judged fuller. In the five equality pairs, three pairs (numbers 3, 9 and 18) have glasses with differing volumes full (V.F.) and two (numbers 1 and 15) have equal V.F. Therefore, because almost as many pairs have equal V.F. as different V.F., we should not expect a significantly high correlation between the judgment of equally full and the influence of perceptual cues. On the other hand, however, twelve of the remaining nineteen pairs (numbers 4, 5, 7, 8, 10, 13, 14, 17 and 19 to 22) have the fuller glass containing the greater V.F.; four pairs (numbers 2, 12, 16, 23) have the fuller glass with equal V.F., and only three pairs (numbers 6, 11, 24) have the fuller glass containing less V.F. Therefore we should expect a very high correlation in these pairs. Indeed, even in the three pairs where the fuller glass contains less water the three perceptual cues are arranged in such a way as to facilitate judgment somewhat: in pair 6, while H.G. and D.G. militate against the correct solution W.L. is equal; in pair 11 H.G. is equal and D.G. militates against a correct judgment, but W.L. favours the right solution; in pair 24 W.L. also biases in favour of the correct response although H.G. and D.G. militate against. Thus pairs 11 and 24 especially might be said to be perceptually easy.

Of further note is the fact that of the six pairs chosen randomly within each 'perceptual group' (as explained in the Method) for the demonstration the one equal pair has two identical glasses containing equal amounts of water while of the remaining five pairs, four have the fuller glass with greater V.F. Indeed, the only pairs which evoked any spoken disagreement from the subjects during the demonstration were the fourth and fifth pairs: in pair 4 the glasses look as though they might contain equal volumes of liquid, and in pair 5, of course, W.L. is equal and the fuller glass has less V.F. Thus, in both the demonstration and the testing pairs the fuller glass in each pair in most instances does in fact contain more water, although no comparison is made across glasses during the demonstration; the judgment is made for each container individually and then it is paired with a second glass.

In conclusion, it may be stated that in a pair of glasses one of which is actually fuller than the other, the fuller glass does tend to contain a greater volume of liquid. This makes it difficult for the subject to learn that this is not always the case and that some other rule governs the decision as to which is the fuller glass. Therefore, while all except one of the correlations are significant at the 0.05 level of rejection, and of these all excluding three at $\alpha = .01$, it is worthwhile to repeat that, except for the five and fifteen year old subjects, those subjects who were given the demonstration appear to be influenced less by visual cues than those who did not participate in the demonstration.

(d) The Influence of the experimenter's verbal instructions on the subjects' reasoning

Close examination of the subjects' protocols reveals the influence the verbal instructions and language used during the demonstration exerted on the reasons given to substantiate the judgement of fuller.

The experimenter pinpoints two aspects involved in computing the relevant ratios: (1) whether each glass contains more water than unfilled space or vice versa, and (2) the extent to which each container has been filled. The latter is essentially only a concise mathematical way of formulating the former and answers the implicit question 'How full is each glass?' No comparison of the volumes of water contained was made across glasses and the only fraction mentioned is the 'easy' half.

As seen from Table 28, approximately one fifth of the non-demonstration subjects named fractions: twenty-five of these restricted themselves to the fraction "half" and the same number used other proportions. On the other hand, more than two fifths of the demonstration subjects used fractions to explain their choice and of these, seventy-five mentioned only "half" and thirty-two named other ratios. In both instances the different

TABLE 28

NUMBER OF SUBJECTS IN BOTH STUDIES WHO MAKE USE OF FRACTIONS DURING THE VERBAL REASONING

	Age in Years												
The fraction ½	5	6	7	8	9	10	11	12	13	14	15	Adults	Totals
original study	1	1	4	3	3	0	3	1	1	1	2	3	25
demonstration study	0	5	4	5	5	7	5	10	9	9	9	7	75
Other fractions													
original study	0	0	2	0	1	2	5	2	1	2	2	8	25
demonstration study	0	0	1	2	2	3	6	0	4	5	3	6	32

TABLE 29

NUMBER OF SUBJECTS IN BOTH STUDIES WHO USE EITHER THE WORDS 'AIR' OR 'SPACE' DURING THEIR EXPLANATIONS

Age	5	6	7	8	9	10	11	12	13	14	15	Adults	Totals
original subjects	0	0	0	0	0	0	1	7	0	3	2	2	15
demonstration subjects	0	2	4	6	6	9	7	16	17	12	12	13	104

TABLE 30

NUMBER OF SUBJECTS IN BOTH STUDIES WHO REFER TO POURING THE CONTENTS FROM ONE GLASS TO THE OTHER DURING THE JUSTIFICATION OF THEIR RESPONSES

Age	5	6	7	8	9	10	11	12	13	14	15	Adults	Totals
original subjects	1	0	2	5	6	3	5	7	3	1	1	2	36
demonstration subjects	0	0	0	0	1	3	5	0	1	0	1	0	11

proportions referred to were thirds, quarters, fifths, sixths, eighths, and tenths, with differentiation increasing with age. Of note is the fact that most subjects in both studies who refer to fractions other than half also use that ratio as well.

The number of subjects in the two studies who utilized either the words "space" or "air", was also calculated and is shown in Table 29. It is significant that one hundred and four of the two hundred and forty demonstration subjects (approximately 2/5ths) followed E's example to the extent of using similar words in comparison to the fifteen of the two hundred and forty original subjects (1/16th). While the demonstrator compared only the proportion of air or space to water in each individual glass, a large majority of the subjects compared these volumes across the two glasses of each pair. Also, many subjects who consider the air or space in the glasses also take into account the proportion filled.

Thus it is clear that when given examples of the verbal reasoning involved in the problem, more subjects follow suit than when left without aids. However, do the subjects use any other lines of reasoning not employed in the demonstration, and, if so, how do they compare with the non-demonstration group?

One type of reasoning encountered is a reference to how the glasses of a pair would compare if the contents of one were poured either into the other pair member when empty or into another glass identical with the second glass. Thirty-six original subjects consider this state of affairs while only eleven demonstration subjects think in this way (Table 30). Here again the categories are not mutually exclusive and a child may vary his reasoning throughout the judging session.

Other modes of reasoning encountered less frequently were: a comparison of the volumes of unfilled space in the two glasses of each pair; a comparison of the size of the glasses; a comparison of the water levels, including whether the contents of one glass were nearer the top of the glass than the other.

In spite of these different types of verbal reasoning presented by many subjects an attempt was made to decide on the principal strategy determining the performance of each individual subject. This was difficult in some instances as the governing principle varied throughout the task with some subjects, but the categories most prevalent (as shown in Table 31) were as follows: (I) a comparison of the proportion each glass in the pair was filled; (II) whether one glass contained more water relative to space than the other; (III) whether one glass contained more water than the other; (IV) whether one glass contained more unfilled space than the other; (V) a comparison of the size of the two glasses; (VI) a comparison of the water levels; (VII) a comparison of the total capacities of the glasses involving the state of affairs if the contents of one glass were poured into the other; (VIII) whether the contents of one glass was 'nearer the top' than the other; and finally (IX) those young subjects who were unable to substantiate their choices verbally. Only I and II occur in the demonstration.

It is clear from this table that the types of reasoning in order of frequency for both studies are III, I, II, V, VI, IV, IX, VIII, VII. For the original subjects in the non-demonstration study the order is III, VI, V, I and VII, VIII and IX, then II and IV not occurring at all. For the demonstration subjects, however, the order of frequency becomess III, I, II, IV, V, VI, IX, VIII with no instances of VII.

These results again reflect what has already been stated: even when subjects participate in a demonstration where the strategy determing the judgment of fullness is explicitly verbalized, there remains the idea that the fuller container is the one containing more water. Also of significance in this table is the difference in I, II and also III and V in favour of the demonstration subjects and the difference in favour of the non-demonstration subjects in VI.

In conclusion, it may therefore be stated that the verbalization by the experimenter in the second study influenced a considerable number of subjects in favour of utilizing the correct strategy, but the demonstration in toto was insufficiently effective to persuade about one third of the subjects not to choose the glass containing the greater volume of liquid.

TABLE 31

THE DIFFERENT TYPES OF VERBAL REASONING EMPLOYED BY THE ORIGINAL AND DEMONSTRATION SUBJECTS

Age		5	6	7	8	9	10	11	12	13	14	15	Adults	Totals	
I Proportions filled compared	O						1	3					8	12	57
	D		1	1	3	1	3	8	4	3	9	4	8	45	
II Water compared with space in each glass	O												1	1	45
	D		1	1	2	2	3	1	5	7	5	7	10	44	
III Volume of water in the 2 glasses compared	O	16	14	17	18	17	18	17	11	13	15	11	7	174	261
	D	13	12	10	10	11	8	5	6	6	1	3	2	87	
IV Volune of space in the 2 glasses compared	O													0	24
	D		1	2		1	2	3	3	4	4	4		24	
V Dimensions of the 2 glasses compared	O		1						4	5	1	6		17	38
	D		1	4	3	5	4	1	1		1	1		21	
VI Water levels compared	O		4	2	2	2	1		4	2	3	2	2	24	33
	D	3	3		1				1			1		9	
VII Capacities of the glasses (involving pouring)	O	1				1							2	4	4
	D													0	
VIII 'Nearer the top'	O			1					1		1	1		4	8
	D			1	1			2						4	
IX No reason given	O	3	1											4	10
	D	4	1	1										6	
TOTALS	O	20	20	20	20	20	20	20	20	20	20	20	20	240	480
	D	20	20	20	20	20	20	20	20	20	20	20	20	240	

CHAPTER X

DISCUSSION OF THE DEMONSTRATION STUDY

The principal intention of the second study was to determine how subjects would perform in the task of choosing which of two glasses was fuller, if the problem were communicated to them more precisely by means of a demonstration. The demonstration preceded the original task and involved both verbal and perceptual factors. It is interesting, therefore, to consider how the influence these factors exerted on the subjects' behaviour compares with the findings of the studies cited earlier.

It is noteworthy, in view of Piaget's contention that perceptions precede language developmentally, that some subjects in the original task responded correctly in terms of choosing the correct and fuller container, but when required to verbalize the reasons for their choice either had difficulty in doing so or gave the wrong reason. The very young subjects, true to their interpretation of the task, were capable of identifying the glasses containing the greater volume of water, but in many instances were unable to substantiate their response. As Mermelstein and Shulman (1967) ask, which do we accept as the true acquisition of the concept - the overt correct performance or the ability to manifest it in verbal communication? Also of note is their suggestion that it is conceivable that, due to the syncretic nature of egocentric thought, specific training in language does not ensure analytic understanding of the questions used prior to nine years of age. Our demonstration study indicates that in certain cases this age limit may be extended upwards.

The lack of understanding of the problem which was so evident in the first study suggested that the concept of fullness may be generally misunderstood in everyday life, and therefore prompted the second experiment. It was generally shown to be true and is, indeed, given support by other studies where similar questions involving commonly used words and concepts were also misunderstood. In one study on the development of size conservation (Braine and Shanks, 1965b), for example, the question "Which is bigger?" was found to be ambiguous; it was interpreted as "Which looks bigger?" or, "Which is really bigger?" The experimenters in this study therefore used these two questions and found that below seven years of age children tend to construe questions containing the word "bigger" as questions about phenomenal size unless feedback information forces a "reality" interpretation. Similarly Cohen (1967) eliminated words like "more", "bigger" etc. from both instructions and responses in her study on quantity conservation because of their ambiguity.

Furthermore, it has also become clear that non-conserving children tend to make certain irrelevant interpretations of the words "same" and "more" when these are used in conservation of number problems. Braine and Shanks (1965a, b) have suggested that nonconserving children interpret "same" as meaning "look alike" rather than numerically equivalent. The former interpretation of "same number" was also seen in this study in the extensive use of the matching category when the nonconserving subjects were asked to make the two arrays have the same number of objects. Hood (1962) has suggested that the word "more" may mean to the nonconserving child only that the shape of the set is changed and that the space it occupies is greater than it was before. This finding is corroborated by the earlier study of Estes (1956), where the perceptual "more" and "numerical equal" were seen to co-exist without adequate differentiation in the cognition of children at a certain level, but where it was also seen that an experimenter can bring either one to the foreground by appropriate questioning.

These interpretations of the words "same" and "more" appear in solving conservation problems even after appropriate understanding of these words in other, nonconservation situations has been demonstrated. Indeed, Rothenberg (1969) goes as far as to suggest that stage 1, defined by Piaget (1952) as the absence of conservation, might be more meaningfully divided into substages. These substages would be: (a) lack of understanding of the conservation questions, as seen among ~~the~~ inconsistent nonconservers, and (b) understanding of the language of the questions with, however, no conservation of any transformations, as seen among ~~the~~ consistent nonconservers.

The verbalization by the experimenter during the demonstration was intended to direct the subjects' attention away from irrelevant factors towards the significant dimensions. Other research (Hall and Kingsley, 1968) has also shown that children in the intuitive thought stage do not necessarily rely on visual cues but are quite dependent on the experimenter's suggestion which is seen to systematically bias the results. On the other hand, while Wolff (1967) claims that verbalization by the subject has a positive effect, it is perhaps more relevant to repeat Phelan's (1965) suggestion that concepts are formed without verbalization and that unsuccessful attempts to put concepts into words can destroy or distort the concepts themselves. Support for his position may be claimed from the present study when he reports that spurious rules can be developed from inadequate verbalization and these can be substituted for the incorrect but efficiently functioning concept and so lead to maladaptive problem solving behaviour: the concept of fullness as a ratio would seem to be generally misunderstood and verbalization by the subject revealed this misinterpretation during the experiment.

Although it has been shown that conservation is positively correlated with intelligence quotient, mental age and verbal ability (Goldschmid, 1967) the subjects of secondary school age in this study on the fullness concept did not differ significantly in their performance in the original task and the grammar school pupils showed a marked superiority only in the demonstration study. Of relevance here also, is the investigation by Osler and Weiss (1962) where it was seen that more intelligent children were superior in the problem finding phase but not in problem solving.

Providing the subjects with the principle involved is important

(Klausmeier and Meinke 1968) but may be inferior to perceptual confirmation training (Beilin et al 1966). The demonstration incorporated instructions about the principle and also to a limited extent involved perceptual confirmation, but, as already indicated, it may not have provided sufficient examples of the few cases where the fuller glass does not contain the greater volume of water. It also gave the subjects an opportunity to select the relevant factors from the irrelevant ones although as seen earlier some subjects err in deciding which are the correct factors and make what Donaldson (1963) calls a structural error.

Also of note is the memory load involved (Denny and Gamblin 1965). The information required to perform correctly was provided prior to usage and once given was not repeated during the actual testing. Indeed, the subjects were given no real opportunity, once the demonstration was given, of testing their hypotheses. Unlike the study by Wallach, Wall and Anderson (1967) where children were induced into conservation by a reversibility procedure, the subjects in the present study were not forced to stop using misleading perceptual cues, either by the method used or by feedback. Also, no use could be made by the subject of the negative instances except where corrected in his use of phrases such as the "bigger glass" or "the glass with more water" etc. when giving his choice and was instead asked if one glass were <u>fuller</u> than the other, thus implying that a difference existed.

It may be felt that lack of active participation during the demonstration may also have been detrimental to the maximum improvement possible, but a very recent experiment on training in the conservation of weight, suggests that this would not have made a difference (Overbeck and Schwartz, 1970). Cohen (1967) does, however, consider the participation factor to play an important role in her study on the conservation of quantity.

In view of Bruner and Kenney's findings, it is somewhat surprising to find that some adults, even after the demonstration, are not fully convinced of the principle involved. However the work by Lovell (1968) on the schema of proportion and at least two experiments on conservation of volume, a less difficult concept, support the present results. In his twenty tasks covering both the schema of proportion and of ratio Lovell (1968) has corroborated earlier studies which demonstrate that the move towards understanding proportionality is slow well into high school. Even at fifteen years of age the number of responses at or near the stage of formal thought in six of his tasks (the calculation of missing numbers in a series, the rings and shadow experiment, the balance experiment, the relation between the size of the external angle of a regular polygon and the number of sides of the polygon, the ratio of the areas of similar triangles given the dimensions of one pair of corresponding sides, and verbal analogies), is still a little under 50 per cent.

In one study (Elkind 1961a, 1962) on the conservation of volume only 47 per cent of the junior and high school students tested had an abstract conception of volume, and the percentage of students having a true concept increased significantly between the ages of twelve and eighteen. The 75 per cent level for conservation of volume occurred about age fifteen. Furthermore, Hall and Kingsley (1968) found that, not only do adults fail to conserve quickly in many cases, but the majority also extinguish when faced with empirical evidence contrary to their expectations.

CHAPTER XI

CONCLUSION

An investigation was made of the manner in which the concept of proportion, as embedded in the idea of fullness, grows in the child's thinking. What emerges is the importance of the subject's method of dealing with the perceptual features of the task and the finding that the notion of fullness as a ratio is generally misunderstood even by adults.

The child begins with a discrete, almost binary, rendering of the concept of fullness in terms of the two extreme states, empty and full. Early in his development he accomplishes this by use of highly enactive definitions - 'full' is 'when it is about to spill' and 'empty' is, perhaps, 'turn-over-able after drinking'. Gradually there is movement to perceptual definition of the two terms - 'full' means 'much water', 'empty' means 'little water' - and soon there is a separation of the attributes used for dealing with this contrast pair. At this stage the comparative terms are reasoned similarly - 'fuller' means 'greater volume of liquid' and 'emptier' means 'lesser volume of liquid'.

Progression to the next stage does not involve simply attending to more cues but rather using a new computing of relationships and the child must eventually learn to form a ratio idea into which previously dominating perceptual cues can be fitted and put to a new use. Without an explanation of the exact relationships involved, however, both the child and adult do not realize that the task requires the volume of liquid contained to be related to the total volume of the container.

Piagetian theory holds that the child is ready for attaining the abstract concept of volume about age eleven or twelve. Readiness, either maturational or experiential, assumes that the child is beginning to attain the concept but, as supported by the conservation of volume studies already cited (especially those by Elkind (1962) and Rothenberg (1969)), it does not imply that he has already achieved it. At the same age the child is developing formal mental operations which are concered with constructing systems and theories. It is not until this ability develops that he can cope with the multiplicative compensations and proportions involved in the task of conserving volume.

In view of this background it is therefore not surprising to find that the child up to fifteen years of age and also the adult do not fully understand the ratios involved in the task of judging the fuller of two containers. In order to judge correctly, by reasoning correctly, which of two glasses is fuller one requires much more than an understanding of the proportions which occur, for example, when liquid is poured from one container to another. By eleven years of age the child can cope with fractions and can estimate the proportions or ratios existing between two volumes of liquid in containers but the fifteen year old and even the adult are still not aware of the full implications of the task.

When the problem is communicated more precisely by means of a demonstration, involving perceptual and linguistic aids, the child shows a marked improvement at all age levels from five to fifteen years, and the adult also reveals superior performance. Performance of the task following the demonstration is, however, still not perfect even for the adult. A person may persist in the belief that he need only compare the volumes of liquid contained and therefore prove resistant to the suggestion that in judging fullness he must also take into account the total volume of the containers; that is, he must not only estimate the volume of the glass that is filled but also relate this volume to the total volume of the container and finally compare the two ratio measures obtained.

The concept of fullness as a ratio, when measured in an exact and comprehensive manner, is seen to be generally confused and it is just as much a question of learning what the task requires as learning how to perform it.

BIBLIOGRAPHY

ALMY, M. et al (1966) Young Children's Thinking: Studies of some aspects of Piaget's theory
New York: Teachers' College, Columbia University, 1966.

AMSTER, H. & MARASCUILO (1965) Children's Concept Learning
J. Expt. Child Psych. 1965, 2, 192-204.

ANDERSON, R.C. (1965) Can first graders learn an advanced problem-solving skill?
J. Educ. Psych. 1965, 56(6), 283 to 294.

ANDERSON, R.C. & GUTHRIE, J.T. (1966) Effects of some sequential manipulations of relevant and irrelevant stimulus dimensions on concept learning
J. Expt. Psych. 1966, 72(4), 501-4.

ARCHER, E.J. (1962) Concept identification as a function of obviousness of relevant and irrelevant information
J. Expt. Psych. 1962, 63(6), 616-20.

ARCHER, E.J., BOURNE, L.E. Jr. & BROWN, F.G. (1955) Concept identification as a function of irrelevant information and instructions
J. Expt. Psych. 1955, 49, 153-64.

BEILIN, H. (1965) Learning and operational convergence in logical thought development
J. Expt. Child Psych. 1965, 2, 317-39.

BEILIN, H. (1966) Feedback and Infralogical Strategies in Invariant Area Conceptualization
J. Expt. Child Psych. 1966, 3, 267-78.

BEILIN, H. & FRANKLIN, Irene C. (1962) Logical operations in area and length measurement: age and training effects
Child Dev. 1962, 33(3), 607-18.

BEILIN, H., KAGAN, J., RABINOWITZ, R. (1966) Effects of verbal and perceptual training on water level representation
Child Dev. 1966, 37, 317-29.

BITTNER, A.C. & SHINEDLING, M.M. (1968) Methodological investigation of Piaget's concept of conservation of substance
Genetic Psych. Monogs. 1968, 77, 135-165.

BRAINE, M.D.S. & SHANKS, B.L. (1965 a) The conservation of a shape property and a proposal about the origin of the conservations
Canadian J. Psych. 1965, 19, 197-207 (a).

BRAINE, M. & SHANKS, B.L. (1965 b) Development of conservation of size
J. Verb. Learn. Verb. Beh. 1965, 4, 227-242 (b).

BRALEY, L.S. (1963) Strategy selection and negative instances in concept learning
J. Educ. Psych. 1963, 54(3), 154-9.

BRESSON, F. & M. de MONTMOLLIN (eds.) (1966) Psychologie et épistémologie génétiques: Thèmes piagetiens
Paris, France, Dunod. 1966, 211-222.

BRISON, D.W. (1966) Acceleration of Conservation of Substance
J. Genetic Psych. 1966, 109, 311-22.

BROWN, F.G. & ARCHER, E.J. (1956) Concept identification as a function of task complexity and distribution of practice
J. Exptal. Psych. 1956, 52, 316-21.

BROWN, B.R., WALKER, D.W., EVANS, S.H. (1968) Schematic concept formation as a function of constraint redundancy and knowledge of results
Psychon. Sci. 1968, 11(2), 75-6.

BRUNER, J.S. (1964) The Course of Cognitive Growth
American Psychol. 1964, 19(1), 1-15.

BRUNER, J.S., GOODNOW, J.J. & AUSTIN, G.A. (1956) A Study of Thinking
New York: Wiley, 1956.

BRUNER, J.S., OLVER, Rose R., GREENFIELD, Patricia M. et al (1966)
Studies in Cognitive Growth
New York: Wiley, 1966.

BYERS, J.L. (1963) Strategies and Learning set in concept attainment
Psych. Reports 1963, 12(3), 623-4.

CALLANTINE, Mary F. & WARREN, J.M. (1955) Learning sets in human concept formation
Psychol. Rep. 1955, 1, 363-7.

CARPENTER, T.E. (1955) A pilot study for a quantitative investigation of Jean Piaget's original work on concept formation
Educ. Rev. 1955, 7(2), 142-9.

COHEN, Gillian M. (1967) Conservation of quantity in children: the effect of vocabulary and participation
Quant. J. Exptal. Psych. 1967, 19(2), 150-4.

CRAIG, R.C. (1956) Directed versus independent discovery of established relations
J. Educ. Psych. 1956, 47, 223-34.

CRANE, N.L. & ROSS, L.E. (1967) A developmental study of attention to cue redundancy introduced following discrimination learning.
J. Expt. Child Psych. 1967, 5, 1-15.

DENNY, J.P. & GAMBLIN, P. (1965) Memory load and form of information input as determinants of proficiency in concept formation
Proceedings of the 73rd Annual Convention of the American Psych. Assocn. 1965, 93-4.

DETAMBEL, M.H. & STOLUROW, L.M. (1956) Stimulus sequence and concept learning
J. Expt. Psych. 1956, 51, 34-40.

DODWELL, P.C. (1960) Children's understanding of number and related concepts.
Canad. J. Psych. 1960, 14(3), 191-205.

DODWELL, P.C. (1961) Children's understanding of number concepts: characteristics of an individual and of a group test
Canad. J. Psych. 1961, 15(1), 29-36.

DONALDSON, Margaret (1963) A Study of Children's Thinking
Tavistock Publications, 1963.

DONALDSON, Margaret & BALFOUR, G. (1968) Less is more: A study of language comprehension in children.
Brit. J. Psych. 1968, 59(4), 461-471.

DRUKER, J.F. & HAGEN, J.W. (1969) Developmental Trends in the processing of task relevant and task irrelevant information
Child Dev. 1969, 40(2), 371-82.

DUNCKER, K. (1945) On problem solving
Psych. Mongs. 1945, 270, 58, 114.

ELKIND, D. (1961 a) Quantity conceptions in junior and senior high school students
Child Dev. 1961 (a), 32, 551-60.

ELKIND, D. (1961 b) Development of quantitative thinking: a systematic replication of Piaget's studies
J. Genet. Psych. 1961 (b), 98, 37-46.

ELKIND, D. (1961 c) Children's discovery of the conservation of mass, weight and volume: Piaget replication study II
J. Genet. Psych. 1961 (c), 98, 219-27.

ELKIND, D. (1962) Quantity conceptions in college students
J. Soc. Psych. 1962, 57, 459-465.

ERVIN, S.M. (1960) Transfer effects of learning on verbal generalization
Child Dev. 1960, 31, 537-554.

ESTES, B.W. (1956) Some mathematical and logical concepts in children
J. Genet. Psych. 1956, 88, 219-222.

EWERT, P.H. & LAMBERT, J.F. (1932) Part II The effect of verbal instructions upon the formation of a concept
J. Genet. Psych. 1932, 6, 400-413.

FEIGENBAUM, K.D. (1963) Task complexity and I.Q. as variables in Piaget's problem of conservation
Child Dev. 1963, 34, 412-432.

FLAVELL, J.H. (1966) The Developmental Psychology of Jean Piaget
J. Van Nostrand Co. Inc. 1966 edition.

FLEISHMAN, Barbara, GILMORE, Silvia & GINSBURG, H. (1966) The strength of non-conservation
J. Exptal. Psych. 1966, 4, 353-68.

FORGUS, R.H. & SCHWARTZ, R.J. (1957) Efficient retention and transfer as affected by learning method
J. Psych. 1957, 43, 135-9.

FOWLER, H.L. (1931) The development of concepts: an investigation into methods of teaching
Brit. J. Educ. Psych. 1931, 1, 13-40.

FURTH, H.G. (1961) The influence of language on the development of concept formation in deaf children
J. Abnorm. Soc. Psych. 1961, 63(2), 386-389.

GAGNÉ, R.M. (1962) Military training and principles of learning
American Psychologist 1962, 17, 83-91.

GAGNÉ, R.M. (1963) The learning requirements for enquiry
Journal of Research in Sciences Teaching, 1963, 1, 144-53.

GAGNÉ, R.M. (1965) The Conditions of Learning
New York: Holt, Rinehart & Winston, 1965.

GAGNÉ, R.M. & PARADISE, W.E. (1961) Abilities and learning sets in knowledge acquisition
Psychological Monographs 1961, 75, No. 14
(Whole No. 518).

GENGERELLI, J.A. (1927) Mutual interference in the evolution of concepts
Amer. J. Psych. 1927, 38, 639-46.

GINSBURG, H. (1967) Children's estimates of simultaneously presented proportions
Merrill-Palmer Quarterly 1967, 13(2), 151-7.

GINSBURG, H. & RAPOPORT, A. (1967) Children's estimates of proportions
Child Dev. 1967, 38(1), 205-12.

GOLDSCHMID, M.L. (1967) Different types of conservation and nonconservation and their relation to age, sex, I.Q., M.A. and vocabulary
Child Dev. 1967, 38(4), 1229-46.

GOODENOUGH, F. (1930) Inter-relationships in the behaviour of young children
Child Dev. 1930, 1, 29-47.

GOODNOW, Jacqueline J. & BETHON, G. (1966) Piaget's tasks: effects of schooling and intelligence
Child Dev. 1966, 37, 573-82.

GRIFFITHS, J.A., SHANTZ, C.A. & SIGEL, I.E. (1967) A methodological problem in conservation studies: the use of relational terms
Child Dev. 1967, 38, 841-8.

GRUEN, G.E. (1965) Experiences affecting the development of number conservation in children
Child Dev. 1965, 36, 4.

GRUEN, G.E. (1966) Note on conservation: methodological and definitional considerations
Child Dev. 1966, 37, 977-983.

HAGEN, J.W. (1967) The effect of distraction on selective attention
Child Dev. 1967, 38, 685-94.

HAGEN, J.W. & SABO, R. (1967) A developmental study of selective attention
Merrill-Palmer Quarterly 1967, 13, 159-72.

HALFORD, G.S. (1968) An experimental test of Piaget's notions concerning conservation of quantity in children
J. Exptal. Child Psych. 1968, 6, 33-43.

HALL, V.C. & KINGSLEY, R. (1968) Conservation and equilibration theory
J. Genet. Psych. 1968, 113(2), 195-213.

HANLON, Elizabeth P. (1967) How the child acquires the concept of ratio through the idea of fullness
Thesis submitted in part-fulfilment of the requirements for the degree of Bachelor of Arts in Psychology, in the Queen's University of Belfast, 1967.

HASERLUD, G.M. & MEYERS, S. (1958) The transfer value of given and individually derived principles
J. Educ. Psych. 1958, 49, 293-8.

HATANO, G. (1966) J. Abstract - "Development of the ability for concept learning" (trans.)
Japanese J. Psych. 1966, 37(2), 86-95.

HENDRICKSON, G. & SCHROEDER, W.H. (1941) Transfer of training in learning to hit a submerged target
J. Educ. Psych. 1941, 32, 205-13.

HILGARD, E.R., IRVINE, R.P. & WHIPPLE, J.E. (1953) Rote memorization, understanding, and transfer: An extension of Katona's card-trick experiments
J. Expt. Psych. 1953, 46, 288-92.

HOOD, H.B. (1962) An experimental study of Piaget's theory of the development of number in children.
Br. J. Psych. 1962, 53, 273-286.

HOVLAND, C.I. & WEISS, W. (1953) Transmission of information concerning concepts through positive and negative instances
J. Expt. Psych. 1953, 45(3), 175-82.

HUTTENLOCHER, J. (1962) Effects of manipulation of attributes on efficiency of concept formation
Psych. Rep. 1962, 10(2), 503-9.

HYDE, D.M. (1959) An investigation of Piaget's theories of the development of the concept of number.
Unpubd. doctoral dissertation, Univ. of London, 1959.

INHELDER, Barbel & PIAGET, J. (1958) The Growth of Logical Thinking from
Childhood to Adolescence
London, Routledge & Kegan Paul, 1958.

JACKSON, S. (1963) The growth of Logical Thinking in Normal and
Subnormal children
Unpubd. M.Ed. thesis, Univ. Manchester, Dept.
Educ.

JUDD, C.H. (1908) The relation of special training to general
intelligence.
Educ. Rev. 1908, 36, 28-42.

JUZAK, T. (1955) The effects of praise and reproof on the
generalization of learned concepts.
J. Psych. 1955, 39, 329-40.

KATES, S.L. (1962) Concept attainment by deaf and hearing adolescents
J. Educ. Psych. 1962, 53(3), 119-126.

KATZ, Phyllis A. (1968) Role of irrelevant cues in the formation of
concepts by lower-class children
J. Educ. Psych. 1968, 59(4), 233-8.

KEASEY, C.T. & CHARLES, D.C. (1967) Conservation of substance in normal
and mentally retarded children.
J. Genet. Psych. 1967, 111, 271-9.

KENDLER, H.H. & VINEBERG, R. (1954) The acquisition of compound concepts
as a function of previous training
J. Exp. Psych. 1954, 48, 252-8.

KINGSLEY, R.C. & HALL, V.C. (1967) Training conservation through the
use of learning sets
Child Dev. 1967, 38(4), 1111-26.

KLAUSMEIER, H.J. & MEINKE, D.L. (1968) Concept attainment as a function
of instructions concerning the stimulus material,
a strategy, and a principle for securing information
J. Educ. Psych. 1968, 59 (3), 215-22.

KOFSKY, Ellin (1967) The effect of verbal training on concept
identification in disadvantaged children
Psychonomic Sc, 1967, 7(10), 365-6.

KURTZ, K.H. & HOVLAND, C.I. (1956) Concept learning with differing
sequences of instances
J. Expt. Psych. 1956, 51, 239-43.

LISTER, Caroline M. (1969) The development of the concept of weight
conservation in E.S.N. children
Brit. J. Educ. Psych. 1969, 39(3).

LISTER, Caroline M. (1970) The development of the concept of volume
conservation in E.S.N. children
Brit. J. Educ. Psych. 1970, 40(1).

LONG, L. (1939) Concept formation in children
Psych. Bull. 1939, 36, 550-1.

LONG, L. & WELCH, L. (1941) Reasoning ability in young children
J. Psych. 1941, 12, 21-44.

LONG, L. & WELCH, L. (1942) Influence of levels of abstractness on reasoning ability.
J. Psych. 1942, 13, 41-59.

LOVELL, K. (1961) A follow up study of Inhelder and Piaget's 'The Growth of Logical Thinking'
Brit. J. Psych. LII, 2. 143-153.

LOVELL, K. (1968) Some Recent Studies in Cognitive and Language Development
Merrill-Palmer Quarterly of Behaviour and Development 1968, 14(2), 123-138.

LOVELL, K. & OGILVIE, E. (1960) A study of the conservation of substance in junior school children
Br. J. Educ. Psych. 1960, 30, 109-18.

LOVELL, K. & OGILVIE, E. (1961) Growth of the concept of volume in junior school children
J. Child Psych. Psychiat. 1961, 2, 118-26.

LUMSDEN, E.A. Jr. & POTEAT, Barbara W.S. (1968) The salience of the vertical dimension in the concept of "bigger" in five and six year olds.
J. Verbal Learning and Verbal Beh. 1968, 7, 404-8.

LUNZER, E.A. (1956) A pilot study for a quantitative investigation of Jean Piaget's original work on concept formation
Educ. Rev. 1956, 8(3), 193-200.

LUNZER, E.A. (1960 a) Recent studies in Britain based on the work of Jean Piaget
London, Nat. Found. Educ. Res., England, Wales, 1960 (a).

LUNZER, E.A. (1960 b) Some points of Piagetian theory in the light of experimental criticism
J. Child Psych. Psychiat. 1960 (b), 1, 191-202.

LUNZER, E.A. (1965) Problems of formal reasoning in test situations. in Mussen, P. (Edit.) European Research in Cognitive Development: Mon. Soc. Res. Child Dev. 100, Univ. Chicago Press.

LUNZER, E.A. & PUMFREY, P.D. (1966) Understanding proportionality
Mathematics Teaching, 1966, 34, 7-12.

MACCOBY, E. & HAGEN, J.W. (1965) Effects of distraction upon central versus incidental recall: developmental trends
J. Expt. Child Psych. 1965, 2, 280-9.

MAIER, N.R.F. (1931) Reasoning in humans II. The Solution of the problem and its appearance in consciousness
J. Comp. Psych. 1931, 12, 181-94.

MARTIN, W.E. (1951) Quantitative expression in young children
Genet. Psych. Monographs. 1951, 44, 147-219.

MERMELSTEIN, E. & MEYER, E. (1969) Conservation training techniques and their effects on different populations
Child Dev. 1969, 40(2), 471-90.

MERMELSTEIN, E. & SHULMAN, L. (1967) Lack of formal schooling and acquisition of conservation
Child Dev. 1967, 38, 39-52.

MEYER, E. & MERMELSTEIN, E. (1968) Sequence and acquisition of conservation
Paper presented to American Educational Research Association, Chicago, Feb. 1968.

MILGRAM, N.A. & NOCE, J.S. (1968) Relevant and irrelevant verbalization in discrimination and reversal learning by normal and retarded children
J. Educ. Psych. 1968, 59, 169-175.

MILLER, G.A. (1951) Speech and language
p. 789-810 Handbook of Experimental Psych. edit. S.S. Stevens, John Wiley & Sons, London, 1951.

MILLER, G.A., GALANTER, E. & PRIBRAM, K.H. (1960) Plans and the Structure of Behaviour
New York: Holt, 1960.

MOORE, M.F. and McWHIRTER, Elizabeth P. (in press) The development of the child's understanding of fullness as a ratio
Brit. J. Educ. Psych. in press.

MURRAY, F. (1968) Cognitive conflict and reversibility training in the acquisition of length conservation.
J. Educ. Psych. 1968, 59, 82-7.

NEWMAN, S.E. (1956) Effects of contiguity and similarity on the learning of concepts.
J. Expt. Psych. 1956, 52, 349-53.

OSLER, Sonia F. & WEISS, Sandra R. (1962) Studies in concept attainment: III The effect of instructions at two levels of intelligence
J. Exptal. Psych. 1962, 63(6), 528-33.

OVERBECK, C. & SCHWARTZ, M. (1970) Training in conservation of weight
J. Exptal. Child Psych. 1970, 9, 253-64.

PETRE, R.D. (1966) Effect of amount of pretraining with identical and dissimilar stimuli on concept learning
J. Exptal Psych. 1966, 71(3), 472-4.

PHELAN, J.B. (1965) A replication of a study on the effects of attempts to verbalize on the process of concept attainment
J. Psych. 1965, 59(2), 283-293.

PIAGET, J. (1946) Les notions de mouvement et de vitesse chez l'enfant.
Paris, Presses Univer. France, 1946.

PIAGET, J. (1947) The Psychology of Intelligence
New York: Harcourt Brace, 1947.

PIAGET, J. (1952) The Child's Conception of Number
Routledge and Kegan Paul, 1952.

PIAGET, J. (1952) Origins of Intelligence in Children
New York, International Univer. Press, 1952.

PIAGET, J. (1954) Construction of Reality in the Child
New York, Basic Books, 1954.

PIAGET, J. & INHELDER, B. (1948) The Child's Conception of Space.
Routledge and Kegan Paul, 1948.

PIAGET, J. & INHELDER, B. (1951) L'idée de Hasard chez L'enfant
Paris, Presses Univer. 1951.

PIAGET, J. & INHELDER, B. (1962) Le Développement des Quantités
Physiques Chez L'enfant.
Neuchâtel, Delachaux et Niestlé, 1962 edition.

PIAGET, J., INHELDER, B. & SZEMINSKA, A. (1960) The Child's Conception
of Geometry
New York: Basic Books, 1960.

PIAGET, J. & SZEMINSKA, A. (1939) Quelques expériences sur la conservation
des quantités continues chez l'enfant
Journal de Psychologie, 1939, 36, 36-65.

PRATOOMRAJ, S. & JOHNSTON, R.C. (1966) Kinds of questions and types of
conservation tasks as related to children's
conservation responses.
Child Dev. 1966, 37, 343-53.

PREHM, H.J. (1966) Concept learning in culturally disadvantaged
children as a function of verbal pretraining
Exceptional Children, 1966, 32(9), 599-604.

PRICE-WILLIAMS, D.R. (1962) Abstract and concrete modes of classification
in a primitive society
Br. J. Educ. Psych. 1962, 32, 50-61.

PUMFREY, P.D. (1968) The Growth of the Schema of Proportionality
Br. J. Educ. Psych. 1968, 38(2), 202-5.

RASMUSSEN, E.A. & ARCHER, E.J. (1961) Concept identification as a
function of language pretraining and task
complexity
J. Expt. Psych. 1961, 61, 437-441.

REED, H.B. (1946) Factors influencing the learning and retention
of concepts. I: The influence of set.
J. Exp. Psych. 1946, 36, 71-87.

ROMMETVEIT, R. & KVALE, S. (1965) Stages in concept formation: III
Further inquiries into the effects of an extra
intention to verbalize; IV A temporal analysis
of the effects of an extra intention to verbalize
Scand. J. Psych. 1965, 6(2) III: 65-74
IV: 75-79

ROSENBAUM, M.E. & ARENSON, S.J. (1967) Einstellung by Observation
Psych. Reports, 1967, 21(2), 345-52.

ROTHENBERG, Barbara B. (1969) Conservation of number among four- and five-year old children: some methodological considerations
Child Dev. 1969, 40(2), 383-406.

SASSENRATH, J.M. (1959) Learning without awareness and transfer of learning sets.
J. Educ. Psych. 1959, 50, 205-12.

SECHREST, L. & WALLACE, J. (1962) Assimilation and utilization of information in concept attainment under varying conditions of information presentation
J. Educ. Psych. 1962, 53(4), 157-64.

SIEGEL, A.W. & STEVENSON, H.W. (1966) Incidental learning: a developmental study
Child Dev. 1966, 37, 811-7.

SIGEL, I.E., ROEPER, A., HOOPER, F.H. (1966) A training procedure for the acquisition of Piaget's conservation of quantity: a pilot study and its replication
Brit. J. Educ. Psych. 1966, 36, 301-11.

SLATER, C.W. (1958) A study of the influence which environment plays in determining the rate at which the child attains Piaget's "operational" level in his early number concepts.
Unpubd. dissertation, Birmingham Univ., 1958.

SMEDSLUND, J. (1961 a) Acquisition of conservation of substance and weight in children.
Scand. J. Psych. 1961,(a), 2, 71-84.

SMEDSLUND, J. (1961 b) External reinforcement of conservation of weight and of the operations of addition and subtraction
Scand. J. Psych. 1961,(b), 2, 71-84.

SMEDSLUND, J. (1961 c) Extinction of conservation of weight acquired normally and by means of empirical controls on a balance
Scand. J. Psych. 1961,(c), 2, 85-7.

SMEDSLUND, J. (1961 d) Attempt at extinction of the visual components of the weight concept
Scand. J. Psych. 1961,(d), 2, 153-5.

SMEDSLUND, J. (1961 e) Practice in conflict situations without external reinforcement
Scand. J. Psychol. 1961,(e), 2, 156-60.

SMEDSLUND, J. (1962 a) Practice on continuous versus discontinuous material in problem situations without external reinforcement
Scand. J. Psych. 1962,(a), 2, 203-10.

SMEDSLUND, J. (1962 b) Conservation of discontinuous quantity and the operations of adding and taking away
Scand. J. Psych. 1962(b), 3, 69-77.

SMITH, I.D. (1968) The effects of training procedures upon the conservation of weight
Child Dev. 1968, 39, 515-26.

SMOKE, K.L. (1932) An objective study of concept formation
Psych. Monog. 1932, 42(4), 46

SMOKE, K.L. (1933) Negative instances in concept learning
J. Expt. Psych. 1933, 16, 583-8.

SPEARMAN, C.E. (1923) The nature of 'intelligence' and the principles of cognition
London: Macmillan, 1923.

STERN, Carolyn (1965) Labelling and variety in concept identification with young children.
J. Educ. Psych. 1965, 56(5), 235-40.

STERN, Carolyn (1967) Acquisition of problem-solving strategies in young children and its relation to verbalization
J. Educ. Psych. 1967, 58(4), 245-52.

STERN, Carolyn & KEISLAR, E. (1965) Acquisition by young children of strategies for concept identification problems
Proceedings of the 73rd Annual Convention of the American Psychological Assocn. 1965, 321-2.

STERN, C. & KEISLAR, E.R. (1966) Acquisition of problem solving strategies by young children and its relation to mental age.
Am. Educ. Res. J., 1966, 4, 1-12.

WALLACE, J. (1964) Concept dominance, type of feedback and intensity of feedback as related to concept attainment
J. Educ. Psych. 1964, 55(3), 159-66.

WALLACE, J.G. (1966) Some issues raised by a non-verbal test of number concepts
Educ. Rev. 1966, 18, 122-35.

WALLACH, L. & SPROTT, R.L. (1964) Inducing number conservation in children
Child Dev. 1964, 35, 1057-71.

WALLACH, Lise, WALL, A.J. and ANDERSON, L. (1967) Number conservation: the roles of reversibility, addition-subtraction and misleading perceptual cues
Child Dev. 1967, 38, 425-42.

WERTHEIMER, M. (1945) Productive thinking
New York: Harper, 1945.

WHORF, B.L. (1956) Language, thought and reality: selected writings.
Ed. J.B. Carroll
Cambridge, Mass., Mass. Institute of Technology, 1956.

WINCH, W.H. (1913) Should young children be taught Arithmetical Proportion?
J. Exp. Ped. 1913, 2, 79-88, 319-29, 406-20.

WOHLWILL, J. (1960 a) Absolute vs. relational discrimination on the dimension of number
J. Genet. Psych. 1960 (a), 96, 353-63.

WOHLWILL, J. (1960 b) A study of the development of the number concept by scalogram analysis
J. Genet. Psych. 1960, 97, 345-77.

WOHLWILL, J.F. & LOWE, R.C. (1962) Experimental Analysis of the Development of the Conservation of Number
Child Dev. 1962, 33, 153-67.

WOLFF, J.L. (1967) Effects of verbalization and pretraining on concept attainment by children in two mediation categories
J. Educ. Psych. 1967, 58(1), 19-26.

ZIMILES, H. (1966) The development of conservation and differentiation of number
Monogr. Soc. Res. Child Dev. 1966, 31, (6, Serial No. 108).