Essays on Realised Volatility Forecasting for International Stock Markets

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Abstract

Modelling and forecasting market volatility is an important topic within finance research, with the aim of producing accurate forecasts, as confirmed by the plethora of academic papers written over the past few decades. Understanding volatility is crucial for market participants such as investors, policymakers, and academics. The linear Heterogeneous Autoregressive (HAR) model currently dominates the volatility models for forecasting Realised Volatility (RV). This thesis enters the ongoing volatility forecasting debate by developing further the HAR model. First, within the HAR setting volatility jumps, realised semi-variance and the leverage effect are added. With the use of a selection of loss functions and forecasting comparisons it is found that adding the leverage effect into the HAR model can produce the most accurate forecasts over daily, weekly, and monthly horizons. Second, this thesis compares the foresting ability of the Autoregressive (AR) model with flexible lags, generated by the Least Absolute Shrinkage & Selection Operator (Lasso) approach (es), to the HAR model with a fixed lag structure. In-sample results show the Lasso approach to improve the model fitness, and the out-of-sample results indicate a more flexible lag structure is preferred, especially the ordered Lasso performs the best. Third, this thesis incorporates the Smooth Transition and Markov-switching approaches with the linear HAR model in a further forecasting exercise. In-sample results show that the regime-switching models provide better estimation accuracy than the linear HAR model. For the out-of-sample results, although the regime-switching models have limited forecasting ability over the daily horizon, these do outperform the linear HAR model over weekly and monthly horizons. The Markov-switching model is found to be the best, by consistently exhibiting the most accurate forecasts over time. All the above findings have been evaluated within a risk management setting (Value at Risk & Expected Shortfall).

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Most importantly, I would like to thank my parents for all their love and understanding through all these years. My parents do not speak English and have no idea what finance really is, but they try their best to support my education and encourage me to do what I want. They cultivate me from a child who lives in a small town in China to a person who completes a PhD thesis in the UK. In the twenty years of studying, I have walked a long way and suffered a lot to present this PhD thesis to you.

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Chapter 1 Introduction

1.1 Motivations

The stock market is at the core of the national economy. It associates with the whole financial market. The stock market volatility has always been treated as an attractive and meaningful research setting in time series econometrics. After the stock market crash in 1987, the stock price of the Standard & Poor's (S&P) composite portfolio decreased by 20.4% (Schwert, 1990), it is can be seen that economic globalization inevitably made major financial events riskier to global markets. With the bankruptcy of Lehman Brothers in 2008, the Dow Jones Industrial Average declined by more than 500 points by the end of the trading session of the day (Johnson and Mamun, 2012), then the financial crisis swept the global stock market. In 2015, Greece defaulted on its debt that triggered a European sovereign debt crisis, the sovereign debt crisis had a deep impact on the value of the common currency and challenged the stability of the monetary union (Kräussl et al. 2016). In recent years, the potential influence of Brexit spread throughout the EU and the economic recession brought from the COVID-19 uncertainty that exacerbates the variation in the stock market (Li, 2020 and Corbet et al. 2021). In general, the term volatility is associated with risk in financial assets within a certain period (Figlewski, 2004), high volatility is considered a phenomenon of market disruption, which means assets and securities are not fairly priced. The above examples emphasise the importance of volatility for investing activities, which take risk into account. To minimize the market risk, investors can use predicted volatility to change their investment strategies toward less risky assets.

The impact of intraday volatility is partly due to transitions. The attention of regulators has also enhanced the potential negative impact of such trading in the financial market. High frequency data can increase the spread of shocks across different markets, thereby increasing systemic risk. In May 2010, an example of a faster transmission of shocks was the Flash Crash which triggered the sudden drop of the S&P 500, allegedly caused by an automated trading algorithm that works on intraday intervals (Vuorenmaa, and Wang, 2014). A similar incident of trading errors occurred again two years later. Knight Capital, the largest US trader, suffered considerable losses in 2012, causing severe stock market turmoil (Popper, 2012). Due to Flash Crash and algorithmic trading system errors, regulators should closely examine the connection between high frequency trading and intraday volatility to reduce potential risks. Regulators and policymakers also need to pay attention to intraday volatility and implement regulatory reforms to assist financial markets in adapting to higher frequency volatility.

It can be seen that it is critical for everyone involved in the financial markets to forecast volatility accurately. As recent examples have shown, in today's financial markets, especially in periods of instability, accurate modelling and forecasting volatility are becoming ever more essential since regulators, individual and institutional investors all grapple with greater risks and increasing volatility. The ability to accurately predict volatility might be the determining factor in benefiting from the period of turmoil and stability. This benefit extends to anyone who can successfully apply future volatility – this is the motivation for this thesis.

It is evident that the need of getting accurate predictions is important for every market participant. Specifically, understanding volatility is crucial to investors in regard of making improved portfolio allocation and market timing decisions; at the same time, regulators and policymakers can also seek to implement policies to stabilize market risks. Therefore, modelling and forecasting volatility is an ongoing task in current years. Although a large number of volatility models have been proposed, no conclusion has been reached on which model generates the most accurate volatility estimation and prediction. The contribution of this thesis is to analyse the predictive power of alternative volatility models and propose novel developments on current models to improve the predictive performance of these models.

1.2 Outline of Thesis

The current literature focuses mainly on intraday volatility. Although excessive volatility models have been proposed, there is no evidence indicating which volatility model generates the most accurate forecasts. Current research is continuously improving forecasting performance. As the heterogeneous autoregressive (HAR) (developed by Corsi, 2009) type models are leading the current research trends, it is able to replicate the volatility persistence using the aggregated volatility at different interval sizes. This thesis has three main contributions. First, the previous volatility models incorporate volatility various characteristics to provide more accurate predictions, for example volatility clustering, long-memory and leverage effect. Chapter 3 reports several loss functions and forecasting tests to indicate that adding leverage effect into the HAR models produces superior forecasts to volatility jumps and realised semi-variance over daily, weekly and monthly horizons. Second, as the HAR model has a fixed lag structure and is considered a restricted AR(22) model, Chapter 4 finds the parsimonious lags in the AR models generated by the Lasso approach could provide more accurate forecasts over the HAR model. Third, sudden market changes can affect volatility persistence, therefore the persistence is not always consistent. Chapter 5 finds that the HAR model incorporating a regime-switching framework can improve forecasting accuracy. More specifically it is found that the Markov-switching HAR model performs best.

In addition to the main contributions, two further aspects are examined in this thesis. First, due to data availability, the sample is selected from developed and emerging countries to identify any different patterns between them, which allows the results in this thesis to be adopted by a wider range of market participants[1]. Second, the findings of this thesis are also confirmed within a practitioner's setting with the use of the Value at Risk and Expected

[1] Due to database limitation, only four emerging markets are available from the Oxford-Man Institute of Quantitative Finance. This thesis also selects four developed markets for comparison purposes. Therefore, this thesis employs the realised volatility of eight international indices, including four developed markets: the UK, Japan, the US, Germany; and four emerging markets: China, India, Brazil and Mexico.

Shortfall settings. This enables the risk managers, bankers and regulators to use these results and to manage risk directly.

Accurately modelling and forecasting volatility is of importance to all market participants. This thesis will rely on past and current research and propose improvements to existing models. The rest of the thesis is organized as follows.

Chapter two comprehensively introduces the literature review covering several aspects related to forecasting volatility models. This chapter starts by reviewing the GARCH type model. More recently, due to the availability of high frequency data opportunities for improving forecast accuracy arose from the work of Andersen and Bollerslev (1998). This chapter also addresses the issue of microstructure noise caused by the intraday data and discusses the alternative measures used to deal with this problem. This chapter introduces realised volatility as it is regarded as a less noisy proxy than the squared returns. The stylised characteristics of volatility are discussed, some of which are also detected in intraday data. The HAR model by Corsi (2009), is then introduced motivated by the Heterogeneous Market Hypothesis (HMH) and setting the scene for the rest of the thesis.

In Chapter 3 the traditional HAR model is modified to include three additional components, namely: volatility jumps, realised semi-variance and leverage effect. The forecasting performance of the models are assessed on a number of loss functions, including symmetric loss functions, asymmetric loss functions, pairwise comparisons and equal predictive ability tests. The main conclusion of this Chapter is that the HAR-RV model taking into account the leverage effect produces the top performance over daily, weekly and monthly horizons, this finding consistent with recent studies (Buncic & Gisler, 2017 and Horpestad et al., 2019). When considering more sophisticated HAR models with realised semi-variance, as well as the signed jumps, the forecasting accuracy is also improved, but the performance is not as good as the leverage effect. This chapter also notes the HAR models with volatility jumps have limited forecasting information compared with the leverage effect and realised semi-variance.

Due to the lag structure in the basic HAR model being fixed it is regarded as a restricted AR(22) model (Cosri 2009). Chapter 4 explores whether the parsimonious lag models generated from the Lasso-based approach can improve volatility accuracy over the HAR model. Specifically, this chapter employs four Lasso type-based methods to generate the parsimonious lags in the AR model to compare their forecasting ability against two HAR models with fixed lag structures. An AR(22) with the same lag structure to HAR model and an AR(100) with longer lags, based on the work by Audrino and Knaus (2016) and Croux et al. (2018). It is found that the model's flexible lags can improve the in-sample fit over the HAR models and the lags beyond 22 still contain efficient forecasting information. The out-of-sample results indicate that the AR(100) ordered Lasso performs the best for daily forecasts. In contrast, the AR(22) ordered Lasso dominates the weekly and monthly forecasts. These results are confirmed within a Value at Risk setting. This chapter has been published[2] in the Journal of International Financial Markets, Institutions and Money, this paper can be found in the Appendix 4 of this thesis.

Chapter 5 investigates whether and which nonlinear regime-switching approach can improve the forecasting performance of the HAR model, due to the persistence of RV being nonlinear. Two approaches are considered, the Smooth Transition and the Markov-switching. This chapter extends the Markov-switching model by considering the time-varying transition probability to examine the time-variation in the regime transition process and considers the variance shifts between regimes by implying the heteroscedasticity in the Markov-switching dynamics. Both the in-sample and out-of-sample results indicate the regime-switching HAR

[2] Reference: Ding, Y., Kambouroudis, D. and McMillan, D.G., 2021. Forecasting Realised Volatility: Does the LASSO approach outperform HAR?. *Journal of International Financial Markets, Institutions and Money*, p.101386.

models are preferred over the linear HAR model. The Markov-switching method performs better than two types of smooth transition approaches. For the out-of-sample results, although the regime-switching models have limited forecasting ability over the daily horizon, the Markov-switching HAR model is the best and consistently exhibits the most accurate forecasts over time, weekly and monthly horizons. As before these results are also confirmed within a risk management setting with the application of Value at Risk and Expected Shortfall techniques.

Chapter 2 A Review of Forecasting Models

2.1 Introduction

The volatility describes the fluctuating pattern of assets in the financial market over time. In the mathematical or statistical sphere, it is conventionally associated with the variance of the asset returns. Since the ARCH/GARCH model (Engle, 1982 and Bollerslev, 1986) are proposed, the GARCH model is used to estimate volatility by providing a model that more closely resembles real markets. Thus, the time-series volatility has been modelled using econometric techniques. Subsequently, the GARCH model and its extensions have made remarkable achievements in predicting volatility. The GARCH-type models dominated volatility modelling in the past several decades. The convenience of requiring high frequency financial data has stimulate many activities in the field of volatility modelling. In particular, the understanding of high frequency volatility and its dynamic characteristics has benefited greatly from high frequency financial data availability. Andersen and Bollerslev (1998) use 5-minute returns to construct the realised variance and show that the standard volatility models deliver better forecasting performance when using the high frequency data. The high frequency data contain more transition information and microeconomics changes. Therefore, the recent works of forecasting volatility increasingly concentrate on high frequency data.

The rest of this chapter is shown as follows. In this survey, this chapter starts with a summary of GARCH-type volatility models. Besides the review of the GARCH-type volatility models, this chapter introduces the microstructure noise of high frequency data and the different existing measures to reduce the influence of microstructure noise, in which I highlight the RV. Next, I outline the empirical feature of high frequency data. The volatility models in the later chapters focus on those features to provide more accurate predictions. In addition, I highlight the heterogeneous market hypothesis (HMH) and the HAR model, which is the main direction for this thesis.

2.2 Previous GARCH models

In previous works, the volatility is constructed by the daily squared returns, which is inherently latent. After the traditional approaches of several simple models[3], the GARCH models have been developed to model and forecast volatility over the past years. Compared to the whole world of daily or low-frequency volatility models created over the past several decades, there is something in common with modelling high-frequency volatility. Therefore, it is still valuable to look back to the previous GARCH models, which provide common ideas to bring into the high-frequency field. In this section, the various GARCH models are presented, which are widely used for latent volatility[4].

2.2.1 GARCH Models

ARCH model

Engle (1982) first proposes the ARCH model to test the UK's conditional variance of the inflation rate. The ARCH model suggests the present conditional variance depends on the past squared error term rather than constant. In a more detailed way, assume the simple model:

$$\mathrm{Y_t} = \alpha + \beta \mathrm{X_t} + \mathrm{u_t} \tag{2.1}$$

where $\mathrm{Y_t}$ is a dependent variable, $\mathrm{X_t}$ is an explanatory variable and β is a coefficient. Normally, $\mathrm{u_t}$ is the independently normal distribution with a zero mean and conditional variance, $\mathrm{h_t}$, shown as:

$$\mathrm{u_t}|\Omega_\mathrm{t} \sim \mathrm{iid\ N}(0, \mathrm{h_t}) \tag{2.2}$$

[3] Some simply models are traditionally used in the past techniques. For example: the future volatility is the mean of all past volatility in the historical average approach, and the moving average method uses the averaged volatility within a fixed interval to generate the forecasts.

[4] In this section, this thesis provides an extensive review of the GARCH-type models, especially asymmetric and long-memory GARCH models. Asymmetry and long memory are two common features for modelling high-frequency volatility that have been extensively modelled so far. Some empirical models used in Chapter 2 are relevant to asymmetry and the long lag length AR models in Chapter 3 refer to long memory. Therefore, it is worth to review the literature on asymmetric and long-memory ARCH/GARCH approaches.

where Ω_t is the information set. The ARCH model allows the variance of residuals (h_t) to depend on historical squared error terms, so the ARCH (q) model is given by:

$$h_t = \gamma_0 + \sum_{j=1}^{q} \gamma_t u_{t-j}^2 \tag{2.3}$$

In the ARCH process, the heteroscedasticity of variance will change over time. Moreover, as the variance is positive, the estimated coefficients in the ARCH model must be positive.

GARCH model

Bollerslev (1986) proposes the GARCH model, which extends the ARCH model and lets the conditional variance follow the ARMA process. The GARCH model presents the current volatility depend on both the conditional error term and the historical conditional variance. The GARCH (p, q) models are shown as follows:

$$h_t = \gamma_0 + \sum_{j=1}^{q} \gamma_t u_{t-j}^2 + \sum_{i=1}^{p} \delta_t\, h_{t-i} \tag{2.4}$$

where u_{t-j} is the lagged squared residual terms and h_{t-I} is the past value of itself. It is clear that for $p = 0$ the model can be reduced to ARCH (p).

GARCH-in-Mean model

In the real financial data, the conditional mean is not always constant. It can be affected by time-varying risk premium during the estimation period. The GARCH-in-Mean (GARCH-M) model is further extended by Engle et al. (1987) that allows the explanatory variables to affect the conditional mean and its conditional variance. Therefore, Engle et al. (1987) add the conditional variance into the conditional mean function of Y_t, the GARCH-M (p, q) model has the following form:

$$\begin{aligned} Y_t &= \alpha + \beta X_t + \theta h_t + u_t \\ u_t|\Omega_t &\sim \text{iid } N(0, h_t) \end{aligned} \tag{2.5}$$

$$h_t = \gamma_0 + \sum_{j=1}^{q} \gamma_t u_{t-j}^2 + \sum_{i=1}^{p} \delta_t \, h_{t-i} \tag{2.6}$$

Another form of the GARCH-M model to capture the risk premium is to utilize standard deviation rather than unconditional mean. So this specification of the GARCH-M model can be shown as follow:

$$\begin{aligned} Y_t &= \alpha + \beta X_t + \theta\sqrt{h_t} + u_t \\ u_t|\Omega_t &\sim \text{iid } N(0, h_t) \end{aligned} \tag{2.7}$$

$$h_t = \gamma_0 + \sum_{j=1}^{q} \gamma_t u_{t-j}^2 + \sum_{i=1}^{p} \delta_t \, h_{t-i} \tag{2.8}$$

The GARCH-M model adjusts the conditional mean with market time-varying risk premium. Hall et al. (1990) combine the GARCH-M model and capital assets pricing model (CAPM) to apply with financial data.

2.2.2 Asymmetric GARCH Models

One of the restrictions of the GARCH model is that it cannot describe the asymmetry affected by positive and negative news; the GARCH model treats news equally. Market volatility has a negative relationship with stock returns (Black, 1976 and Christie, 1982). The high volatility leads to a negative return, whereas the stock price grows when the market experiences low volatility. To capture the asymmetric effect, numerous extensions of the GARCH models are developed.

EGARCH model

The exponential GACRCH (EGARCH) model is first proposed by Nelson (1991). There are several ways to express this equation, one of the conditional variance equations is given by:

$$\log(h_t) = \gamma_0 + \sum_{j=1}^{q} \zeta_j \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \sum_{j=1}^{q} \xi_j \frac{u_{t-j}}{\sqrt{h_{t-j}}} + \sum_{i=1}^{p} \delta_i \log(h_{t-i}^2) \tag{2.9}$$

where the coefficient ξ_j tests the asymmetric effect of market news. When $\xi_j < 0$, the positive news generates less volatility than negative news. In addition, there are positive and negative news, which is $u_{t-j} > 0$ or $u_{t-j} < 0$, to describe the asymmetric effect, so another form of EGARCH model shown as:

$$\begin{cases} \log(h_t) = \gamma_0 + \sum_{j=1}^{q} (\zeta_j - \xi_j) \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \sum_{i=1}^{p} \delta_i \log(h_{t-i}^2), & \text{when } u_{t-j} < 0 \\ \log(h_t) = \gamma_0 + \sum_{j=1}^{q} (\zeta_j + \xi_j) \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \sum_{i=1}^{p} \delta_i \log(h_{t-i}^2), & \text{when } u_{t-j} > 0 \end{cases} \tag{2.10}$$

If $\xi_j = 0$, this conditional variance equation is symmetric. When $\xi_j < 0$, then negative news has a greater impact than positive news. The term $\sum_{i=1}^{p} \delta_i \log(h_{t-i}^2)$ is used to capture the volatility clustering. Furthermore, on the left-hand side, to guarantee the conditional variance to be positive, it uses exponential measurement instead of quadratic.

TGARCH model

Zakoian (1990) provides another model to capture asymmetry: the Threshold-GARCH (TGARCH) model. Similarly, Glosten et al. (1993) propose the GJR-GARCH model, which is closely related to the TGARCH model. The main idea of the TGARCH model is to set up a dummy variable as a threshold, which captures the leverage effect of positive and negative news. The TGARCH model can be shown as follows:

$$h_t = \gamma_0 + \sum_{j=1}^{q} \gamma_j u_{t-j}^2 + \sum_{i=1}^{p} \delta_i h_{t-i} + \sum_{j=1}^{q} \theta_j u_{t-j}^2 I_{t-j} \tag{2.11}$$

where the I_{t-1} is set as a dummy variable and shown as:

$$I_{t-1} = \begin{cases} 0 & \text{if } u_{t-j} > 0 \\ 1 & \text{if } u_{t-j} < 0 \end{cases} \tag{2.12}$$

where the I_{t-1}takes the value 1 for the bad news and 0 for the good news. Thus, the positive and negative news have different coefficients in the TGARCH model. For the positive news, the coefficients are γ_j, while $(\gamma_j + \theta_j)$ are the coefficients of negative news. When $\theta_j > 0$, this conditional variance equation is asymmetric, and the negative news has larger impacts than positive news.

2.2.3 Long-Memory GARCH Models

As Ding et al. (1993) show that asset returns have a long-memory feature, the lags of absolute and squared returns decay exponentially in their autocorrelation function. To capture the long-memory feature, the IGARCH model and CGARCH model are described below.

IGARCH model

Engle and Bollerslev (1986) develop the Integrated GARCH (IGARCH) model as an extended form of the GARCH model with infinity memory. Though empirical results report that the summary of two parameters in the GARCH model is closely equal to one, Engle and Bollerslev (1986) directly set the IGARCH model follows the condition of $\sum_{j=1}^{q} \gamma_t + \sum_{i=1}^{p} \delta_t = 1$ for conditional variance. Thus, the $\gamma_t = 1 - \delta_t$ in the IGARCH model, the specification of the IGARCH model shown as follows:

$$h_t = \gamma_0 + \sum_{j=1}^{q} (1 - \delta_t) u_{t-j}^2 + \sum_{i=1}^{p} \delta_t h_{t-i} \tag{2.13}$$

where, the summary of parameters in the IGARCH model is equal to one. To capture volatility persistence, the mean of volatility is ultimately stationary and reverts to a constant value in IGARCH model.

CGARCH model

In contrast with the GARCH model, the unconditional variance is constant at all time, which is mean reversion. Engle and Lee (1993) develop the component GARCH (CGARCH) model to measure conditional variance by decomposing it into long-run and short-run components, which allows the mean reversion to a time-varying trend. The specifications of the CGARCH model shown as follow:

$$h_t = q_t + \alpha(u_{t-1}^2 - q_{t-1}) + \beta(h_{t-1} - q_{t-1}) \tag{2.14}$$

$$q_t = \omega + \emptyset(u_{t-1}^2 - h_{t-1}) + \rho q_{t-1} \tag{2.15}$$

where the conditional variance h_t is mean-reversion with a long-run component q_t, and the long-run component q_t is time-varying and determined by the forecasting error $(u_{t-1}^2 - h_{t-1})$. The speed of the coefficients $\emptyset$ and ρ drive the mean-reversion of trend. When $\rho > \alpha + \beta$, the q_t represents the conditional volatility has a long-run trend. The short-run component of conditional variance is the difference between conditional variance and long-run trend, $(h_{t-1} - q_{t-1})$. If the long-run component q_t is constant, the CGARCH model becomes the classical GARCH model.

2.2 Microstructure Noise

The paper of Andersen and Bollerslev (1998) is a response to the mistrust of the GARCH model. Several papers point out that although the GARCH model has a good in-sample estimation, it cannot account for the variability of daily squared returns when performing out-of-sample evaluation (see, e.g., Jorion, 1995 and Figlewski, 1997). This wrong conclusion has been refuted by using the high frequency data, which provides more accurate forecasting information than daily squared returns. Andersen and Bollerslev (1998) show that the standard GARCH model delivers accurate forecasts using the realised variance. Therefore, the obvious poor performance is attributed to the fact that the daily squared returns lack forecasting information, and it is a very noisy proxy of conditional variance. In the works of Andersen and Bollerslev (1998), the RV can be constructed through the sum of squared intraday returns. Theoretically, this approach, which reduces the data frequency from daily interval to infinitesimal interval, can converge to a true measure of latent volatility. In practice, this would be infeasible due to data storage limitations and the market microstructure noise.

In theory, traditional economic theory holds that the financial markets are efficient, and the prices of the assets reflect all information, which is the true price of the asset (Fama, 1970). However, the efficient market hypothesis is not enough to explain the intraday trading behaviour in the real financial market. There is a difference between the market intraday prices and true prices, which is called the market microstructure noise. The microstructure noise in the financial markets can be affected by the discrete asset price (Harris, 1990), bid-ask spread (O'Hara, 1995). For the low frequency data (such as daily, weekly or monthly data), the market microstructure noise has a negligible impact on volatility. However, for the high frequency data, this effect cannot be ignored.

Assume $p_{t,I}$ is the asset price, which is affected by the microstructure noise, and $p^*_{t,i}$ is the real asset price, so the $p_{t,I}$ can be shown as the $p^*_{t,i}$ and the microstructure noise term, $\varepsilon_{t,i}$:

$$p_{t,i} = p^*_{t,i} + \varepsilon_{t,i} \tag{2.16}$$

Then, the asset return is given as follow:

$$\begin{aligned} r_{t,i} &= p_{t,i}^* - p_{t,i-1}^* + \varepsilon_{t,i} - \varepsilon_{t,i-1} \\ &= r_{t,i}^* + e_{t,i} \end{aligned} \tag{2.17}$$

where $e_{t,I}$ is the microstructure noise of asset return. Furthermore, the squared intraday return with microstructure noise is:

$$r_{t,i}^2 = \sum_{i=1}^{n_t} \left(r_{t,i}^*\right)^2 + 2\sum_{i=1}^{n_t} r_{t,i}^* \, e_{t,i} + \sum_{i=1}^{n_t} \left(e_{t,i}\right)^2 \tag{2.18}$$

when $n \to \infty$, due to the markets with microstructure noise term, $\sum_{i=1}^{n_t}\left(e_{t,i}\right)^2$ shows the squared intraday return cannot converge to the true volatility. Consequently, the influence of microstructure noise in the high frequency data becomes more significant as the sampling frequency increases.

The reason why high frequency data perform better is that high frequency data could contain more market information; the higher frequency is used, the more information is included. However, with the frequency increases, the intraday data will diverge to infinity rather than converge to the true volatility (Bandi and Russell, 2008). On other words, because of a host of practical market microstructure noise, the RV suffers from a bias problem as sampling frequency of intraday data increase. After Andersen and Bollerslev (1998) propose the RV, the widespread measurement is selected the suitable frequency sample to trade off the microstructure noise and the interval of intraday data. In order to get a more accurate RV estimator, Andersen et al. (2000, 2003) recommend using the sampling frequency from 5-minute to 30-minute to reduce the influence of microstructure noise. Aït-Sahalia et al. (2005) discuss the optimal sample frequency in different hypotheses of microstructure noise. Further investigation of Liu et al. (2015) finds that the other sampling frequency could not beat the 5-minute RV.

At present, besides the RV, four main realised measures are computed by intraday data to deal with the influence of microstructure noise. First, the RV is usually calculated based on intraday data sampled moderately at a certain frequency, humans subjectively determine the selection of the data frequency. Zhang et al. (2005) propose subsampling ways to calculate the average RV. This is a trade-off between bias and variance when choosing a sampling frequency. Second, Martens and van Dijk (2006) put forward the realised range-based volatility computed by the intraday interval of the high-low range. They show the realised range-based volatility has better forecasting ability than RV when using the same sample frequency. Then, the Quantile-based realised variance is developed by Christensen et al. (2010); they show the Quantile-based realised variance is robust to both jumps, outliers, and microstructure noise. Last, Hansen and Lunde (2004, 2006) propose an unbiased estimator that is Realised Kernel and Barndorff-Nielsen et al. (2008) improve the Realised Kernel and proposed the Flat-top Realised Kernel. These two empirical works show the kernel-based estimators are more accurate than the RV. So far, it is still a controversy for intraday volatility to diminish the influence of microstructure noise completely. There is no convincing method to solve this problem.

2.3 Realised volatility

An important part of accurately modelling volatility is to measure the “true” volatility. As volatility is latent, the daily squared returns lack intraday transaction information and microeconomics change. As mentioned in the previous section, the volatility measures tend to increase the data frequency to improve forecasting accuracy. In the work of Andersen and Bollerslev (1998), the RV is firstly used to construct a new volatility measure, calculated by the sum of squared intraday returns. They point out that this measure is less noisy relative to the squared return. Since then, the RV dominates the research of high frequency data in estimating and forecasting intraday volatility.

Specifically, denote the logarithm of an asset price p_t at the time t, and assume p_t follows a continuous-time stochastic volatility diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_t \tag{2.19}$$

where μ_t and σ_t represent the drift term and the instantaneous volatility, respectively; W_t is the standard Brownian motion. Then, assume the $r_{t,i} = p_{t,i} - p_{t,i-1}$, $(t = 1, \cdots, T;\ i = 1, \cdots, N)$, where T means the total trading days, $p_{t,I}$ means the log assets price at day t and ith intraday interval, thus, $r_{t,I}$ means the log assets return at day t and ith intraday interval. So the RV is the sum of intraday return:

$$RV_t = \sum_{i=1}^{N} \left(p_{t,i} - p_{t,i-1}\right)^2 = \sum_{i=1}^{N} \left(r_{t,i}\right)^2 \tag{2.20}$$

Andersen et al. (2003) notice the log of assets return is a local martingale process that integrated volatility is the limitation of RV probability if the time intervals are close to zero, the RV is the unbiased estimator of the true volatility and the error of RV is close to zero. As Andersen et al. (2003) presented:

$$r(t + h, h)|\sigma\{\mu_{t+s}, \sigma_{t+s}\}_{s\in[0,h]} \sim N\left(\int_0^h \mu_{t+s} ds\, , \int_0^h \sigma^2{}_{t+s} ds\right) \tag{2.21}$$

where, $\sigma\{\mu_{t+s}, \sigma_{t+s}\}_{s\in[0,h]}$ is the σ-field generated by the $\{\mu_{t+s}, \sigma_{t+s}\}_{s\in[0,h]}$. And $\int_0^h \sigma^2{}_{t+s} ds$ is the Integrated Volatility (IV). Consequently, without markets microstructure noise and the jumps, the RV converges to integrated volatility, which means RV $\xrightarrow{p}$ IV.

Apart from RV, the daily square returns have been previously used to measure volatility, it is computed by the log difference of daily prices. However, Andersen and Bollerslev (1998) point out the poor performance of forecasting model is not a failure of the model itself, but a failure to correctly specify the measure of the true measure, so the daily square return is considered as a noisy volatility estimator. Alternatively, another measure for modelling volatility is the implied volatility, which is obtained from the Black-Scholes option pricing model (Black and Scholes, 1973). The market price of an option is observable, the Black-Scholes model can be solved backward from the observed prices to derive or imply what the market volatility should be. This measure of volatility is called implied volatility, and it is usually used as the market's expectation of the volatility of the option maturity date. While implied volatility is low frequency volatility, the importance of implied volatility in volatility forecasting is found in recent empirical works. (Busch et al. 2011; Oikonomou et al. 2019. Jeon et al. 2020 and Kambouroudis, D.S., 2021.)

2.4 Characteristics of Intraday Volatility

After resolving the preliminary problems of microstructure noise, empirical works try to clarify the influential factors to model the intraday data and accommodate regularities. The present volatility models are developed based on the basic volatility stylized features. Nonetheless, it is still worth concentrating on the daily squared returns, which provides an opportunity to bring the traditional volatility model into the high frequency sphere. This section discusses several characteristics that are the volatility models frequently consider.

Volatility Clustering

This characteristic of volatility is visible that can be observed when plotting all data through time. A large stock change follows the large stock volatility for a certain time and the low volatility is followed by the low volatility (Mandelbort, 1963). Moreover, volatility clustering is closely related to other features, which are long memory and high persistence. So, the volatility clustering can also be explained by the fact that it can continuously affect the future mean for a long time. The volatility clustering motivates the ARCH and GARCH models (Engle, 1982, and Bollerslev, 1986), which capture the feature that volatility persists for more than one period.

Asymmetric Volatility Effects

The asymmetric volatility effects have been firstly noticed by Black (1976), which refers to the volatility of the stock market has a negative relationship with stock return. The negative returns lead to high volatility in the stock market. In contrast, the stock price grows when the market has low volatility. There are three main explanations for this asymmetric effect: the leverage effect, the feedback effect, and the investors' behaviour effect.

First, Black (1976) and Christie (1982) state that this phenomenon can be explained by the firm's leverage effect, which is a primary theory to illustrate the asymmetry. When the stock prices of firms decline, the firms with debt and equity will become highly leveraged. In other words, the firms become riskier that leads to the value of firms usually falls. Dutt and Humphery-Jenner (2013) also indicate that firms with lower volatility have higher operating returns, partly explaining why lower volatility provides higher value for firms. In addition, Baur and Dimpfl (2019) note that positive and negative market returns could increase volatility. However, the negative return plays a stronger role than a positive return. Second, the volatility feedback effect suggests a high rate of asset return in the market only occurs when the asset price drops, leading to the volatility increase (Engle et al., 1987 and Pindyck, 1984). Third, Avramov et al. (2006) indicate the asymmetric volatility is caused by the investment behaviour of traders who have not been informed. The herd mentality of non-informed investors can easily lead to blind investment, which can increase the volatility when assets price drop, while the rational investment behaviour of informed investors reduces the volatility when asset prices increase. Empirically, Xiang and Zhu (2014) also find that the behaviours of investors play a dominant role in asymmetric volatility.

Leptokurtic

The distribution of asset return is leptokurtic (Mandelbort, 1963; Fama, 1965). This phenomenon is also called the fat-tail or thick-tail distribution. The skewness and kurtosis of the standard normal distribution are 0 and 3, respectively, but the distribution of asset return has higher kurtosis and fatter skewness than the normal distribution. Moreover, this feature is suitable for high-frequency data as well. Andersen et al. (2001) examine high-frequency data and find the asset return and realised variance has a fatter tail. It is right-skewed, but the logarithmic standard deviation is an approximately standard normal distribution.

Long Memory and Volatility Persistence

According to Ding et al. (1993) and Bollerslev and Mikkelsen (1996), volatility persistence is usually described as the long memory feature. The volatility of asset returns has high autocorrelation for long lags. The increasing lags decay exponentially in autocorrelation function (ACF). Due to varying degrees of the measurement errors in the different asset return, there are slight differences for the short-term lags. However, those differences will be diminished along with the increase of lags and will show similar patterns and decay rate for the long-term lags (Hansen and Lunde, 2014; Tim Bollerslev et al. 2018). After realizing the proposed volatility, Andersen et al. (2003) and Lieberman and Philips (2008) consider the long memory and high persistence as the fundamental feature of RV. Many empirical works model the RV based on this feature (see, e.g., Hol and Koopman, 2002; Andersen et al., 2004 and Corsi, 2009).

Macroeconomic Variation

An assumption of volatility is mean-reversion; the volatility is ultimately stationary and will revert to a constant mean (Ding et al., 1993). However, this assumption is controversial.

Granger and Ding (1996) and Longin (1997) provide evidence that the persistence of volatility is not always consistent. In addition, Granger and Hyung (2004) present evidence that occasional breaks affect long memory within the volatility series. The one explanation is associated with the business cycle (Hamilton, 1989). The economic recession and expansion lead to the cyclical variation in volatility series. Another more convincing interpretation is that the mean of volatility change when the macroeconomic policy has been announced and the major financial events; thus, there are short-lived high volatility, and the low volatility has longer persistence (McAleer and Medeiros, 2008). Therefore, the stylized fact of macroeconomic variation empirically motivates the volatility models to incorporate with the structure break and regime-switching (Rapach and Strauss, 2008 and McAleer and Medeiros, 2008).

Intraday periodicity

Along with the empirical features usually exhibited in the low frequency data, the intraday volatility shows a unique property. The high-frequency volatility is strongly affected by intraday periodicity (Andersen and Bollerslev, 1997). More specifically, intraday periodicity refers to that, in one trading date, there are more frequent transactions in a short time after trading starts and before trading ends. This phenomenon causes the volatility in these two periods to be greater than others. So the periodicity of intraday volatility in one day is a U-shape pattern. However, for intraday data, Martens et al. (2002) suggest that the traditional GARCH-type models are not directly applicable to intraday data because the GARCH-type models easily distort the periodic pattern of intraday data.

2.5 Heterogeneous Market Hypothesis

After Fama (1970) proposes the Efficient Markets Hypothesis (EMH), which provides a theoretical basis for the expected changes of return. According to EMH, all information in the market, including historical information, internal information, and public information, can be reflected immediately by the asset prices. When new information is released, the asset pricing changes depend on investors adjust their estimates of asset prices rationally. However, the EMH is a debate currently. Many empirical results cannot be explained reasonably. Bondt and Thaler (1975, 1978) use experimental psychology to propose the Overreaction Hypothesis. They indicate that most investors tend to overreact to the unexpected and dramatic good news. The post losers outperform the market in the following years. Moreover, Jegadeesh and Titman (2001) propose the Momentum Strategies that find buying high-return stocks over the past 3 to 12 months and selling low-return stocks during the same period will perform well in the next 12 months. Both the Overreaction Hypothesis and Momentum Strategies indicate that the EMH cannot explain the real market change very well.

Notably, Müller et al. (1993) research foreign exchange volatility with different time horizons from short to long term and find the different traders have their own expected goals in different time horizons. According to those heterogeneous trading behaviours that reflect assets prices in the market, Müller et al. (1993) propose the heterogeneous market hypothesis (HMH). Since traders in the market dominate market prices, traders have different expectations of assets return in the market. They will make different transaction decisions on trading price, trading volume, and trading frequency, depending on their trading behaviour. Short-term traders react to every market change in real-time, so their trading frequency is very high. Mid-term traders focus on daily or weekly information to make trading decisions. And long-term traders only make adjustments to trading strategies for important news. Therefore, the trader's heterogeneous trading behaviour on market information determines the market volatility.

According to the HMH by Müller et al. (1993), volatility in different periods reflects the different market expectations and the composition of investors' behaviours. The different periods of volatility should be utilized to reveal the dynamic changes in the market. Corsi (2009) considers the fractional integrated models cannot completely reflect market information and proposes the Heterogeneous Autoregression Realised Volatility (HAR-RV) model with daily, weekly and monthly horizons of RV, which correspond to different trading behaviours. The HAR-RV model is given as:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + u_t \tag{2.22}$$

where the coefficient, β_d, is the daily lag of RV, β_w and β_m are the weekly and monthly average lags, calculated as follow:

$$RV_{t+1-h:t} = \frac{RV_t + RV_{t-1} + RV_{t-2} + \cdots + RV_{t-h+1}}{h} \tag{2.23}$$

The long-memory pattern and heterogeneous trading behaviours in the HAR-RV model are reproduced by the sum of the daily, weekly, and monthly volatility components.

2.6 Conclusion and Summary

As it can be seen, modelling volatility plays an important role in understanding market risk. This section reviews various forecasting models, ranging from the basic GARCH model to the HAR model and the stylized features of volatility. This review also shows that numerous ways of modelling volatility features have their own superiorities. There is no consensus on how to obtain the best forecasting model, which leaves gaps for new exploration. The previous works of intraday volatility models demonstrate the intraday volatility contains much more market information. The RV has replaced daily squared return to model volatility due to it can achieve better forecasting performance.

As discussed above, forecasting RV has become the main direction of current academic research, however, so far no generally conclusion has been accepted. With the popularity of the HAR model in recent years, this thesis adopts the stylized features of volatility and extends the HAR model to generate forecasting models with more accurate forecasts. Specifically, this thesis expands the existing literature in three aspects. First, to improve the forecasting ability, the HAR model is used to accommodate more volatility components. However, we do not know which one can provide the optimal results. Second, the HAR model has three fixed lag structures to capture the volatility persistence. Whether the flexible lags structure or the long lags may improve forecasts? Third, macroeconomic variation affects the volatility persistence, but most forecasting models are single regime models. Whether the forecasting models that allow the regime switch between low and high volatility could be more suitable for the real market?

Chapter 3 Foresting Realised Volatility: the HAR model with Volatility Jumps, Realised Semi-Variance and Leverage Effect

3.1 Introduction

The volatility of financial assets has received great attention over recent decades as forecasting volatility is an essential part of financial and econometric research. The undertaken research reflects that volatility is associated with risk and therefore is crucial to understand within business and financial activities. In general, to reduce the investment risk in the financial market, investors can use the predicted volatility to adjust their trading strategies and asset allocation, while regulators also need to understand potential future volatility to formulate appropriate macro-policies. Therefore, modelling and predicting volatility is one of the core elements of financial market research.

After the achievements of the ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) model on low-frequency data, the Realised Volatility (RV) model based on high-frequency data was provided by Andersen and Bollerslev (1998) as an unbiased estimator. The high-frequency data could contain more information than daily data, although the high-frequency data could be affected by the microstructure noise. Corsi (2009) propose the Heterogeneous Autoregressive of Realised Volatility (HAR-RV) model to forecast RV, which could capture the long-memory and fat-tail characteristics of volatility.

Consequently, the HAR-RV model is used in recent research widely to forecast RV. A growing amount of empirical literature has concentrated on investigating the forecasting accuracy of RV components in the HAR model, and thus many extension HAR models that depend on characteristics of RV have been generated. The most common components include discontinuous jump, which is influenced by the market news. As a result, Andersen et al. (2007) and Corsi et al. (2010) add the jump and continuous components into the HAR model. To capture the asymmetric effect of RV, Barndorff-Nielsen et al. (2008) and Patton and Sheppard

(2015) propose the signed jump variance and added signed parameters into the HAR model. Apart from the signed jump, Corsi et al. (2012) add negative returns into the HAR model to investigate the asymmetric effect.

Several issues arise from the existing empirical papers. First, there is no clear evidence across international indices to show which volatility components contain superior forecasting information of RV models. Second, the different horizons of investment decisions, which volatility components are appropriate over different forecasting horizons, corresponding to the long-term and short-term investing strategies. Third, the question of whether the superior volatility model can remain unchanged for the choice of loss function used in forecast evaluation.

Therefore, the aim of this chapter is to make an empirical comparison of three additional components in the basic HAR model, namely volatility jumps, realised semi-variance, and leverage effect. These three components are reported to have good forecasting performance in separate previous works (e.g. Andersen et al., 2007; Patton and Sheppard, 2015 and Corsi et al., 2012). To obtain a more specific assessment of each volatility component, this chapter extends three different models for every component based on existing works[5]. Further, this chapter employs various forecasting evaluations to assess the forecasting performance of the models, including symmetric loss functions, asymmetric loss functions, pairwise comparisons and equal predictive ability tests.

In the preview of the results, this chapter highlights that the leverage effect incorporated into the HAR-RV model can provide superior performance over daily, weekly and monthly horizons. There seems to be a slight improvement in considering more sophisticated HAR-RV models with realised semi-variance, as well as the signed jumps. However, taking into account

[5] To the best of my knowledge previous studies that those extensions are the most commonly used in the empirical works of these three additional components. (e.g., Sevi, 2014; Bucic and Gisler, 2017 and Horpestad et al., 2019.)

the jumps with the HAR models, the forecasting performance is the worst, which means the volatility jumps contain limited forecasting information. Moreover, for the overpredictions and underpredictions in each model, the asymmetric forecasting errors are broadly mixed, and there are no volatility components that can dominate.

The rest of the chapter proceeds as follows. Section 2 provides a review of relevant literature. Section 3 introduces the HAR-type models considered in this chapter and the methodology of forecasting evaluation. The data description and empirical findings of in-sample estimation and out-of-sample forecasting are presented in section 4 and 5. The conclusions are provided in section 6.

3.2 Literature Review

It is widely acknowledged that the daily returns of financial assets are difficult to predict, but the volatility of the assets returns are relatively predictable. Volatility is not observed directly. Thus the volatility forecasting models are based on the characteristics of volatility, such as volatility clustering, asymmetric effects and long memory. Since the ARCH (Engle, 1982) and GARCH (Bollerslev, 1982) models have been introduced, the GARCH model and its extensions obtained remarkable achievement in modelling and forecasting the volatility.

Realised Volatility

One of the main drawbacks of using daily squared returns is that they are noisy, to solve this shortage, Andersen and Bollerslev (1998) first propose the RV by using high-frequency data, conducting their calculations using the sum of squared intraday returns in a trading day. Moreover, based upon the evidence of Meddahi (2002) and Andersen et al. (2003), they notice that without market microstructure noise and the jumps, the RV converges to integrated volatility. Following this finding, the empirical evidence is increasingly concentrated on RV.

The high-frequency data could contain more information than daily data, although the high-frequency data could be affected by the microstructure noise. The widespread measurement to reduce the influence of microstructure noise is to select the optimal sampling frequency. Andersen et al. (2000, 2003) and Bandi et al. (2008) recommend using the sampling frequency from five-minutes to 30-minutes. Further investigation of Liu et al. (2015) finds that other frequency RV could not beat five-minute frequency RV. In addition to the deal with microstructure noise, there are other approaches for estimating integrated volatility such as the realised kernel (Hansen and Lunde, 2004, 2006; and Barndorff-Nielsen et al., 2008), realised range-based volatility (Martens and van Dijk, 2007), subsampling ways of realised volatility (Zhang et al., 2005) and quantile-based realised variance (Christensen et al., 2010). [6]

The HAR Model

According to the Heterogeneous Market Hypothesis of Müller et al. (1997), the volatility of assets return is affected by different trading behaviours and trading expectations at different investment horizons, which could reveal the dynamic change in the market. Given this, Corsi (2009) proposes the HAR model, which assumes that there are three fundamental trading horizons based upon daily, weekly and monthly periods, which correspond to short-, medium- and long-term investing behaviours. Thus, each trading behaviour affects volatility and can capture the long memory feature of volatility. The HAR model has a simple auto-regressive structure of RV with economically meaning. Corsi et al. (2008) compare the HAR model with other long memory models, ARFIMA model, and consider that the fractional integrated models cannot completely reflect market information and the HAR model might be more suitable for the long memory feature of RV. The HAR model has been widely employed in recent years because not only can it be simply estimated using the OLS approach, but it reproduces the

[6] More details can be found in Section 2.2.

persistence and incorporates the long-memory feature with economic meanings. Therefore, the basic HAR model allows for the various extensions to accommodate volatility characteristics.

Volatility Jumps

The assets price occurs in large changes in a short period of time in some cases, which is called volatility jumps, and the volatility jumps are usually related to the news announcement in the market. The normal news and important news have different influences on the volatility of assets return: the normal news makes the assets return changes in the markets smoothly, but the important news causes unusual huge changes in the market that are the volatility jumps (Maheu and McCurdy, 2004). Since Aït-Sahalia (2004) considers that the continuous-time model of volatility is composed of continuous parts and discrete finite large jumps, as per the volatility measurement of Bipower Variation (BPV) proposed by Barndorff-Nielsen and Shephard (2004, 2006), the jump components are computed as the positive part of the difference between RV_t and BPV_t. In order to detect the volatility jump-diffusion, the Z Test (Huang and Auchen, 2005) and C_TZ Test (Corsi et al., 2010) are developed to identify the discontinuous jump variation. The Z test and the C_TZ test have the same predictive ability when there is no jump, but the Z test does not recognize the jump if the jump is continuous. Therefore, some parts of the continuous jump are included in the continuous component of the volatility that leads the Z test to underestimate the predictive power (Corsi et al., 2010).

For the empirical works of volatility jump in the HAR model, Andersen et al. (2007) initially propose the HAR-RV-J model and HAR-RV-CJ model, which incorporated the jump and continuous components. They employ those two models on three different financial assets, and both of those models outperform the basic HAR-RV model. Andersen et al. (2011) consider the jump and continuous component, as well as the overnight return variance following the GARCH process. The evidence surrounding volatility jumps is controversial.

Some evidence finds the jumps have a statistically significant impact on the estimation and prediction of volatility, which can improve the forecasting performance of RV (e.g. Andersen et al., 2007; Dumitru and Urga 2012; Maneesoonthorn et al. 2017; Liu et al., 2018); however, some also find that the persistence of volatility is not affected by the jumps and has limited forecasting ability (e.g. Sevi, 2014; Prokopczuk et al., 2016; Bucic and Gisler, 2017; Baur and Dimphl, 2019). In particular, Bucic and Gisler (2017) find the jump and continuous component is only important for the American market and has limited predictive power for many international equity markets.

Realised Semi-Variance

Although the continuous parts and discrete finite jumps constitute the volatility in the markets (Aït-Sahalia, 2004), the jumps cannot capture the significant effect of returns. Ang et al. (2006) consider the semi-variance in the asset pricing model and illustrate its relevance in financial asset pricing. To capture the sign effect of intraday returns, Barndorff-Nielsen et al. (2008) refer to the works of Aït-Sahalia (2004) to decompose the RV into downside realised semi-variance and upside realised semi-variance, which depends on the sign of intraday returns. And they find the downside RV could provide more informative forecasts than the normal RV. Taking the analysis of Barndorff-Nielsen et al. (2008) one step further, Patton and Sheppard (2015) propose the signed jump variation which is calculated by the difference between positive and negative semi-variance, as well as decomposed the signed jump into negative and positive parts, finding that volatility has a strong relationship with negative jumps and that negative jumps cause higher volatility than positive jumps.

Therefore, those signed parameters have provided new insight into sign effect for forecasting volatility. As for the empirical evidence of semi-variance and signed jump, those new parameters having better forecasting estimation depends on the in-sample forecast. Recent papers indicate that the realised semi-variance and the signed jump contain more information for forecasting the volatility than the discontinuous jump (Sevi, 2014; Wen et al., 2016; Fang et al., 2017; Kilic and Shaliastovich, 2019). Furthermore, the negative realised semi-variance has better predictive power than positive semi-variance (Patton and Sheppard, 2015)

Leverage Effect

The asymmetric volatility effects first noticed by Black (1976) and Christie (1982) and refer to the volatility of the stock market having a negative relationship with the stock return. In other words, when the stock prices of a firm decline, the firm's debt and equity will become highly leveraged and riskier, leading to the value of the firm typically falling. Moreover, the other two explanations of the leverage effect are the feedback effect and the investors' behaviours: first, Engle et al. (1987) and Pindyck (1984) note the volatility feedback effect that suggests there is a high rate of asset return in the market that only occurs when the asset price drop, so that this effect leads to volatility increasing; and second, Avramov et al. (2006) posit that the main reason for asymmetric volatility is the investment behaviour of traders who have not informed.

Regarding recent literature on RV, Christensen et al. (2015) and Baur and Dimpfl (2019) note that negative return appears to be strongly affected by the positive return. Due to one of the main volatility features being asymmetric volatility effects, the empirical evidence of leveraged HAR model is a simplified modification that incorporates only the negative returns. Corsi et al. (2012), Asai et al. (2012) and Patton and Sheppard (2015) provide the Leveraged HAR-RV model, not only for the lagged average RV at the different horizons, but also the lagged negative returns over each horizon. Those leveraged models do improve the forecasting capacity and show that leveraged parameters have a negative relationship with RV and strongly improve the forecasting ability. Empirically, the leveraged HAR model leads to more accurate volatility forecasts (e.g. Souček and Todorova, 2014; Buncic and Gisler, 2017; Horpestad et al., 2019).

3.3 Empirical Methodology

The calculations of volatility components and all the forecasting models are presented in this section. This section starts with a description of the basic HAR model, then introduce the extension models with the jump and continuous components, realised semi-variance and the leverage effect. To examine the forecasting accuracy, the symmetric loss function, asymmetric loss functions, the Diebold-Mariano test and Model Confidence Set test are introduced at the end.

3.3.1 Empirical Models

HAR model

According to the calculation method of the RV by Andersen and Bollerslev (1998), assume the $\mathrm{r_{t,i}} = \mathrm{p_{t,i}} - \mathrm{p_{t,i-1}}$, $(\mathrm{t} = 1, \cdots, \mathrm{T};\ \mathrm{i} = 1, \cdots, \mathrm{N})$, where T means the total trading days, $\mathrm{p_{t,I}}$ means the log assets price at day t and ith intraday interval, thus, $\mathrm{r_{t,I}}$ means the log assets return at day t and ith intraday interval. Therefore, the RV on trading day t ($\mathrm{RV_t}$) can be calculated as:

$$\mathrm{RV_t} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(\mathrm{r_{t,i}}\right)^2 \tag{3.1}$$

where the n is the number of intraday returns in day t. Based on the heterogeneous market hypothesis, the HAR-RV model (Corsi, 2009) is given as:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t \tag{3.2}$$

where the coefficients, β_d, β_w and β_m, can be estimated using the OLS method, in which the weekly and monthly averages of RV are calculated as:

$$RV_{t-1:t-5} = \frac{1}{5}\sum_{i=1}^{5} RV_{t-i} \tag{3.3}$$

$$RV_{t-1:t-22} = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i} \tag{3.4}$$

The HAR model predicts future volatility using daily, weekly and monthly lagged average of RV. Apart from the original HAR model, the chapter employs several extensions of the HAR-RV model to carry out the modelling estimation and volatility comparison.

HAR model with jumps and continuous components

To obtain the robust jumps, this chapter follows the jump-diffusion process of Barndorff-Nielsen and Shephard (2004, 2006). The logarithm of an asset price is denoted by p_t at the time t, while p_t is assumed to follow a continuous time stochastic volatility diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_t + k_t dq_t \tag{3.5}$$

where μ_t and σ_t are the drift and instantaneous volatility, W_t is standard Brownian motion, q_t is a counting process with time-varying intensity and $k_t = p_t - p_{t-1}$ refers to the size of the discrete jump process. The Quadratic Variation (QV) of the assets price p_t is shown as follows:

$$QV = \int_0^t \sigma^2(s)ds + \sum_{0<s\ll t} k^2(s) \tag{3.6}$$

where $\int_0^t \sigma^2(s)ds$ on the right side is the integrated variance of the continuous components, and the second term, $\sum_{0<s\ll t} k^2(s)$, is the squared jump component between 0 to t. In other words, the QV is separated into its continuous and jump components. Meanwhile, Barndorff-Nielsen and Shephard (2004, 2006) note that, when $n \rightarrow \infty$, the RV is a consistent estimator of QV which can be written as:

$$RV_t = \sum_{i=1}^{n} \left(r_{t,i}\right)^2 \rightarrow \int_0^t \sigma^2(s)ds + \sum_{0<s\ll t} k^2(s)\,, \qquad when\, n \rightarrow \infty \tag{3.7}$$

Then, they set the continuous components in the QV which can be estimated by the Bi-power Variation (BPV). The BPV is shown as:

$$BPV_t = \sqrt{\frac{\pi}{2}} \sum_{i=2}^{n} |r_{t,i}||r_{t,i-1}| \to \int_0^t \sigma^2(s)ds\,, \qquad when\; n \to \infty \tag{3.8}$$

So the jump components can be calculated using the difference between the RV_t and BPV_t and due to the difference between two estimators potentially being negative, Nielsen and Shephard (2004) and Andersen et al. (2007) suggest using the positive value as the jumps. Therefore, the jumps in the RV can be calculated as:

$$\begin{aligned} J_t &= RV_t - BPV_t \\ &= \max(RV_t - BPV_t, 0) \end{aligned} \tag{3.9}$$

Subsequently, as the volatility is composed of continuous parts and discrete finite jumps, Andersen et al. (2007) decompose the RV into jump variation and continuous variation, which is $RV_t = J_t + C_t$. Therefore, the continuous components can be calculated using:

$$\begin{aligned} C_t &= RV_t - J_t \\ &= RV_t - \max(RV_t - BPV_t, 0) \end{aligned} \tag{3.10}$$

To incorporate the jumps with the HAR model, Andersen et al. (2007) add the one lagged jump component into the HAR-RV model as an explanation estimator and proposed the HAR-RV-J model. They find the jump component in this model negative and statistically significant. The model is shown as follows:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + \beta_j J_{t-1} + u_t \tag{3.11}$$

Furthermore, the HAR-RV-CJ model is also proposed by Andersen et al. (2007). They add the jump and continuous components, respectively, and separate at daily, weekly and monthly horizons. So the HAR-RV-CJ model is given by:

$$RV_t = \beta_0 + \beta_{cd}C_{t-1} + \beta_{cw}C_{t-1:t-5} + \beta_{cm}C_{t-1:t-22} + \beta_{jd}J_{t-1} + \beta_{jw}J_{t-1:t-5} + \beta_{jm}J_{t-1:t-22} + u_t \tag{3.12}$$

Inspired by the HAR-RV-J model, the HAR-CJ model is the new specification where this chapter considers that the jump component is only provided by the daily horizon, which is integrated the main content of HAR-RV-J and HAR-RV-CJ model. So the HAR-CJ model is shown as:

$$RV_t = \beta_0 + \beta_{cd}C_{t-1} + \beta_{cw}C_{t-1:t-5} + \beta_{cm}C_{t-1:t-22} + \beta_{jd}J_{t-1} + u_t \tag{3.13}$$

HAR model with realised semi-variance

To examine the potential asymmetric effect of RV, Barndorff-Nielsen et al. (2008) decompose the realised variance into downside realised semi-variance (RSV^-) and upside realised semi-variance (RSV^+) corresponding to the daily bad and good RV, and thus the RSV^- and RSV^+ are calculated as the sum of positive and negative intraday returns in a day, respectively, which is given as:

$$RV = RSV^+ + RSV^- \tag{3.14}$$

$$RSV^- = \sum_{i=1}^{n} {r_{t,i}}^2 I(r_{t,i} < 0) \tag{3.15}$$

$$RSV^+ = \sum_{i=1}^{n} {r_{t,i}}^2 I(r_{t,i} > 0) \tag{3.16}$$

where $I(.)$ denotes the indicator variable. Besides the RSV, Barndorff-Nielsen et al. (2008) defined the signed jump variance (SJV), which is calculated by the difference between the RSV^- and RSV^+:

$$\mathrm{SJV}^2 = \mathrm{RSV}^+ - \mathrm{RSV}^- \tag{3.17}$$

Moreover, Patton and Sheppard (2015) divide the signed jump variance and propose the positive jumps and negative jumps. The signed jumps variances are provided as follows:

$$SJV^{2+} = (RSV^{+} - RSV^{-})I\{(RSV^{+} - RSV^{-}) > 0\} \tag{3.18}$$

$$SJV^{2-} = (RSV^{+} - RSV^{-})I\{(RSV^{+} - RSV^{-}) < 0\} \tag{3.19}$$

For the empirical models with RSV, Patton and Sheppard (2015) incorporate the RSV to HAR model and provide the HAR-PS model, which only decomposed the one-lagged RV into the negative semi-variance and positive semi-variance, shown as:

$$RV_t = \beta_0 + {\beta_d}^{+}{RSV_{t-1}}^{+} + {\beta_d}^{-}{RSV_{t-1}}^{-} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t \tag{3.20}$$

For the HAR model with signed jump variance, Patton and Sheppard (2015) also provide the HAR-RV-SJV model. It decomposes the daily horizon RV into one-lagged negative and positive signed jumps variances and the continuous component.

$$\begin{aligned} RV_t = \beta_0 + {\beta_j}^{+}{SJV_{t-1}}^{+} + {\beta_j}^{-}{SJV_{t-1}}^{-} + \beta_c C_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} \\ + u_t \end{aligned} \tag{3.21}$$

Additionally, to capture the positive and negative semi-variance with different horizons based on the heterogeneous structure of the market, Patton and Sheppard (2015) decompose semi-variances into different lags, which is HAR-RSV shown as follows:

$$\begin{aligned} RV_t = \beta_0 + {\beta_d}^{+}{RSV_{t-1}}^{+} + {\beta_d}^{-}{RSV_{t-1}}^{-} + {\beta_w}^{+}{RSV_{t-1:t-5}}^{+} + {\beta_w}^{-}{RSV_{t-1:t-5}}^{-} \\ + {\beta_m}^{+}{RSV_{t-1:t-22}}^{+} + {\beta_m}^{-}{RSV_{t-1:t-22}}^{-} + u_t \end{aligned} \tag{3.22}$$

HAR model with leverage effect

Asai et al. (2012) mention the leveraged HAR-RV model, which is based on the HAR model of Corsi (2009), they add the negative value of daily return ${r_t}^{-}$ as a leveraged parameter into HAR-RV model and separated it at the different horizons. Furthermore, inspired by Horpestad et al. (2019), who only use the one-lagged negative return as the leverage factor, this chapter generates another new specification of LHAR-RV with only a one-lagged negative return. Therefore, this chapter classifies these two models as LHAR-RV1 and LHAR-RV2 model, which are shown as follows:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + \beta_{ld} r_{t-1}{}^{-} + u_t \tag{3.23}$$

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + \beta_{ld} r_{t-1}{}^{-} + \beta_{lw} r_{t-1,t-5}{}^{-} + \beta_{lm} r_{t-1,t-22}{}^{-} + u_t \tag{3.24}$$

And Corsi et al. (2012) provide the LHAR-RV-CJ model which separates the jump, continuous and leveraged components, respectively, with each three-lagged components having different forecasting horizons:

$$RV_t = \beta_0 + \beta_{cd} C_{t-1} + \beta_{cw} C_{t-1:t-5} + \beta_{cm} C_{t-1:t-22} + \beta_{jd} J_{t-1} + \beta_{jw} J_{t-1:t-5} + \beta_{jm} J_{t-1:t-5} + \beta_{ld} r_{t-1}{}^{-} + \beta_{lw} r_{t-1,t-5}{}^{-} + \beta_{lm} r_{t-1,t-22}{}^{-} + u_t \tag{3.25}$$

The Appendix 1 from Appendix provides a table summing up the all model specifications in this chapter.

3.3.2 Evaluation Methodology

As for the loss function, there are many ways to evaluate and compare the accuracy of the different forecasting models. However, it is not obvious which loss function is more appropriate. Thus, the four loss functions which are commonly used in the empirical research are employed in this chapter, including Mean Absolute Error (MAE), Mean Squared Error (MSE), Quasi-Likelihood (QLIKE) and the adjusted R^2 of Mincer-Zarnowitz regressions. In addition, Patton (2011) indicates that QLIKE and MSE are the most robust loss functions for heteroscedasticity. The four loss functions this chapter uses are shown as follows:

$$MAE = \frac{1}{n}\sum_{t=1}^{n} \left| RV_t - \widehat{RV_t} \right| \tag{3.26}$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n} \left(RV_t - \widehat{RV_t} \right)^2 \tag{3.27}$$

$$QLIKE = \frac{1}{n}\sum_{t=1}^{n} \left(\log\left(\widehat{RV_t} \right) + \frac{RV_t}{\widehat{RV_t}} \right) \tag{3.28}$$

$$RV_t = a_0 + \ a_1\widehat{RV}_t + \epsilon_t \tag{3.29}$$

The actual volatility is denoted as RV_t, and the volatility forecast obtained is indicated by $\widehat{RV}_t$.

However, these errors of loss functions above are assumed to be symmetric. In practice, not all investors need to treat the underpredictions and overpredictions of volatility equally. For example, as the volatility of the underlying asset is positively correlated with the call option price, the underpredicted volatility will lead to the decline of the call option price, and so the sellers pay more attention to the underpredictions. In contrast, the overpredicted volatility is more likely to attract the attention of the buyer than seller. To account for the potential asymmetry of error statistics in the loss function, this chapter also employs the mean mixed error (MME) of underpredictions and overpredictions (Brailsford and Faff, 1996 and McMillan et al., 2000).

$$MME(U) = \frac{1}{n}\left[\sum_{t=1}^{O}\left|RV_t - \ \widehat{RV}_t\right| + \sum_{t=1}^{U}\sqrt{\left|RV_t - \ \widehat{RV}_t\right|}\right] \tag{3.30}$$

$$MME(O) = \frac{1}{n}\left[\sum_{t=1}^{O}\sqrt{\left|RV_t - \ \widehat{RV}_t\right|} + \sum_{t=1}^{U}\left|RV_t - \ \widehat{RV}_t\right|\right] \tag{3.31}$$

where the O means the number of over-prediction observations and U means the number of underprediction observations among the out-of-sample forecasts. The underpredictions are penalized more heavily in MME (U) and over-predictions are penalized heavily in MME (O).

Next, this chapter employs the pairwise comparison of the Diebold-Mariano (DM) test (Diebold and Mariano, 1995). The use of this forecasting evaluation test is to compare the forecasting ability of two forecasting models, which requires a loss function that is a measure of the difference between the RV and the forecast value in the out-of-sample period. Harvey et al. (1997) propose the modified DM test as it is an approximately unbiased measurement beyond one-step ahead forecasting in terms of mean squared prediction error. Notably, Diebold (2015) indicates that the estimation errors of the DM test decrease with the expanding out-of-sample size, which generate the pseudo-out-of-sample. Nonetheless, the original DM test fared well as it provides direct comparative information from historical predictive performance. The loss function of the DM-test is defined as:

$$X_{t;j}^{(A,B)} = L_{t;l}^{(A)} - L_{t;l}^{(B)} \tag{3.32}$$

where the A, B are two compared models, and $L_{t;l}^{(A)}$ and $L_{t;l}^{(B)}$ are the loss function of each model. Then the DM statistic is given by:

$$DM = \frac{\bar{X}_{t=1,2,\dots,t;l}^{(A,B)}}{\frac{\widehat{\sum\tau}}{\sqrt{n}}} \tag{3.33}$$

The $\widehat{\sum\tau}$ is the standard deviation of $\bar{X}_{t=1,2,\dots,t;l}^{(A,B)}$. The statistics follow a standard normal distribution and allow for easy comparison of model pairs at each horizon. Thus this chapter compares the forecasting ability of every two HAR-type models, and the loss function used is the mean squared error.

Selecting the best optimal model which could adequately describe the data generating process from several alternative models produces the issue about selecting the optimality criterion. Besides loss function, Hansen and Lunde (2005) propose the superior predictive ability test to compare the model's model accuracy. However, due to the superior predictive ability test having to select a model as a benchmark, the benchmark selection would affect the comparison results directly. Thus, this chapter considers the Model Confidence Set (MSC) test developed by Hansen et al. (2011). The MSC test utilizes the bootstrap implementation and removes the worst model sequentially according to the rejection of the null hypothesis of equal predictive ability (EPA). The specific process of MCS is introduced as follows. First, assume there are m_0 alternative forecasting models to be tested, so $M_0 = \{1, 2, \cdots, m_0\}$. Let $d_{ij,t}$ demote the loss function differences between any two models at time t:

$$d_{ij,t} = l_{i,t} - l_{j,t} \quad (i, j \in M_0) \tag{3.34}$$

Second, the MSC test is a process that sequentially removes the worst forecasting model from M_0. Thus, in each step, the null hypothesis is set as any two models that have EPA:

$$H_{0,M}: E\left(d_{ij,t} = 0\right),\ \ for\ all\ \ i,j\ \in M_0 \tag{3.35}$$

$$H_{A,M}: E\left(d_{ij,t} \neq 0\right),\ \ for\ some\ \ i,j\ \in M_0 \tag{3.36}$$

Third, in each step of the MSC test, if the null hypothesis of EPA is rejected at a certain significant level, the worst foresting model would be removed sequentially until the null hypothesis of EPA is accepted. However, there is one drawback in this test: the prediction ability of any two forecasting models need to recalculate the test statistics at every step of the process. In order to overcome this shortage, Hansen et al. (2011) construct the Range Statistics and Semi-Quadratic Statistics to test the hypotheses above, and the two tests are shown as follows:

$$T_R = \max_{i,j\in M_0}\left|\frac{\bar{d}_{i,j}}{\sqrt{\widehat{var}\left(\bar{d}_{i,j}\right)}}\right|\ \ and\ \ T_{SQ} = \sum_{i,j\in M_0}\frac{\left(\bar{d}_{i,j}\right)^2}{\widehat{var}\left(\bar{d}_{i,j}\right)} \tag{3.37}$$

where $\bar{d}_{i,j}$ is the mean value of the loss functions difference, calculated as $\bar{d}_{i,j} = \frac{1}{M}\sum d_{ij,t}$. Finally, though sequentially removing the worst model, the $\widehat{M}_0$ is a subset of models which contains the surviving models from M_0.

3.4 Data

All the high frequency data and daily returns are obtained from the Oxford-Man Institute of Quantitative Finance. To deal with the influence of microstructure noise, this chapter employs the five-minute RV data for the eight international indices[7], including the UK (FTSE), Japan

[7] This chapter relies on a common 5-minute sampling frequency for all indices. This choice directly mirrors the sampling frequency used in much of the existing realized volatility literature. The empirical study by Liu et al. (2015), comparing more than 400 different RV estimators across multiple asset, concludes that it is difficult to significantly beat 5-mintue RV.

(N225), the US (SPX), Germany (DAX), China (SSEC), India (NSEI), Brazil (BVSP) and Mexico (MXX). The data sample includes both developed countries and emerging countries, since the majority of empirical work concentrates on developed markets. The RV and returns are obtained over the 15-year sample size ranging from 1st January 2003 to 31st December 2017. The total data are divided into the five-year in-sample estimation period (1st January 2003 to 31st December 2007) and 10-year out-of-sample forecasting period (1st January 2008 to 31st December 2017).

Table 3.1 is the descriptive statistics results of all variables for eight indices, including daily realised volatility (RV), jump and continuous components (J and C), negative and positive realised semi-variance (RSV^- and RSV^+), negative and positive signed jump variance (SJV^- and SJV^+), and daily negative return (R^-). The RV of all indices exhibit a non-normal distribution with excess kurtosis and are right-skewed. The one special series is the RV of SSEC with the highest skewness and the smallest kurtosis exhibiting a fatter tail than other series. The statistics of the Jarque-Bera test for all variables are statistically significant at the 1% level, which shows all the data do not follow a normal distribution. The last column presents the results of the augmented Dickey-Fuller test, which shows that all variables significantly reject the null hypothesis of a unit root at a confidence interval of 99%, and thus every series is stationary and allows for further modelling analysis.

3.5 Empirical Results

The forecasts are generated from all considered forecasting models under both rolling window and recursive approaches. The out-of-sample period is from 1st January 2008 to 31st December 2017. All forecasting models produce the RV forecasts over the daily, weekly, and monthly horizons. To produce the multi-step-ahead forecast for the long-term horizon, this chapter simply replaces the data frequency of the volatility model. In other words, to replace RV_{t+1} on the lift-head side over the forecasting horizon h, say $RV_{t+h}^{h} = \frac{1}{h}\sum_{i=1}^{h} RV_{t-h+i}$, thus $h = 1, 5\ and\ 22$[8]. First, the forecasts are evaluated by the symmetric loss functions, including MSE, MAE, QLIKE and the adjusted R^2 of Mincer-Zarnowitz regressions, as well as the MME (O) and MME (U) to account the potential asymmetry of forecasting error. Second, the DM test using MSE criteria is employed to compare the forecasting performance between every two models. Last, the MCS in terms of MSE criteria is used to select the optimal models with EPA.

3.5.1 Symmetric Forecast Error Results

Table 3.2, 3.3, 3.4 and 3.5 report the statistics of the MSE, MAE, QLIKE and the adjusted R^2 of Mincer-Zarnowitz regressions using the rolling window, respectively, for all indices sampled over daily, weekly, and monthly frequencies (h = 1, 5 and 22). Overall, according to the results of symmetric error, the rolling window approach obtains more accurate forecasts than the recursive method.

Table 3.2 shows the MSE comparison results. For the one-day-ahead forecast, the LHAR-RV-CJ model performs the best for almost every series, except for the BVSP. The HAR-RV-J model performs worse than the basic HAR model. For the one-week-ahead forecast, the LHAR-RV2 model produces the best forecasting performance for three indices, FTSE, SPX and DAX. The LHAR-RV1 model and HAR-RSV model also have good forecast ability. Again, the LHAR-RV2 model performs the best for the one-month-ahead forecast. Table 3.3 presents the forecasting error of MAE; the results are mixed. For four of the eight indices at the one-day-ahead forecasting horizon, the LHAR-RV-CJ model yields the lowest forecasting error.

[8] This chapter uses a simple approach for constructing multi-day-ahead forecasts by replacing the daily RV on the left-hand side of forecasting models. For instance, to generate weekly forecasts, I compute the average of five daily RVs and replace these 5 daily RVs with five identical averages. The approach of monthly forecast is the same. In the forecasting literature, this approach is commonly referred to as direct forecasts and has been extensively studied in the literature of Ghysels et al. (2009). Chapters 4 and 5 use the same method to generate weekly and monthly forecasts.

The HAR-RSV model provides the best performance for the N225, DAX and MXX at the weekly horizon. For the one-month-ahead forecast, both the HAR-RSV and LHAR-RV2 models are preferred, whilst the three HAR models with jumps perform poorly. In Table 3.4 of the QLIKE loss function, the superiority of the LHAR-RV2 model is evident for four of the eight indices over the daily horizon. At weekly and monthly horizons, the LHAR-RV2 model gains a leading role compared with others. In Table 3.5 of adjusted R squared, the best performance is given by the LHAR-RV-CJ model for almost all indices over the daily horizon, whereas the LHAR-RV2 model provides the best performance over the weekly and monthly horizons.

In terms of the recursive approach results, Table 3.6, 3.7, 3.8 and 3.9 present the forecast error for loss functions using over three different forecasting horizons. The four recursive approach tables present roughly consistent results, in which the jumps cannot provide enough forecasting information in the HAR model compared with semi-variance and leverage effect. According to the MSE in Table 3.6, the LHAR-RV-CJ model performs the best with the lowest error on almost all indices at the daily forecasting horizon, except for the N225 and BVSP. The LHAR-RV2 model outperforms other models over the daily and monthly horizons. For Table 3.7 of MAE results, the LHAE-RV-CJ model also offers top performance at the daily horizon. For a one-week-ahead forecast, the HAR-RSV model can provide more accurate forecasts than others. The LHAR-RV2 model is preferred over the monthly horizon. Table 3.8 and Table 3.9 obtain the same results as Table 3.6. Again, the LHAR-RV-CJ model outperforms at the daily horizon and the LHAR-RV2 model at weekly and monthly horizons.

3.5.2 Asymmetric Forecast Error Results

To account for the asymmetric error of considered models, the MME (U) and MME (O) under both rolling window and recursive approaches are presented in Table 3.10 to Table 3.13. The results of the MME (U) loss function penalizes the underpredictions more heavily, which means the models with lower statistics provide fewer underpredicted forecasts. In contrast, the models with lower MME (O) statistics generate fewer overpredictions.

Generally, the results of asymmetric forecast error are broadly mixed and the differences between all considered models are small, and the best model could outperform on several indices rather than dominating the asymmetric performance. For the rolling window method of MME (O) in Table 3.10, the LHAR-RV2 model has the best performance, closely followed by the HAR-PS model at the daily horizon. The HAR-RV model produces fewer overpredictions for the one-week-ahead forecast, while the HAR-CJ and LHAR-RV model also perform well. For monthly prediction, the HAR-RV, LHAR-RV1 and LHAR-RV2 models perform equivalently to obtain fewer overpredictions. For the MME (U) under the rolling window method in Table 3.11, the LHAR-RV-CJ model is the best model if underpredictions are penalized more heavily over the daily horizon. The HAR-PS model provides the best forecasting performance with fewer underpredictions over the weekly forecast. The indistinguishable evidence at the monthly horizon indicates no model performs better than any other.

Considering the recursive method in Table 3.12 and 3.13 of MME (O) and MME (U), as with the rolling window approach, the statistics are mixed. Table 3.12 shows the MME(O) comparison results, demonstrating that the HAR-PS model has fewer overpredictions at the daily horizon, and that the HAR-CJ and LHAR-RV2 models also perform well. For the weekly forecasting series, the HAR-CJ model is preferred. The HAR-CJ, LHAR-RV1 and LHAR-RV2 model display the same ability to obtain the lowest results over the daily horizon. Notably, for Table 3.13 of MME (U), the results are much less obvious, and no one model can perform well on more than three indices over daily, weekly and monthly forecasting horizons.

3.5.3 Testing For Differential Predictive Accuracy

To compare the difference in predictive accuracy between every two HAR models, Table 3.14, 15 and 16 offer the statistics and p-values from the DM test in terms of MSE criteria over daily, weekly and monthly forecasting horizons. In the three tables, the rolling window results are reported in the cells above the main diagonal; below the main diagonal, recursive forecasting results are presented. The results with a p-value below the 5% significant level mean the null hypothesis of identical predictive performance between two models is rejected, while a positive value indicates that the model in the row outperforms the model in the column.

Table 3.14 presents the DM test of the daily forecasting horizon. As the results of basic HAR-RV for all indices, besides the SPX, are negative, the extensions of the HAR model can significantly improve the forecasting performance. Then, for the HAR model with jumps and continuous components, the HAR-RV-J model significantly performs the worst for all indices with the exception of SSEC. The HAR-CJ model and HAR-RV-CJ model have positive results on the N225, DAX, NSEI and BVSP, whilst those two models perform as poorly as the HAR-RV-J model on other indices. In terms of the HAR models with realised semi-variance, the results are mixed: the HAR-RSV model provides significantly better results than other models for FTSE, SPX and SSEC. For the leveraged HAR models, the LHAR-RV-CJ model is only superior to other models on the DAX and BSVP. Notably, in the case of the MXX, the LHAR-RV1 model and LHAR-RV2 model perform well with significantly positive value. But, for the three leveraged HAR models for the FTSE and NSEI, the results are statistically insignificant, meaning the leverage effect does not contain competitive forecasting information on those two indices.

The DM test for the one-week-ahead forecast is displayed in Table 3.15. In most cases, the null hypothesis of equal predictive accuracy test between the HAR model and other sophisticated models is significantly negative, and therefore the HAR model performs worst for the one-week-ahead forecast. For the HAR models with jumps, the HAR-CJ model significantly improves the forecasting ability only for the DAX and NSEI, whilst the HAR-RV-CJ outperforms other models for the N225, DAX and BVSP. The results are mixed for the HAR models with realised semi-variance, and only the HAR-RSV is preferred to the SSEC. As with the daily forecasts, the leveraged HAR models do not perform well for the FTSE and NSEI. However, the LHAR-RV2 model performs the best for the SPX and BVSP, and the LHAR-RV-CJ model performs best for the N225, MXX, respectively.

Table 3.16 provides the DM test results of the one-month-ahead forecast. Table 3.16 produces roughly similar results to Table 3.14 and 3.15. Again, the HAR model performs worse than the extension HAR models. For the HAR models with jumps, almost all indices showed significantly negative value except for the DAX and NSEI. The results obtained for the HAR models with realised semi-variance in Table 16 are the same as that of Table 3.15, they are both mixed and only the HAR-RSV is the best model for the SSEC. The LHAR-RV-CJ model performs the best among all considered models, which accounts for the vast majority of indices, including SPX, N225, DAX and BSVP.

3.5.4 Testing Models With Equal Predictive Ability

Moreover, the MSC test results in terms of MSE metrics for rolling window and recursive approaches are reported in Table 3.17 and 3.18. In the MCS test, the optimal model is chosen by the value 1 means, whereas the value 0 means the model is eliminated. The MCS test selects a subset of models with EPA at the 75% confidence level.

Generally, the HAR model incorporated with jumps performs worse than the HAR model with semi-variance and leverage effect, the basic HAR model also performs poorly. Specifically, as for the rolling window results in Table 3.17, the LHAR-RV-CJ model performs best for most series on the daily horizon. The LHAR-RV1 also provides good forecasting performance on the N225, SSEC and NSEI. For the weekly forecasting, the HAR-RSV model is preferred, while the LHAR-RV1 and LHAR-RV2 model also have competitive forecasting performance for two series, respectively. At the monthly horizon, the LHAR-RV1 and LHAR-RV2 model yield equivalent forecasting performance and outperform others.

Table 3.18 reports the MCS test under the recursive window approach. In comparison with Table 3.17 and Table 3.18, they obtain similar results and show the HAR model with jumps does not produce accurate forecasts compared with semi-variance and lever. At the daily forecasting horizon, the LHAR-RV-CJ model is the best model, while the other two leveraged HAR models, the LHAR-RV1 and LHAR-RV2 models, have good forecasting ability as well. The one-week-ahead forecasting results indicate that LHAR-RV2 is the best performing forecast model and the HAR-RSV model also performs well on the N225, BVSP and MXX. The LHAR-RV-CJ dominates all the models for all indices, except for the FTSE, over the monthly horizon.

To summarise, according to the statistics in the symmetric loss functions, DM-test and MCS test above, both the rolling window and recursive approaches indicate that the HAR models with leverage effect have significant forecasting ability, achieving the best performance in most cases over daily, weekly and monthly horizons. Specifically, the LHAR-RV-CJ model is preferred at the daily level, whilst at the weekly and monthly horizons, the LHAR-RV2 dominates. That means the leverage effect in the HAR model contains the most accurate information. This result is consistent with the findings of Buncic and Gisler (2017) and Horpestad et al. (2019). However, the HAR model with jumps performs poorly for all indices, and it indicates that the RV jumps do not have efficient forecasting information. The results also indicate the realised semi-variance in the HAR model can slightly improve the forecasts. Meanwhile, all the forecasting models produce indistinguishable statistics for asymmetric loss functions, which means it is not obvious for all considered models to evaluate the overpredictions and underpredictions.

3.6 Summary and Conclusion

The previous empirical works show that adding the characteristics of RV into the HAR-RV model provides more accurate predictions. To comprehensively compare the volatility components which are commonly used, this chapter investigates the RV forecasting performance of volatility jumps, realised semi-variance and leverage effect in the HAR-RV model. For the purpose of acquiring more specific results, the HAR models with the jumps, realised semi-variance and leverage effect are extended based on recent research. This chapter employs eight international RV indices to generate the forecasts over daily, weekly and monthly horizons using both rolling window and recursive approaches. For the forecasting evaluations, this chapter reports various loss functions to assess forecasting error, including the widely used symmetric loss functions, asymmetric loss functions, pairwise comparison and equal predictive ability test.

In summary of the results, the main finding is that this chapter provides strong evidence that the HAR-RV model with leverage effect produces the top performance over daily, weekly and monthly horizons. This finding is consistent with recent studies (Buncic and Gisler, 2017 and Horpestad et al., 2019), which considers that the asymmetric effect improves forecasting performance. When considering more sophisticated HAR models with realised semi-variance, as well as the signed jumps, the forecasting accuracy is improved, which is the same results as Sevi (2014) and Kilic and Shaliastovich (2019). But the performance is not as good as the leverage effect. This chapter also notes the HAR models with volatility jumps are merely better than the HAR-RV model and the jumps have limited forecasting information compared with the leverage effect and realised semi-variance. This evidence directly conflicts the results of Maneesoonthorn et al. (2017) and Liu et al. (2018).

The current literature extends the HAR-RV model (Corsi, 2009) by adding specific components, which are based on the features of RV, to improve the forecasting ability. Overall, this chapter shows a generally greater number of situations in which the leverage effect in the HAR model displays the best forecasting performance, and the results are consistent for the different forecasting horizons. The results of this chapter emphasise the priority of negative returns in forecasting RV for both developed countries and emerging countries. Although this chapter does not find the best model in asymmetric loss functions, it is still worth it for future works to consider the asymmetry of forecasts for different investing purposes.

Table 3.1Summary statistic for all variables

	Mean	Median	Std.dev	Skewness	Kurtosis	Jarque-Bera	ADF test
				FTSE			
RV	$1.13*10^{-04}$	$5.19*10^{-05}$	$2.85*10^{-04}$	18.535	554.22	$4.81*10^{7}$***	-8.2131***
J	$3.02*10^{-05}$	$8.70*10^{-06}$	$1.12*10^{-04}$	31.186	1404.4	$3.10*10^{8}$***	-11.831***
C	$8.29*10^{-05}$	$3.90*10^{-05}$	$2.01*10^{-04}$	13.140	253.81	$1.00*10^{7}$***	-7.6676***
RSV^-	$5.80*10^{-05}$	$2.23*10^{-05}$	$2.06*10^{-04}$	26.831	1007.8	$1.60*10^{8}$***	-9.4094***
RSV^+	$5.51*10^{-05}$	$2.52*10^{-05}$	$1.18*10^{-04}$	9.5582	137.17	$2.90*10^{6}$***	-6.9296***
SJV^-	$-2.32*10^{-05}$	0.0000	$1.56*10^{-04}$	-32.793	1362.1	$2.92*10^{8}$***	-11.388***
SJV^+	$2.04*10^{-05}$	$9.43*10^{-07}$	$7.94*10^{-05}$	14.307	304.91	$1.45*10^{7}$***	-8.2338***
R^-	$1.47*10^{-04}$	$4.33*10^{-04}$	$1.10*10^{-04}$	-0.1635	11.324	$1.09*10^{4}$***	-16.873***
				SPX			
RV	$1.02*10^{-04}$	$4.01*10^{-05}$	$2.65*10^{-04}$	11.798	236.81	$8.68*10^{6}$***	-6.3651***
J	$2.35*10^{-05}$	$5.17*10^{-06}$	$7.85*10^{-05}$	12.351	236.19	$8.64*10^{6}$***	-7.9335***
C	$7.82*10^{-05}$	$3.05*10^{-05}$	$2.14*10^{-04}$	11.944	224.38	$7.79*10^{6}$***	-6.1354***
RSV^-	$5.13*10^{-05}$	$1.74*10^{-05}$	$1.44*10^{-04}$	10.727	176.94	$4.83*10^{6}$***	-6.7218***
RSV^+	$5.04*10^{-05}$	$1.96*10^{-05}$	$1.39*10^{-04}$	12.972	277.11	$1.19*10^{7}$***	-5.9628***
SJV^-	$-1.72*10^{-05}$	0.0000	$7.32*10^{-05}$	-11.657	202.93	$6.37*10^{6}$***	-7.9827***
SJV^+	$1.63*10^{-05}$	$2.61*10^{-07}$	$6.60*10^{-05}$	11.283	183.92	$5.22*10^{6}$***	-7.5169***
R^-	$2.49*10^{-04}$	$6.31*10^{-04}$	$1.08*10^{-02}$	-0.2816	14.623	$2.13*10^{4}$***	-15.827***
				N225			
RV	$9.70*10^{-05}$	$5.68*10^{-05}$	$1.79*10^{-04}$	8.6577	106.74	$1.69*10^{6}$***	-7.6110***
J	$1.43*10^{-05}$	$6.21*10^{-06}$	$3.40*10^{-05}$	15.139	376.52	$2.15*10^{7}$***	-11.029***
C	$8.27*10^{-05}$	$4.72*10^{-05}$	$1.65*10^{-04}$	9.6100	130.65	$2.55*10^{6}$***	-7.5660***
RSV^-	$5.01*10^{-05}$	$2.60*10^{-05}$	$1.12*10^{-04}$	10.561	153.87	$3.55*10^{6}$***	-7.8068***
RSV^+	$4.69*10^{-05}$	$2.65*10^{-05}$	$8.51*10^{-05}$	8.5629	109.58	$1.78*10^{6}$***	-7.6613***
SJV^-	$-1.68*10^{-05}$	$-1.81*10^{-07}$	$6.99*10^{-05}$	-13.121	222.63	$7.48*10^{6}$***	-9.9579***
SJV^+	$1.35*10^{-05}$	0.0000	$4.72*10^{-05}$	13.133	271.68	$1.11*10^{7}$***	-9.4373***
R^-	$-2.39*10^{-04}$	$-1.08*10^{-04}$	$1.12*10^{-02}$	-0.7191	17.556	$3.27*10^{4}$***	-15.665***
				DAX			
RV	$1.39*10^{-04}$	$7.25*10^{-05}$	$2.57*10^{-04}$	9.9060	166.96	$4.33*10^{6}$***	-7.2686***
J	$1.84*10^{-05}$	$5.60*10^{-06}$	$6.15*10^{-05}$	14.726	331.12	$1.72*10^{7}$***	-8.6111***
C	$1.21*10^{-04}$	$6.32*10^{-05}$	$2.22*10^{-04}$	9.4408	150.31	$3.50*10^{6}$***	-7.3710***
RSV^-	$7.12*10^{-05}$	$3.49*10^{-05}$	$1.38*10^{-04}$	9.8041	174.42	$4.73*10^{6}$***	-7.8558***
RSV^+	$6.80*10^{-05}$	$3.54*10^{-05}$	$1.37*10^{-04}$	12.661	272.74	$1.17*10^{7}$***	-7.0454***
SJV^-	$-1.88*10^{-05}$	0.0000	$6.27*10^{-05}$	-10.689	176.08	$4.83*10^{6}$***	-10.798***
SJV^+	$1.56*10^{-05}$	$1.34*10^{-08}$	$7.11*10^{-05}$	21.584	641.45	$6.50*10^{7}$***	-10.666***
R^-	$1.07*10^{-06}$	$4.10*10^{-04}$	$1.18*10^{-02}$	0.0431	9.5029	$6.72*10^{3}$***	-16.648***

(continued on next page)

Table 3.1 (continued)

	Mean	Median	Std.dev	Skewness	Kurtosis	Jarque-Bera	ADF test
				SSEC			
RV	$1.89*10^{-04}$	$9.17*10^{-05}$	$3.01*10^{-04}$	5.5786	51.041	$3.69*10^{5}$***	-6.5219***
J	$2.13*10^{-05}$	$5.98*10^{-06}$	$5.43*10^{-05}$	8.4829	108.93	$1.74*10^{6}$***	-8.0632***
C	$1.68*10^{-04}$	$8.17*10^{-05}$	$2.71*10^{-04}$	5.8781	57.737	$4.75*10^{5}$***	-6.5888***
RSV^-	$9.27*10^{-05}$	$4.15*10^{-05}$	$1.59*10^{-04}$	5.3981	46.014	$2.98*10^{5}$***	-6.5530***
RSV^+	$9.63*10^{-05}$	$4.68*10^{-05}$	$1.61*10^{-04}$	6.2153	64.007	$5.87*10^{5}$***	-7.0522***
SJV^-	$-2.27*10^{-05}$	0.0000	$6.68*10^{-05}$	-5.9103	48.832	$3.39*10^{5}$***	-8.7285***
SJV^+	$2.63*10^{-05}$	$2.56*10^{-06}$	$7.44*10^{-05}$	8.3349	108.55	$1.73*10^{6}$***	-9.1347***
R^-	$1.36*10^{-03}$	$1.38*10^{-03}$	$1.48*10^{-02}$	-0.2781	6.7102	$2.13*10^{3}$***	-14.658***
				NSEI			
RV	$1.52*10^{-04}$	$7.41*10^{-05}$	$4.61*10^{-04}$	24.467	792.44	$9.71*10^{7}$***	-10.378***
J	$2.57*10^{-05}$	$4.73*10^{-06}$	$2.17*10^{-04}$	37.595	1614.7	$4.04*10^{8}$***	-13.916***
C	$1.26*10^{-04}$	$6.35*10^{-05}$	$2.89*10^{-04}$	15.154	337.37	$1.75*10^{7}$***	-9.0091***
RSV^-	$8.12*10^{-05}$	$3.59*10^{-05}$	$2.87*10^{-04}$	29.557	1156.0	$2.07*10^{8}$***	-10.885***
RSV^+	$7.06*10^{-05}$	$3.47*10^{-05}$	$1.95*10^{-04}$	20.998	641.58	$6.36*10^{7}$***	-10.082***
SJV^-	$-2.60*10^{-05}$	$-2.52*10^{-06}$	$1.58*10^{-04}$	-44.630	2410.5	$9.01*10^{8}$***	-12.727***
SJV^+	$1.54*10^{-05}$	0.0000	$6.19*10^{-05}$	14.931	309.93	$1.48*10^{7}$***	-11.401***
R^-	$-4.44*10^{-04}$	$-3.22*10^{-04}$	$1.28*10^{-02}$	-0.6809	9.3800	$6.61*10^{3}$***	-13.851***
				BVSP			
RV	$1.57*10^{-04}$	$9.48*10^{-05}$	$2.81*10^{-04}$	8.8839	116.79	$2.05*10^{6}$***	-5.6919***
J	$2.27*10^{-05}$	$6.82*10^{-06}$	$9.41*10^{-05}$	20.556	572.84	$5.04*10^{7}$***	-8.8325***
C	$1.34*10^{-04}$	$8.28*10^{-05}$	$2.25*10^{-04}$	8.1821	98.848	$1.46*10^{6}$***	-5.4013***
RSV^-	$8.00*10^{-05}$	$4.43*10^{-05}$	$1.62*10^{-04}$	11.210	206.22	$6.46*10^{6}$***	-6.1541***
RSV^+	$7.71*10^{-05}$	$4.47*10^{-05}$	$1.51*10^{-04}$	10.344	151.96	$3.49*10^{6}$***	-4.9887***
SJV^-	$-2.50*10^{-05}$	$-7.36*10^{-07}$	$9.87*10^{-05}$	-18.090	496.66	$3.79*10^{7}$***	-8.5745***
SJV^+	$2.21*10^{-05}$	0.0000	$9.18*10^{-05}$	15.082	299.97	$1.38*10^{7}$***	-6.7595***
R^-	$1.97*10^{-04}$	$3.54*10^{-04}$	$1.41*10^{-02}$	-0.0166	8.5211	$4.71*10^{3}$***	-14.856***
				MXX			
RV	$8.14*10^{-05}$	$3.94*10^{-05}$	$1.84*10^{-04}$	13.763	293.85	$1.34*10^{7}$***	-8.1310***
J	$2.37*10^{-05}$	$3.40*10^{-05}$	$1.16*10^{-04}$	19.253	506.10	$4.00*10^{7}$***	-10.592***
C	$5.77*10^{-05}$	$3.26*10^{-05}$	$9.43*10^{-05}$	9.5008	156.09	$3.74*10^{6}$***	-6.8069***
RSV^-	$4.15*10^{-05}$	$1.69*10^{-05}$	$1.35*10^{-04}$	20.222	581.63	$5.29*10^{7}$***	-9.3099***
RSV^+	$3.99*10^{-05}$	$1.89*10^{-05}$	$8.91*10^{-05}$	13.148	280.42	$1.22*10^{7}$***	-6.9439***
SJV^-	$-1.92*10^{-05}$	0.0000	$1.13*10^{-04}$	-23.508	741.37	$8.61*10^{7}$***	-10.928***
SJV^+	$1.76*10^{-05}$	$1.07*10^{-06}$	$7.25*10^{-05}$	15.857	365.63	$2.08*10^{7}$***	-9.3135***
R^-	$4.50*10^{-04}$	$8.77*10^{-04}$	$1.16*10^{-02}$	0.0140	9.8943	$7.47*10^{3}$***	-14.756***

Note: This table reports the summary statistics for all variables of eight indices for whole period from 1st January 2003 to 31st December 2017. *** denotes significant at the 1% level.

Table 3.2: Out-of-sample forecasting performance of MSE under rolling window

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.3466	0.4140	0.3228	0.2839	0.2772	0.2968	0.2718	0.4187
HAR-RV-J	0.3470	0.4180	0.3243	0.2840	0.2783	0.2958	0.2760	0.4249
HAR-CJ	0.3211	0.3843	0.3224	0.2779	0.2757	0.2962	0.2700	0.3964
HAR-RV-CJ	0.3218	0.3856	0.3225	0.2800	0.2764	0.2962	0.2729	0.3991
HAR-PS	0.3240	0.3861	0.3196	0.2743	0.2714	0.2981	0.2695	0.4081
HAR-RSV-SJV	0.3283	0.3842	0.3316	0.2779	0.2713	0.2974	**0.2678***	0.4037
HAR-RSV	0.3197	0.3841	0.3197	0.2719	0.2726	0.2987	0.2690	0.4035
LHAR-RV1	0.3265	0.3995	0.3191	0.2751	0.2688	0.2917	0.2684	0.4132
LHAR-RV2	0.3159	0.3949	0.3191	0.2712	0.2709	0.2925	0.2682	0.4059
LHAR-RV-CJ	**0.3012***	**0.3779***	**0.3190***	**0.2679***	**0.2700***	**0.2911***	0.2707	**0.3885***
				H=5				
HAR-RV	0.1291	0.1706	0.1605	0.1063	0.1015	0.1272	0.0919	0.1526
HAR-RV-J	0.1303	0.1722	0.1615	0.1066	0.1018	0.1271	0.0919	0.1538
HAR-CJ	0.1294	0.1778	0.1651	0.1095	**0.0999***	0.1333	0.0939	0.1517
HAR-RV-CJ	0.1306	0.1792	0.1658	0.1112	0.1006	0.1329	0.0932	0.1508
HAR-PS	0.1234	0.1660	0.1606	0.1033	0.1016	0.1282	0.0916	0.1507
HAR-RSV-SJV	0.1273	0.1672	0.1632	0.1050	0.1011	0.1287	0.0920	0.1514
HAR-RSV	0.1221	0.1637	0.1602	0.1013	0.1025	0.1286	**0.0912***	**0.1495***
LHAR-RV1	0.1242	0.1667	**0.1601***	0.1035	0.1018	**0.1261***	0.0917	0.1514
LHAR-RV2	**0.1196***	**0.1630***	0.1603	**0.1010***	0.1010	0.1266	0.0916	0.1516
LHAR-RV-CJ	0.1263	0.1759	0.1659	0.1051	0.1004	0.1316	0.0931	0.1502
				H=22				
HAR-RV	0.1365	0.1669	0.1693	0.1102	0.1243	0.1152	0.1012	0.1549
HAR-RV-J	0.1367	0.1676	**0.1687***	0.1106	0.1246	0.1150	0.1013	0.1558
HAR-CJ	0.1327	0.1779	0.1715	0.1084	0.1251	0.1197	0.1117	0.1653
HAR-RV-CJ	0.1347	0.1760	0.1708	0.1123	0.1256	0.1181	0.1129	0.1676
HAR-PS	0.1344	0.1648	0.1691	0.1091	**0.1240***	0.1154	0.1009	0.1518
HAR-RSV-SJV	0.1340	0.1648	0.1715	0.1094	0.1243	0.1207	0.1029	0.1560
HAR-RSV	0.1333	**0.1628***	0.1688	0.1087	0.1248	0.1153	0.1002	**0.1509***
LHAR-RV1	0.1341	0.1649	0.1692	0.1088	0.1241	**0.1150***	0.1007	0.1530
LHAR-RV2	**0.1309***	0.1647	0.1693	**0.1078***	0.1250	0.1157	**0.1001***	0.1497
LHAR-RV-CJ	0.1315	0.1764	0.1709	0.1091	0.1267	0.1186	0.1135	0.1634

Note: This table reports the symmetric forecasting evaluation (MSE) of eight RV indices for all forecasting models considered using rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.3Out-of-sample forecasting performance of MAE under rolling window method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.4489	0.5006	0.4249	0.4091	0.3991	0.4072	0.3949	0.4944
HAR-RV-J	0.4473	0.4981	0.4249	0.4090	0.3996	0.4064	0.3953	0.4957
HAR-CJ	0.4303	0.4812	0.4250	0.4050	0.3986	0.4069	0.3948	0.4801
HAR-RV-CJ	0.4307	0.4816	0.4244	0.4059	0.3985	0.4071	0.3965	0.4823
HAR-PS	0.4331	0.4828	0.4206	0.4000	0.3924	0.4072	0.3949	0.4877
HAR-RSV-SJV	0.4334	0.4816	0.4255	0.4030	0.3938	0.4101	**0.3926***	0.4846
HAR-RSV	0.4304	0.4816	0.4197	**0.3969***	0.3953	0.4087	0.3948	0.4852
LHAR-RV1	0.4349	0.4906	0.4209	0.4018	0.3913	**0.4037***	0.3933	0.4916
LHAR-RV2	0.4317	0.4895	**0.4205***	0.3994	0.3929	0.4046	0.3931	0.4889
LHAR-RV-CJ	**0.4173***	**0.4768***	0.4208	0.3971	**0.3916***	0.4038	0.3948	**0.4771***
				H=5				
HAR-RV	0.2531	0.2998	0.2704	0.2313	0.2379	0.2516	0.2118	0.2688
HAR-RV-J	0.2535	0.3005	0.2712	0.2319	0.2379	**0.2514***	0.2120	0.2702
HAR-CJ	0.2569	0.3092	0.2794	0.2370	**0.2349***	0.2591	0.2142	0.2721
HAR-RV-CJ	0.2575	0.3092	0.2747	0.2368	0.2363	0.2588	0.2156	0.2689
HAR-PS	**0.2491***	0.2972	0.2704	0.2289	0.2381	0.2534	0.2119	0.2674
HAR-RSV-SJV	0.2508	**0.2946***	0.2710	0.2302	0.2375	0.2567	0.2120	0.2677
HAR-RSV	0.2473	0.2951	**0.2695***	**0.2251***	0.2393	0.2540	0.2117	**0.2672***
LHAR-RV1	0.2499	0.2970	0.2698	0.2293	0.2384	0.2517	0.2115	0.2679
LHAR-RV2	0.2461	0.2950	0.2716	0.2274	0.2376	0.2521	**0.2109***	0.2707
LHAR-RV-CJ	0.2540	0.3083	0.2754	0.2339	0.2353	0.2586	0.2151	0.2708
				H=22				
HAR-RV	0.2499	0.2834	0.2796	0.2245	0.2562	0.2379	0.2320	0.2816
HAR-RV-J	0.2498	0.2842	0.2791	0.2246	0.2566	0.2377	0.2322	0.2822
HAR-CJ	0.2486	0.2989	0.2851	0.2277	0.2583	0.2424	0.2393	0.2924
HAR-RV-CJ	0.2505	0.2982	0.2841	0.2285	0.2585	0.2390	0.2404	0.2934
HAR-PS	0.2479	0.2822	0.2796	0.2232	0.2558	0.2381	0.2322	0.2792
HAR-RSV-SJV	0.2465	0.2815	0.2827	0.2238	0.2558	0.2394	0.2318	0.2799
HAR-RSV	0.2485	**0.2789***	**0.2788***	**0.2231***	0.2584	0.2378	0.2314	0.2780
LHAR-RV1	0.2480	0.2824	0.2795	0.2238	**0.2551***	**0.2376***	0.2315	0.2816
LHAR-RV2	**0.2449***	0.2834	0.2789	0.2250	0.2564	0.2379	**0.2310***	**0.2791***
LHAR-RV-CJ	0.2479	0.3000	0.2842	0.2283	0.2593	0.2394	0.2397	0.2913

Note: This table reports the symmetric forecasting evaluation (MAE) of eight RV indices for all forecasting models considered using rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.4: Out-of-sample forecasting performance of QLIKE under rolling window method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.002036	0.002235	0.001833	0.001678	0.001803	0.001737	0.001688	0.002336
HAR-RV-J	0.002047	0.002257	0.001842	0.001682	0.001814	0.001726	0.001716	0.002366
HAR-CJ	0.001900	0.002061	0.001828	0.001639	0.001794	0.001729	0.001716	0.002203
HAR-RV-CJ	0.001906	0.002077	0.001831	0.001659	0.001801	0.001730	0.001754	0.002224
HAR-PS	0.001909	0.002079	**0.001812***	0.001622	0.001764	0.001741	0.001671	0.002276
HAR-RSV-SJV	0.001999	0.002093	0.001904	0.001683	0.001761	0.001771	0.001714	0.002255
HAR-RSV	0.001921	0.002154	0.001816	0.001625	**0.001734***	**0.001695***	0.001658	0.002304
LHAR-RV1	0.001887	0.002131	0.001813	0.001599	0.001752	0.001700	**0.001658***	0.002254
LHAR-RV2	**0.001789***	**0.002062***	0.001816	**0.001582***	0.001750	0.001686	0.001749	**0.002155***
LHAR-RV-CJ	0.001836	0.002155	0.001822	0.001598	0.001768	0.001704	0.001776	0.002302
				H=5				
HAR-RV	0.000788	0.000943	0.000933	0.000664	0.000671	0.000799	0.000607	0.000878
HAR-RV-J	0.000802	0.000959	0.000942	0.000668	0.000673	0.000796	0.000607	0.000888
HAR-CJ	0.000930	0.000985	0.000965	0.000700	**0.000658***	0.000864	0.000660	0.000870
HAR-RV-CJ	0.000940	0.001005	0.000977	0.000725	0.000665	0.000867	0.000630	0.000866
HAR-PS	0.000753	0.000915	0.000933	0.000645	0.000670	0.000804	**0.000604***	0.000867
HAR-RSV-SJV	0.000854	0.000934	0.000961	0.000671	0.000667	0.000834	0.000616	0.000875
HAR-RSV	0.000761	0.000925	0.000931	0.000647	0.000673	**0.000792***	0.000605	0.000871
LHAR-RV1	**0.000741***	**0.000902***	**0.000928***	**0.000626***	0.000670	0.000796	0.000605	0.000869
LHAR-RV2	0.000978	0.001000	0.000976	0.000670	0.000667	0.000848	0.000627	0.000858
LHAR-RV-CJ	0.000878	0.000983	0.000956	0.000668	0.000683	0.000798	0.000606	**0.000865***
				H=22				
HAR-RV	0.000841	0.000947	0.000987	0.000698	0.000815	0.000737	0.000647	0.000891
HAR-RV-J	0.000846	0.000952	**0.000983***	0.000703	0.000818	**0.000735***	0.000648	0.000899
HAR-CJ	0.000823	0.001002	0.001003	0.000690	0.000821	0.000763	0.001248	0.001091
HAR-RV-CJ	0.000837	0.001003	0.001001	0.000721	0.000826	0.000755	0.001009	0.001092
HAR-PS	0.000829	0.000935	0.000986	0.000692	0.000813	0.000738	0.000644	0.000872
HAR-RSV-SJV	0.000847	0.000938	0.001009	0.000697	0.000813	0.000891	0.000730	0.000973
HAR-RSV	0.000829	0.000939	0.000989	0.000691	**0.000812***	0.000736	0.000643	0.000879
LHAR-RV1	**0.000808***	**0.000934***	0.000986	**0.000680***	0.000821	0.000741	**0.000638***	**0.000853***
LHAR-RV2	0.000818	0.001012	0.001002	0.000694	0.000834	0.000759	0.001145	0.001041
LHAR-RV-CJ	0.000820	0.000946	0.000987	0.000699	0.000817	0.000739	0.000987	0.000911

Note: This table reports the symmetric forecasting evaluation (QLIKE) of eight RV indices for all forecasting models considered using rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.5: Out-of-sample forecasting performance of MZ regression adjusted R^2 under rolling window method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.6785	0.7444	0.6675	0.7209	0.7634	0.7212	0.6507	0.5269
HAR-RV-J	0.6780	0.7415	0.6659	0.7208	0.7625	0.7218	0.6452	0.5199
HAR-CJ	0.7022	0.7636	0.6681	0.7271	0.7650	0.7214	0.6531	0.5517
HAR-RV-CJ	0.7014	0.7627	0.6680	0.7251	0.7643	0.7213	0.6500	0.5486
HAR-PS	0.6995	0.7617	0.6709	0.7303	0.7685	0.7203	0.6537	0.5388
HAR-RSV-SJV	0.6955	0.7625	0.6586	0.7268	0.7687	0.7208	0.6558	0.5437
HAR-RSV	0.7035	0.7633	0.6707	0.7327	0.7679	0.7198	0.6545	0.5441
LHAR-RV1	0.6973	0.7533	0.6715	0.7296	**0.7707***	0.7263	0.6551	0.5332
LHAR-RV2	0.7071	0.7567	0.6716	0.7335	0.7688	0.7259	**0.6555***	0.5415
LHAR-RV-CJ	**0.7206***	**0.7679***	**0.6717***	**0.7368***	0.7698	**0.7264***	0.6528	**0.5607***
				H=5				
HAR-RV	0.8574	0.8772	0.7757	0.8745	0.8941	0.8629	0.8304	0.7413
HAR-RV-J	0.8561	0.8759	0.7743	0.8740	0.8938	0.8628	0.8304	0.7389
HAR-CJ	0.8573	0.8732	0.7693	0.8711	0.8961	0.8561	0.8270	0.7415
HAR-RV-CJ	0.8561	0.8722	0.7684	0.8694	0.8954	0.8564	0.8281	0.7432
HAR-PS	0.8638	0.8806	0.7755	0.8780	0.8941	0.8619	0.8311	0.7445
HAR-RSV-SJV	0.8596	0.8797	0.7721	0.8760	0.8947	0.8608	0.8301	0.7430
HAR-RSV	0.8652	0.8826	0.7760	0.8803	0.8941	0.8615	**0.8317***	**0.7469***
LHAR-RV1	0.8629	0.8801	**0.7763***	0.8778	0.8938	**0.8642***	0.8309	0.7433
LHAR-RV2	**0.8679***	**0.8831***	0.7761	**0.8807***	0.8945	0.8636	0.8312	0.7431
LHAR-RV-CJ	0.8611	0.8748	0.7683	0.8760	**0.8953***	0.8579	0.8284	0.7441
				H=22				
HAR-RV	0.8301	0.8621	0.7960	0.8496	0.8818	0.8655	0.8421	0.7795
HAR-RV-J	0.8298	0.8614	0.7967	0.8490	0.8815	0.8656	0.8419	0.7783
HAR-CJ	0.8350	0.8545	0.7936	0.8525	0.8812	0.8604	0.8260	0.7645
HAR-RV-CJ	0.8327	0.8562	0.7944	0.8472	0.8807	0.8623	0.8247	0.7616
HAR-PS	0.8327	0.8638	0.7962	0.8510	**0.8821***	0.8653	0.8247	0.7839
HAR-RSV-SJV	0.8332	0.8637	0.7933	0.8506	0.8818	0.8586	0.8394	0.7782
HAR-RSV	0.8341	**0.8651***	**0.7965***	0.8517	0.8817	0.8654	0.8438	0.7854
LHAR-RV1	0.8331	0.8637	0.7962	0.8515	0.8820	**0.8658***	0.8430	0.7823
LHAR-RV2	**0.8371***	0.8643	0.7961	**0.8530***	0.8811	0.8649	**0.8441***	**0.7870***
LHAR-RV-CJ	0.8365	0.8561	0.7944	0.8514	0.8797	0.8615	0.8240	0.7675

Note: This table reports the symmetric forecasting evaluation (MZ regression adjusted R^2) of eight RV indices for all forecasting models considered using rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.6: Out-of-sample forecasting performance of MSE under recursive method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.3472	0.4143	0.3242	0.2826	0.2760	0.3010	0.2714	0.4205
HAR-RV-J	0.3501	0.4159	0.3245	0.2820	0.2759	0.3002	0.2768	0.4200
HAR-CJ	0.3213	0.3853	0.3242	0.2792	0.2750	0.3010	0.2703	0.3991
HAR-RV-CJ	0.3222	0.3865	0.3245	0.2799	0.2756	0.3005	0.2730	0.4013
HAR-PS	0.3242	0.3867	0.3206	0.2736	0.2710	0.3020	0.2694	0.4098
HAR-RSV-SJV	0.3267	0.3845	0.3300	0.2769	0.2693	0.3017	0.2667	0.4065
HAR-RSV	0.3194	0.3840	0.3208	0.2706	0.2714	0.3023	0.2686	0.4040
LHAR-RV1	0.3277	0.3991	0.3203	0.2731	0.2674	0.2956	0.2678	0.4153
LHAR-RV2	0.3157	0.3922	**0.3196***	0.2681	0.2689	0.2945	**0.2674***	0.4079
LHAR-RV-CJ	**0.3013***	**0.3755***	0.3205	**0.2664***	**0.2688***	**0.2936***	0.2704	**0.3895***
				H=5				
HAR-RV	0.1290	0.1701	0.1596	0.1056	0.0989	0.1291	0.0923	0.1536
HAR-RV-J	0.1303	0.1712	0.1594	0.1056	0.0986	0.1289	0.0924	0.1542
HAR-CJ	0.1306	0.1779	0.1631	0.1097	**0.0972***	0.1363	0.0953	0.1574
HAR-RV-CJ	0.1318	0.1801	0.1631	0.1110	0.0977	0.1357	0.0945	0.1564
HAR-PS	0.1231	0.1654	0.1595	0.1029	0.0990	0.1300	0.0921	0.1516
HAR-RSV-SJV	0.1256	0.1663	0.1612	0.1041	0.0984	0.1323	0.0925	0.1522
HAR-RSV	0.1215	0.1618	0.1594	0.1009	0.0995	0.1306	0.0916	**0.1493***
LHAR-RV1	0.1240	0.1665	0.1590	0.1027	0.0992	0.1281	**0.0920***	0.1523
LHAR-RV2	**0.1189***	**0.1617***	**0.1588***	**0.0996***	0.0991	**0.1276***	0.0922	0.1509
LHAR-RV-CJ	0.1274	0.1756	0.1624	0.1046	0.0981	0.1339	0.0943	0.1530
				H=22				
HAR-RV	0.1368	0.1647	0.1688	0.1091	0.1231	0.1196	0.1013	0.1557
HAR-RV-J	0.1373	0.1654	**0.1680***	0.1093	0.1229	0.1194	0.1013	0.1558
HAR-CJ	0.1322	0.1767	0.1721	0.1087	0.1238	0.1244	0.1128	0.1646
HAR-RV-CJ	0.1339	0.1758	0.1726	0.1096	0.1240	0.1229	0.1137	0.1665
HAR-PS	0.1346	0.1627	0.1686	0.1082	0.1229	0.1198	0.1012	0.1523
HAR-RSV-SJV	0.1343	0.1628	0.1711	0.1081	0.1229	0.1250	0.1032	0.1540
HAR-RSV	0.1333	0.1618	0.1685	0.1078	0.1231	0.1190	0.1005	0.1506
LHAR-RV1	0.1344	0.1629	0.1689	0.1075	**0.1227***	0.1195	0.1007	0.1535
LHAR-RV2	0.1311	**0.1609***	0.1683	**0.1060***	0.1233	**0.1188***	**0.1001***	**0.1498***
LHAR-RV-CJ	**0.1307***	0.1747	0.1723	0.1063	0.1243	0.1222	0.1141	0.1609

Note: This table reports the symmetric forecasting evaluation (MSE) of eight RV indices for all forecasting models considered using recursive window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.7: Out-of-sample forecasting performance of MAE under recursive method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.4483	0.5004	0.4256	0.4078	0.3986	0.4102	0.3953	0.4949
HAR-RV-J	0.4489	0.4980	0.4250	0.4076	0.3984	0.4094	0.3966	0.4950
HAR-CJ	0.4294	0.4810	0.4264	0.4068	0.3960	0.4107	0.3956	0.4772
HAR-RV-CJ	0.4300	0.4819	0.4263	0.4071	0.3964	0.4103	0.3974	0.4785
HAR-PS	0.4327	0.4825	0.4207	0.3998	0.3930	0.4106	0.3950	0.4882
HAR-RSV-SJV	0.4331	0.4819	0.4253	0.4037	0.3918	0.4139	**0.3923***	0.4847
HAR-RSV	0.4292	0.4814	0.4201	0.3966	0.3939	0.4113	0.3944	0.4851
LHAR-RV1	0.4351	0.4904	0.4210	0.4001	**0.3911***	0.4071	0.3932	0.4927
LHAR-RV2	0.4302	0.4881	**0.4202***	**0.3964***	0.3926	0.4060	0.3927	0.4899
LHAR-RV-CJ	**0.4156***	**0.4748***	0.4221	0.3966	0.3901	**0.4053***	0.3948	**0.4731***
				H=5				
HAR-RV	0.2520	0.2963	0.2657	0.2305	0.2322	0.2557	0.2146	0.2653
HAR-RV-J	0.2526	0.2969	0.2656	0.2308	0.2319	0.2552	0.2149	0.2662
HAR-CJ	0.2565	0.3084	0.2750	0.2387	**0.2261***	0.2658	0.2184	0.2708
HAR-RV-CJ	0.2571	0.3091	0.2724	0.2389	0.2286	0.2645	0.2208	0.2682
HAR-PS	0.2473	0.2935	0.2651	0.2280	0.2325	0.2574	0.2148	0.2636
HAR-RSV-SJV	0.2482	0.2916	0.2661	0.2298	0.2317	0.2605	0.2147	0.2638
HAR-RSV	**0.2452***	**0.2903***	0.2650	**0.2246***	0.2319	0.2586	0.2147	**0.2612***
LHAR-RV1	0.2478	0.2935	**0.2648***	0.2281	0.2327	0.2561	0.2143	0.2641
LHAR-RV2	0.2433	0.2901	0.2650	0.2248	0.2349	**0.2546***	**0.2141***	0.2630
LHAR-RV-CJ	0.2530	0.3070	0.2714	0.2329	0.2302	0.2635	0.2196	0.2655
				H=22				
HAR-RV	0.2469	0.2778	0.2773	0.2232	0.2548	0.2466	0.2321	0.2806
HAR-RV-J	0.2475	0.2783	0.2767	0.2231	0.2547	0.2463	0.2320	0.2810
HAR-CJ	0.2447	0.2964	0.2843	0.2299	0.2550	0.2516	0.2409	0.2920
HAR-RV-CJ	0.2457	0.2982	0.2845	0.2291	0.2552	0.2479	0.2423	0.2926
HAR-PS	0.2450	0.2763	0.2772	0.2220	0.2548	0.2470	0.2324	0.2778
HAR-RSV-SJV	0.2435	0.2763	0.2805	0.2224	**0.2533***	0.2481	0.2322	0.2780
HAR-RSV	0.2444	**0.2753***	0.2772	0.2217	0.2553	0.2448	0.2315	**0.2762***
LHAR-RV1	0.2448	0.2767	0.2772	0.2218	0.2537	0.2466	0.2315	0.2800
LHAR-RV2	**0.2424***	0.2761	**0.2761***	**0.2213***	0.2558	**0.2424***	**0.2309***	0.2766
LHAR-RV-CJ	0.2432	0.2984	0.2842	0.2264	0.2560	0.2443	0.2414	0.2887

Note: This table reports the symmetric forecasting evaluation (MAE) of eight RV indices for all forecasting models considered using recursive window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.8: Out-of-sample forecasting performance of QLIKE under recursive method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.002040	0.002236	0.001842	0.001670	0.001791	0.001761	0.001688	0.002348
HAR-RV-J	0.002066	0.002248	0.001845	0.001667	0.001792	0.001752	0.001724	0.002348
HAR-CJ	0.001913	0.002067	0.001842	0.001647	0.001785	0.001756	0.001719	0.002216
HAR-RV-CJ	0.001921	0.002079	0.001844	0.001653	0.001791	0.001754	0.001758	0.002234
HAR-PS	0.001909	0.002083	0.001820	0.001616	0.001758	0.001764	0.001673	0.002287
HAR-RSV-SJV	0.001990	0.002101	0.001878	0.001643	0.001742	0.001747	0.001703	0.002275
HAR-RSV	0.001884	0.002073	0.001820	0.001600	0.001760	0.001765	0.001668	0.002253
LHAR-RV1	0.001922	0.002146	0.001823	0.001609	**0.001721***	0.001717	0.001657	0.002318
LHAR-RV2	0.001865	0.002102	**0.001814***	0.001573	0.001735	0.001713	**0.001655***	0.002269
LHAR-RV-CJ	**0.001804***	**0.002019***	0.001822	**0.001565***	0.001736	**0.001701***	0.001749	**0.002159***
				H=5				
HAR-RV	0.000787	0.000942	0.000928	0.000659	0.000651	0.000810	0.000609	0.000886
HAR-RV-J	0.000800	0.000953	0.000927	0.000658	0.000649	0.000807	0.000611	0.000891
HAR-CJ	0.001065	0.000980	0.000948	0.000696	**0.000639***	0.000881	0.000667	0.000894
HAR-RV-CJ	0.001077	0.001008	0.000952	0.000712	0.000646	0.000883	0.000638	0.000888
HAR-PS	0.000751	0.000913	0.000927	0.000642	0.000651	0.000814	0.000608	0.000874
HAR-RSV-SJV	0.000860	0.000930	0.000946	0.000655	0.000648	0.000823	0.000619	0.000878
HAR-RSV	0.000744	0.000897	0.000925	0.000630	0.000653	0.000818	**0.000602***	0.000857
LHAR-RV1	0.000757	0.000924	0.000925	0.000641	0.000653	0.000804	0.000608	0.000878
LHAR-RV2	**0.000732***	**0.000892***	**0.000921***	**0.000615***	0.000655	**0.000803***	0.000609	**0.000869***
LHAR-RV-CJ	0.001169	0.000989	0.000947	0.000660	0.000650	0.000861	0.000635	0.000866
				H=22				
HAR-RV	0.000846	0.000940	0.000986	0.000690	0.000805	0.000762	0.000648	0.000896
HAR-RV-J	0.000852	0.000945	**0.000980***	0.000692	0.000804	0.000760	0.000648	0.000898
HAR-CJ	0.000824	0.000996	0.001012	0.000689	0.000810	0.000791	0.001244	0.000949
HAR-RV-CJ	0.000837	0.001004	0.001017	0.000696	0.000812	0.000783	0.001013	0.000971
HAR-PS	0.000833	0.000928	0.000986	0.000685	0.000804	0.000764	0.000646	0.000876
HAR-RSV-SJV	0.000858	0.000930	0.001009	0.000684	0.000802	0.000910	0.000726	0.000894
HAR-RSV	0.000824	0.000924	0.000983	0.000682	0.000805	**0.000758***	0.000643	**0.000866***
LHAR-RV1	0.000833	0.000931	0.000990	0.000680	**0.000801***	0.000761	0.000643	0.000883
LHAR-RV2	**0.000811***	**0.000917***	0.000982	**0.000667***	0.000807	0.000759	**0.000638***	0.000856
LHAR-RV-CJ	0.000817	0.001001	0.001014	0.000670	0.000815	0.000780	0.001142	0.000931

Note: This table reports the symmetric forecasting evaluation (QLIKE) of eight RV indices for all forecasting models considered using recursive window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.9: Out-of-sample forecasting performance of MZ regression adjusted R^2 under recursive method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.6779	0.7441	0.6665	0.7221	0.7642	0.7176	0.6513	0.5259
HAR-RV-J	0.6752	0.7427	0.6659	0.7227	0.7642	0.7180	0.6442	0.5259
HAR-CJ	0.7019	0.7625	0.6669	0.7273	0.7649	0.7180	0.6530	0.5517
HAR-RV-CJ	0.7011	0.7616	0.6668	0.7266	0.7644	0.7180	0.6504	0.5493
HAR-PS	0.6992	0.7612	0.6704	0.7311	0.7685	0.7171	0.6541	0.5374
HAR-RSV-SJV	0.6969	0.7622	0.6608	0.7281	0.7698	0.7186	0.6576	0.5414
HAR-RSV	0.7036	0.7633	0.6701	0.7341	0.7682	0.7165	0.6552	0.5439
LHAR-RV1	0.6959	0.7536	0.6708	0.7314	**0.7715***	0.7235	0.6560	0.5316
LHAR-RV2	0.7071	0.7583	**0.6715***	0.7364	0.7703	0.7235	**0.6566***	0.5396
LHAR-RV-CJ	**0.7205***	**0.7688***	0.6712	**0.7398***	0.7703	**0.7242***	0.6539	**0.5611***
				H=5				
HAR-RV	0.8576	0.8771	0.7770	0.8753	0.8969	0.8615	0.8306	0.7427
HAR-RV-J	0.8561	0.8762	0.7772	0.8753	0.8971	0.8614	0.8302	0.7412
HAR-CJ	0.8560	0.8722	0.7720	0.8719	0.8981	0.8545	0.8252	0.7420
HAR-RV-CJ	0.8548	0.8703	0.7720	0.8703	0.8976	0.8544	0.8273	0.7437
HAR-PS	0.8641	0.8804	0.7772	0.8784	0.8969	0.8608	0.8311	0.7456
HAR-RSV-SJV	0.8614	0.8798	0.7749	0.8771	**0.8973***	0.8564	0.8303	0.7448
HAR-RSV	**0.8658***	0.8831	0.7772	0.8808	0.8962	0.8601	**0.8318***	**0.7496***
LHAR-RV1	0.8631	0.8797	0.7778	0.8786	0.8966	**0.8629***	0.8312	0.7446
LHAR-RV2	0.8687	**0.8833***	**0.7781***	**0.8823***	0.8969	0.8626	0.8308	0.7466
LHAR-RV-CJ	0.8598	0.8737	0.7731	0.8777	0.8972	0.8561	0.8274	0.7473
				H=22				
HAR-RV	0.8300	0.8626	0.7965	0.8511	0.8829	0.8621	0.8423	0.7790
HAR-RV-J	0.8283	0.8621	0.7974	0.8509	0.8831	0.8621	0.8422	0.7787
HAR-CJ	0.8359	0.8534	0.7928	0.8530	0.8820	0.8573	0.8248	0.7698
HAR-RV-CJ	0.8339	0.8535	0.7924	0.8516	0.8818	0.8582	0.8244	0.7675
HAR-PS	0.8327	0.8643	0.7968	0.8523	0.8832	0.8620	0.8426	0.7834
HAR-RSV-SJV	0.8332	0.8643	0.7939	0.8525	0.8829	0.8554	0.8394	0.7816
HAR-RSV	0.8342	0.8650	0.7969	0.8529	0.8830	0.8622	0.8438	0.7858
LHAR-RV1	0.8329	0.8642	0.7965	0.8533	**0.8833***	**0.8624***	0.8432	0.7820
LHAR-RV2	0.8370	**0.8658***	**0.7972***	0.8554	0.8828	0.8606	**0.8443***	**0.7871***
LHAR-RV-CJ	**0.8379***	0.8544	0.7929	**0.8561***	0.8816	0.8572	0.8238	0.7743

Note: This table reports the symmetric forecasting evaluation (MZ regression adjusted R^2) of eight RV indices for all forecasting models considered using recursive window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.10: Mean mixed forecasting error overprediction under rolling window method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.007364	0.006722	0.006623	0.007833	0.009029	0.007177	0.008229	0.006449
HAR-RV-J	0.007388	0.006740	0.006590	0.007827	0.009009	0.007151	0.008231	0.006445
HAR-CJ	0.007404	0.006635	0.006589	0.007849	0.009095	0.007213	0.008231	**0.006391***
HAR-RV-CJ	0.007389	0.006676	0.006605	0.007814	0.009120	0.007241	0.008305	0.006411
HAR-PS	0.007387	0.006674	**0.006566***	0.007743	**0.008950***	0.007189	0.008225	0.006452
HAR-RSV-SJV	0.007383	0.006645	0.006622	0.007806	0.009011	0.007185	0.008221	0.006451
HAR-RSV	0.007415	0.006676	0.006621	0.007741	0.009005	0.007202	0.008193	0.006469
LHAR-RV1	**0.007351***	0.006731	0.006640	0.007791	0.008977	0.007120	0.008201	0.006432
LHAR-RV2	0.007377	0.006681	0.006663	**0.007726***	0.008965	**0.007103***	**0.008140***	0.006441
LHAR-RV-CJ	0.007441	**0.006618***	0.006593	0.007923	0.009063	0.007161	0.008198	0.006441
				H=5				
HAR-RV	**0.007077***	0.006399	**0.006758***	0.007468	0.009278	0.007224	**0.008282***	0.006615
HAR-RV-J	0.007114	0.006360	0.006761	0.007527	0.009291	0.007232	0.008327	0.006618
HAR-CJ	0.007216	**0.006267***	0.006801	0.007744	0.009294	0.007232	0.008359	**0.006601***
HAR-RV-CJ	0.007255	0.006342	0.006806	0.007749	0.009416	0.007174	0.008393	0.006625
HAR-PS	0.007178	0.006376	0.006857	0.007437	0.009292	0.007202	0.008293	0.006622
HAR-RSV-SJV	0.007160	0.006319	0.006830	0.007597	0.009267	0.007201	0.008331	0.006621
HAR-RSV	0.007123	0.006356	0.006793	0.007434	0.009359	0.007188	0.008335	0.006612
LHAR-RV1	0.007178	0.006388	0.006829	0.007493	0.009325	0.007192	0.008296	0.006629
LHAR-RV2	0.007086	0.006391	0.006824	**0.007427***	**0.009132***	0.007174	0.008291	0.006633
LHAR-RV-CJ	0.007241	0.006364	0.006841	0.007669	0.009182	**0.007131***	0.008534	0.006652
				H=22				
HAR-RV	0.007467	0.006706	**0.006577***	0.007822	0.008886	0.007585	**0.007844***	0.006217
HAR-RV-J	0.007430	0.006678	0.006593	0.007832	0.008898	0.007569	0.007844	0.006198
HAR-CJ	0.007454	**0.006544***	0.006593	0.008073	0.008868	0.007565	0.008060	0.006200
HAR-RV-CJ	0.007541	0.006644	0.006620	0.008060	0.008922	0.007676	0.007998	0.006149
HAR-PS	0.007410	0.006651	0.006578	0.007849	0.008875	0.007604	0.007923	0.006181
HAR-RSV-SJV	0.007459	0.006665	0.006588	0.007875	0.008893	0.007605	0.007960	0.006149
HAR-RSV	0.007441	0.006589	0.006586	0.007874	0.008970	0.007603	0.007927	0.006190
LHAR-RV1	**0.007409***	0.006703	0.006594	0.007818	0.008817	**0.007553***	0.007917	0.006253
LHAR-RV2	0.007472	0.006622	0.006578	**0.007715***	**0.008809***	0.007572	0.007917	0.006199
LHAR-RV-CJ	0.007492	0.006630	0.006638	0.007905	0.008894	0.007656	0.008114	**0.006141***

Note: This table reports the asymmetric forecasting evaluation (MME(O)) of eight RV indices for all forecasting models considered using rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.11: Mean mixed forecasting error underprediction under rolling window method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.008402	0.007614	0.007470	0.008921	0.009852	0.007957	0.009208	0.007181
HAR-RV-J	0.008362	0.007574	0.007485	0.008933	0.009877	0.007960	**0.009204***	0.007201
HAR-CJ	0.008289	0.007555	0.007490	0.008878	0.009794	**0.007899***	0.009268	0.007159
HAR-RV-CJ	0.008311	**0.007540***	0.007471	0.008917	0.009771	0.007886	0.009268	0.007170
HAR-PS	0.008281	0.007571	0.007481	0.008931	0.009862	0.007935	0.009215	0.007152
HAR-RSV-SJV	0.008349	0.007629	0.007505	0.008930	0.009805	0.007945	0.009271	0.007143
HAR-RSV	0.008260	0.007557	**0.007410***	0.008905	0.009850	0.007933	0.009243	**0.007130***
LHAR-RV1	0.008344	0.007563	0.007439	0.008881	0.009801	0.007974	0.009216	0.007212
LHAR-RV2	0.008387	0.007648	0.007418	0.008941	0.009829	0.008006	0.009273	0.007195
LHAR-RV-CJ	**0.008208***	0.007615	0.007493	**0.008750***	**0.009706***	0.007933	0.009356	0.007114
				H=5				
HAR-RV	0.007643	**0.007028***	0.006902	0.008120	0.008816	0.007135	0.008034	0.006406
HAR-RV-J	0.007621	0.007092	0.006908	0.008080	0.008790	**0.007128***	0.008005	0.006428
HAR-CJ	0.007686	0.007207	0.006956	**0.007958***	0.008745	0.007228	0.008024	0.006468
HAR-RV-CJ	0.007662	0.007140	0.006898	0.007987	**0.008682***	0.007296	0.008048	0.006394
HAR-PS	**0.007509***	**0.007028***	**0.006795***	0.008145	0.008803	0.007176	0.008028	0.006386
HAR-RSV-SJV	0.007640	0.007070	0.006847	0.008035	0.008822	0.007173	0.008021	0.006396
HAR-RSV	0.007548	0.007032	0.006851	0.008101	0.008726	0.007204	0.007983	0.006393
LHAR-RV1	0.007551	0.007029	0.006821	0.008125	0.008783	0.007173	0.008012	0.006386
LHAR-RV2	0.007622	0.007033	0.006855	0.008143	0.008993	0.007205	0.008014	0.006395
LHAR-RV-CJ	0.007708	0.007174	0.006868	0.008056	0.008910	0.007333	**0.007888***	**0.006366***
				H=22				
HAR-RV	0.007465	**0.006938***	0.006821	0.007946	0.008991	0.007013	0.008463	0.006586
HAR-RV-J	0.007499	0.006980	**0.006797***	0.007934	0.008993	0.007023	0.008462	0.006624
HAR-CJ	0.007521	0.007204	0.006842	**0.007784***	0.009040	0.007053	0.008574	0.006801
HAR-RV-CJ	0.007473	0.007213	0.006815	0.007786	0.008995	**0.006922***	0.008607	0.006869
HAR-PS	0.007498	0.006970	0.006820	0.007895	0.008985	0.006996	0.008384	0.006608
HAR-RSV-SJV	0.007483	0.006982	0.006859	0.007899	0.008962	0.007081	0.008417	0.006716
HAR-RSV	0.007466	0.007019	0.006802	0.007867	**0.008925***	0.006988	**0.008367***	0.006594
LHAR-RV1	0.007532	0.006958	0.006814	0.007956	0.009024	0.007038	0.008379	**0.006577***
LHAR-RV2	**0.007446***	0.007052	0.006800	0.008070	0.009057	0.007023	0.008370	0.006581
LHAR-RV-CJ	0.007527	0.007270	0.006832	0.007957	0.009020	0.006951	0.008533	0.006849

Note: This table reports the asymmetric forecasting evaluation (MME(U)) of eight RV indices for all forecasting models considered using rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.12: Mean mixed forecasting error overprediction under recursive method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.007363	0.006701	0.006608	0.007828	0.009015	0.007214	0.008261	0.006451
HAR-RV-J	0.007368	0.006720	0.006617	0.007872	0.008972	0.007213	0.008272	0.006413
HAR-CJ	0.007358	**0.006612***	0.006688	0.007924	0.008956	0.007216	0.008232	**0.006330***
HAR-RV-CJ	0.007390	0.006644	0.006677	0.007890	0.009023	0.007278	0.008367	0.006336
HAR-PS	0.007389	0.006644	**0.006606***	**0.007791***	**0.008915***	0.007213	0.008268	0.006439
HAR-RSV-SJV	0.007359	0.006641	0.006701	0.007906	0.008958	0.007211	0.008247	0.006443
HAR-RSV	0.007379	0.006685	0.006665	0.007807	0.008938	0.007232	0.008239	0.006464
LHAR-RV1	**0.007287***	0.006702	0.006622	0.007822	0.008989	0.007162	0.008212	0.006429
LHAR-RV2	0.007373	0.006683	0.006681	0.007814	0.008992	**0.007158***	**0.008203***	0.006477
LHAR-RV-CJ	0.007369	0.006626	0.006660	0.007951	0.008949	0.007175	0.008272	0.006381
				H=5				
HAR-RV	0.007036	0.006325	0.006728	**0.007424***	0.009226	0.007325	**0.008406***	0.006446
HAR-RV-J	0.007050	0.006323	0.006727	0.007504	0.009122	0.007309	0.008414	0.006468
HAR-CJ	0.007113	**0.006173***	**0.006698***	0.007918	**0.008971***	0.007357	0.008478	**0.006154***
HAR-RV-CJ	0.007119	0.006199	0.006791	0.007882	0.009141	0.007329	0.008663	0.006183
HAR-PS	0.007052	0.006290	0.006748	0.007463	0.009206	0.007288	0.008429	0.006474
HAR-RSV-SJV	0.007108	0.006234	0.006778	0.007579	0.009171	0.007263	0.008469	0.006454
HAR-RSV	**0.006975***	0.006259	0.006754	0.007489	0.009197	0.007265	0.008521	0.006449
LHAR-RV1	0.007103	0.006354	0.006738	0.007472	0.009235	0.007250	0.008426	0.006442
LHAR-RV2	0.007049	0.006340	0.006746	0.007478	0.009100	0.007282	0.008426	0.006434
LHAR-RV-CJ	0.007165	0.006201	0.006767	0.007839	0.009030	**0.007249***	0.008687	0.006241
				H=22				
HAR-RV	0.007222	0.006523	0.006590	0.007751	0.008908	0.007662	0.007923	0.006179
HAR-RV-J	0.007210	0.006490	0.006586	0.007741	0.008874	0.007622	0.007926	0.006167
HAR-CJ	0.007242	**0.006332***	0.006629	0.008245	**0.008764***	0.007640	0.008065	0.006053
HAR-RV-CJ	0.007322	0.006498	0.006582	0.008193	0.008768	0.007729	0.008089	0.006014
HAR-PS	0.007263	0.006439	0.006564	0.007715	0.008855	0.007671	0.007941	0.006146
HAR-RSV-SJV	0.007238	0.006428	0.006553	0.007739	0.008788	0.007671	0.007997	0.006064
HAR-RSV	0.007281	0.006443	**0.006543***	0.007760	0.008895	0.007651	0.007959	0.006148
LHAR-RV1	**0.007208***	0.006500	0.006578	0.007720	0.008873	0.007669	**0.007887***	0.006212
LHAR-RV2	0.007282	0.006458	0.006577	**0.007695***	0.008849	**0.007590***	0.007966	0.006158
LHAR-RV-CJ	0.007276	0.006458	0.006639	0.008047	0.008893	0.007673	0.008203	**0.005969***

Note: This table reports the asymmetric forecasting evaluation (MME(O)) of eight RV indices for all forecasting models considered using recursive window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.13: Mean mixed forecasting error underprediction under recursive method

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.008398	0.007642	0.007496	0.008916	0.009860	0.007947	0.009192	0.007190
HAR-RV-J	0.008404	0.007602	0.007461	0.008877	0.009899	0.007926	0.009192	0.007237
HAR-CJ	0.008328	0.007565	0.007413	0.008840	0.009892	0.007936	0.009282	0.007171
HAR-RV-CJ	0.008302	0.007556	0.007422	0.008876	0.009841	0.007877	0.009228	0.007188
HAR-PS	0.008273	0.007596	0.007442	0.008888	0.009904	0.007945	**0.009184***	0.007171
HAR-RSV-SJV	0.008376	0.007653	0.007411	0.008820	0.009834	**0.007901***	0.009247	0.007150
HAR-RSV	0.008270	**0.007547***	**0.007374***	0.008842	0.009890	0.007934	0.009206	0.007136
LHAR-RV1	0.008393	0.007593	0.007455	0.008830	0.009783	0.007970	0.009219	0.007231
LHAR-RV2	0.008359	0.007615	0.007386	**0.008806***	**0.009800***	0.007971	0.009219	0.007177
LHAR-RV-CJ	**0.008250***	0.007555	0.007433	0.008729	0.009817	0.007935	0.009287	**0.007122***
				H=5				
HAR-RV	0.007666	0.007071	0.006891	0.008156	**0.008769***	**0.007085***	0.007971	0.006549
HAR-RV-J	0.007669	0.007100	0.006885	0.008085	0.008854	0.007097	0.007976	0.006541
HAR-CJ	0.007826	0.007291	0.007012	**0.007791***	0.008917	0.007178	0.007981	0.006848
HAR-RV-CJ	0.007836	0.007282	0.006891	0.007846	0.008841	0.007208	0.007870	0.006776
HAR-PS	0.007613	0.007083	0.006853	0.008097	0.008794	0.007139	0.007952	0.006502
HAR-RSV-SJV	0.007654	0.007134	0.006853	0.008028	0.008820	0.007137	0.007938	**0.006436***
HAR-RSV	0.007664	0.007095	**0.006845***	0.008034	0.008773	0.007182	0.007862	0.006590
LHAR-RV1	**0.007578***	0.007032	0.006861	0.008118	0.008776	0.007172	0.007945	0.006541
LHAR-RV2	0.007609	**0.007019***	0.006855	0.008059	0.008968	0.007137	0.007940	0.006531
LHAR-RV-CJ	0.007820	0.007291	0.006900	0.007830	0.008970	0.007277	**0.007818***	0.006678
				H=22				
HAR-RV	0.007665	**0.007085***	0.006795	0.007999	0.008958	0.007031	0.008389	0.006609
HAR-RV-J	0.007688	0.007128	**0.006789***	0.007999	0.008997	0.007064	0.008381	0.006641
HAR-CJ	0.007690	0.007390	0.006810	**0.007650***	0.009108	0.007087	0.008577	0.006847
HAR-RV-CJ	0.007622	0.007352	0.006860	0.007669	0.009119	**0.006966***	0.008538	0.006905
HAR-PS	0.007613	0.007152	0.006824	0.008016	0.008999	0.007027	0.008371	0.006623
HAR-RSV-SJV	0.007666	0.007181	0.006879	0.007998	0.009033	0.007110	0.008380	0.006730
HAR-RSV	0.007587	0.007138	0.006843	0.007966	0.008972	0.007011	0.008336	0.006614
LHAR-RV1	0.007693	0.007113	0.006822	0.008022	**0.008948***	0.007023	0.008411	**0.006594***
LHAR-RV2	**0.007598***	0.007164	0.006790	0.008048	0.009027	0.007050	**0.008324***	0.006598
LHAR-RV-CJ	0.007662	0.007416	0.006828	0.007781	0.008984	0.006971	0.008464	0.006913

Note: This table reports the asymmetric forecasting evaluation (MME(U)) of eight RV indices for all forecasting models considered using recursive window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The forecasting model with the best performance is highlighted with *.

Table 3.14: Diebold-Mariano's equal predictive accuracy test of daily forecasting horizon

FTSE	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-4.799***	**-2.027***	**-2.631***	-0.681	**-2.383***	0.012	-0.195	0.434	-1.751
HAR-RV-J	1.447		-0.154	-0.780	1.617	-0.486	**2.016***	**2.001***	**2.030***	-0.433
HAR-CJ	**-2.265***	1.739		**-3.505***	1.506	-0.503	**2.000***	1.612	1.948	-0.495
HAR-RV-CJ	**-2.390***	1.864	0.968		**2.119***	0.464	**2.586***	**2.138***	**2.423***	0.205
HAR-PS	-0.208	-0.752	**-2.562***	**-2.684***		**-2.278***	1.266	0.619	1.042	-1.664
HAR-RSV-SJV	1.243	0.728	**-2.250***	**-2.429***	1.684		**2.669***	**2.335***	**2.433***	-0.129
HAR-RSV	-0.851	-1.303	**-3.081***	**-3.185***	-1.368	**-2.190***		-0.217	0.509	**-2.294***
LHAR-RV1	-0.259	-0.860	**-2.251***	**-2.367***	-0.016	-1.447	0.798		0.851	**-2.013***
LHAR-RV2	-1.228	-1.587	**-2.918***	**-3.000***	-1.135	**-2.137***	-0.507	-1.447		**-3.415***
LHAR-RV-CJ	1.353	1.005	-0.579	-0.796	1.742	0.763	**2.405***	1.685	**3.180***	

SSEC	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-1.715	**2.272***	**2.205***	0.498	1.598	**5.260***	0.343	**-3.192***	-0.840
HAR-RV-J	**-5.424***		**2.991***	**2.839***	1.020	**2.139***	**5.571***	0.930	**-2.455***	-0.355
HAR-CJ	**-11.91***	**10.93***		0.503	-0.948	-0.030	**3.504***	-1.155	**-4.022***	**-2.620***
HAR-RV-CJ	**-11.71***	**10.80***	1.410		-1.032	-0.194	**3.247***	-1.223	**-3.962***	**-2.936***
HAR-PS	-0.362	-1.295	**-8.533***	**-8.563***		0.984	**12.58***	-0.283	**-4.602***	-1.544
HAR-RSV-SJV	**-3.390***	**2.539***	**-7.519***	**-7.471***	**3.421***		**3.698***	-1.511	**-5.313***	**-2.980***
HAR-RSV	-0.694	-1.588	**-8.632***	**-8.690***	-1.189	**-3.626***		**-6.618***	**-10.46***	**-6.636***
LHAR-RV1	0.925	-0.110	**-7.493***	**-7.547***	1.631	**-2.931***	1.975		**-9.650***	-1.615
LHAR-RV2	1.195	0.243	**-6.916***	**-6.995***	1.950	**-2.437***	2.366*	0.886		**2.710***
LHAR-RV-CJ	**8.216***	**7.556***	0.755	0.479	**10.51***	**8.711***	**10.72***	**10.89***	**11.42***	

SPX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-5.258***	-0.951	0.033	-0.794	**-2.480***	1.391	0.031	1.427	-0.073
HAR-RV-J	**7.060***		1.903	**2.881***	**2.396***	0.510	**4.100***	**3.281***	**3.842***	**2.165***
HAR-CJ	**8.418***	**5.425***		**4.505***	0.332	**-2.129***	**2.230***	0.854	1.822	1.130
HAR-RV-CJ	**6.110***	**3.148***	**-16.81***		-0.690	**-3.526***	1.215	-0.011	1.022	-0.168
HAR-PS	1.071	**-2.256***	**-7.299***	**-5.088***		**-2.161***	**4.686***	0.984	**2.134***	0.530
HAR-RSV-SJV	**4.760***	1.154	**-6.035***	**-2.883***	**4.041***		**4.211***	**2.591***	**3.436***	**2.627***
HAR-RSV	-1.518	**-4.354***	**-9.405***	**-7.228***	**-5.085***	**-6.310***		-1.585	0.053	-1.378
LHAR-RV1	-0.741	**-4.766***	**-8.044***	**-5.962***	**-2.046***	**-5.752***	1.256		**2.163***	-0.107
LHAR-RV2	**-2.638***	**-5.630***	**-9.166***	**-7.188***	**-3.541***	**-6.820***	-0.728	**-3.234***		-1.570
LHAR-RV-CJ	**4.720***	**2.233***	**-3.903***	-0.605	**4.649***	**2.005***	**7.372***	**5.962***	**8.795***	

NSEI	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-3.205***	0.310	0.232	**3.623***	**3.383***	**2.165***	1.090	1.717	0.538
HAR-RV-J	**7.171***		1.135	1.030	**4.681***	**4.273***	**3.165***	**2.019***	**2.533***	1.184
HAR-CJ	**-3.834***	**-4.970***		-0.480	1.639	**2.979***	1.104	0.436	0.900	0.463
HAR-RV-CJ	-1.006	**-2.114***	**21.75***		1.655	**2.911***	1.138	0.483	0.935	0.556
HAR-PS	**-5.446***	**-7.498***	1.438	-1.418		0.650	-1.032	-1.774	-0.710	-1.173
HAR-RSV-SJV	**-3.368***	**-4.996***	**2.706***	-2.541	-0.019		-1.077	-1.682	-0.987	-1.729
HAR-RSV	-1.255	**-2.968***	**2.931***	0.232	**3.585***	**2.070***		-0.860	-0.072	-0.757
LHAR-RV1	**-4.555***	**-5.898***	0.429	-1.874	-1.426	-0.915	**-3.322***		1.794	-0.160
LHAR-RV2	**4.151***	**2.647***	**5.788***	**3.469***	**8.277***	**5.810***	**5.704***	**20.71***		-0.804
LHAR-RV-CJ	0.939	0.031	**6.691***	**2.991***	**3.323***	**4.133***	1.799	4.489*	-2.239*	

N225	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-3.151***	0.925	0.492	-0.325	0.055	-1.632	0.499	1.528	1.458
HAR-RV-J	**4.420***		**2.841***	**2.478***	1.698	1.221	0.320	**2.122***	**2.805***	**2.731***
HAR-CJ	**-3.863***	**-5.880***		-1.125	-0.986	-0.576	-1.930	-0.332	0.569	1.027
HAR-RV-CJ	**-5.129***	**-7.133***	**-3.073***		-0.629	-0.271	-1.616	-0.005	0.864	1.466
HAR-PS	-0.544	**-2.922***	**2.624***	**3.500***		0.249	**-2.579***	1.016	**2.053***	1.904
HAR-RSV-SJV	-0.935	**-2.545***	1.722	2.461*	-0.647		-1.116	0.288	1.072	1.416
HAR-RSV	-0.530	**-2.651***	**2.376***	**3.171***	-0.121	0.564		**2.338***	**3.648***	**3.082***
LHAR-RV1	-0.454	**-2.634***	**2.516***	**3.325***	0.097	0.774	0.149		1.925	1.467
LHAR-RV2	-1.629	**-3.442***	1.414	**2.128***	-1.411	-0.268	-1.441	-2.321		0.386
LHAR-RV-CJ	**-4.777***	**-6.184***	**-2.668***	-1.862	**-5.193***	**-4.028***	**-5.363***	**-5.979***	**-5.809***	

BVSP	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		0.692	**4.759***	**4.364***	**2.906***	**2.864***	**3.082***	0.572	1.536	**4.622***
HAR-RV-J	1.893		**3.455***	**3.274***	1.673	1.632	1.988	-0.071	0.703	**3.662***
HAR-CJ	**-3.858***	**-3.883***		-0.446	**-2.838***	**-4.201***	**-2.393***	**-3.848***	**-3.128***	1.168
HAR-RV-CJ	**-7.208***	**-6.709***	**-7.373***		**-2.498***	**-3.033***	**-2.073***	**-3.489***	**-2.795***	1.782
HAR-PS	**-4.279***	**-4.560***	1.437	**4.784***		0.554	1.165	**-2.774***	-1.170	**3.626***
HAR-RSV-SJV	**-3.101***	**-3.099***	**2.527***	**7.076***	0.036		-0.053	**-2.373***	-1.357	**4.117***
HAR-RSV	**-4.856***	**-5.117***	0.715	**4.029***	**-2.372***	-0.909		**-2.928***	-1.797	**3.305***
LHAR-RV1	-1.581	**-2.361***	**2.555***	**5.481***	**2.884***	1.781	**3.645***		**2.208***	**5.027***
LHAR-RV2	**-2.244***	**-2.826***	**2.057***	**4.941***	1.638	1.071	**2.859***	**-2.034***		**4.557***
LHAR-RV-CJ	**-7.191***	**-6.794***	**-6.082***	**-2.068***	**-6.032***	**-7.810***	**-5.441***	**-7.458***	**-7.182***	

DAX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		0.881	**10.30***	**10.04***	1.478	**3.660***	**2.676***	0.214	-0.270	**4.322***
HAR-RV-J	**-3.574***		**9.486***	**9.084***	0.881	**2.614***	**2.082***	-0.233	-0.591	**4.000***
HAR-CJ	**-26.72***	**-24.82***		0.275	**-5.738***	**-7.214***	**-3.929***	**6.326***	**-5.666***	-1.505
HAR-RV-CJ	**-25.43***	**-23.47***	**3.935***		**-5.762***	**-6.994***	**-4.034***	**6.332***	**-5.671***	-1.645
HAR-PS	**-5.035***	**-2.599***	**16.87***	**16.09***		1.603	**2.821***	1.636	-1.599	**3.900***
HAR-RSV-SJV	**-12.15***	**-8.469***	**23.62***	**21.76***	**-5.029***		0.064	**2.990***	**-2.761***	**2.991***
HAR-RSV	**-6.345***	**-4.199***	**13.66***	**13.04***	**-4.110***	**2.412***		**3.051***	**-3.321***	**2.825***
LHAR-RV1	**-0.882***	1.083	**18.74***	**17.98***	**5.513***	**8.903***	**6.958***		-0.709	**5.804***
LHAR-RV2	**-1.862***	-0.245	**15.29***	**14.70***	**2.234***	**5.971***	**4.906***	-1.870		**8.978***
LHAR-RV-CJ	**-17.89***	**-16.61***	-0.149	-0.707	**-16.93***	**-14.85***	**-16.05***	**-22.32***	**-27.24***	

MXX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-2.405***	**-5.097***	**-3.904***	0.134	**-3.013***	0.888	0.521	**2.610***	**2.356***
HAR-RV-J	1.284		**-2.985***	-1.974	**2.005***	-0.762	**2.375***	**2.283***	**3.476***	-0.839
HAR-CJ	**17.52***	**16.88***		**4.598***	**5.249***	**3.957***	**5.570***	**4.938***	**5.841***	**3.494***
HAR-RV-CJ	**17.04***	**16.45***	**-2.267***		**4.043***	**2.185***	**4.383***	**3.825***	**4.805***	1.859
HAR-PS	**-2.807***	**-3.094***	**-19.65***	**-19.15***		**-3.536***	1.284	0.383	**2.591***	**2.750***
HAR-RSV-SJV	**6.122***	**5.447***	**-18.41***	**-17.66***	**9.070***		**3.752***	**3.280***	**4.475***	-0.387
HAR-RSV	**-2.177***	**-2.622***	**-19.14***	**-18.55***	-0.064	**-7.811***		-0.628	1.836	**-3.431***
LHAR-RV1	**-2.475***	**-2.652***	**-17.51***	**-16.98***	0.983	**-7.548***	0.707		**3.061***	**-2.786***
LHAR-RV2	**-6.617***	**-6.027***	**-19.22***	**-18.66***	**-4.426***	**-10.23***	**-4.330***	**-6.783***		**-4.743***
LHAR-RV-CJ	**-11.75***	**-11.33***	**-6.947***	**-6.269***	**14.73***	**10.93***	**15.51***	**13.52***	**18.02***	

Notes: This table presents the values of Diebold and Mariano's test of no difference in predicative accuracy for daily forecasting horizon under both rolling window and recursive forecasting approaches. The positive value indicates that the model in the row outperforms the model in the column. In each panel, in cell above the main diagonal this table reports the rolling window results; below the main diagonal, recursive forecasting results are presented. The value highlighted with * indicates rejection of the null hypothesis below the 5% significant level.

Table 3.15: Diebold-Mariano's equal predictive accuracy test of weekly forecasting horizon

FTSE	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-3.829*	-0.684	-0.228	-0.655	-0.777	0.712	0.115	0.393	-0.309
HAR-RV-J	1.202		0.150	0.597	1.231	0.403	2.195*	1.927	1.596	0.397
HAR-CJ	1.854	1.583		3.602*	0.448	0.213	1.215	0.710	0.892	0.565
HAR-RV-CJ	1.599	1.339	-2.799*		-0.060	-0.504	0.701	0.268	0.462	-0.248
HAR-PS	-0.137	-0.626	-2.136*	-1.850		-0.483	2.165*	0.895	0.887	-0.082
HAR-RSV-SJV	0.724	0.380	-2.262*	-1.851	1.002		1.516	0.888	1.020	0.220
HAR-RSV	-0.825	-1.188	-2.635*	-2.335*	-1.222	-1.524		-0.683	-0.219	-0.789
LHAR-RV1	-0.449	-0.968	-1.988	-1.736	-0.295	-0.979	0.576		0.427	-0.382
LHAR-RV2	-1.326	-1.605	-2.557*	-2.312*	-1.276	-1.684	-0.643	-1.325		-0.694
LHAR-RV-CJ	1.038	0.836	-1.345	-0.843	1.230	0.917	1.715	1.234	2.089*	

SSEC	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-3.312*	1.712	4.188*	0.807	1.171	12.61*	-2.791*	-8.935*	-2.034*
HAR-RV-J	-5.270*		3.630*	5.601*	3.235*	3.667*	12.74*	2.340*	-6.947*	-0.898
HAR-CJ	-17.41*	16.25*		4.581*	-1.343	-1.253	8.135*	-2.157*	-8.512*	-3.777*
HAR-RV-CJ	-10.80*	9.348*	-9.372*		-3.846*	-3.981*	5.605*	-4.540*	-10.17*	-7.623*
HAR-PS	-4.82*	-7.072*	-11.64*	0.000		0.519	12.86*	-2.085*	-9.040*	-2.213*
HAR-RSV-SJV	-5.481*	1.214	-9.583*	7.888*	0.000		11.47*	-1.951	-9.100*	-2.492*
HAR-RSV	0.479	-1.507	-8.090*	1.562	-1.987	0.000		-13.0*	-19.10*	-10.09*
LHAR-RV1	1.932	-4.267*	-10.46*	5.052*	-4.533*	-0.217	0.000		-8.672*	-1.769
LHAR-RV2	0.057	-1.445	-7.332*	0.793	-1.801	-0.295	-0.131	0.000		6.898*
LHAR-RV-CJ	-6.250*	5.191*	-1.525	6.855*	5.069*	6.080*	6.133*	9.483*	0.000	

SPX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-5.145*	2.425*	3.757*	-0.787	-0.153	2.222*	0.239	2.012*	3.429*
HAR-RV-J	-5.976*		3.526*	4.809*	1.811	1.610	4.088*	3.077*	3.750*	4.368*
HAR-CJ	-5.057*	3.889*		5.129*	-3.169*	-4.041*	-1.245	-2.280*	-1.213	2.730*
HAR-RV-CJ	-2.618*	1.525	-14.55*		-4.674*	-6.053*	-2.794*	-3.580*	-2.520*	0.104
HAR-PS	-2.022*	-0.479	-4.664*	-1.970		0.503	4.180*	1.066	2.486*	4.308*
HAR-RSV-SJV	-2.320*	0.385	-5.527*	-1.962	0.990		2.368*	0.287	1.708	5.107*
HAR-RSV	-0.552	-2.327*	-6.119*	-3.339*	-2.936*	-2.832*		-2.184*	-0.320	2.803*
LHAR-RV1	-1.039	-4.299*	-5.282*	-2.865*	-3.156*	-2.953*	0.078		2.385*	3.543*
LHAR-RV2	-3.321*	-5.281*	-6.506*	-4.058*	-4.661*	-4.648*	-2.139*	-3.397*		2.922*
LHAR-RV-CJ	2.375*	1.358	-6.354*	-0.189	1.801	1.683	3.298*	2.741*	4.296*	

NSEI	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-0.120	1.247	1.173	2.545*	3.179*	0.244	1.125	1.089	0.874
HAR-RV-J	9.682*		1.274	1.202	2.509*	3.260*	0.264	1.100	1.061	0.893
HAR-CJ	-5.225*	-6.048*		-0.191	-0.397	1.181	-1.132	-0.794	-0.731	-0.645
HAR-RV-CJ	-2.355*	-3.159*	18.28*		-0.346	1.163	-1.058	-0.738	-0.676	-0.605
HAR-PS	-4.597*	-6.773*	3.872*	0.891		1.968	-2.111*	-1.245	-0.931	0.088
HAR-RSV-SJV	-3.190*	-4.445*	4.975*	0.495	-0.929		-2.792*	-2.371*	-2.188*	-1.369
HAR-RSV	-2.957*	-4.482*	3.750*	0.907	0.132	0.856		0.585	0.734	0.831
LHAR-RV1	-4.536*	-6.584*	3.538*	0.814	-0.018	0.779	-0.114		0.255	0.530
LHAR-RV2	8.441*	6.183*	8.301*	5.482*	12.28*	7.812*	9.991*	23.69*		0.490
LHAR-RV-CJ	-0.274	-1.023	12.59*	5.876*	1.261	2.408*	1.199	1.238	-3.760*	

N225	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-2.401*	0.565	-0.514	0.059	0.616	-4.139*	0.333	2.362*	0.799
HAR-RV-J	5.028*		1.394	0.256	1.734	1.602	-2.968*	1.646	3.054*	1.458
HAR-CJ	-2.782*	-3.974*		-1.148	-0.527	-0.060	-3.172*	-0.391	0.943	0.337
HAR-RV-CJ	-7.334*	-8.665*	-4.981*		0.511	0.928	-2.123*	0.603	1.819	2.037*
HAR-PS	-0.779	-3.704*	2.431*	6.679*		0.630	-4.655*	0.381	2.543*	0.812
HAR-RSV-SJV	-2.176*	-3.637*	1.076	4.769*	-1.939		-3.553*	-0.558	1.370	0.405
HAR-RSV	0.865	-1.292	3.013*	6.705*	1.691	2.504*		4.315*	5.989*	3.238*
LHAR-RV1	-0.512	-2.832*	2.360*	6.329*	0.119	2.426*	-1.287		2.672*	0.729
LHAR-RV2	-1.931	-3.574*	1.383	4.884*	-1.750	0.565	-3.136*	-2.163*		-0.769
LHAR-RV-CJ	-7.554*	-8.570*	-5.343*	-2.500*	-7.744*	-6.328*	-8.399*	-8.021*	-7.944*	

BVSP	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		3.277*	4.584*	3.995*	3.169*	3.519*	4.715*	0.440	3.482*	4.451*
HAR-RV-J	-0.298		4.155*	3.494*	-0.095	2.394*	2.526*	-1.968	1.561	3.978*
HAR-CJ	-3.013*	-3.118*		-0.807	-4.073*	-3.974*	-2.866*	-4.454*	-3.292*	0.022
HAR-RV-CJ	-7.250*	-7.411*	-6.175*		-3.477*	-2.981*	-2.273*	-3.847*	-2.698*	1.941
HAR-PS	-5.156*	-3.656*	2.337*	6.644*		2.107*	3.654*	-2.891*	1.912	4.030*
HAR-RSV-SJV	-3.834*	-4.667*	1.947	6.708*	-1.868		0.344	-3.281*	-0.471	3.582*
HAR-RSV	-4.294*	-3.852*	1.378	5.238*	-2.509*	-0.186		-4.528*	-0.999	2.855*
LHAR-RV1	-1.498	-1.080	2.762*	6.938*	3.172*	3.153*	3.545*		3.965*	4.451*
LHAR-RV2	-0.090	0.065	2.932*	7.017*	3.532*	3.511*	4.345*	1.703		3.445*
LHAR-RV-CJ	-6.099*	-6.237*	-4.336*	5.050*	-5.531*	-5.417*	-4.323*	-5.950*	-6.154*	

DAX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		3.705*	11.50*	10.53*	2.033*	5.248*	2.797*	0.332	-0.781	5.781*
HAR-RV-J	-5.040*		9.933*	9.051*	0.210	3.096*	1.410	-1.437	-1.855	4.787*
HAR-CJ	-27.50*	-25.27*		-0.319	-8.494*	-9.229*	-6.499*	-9.266*	-8.296*	-2.429*
HAR-RV-CJ	-24.97*	-22.61*	7.026*		-7.956*	-8.159*	-6.215*	-8.695*	-7.907*	-2.571*
HAR-PS	-6.291*	-3.221*	21.89*	19.69*		2.926*	1.957	-2.227*	-2.276*	5.110*
HAR-RSV-SJV	-12.92*	-9.495*	25.36*	22.15*	-6.586*		-1.014	-4.885*	-4.281*	3.945*
HAR-RSV	-7.782*	-5.543*	17.76*	15.80*	-4.824*	2.333*		-2.956*	-3.798*	4.428*
LHAR-RV1	-0.930	1.756	23.59*	21.48*	6.881*	11.41*	8.159*		-1.289	6.359*
LHAR-RV2	-1.991	-0.220	19.31*	17.65*	2.121*	6.717*	5.789*	-1.812		9.607*
LHAR-RV-CJ	-20.04*	-18.30*	1.531*	-0.566	-18.45*	-17.51*	-17.39*	-21.50*	-25.99*	

MXX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-1.675	-4.687*	-3.923*	-0.270	-2.221*	0.422	0.356	4.098*	-2.380*
HAR-RV-J	0.956		-3.912*	-3.168*	1.061	-0.655	1.323	1.552	4.243*	-1.777
HAR-CJ	-21.79*	21.23*		3.142*	4.669*	4.450*	4.850*	4.656*	5.916*	4.139*
HAR-RV-CJ	-21.35*	20.75*	-1.615		3.881*	3.519*	4.088*	3.899*	5.263*	2.996*
HAR-PS	-2.595*	-2.541*	-23.02*	-22.50*		-2.154*	0.787	0.728	4.378*	-2.425*
HAR-RSV-SJV	-6.514*	5.899*	-22.28*	-21.70*	9.039*		2.118*	2.311*	4.863*	-1.780
HAR-RSV	1.239	0.569	-22.73*	-22.18*	3.337*	-3.587*		-0.257	3.002*	-2.679*
LHAR-RV1	-2.519*	-2.339*	-21.98*	-21.49*	0.519	-7.558*	-2.569*		4.718*	-2.535*
LHAR-RV2	-7.351*	-6.018*	-23.23*	-22.72*	-5.284*	-10.69*	-5.922*	-8.052*		-4.447*
LHAR-RV-CJ	-18.51*	-18.02*	-7.464*	-7.727*	19.99*	18.58*	19.59*	19.32*	21.19*	

Notes: This table presents the values of Diebold and Mariano's test of no difference in predicative accuracy for daily forecasting horizon under both rolling window and recursive forecasting approaches. The positive value indicates that the model in the row outperforms the model in the column. In each panel, in cell above the main diagonal this table reports the rolling window results; below the main diagonal, recursive forecasting results are presented. The value highlighted with * indicates rejection of the null hypothesis below the 5% significant level.

Table 3.16: Diebold-Mariano's equal predictive accuracy test of monthly forecasting horizon

FTSE	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-3.643***	-0.613	0.568	-0.754	**-2.092***	-0.352	0.066	0.861	0.293
HAR-RV-J	1.224		0.377	1.534	**2.079***	-0.107	1.723	**2.226***	**2.286***	1.153
HAR-CJ	**-2.883***	**2.504***		**5.474***	0.414	-0.561	0.504	0.616	1.054	1.722
HAR-RV-CJ	**-2.327***	1.974	**-4.387***		-0.831	**-2.054***	-0.802	-0.524	-0.046	-0.456
HAR-PS	-0.172	-0.931	**-3.082***	**-2.496***		**-2.116***	0.201	0.861	1.375	0.529
HAR-RSV-SJV	1.079	0.419	**-2.954***	**-2.261***	1.441		1.869	**2.197***	**2.492***	1.513
HAR-RSV	1.147	0.299	**-2.812***	**-2.184***	**2.214***	-0.279		0.414	1.157	0.499
LHAR-RV1	-0.484	-1.088	**-2.978***	**-2.432***	-0.423	-1.415	-1.608		1.092	0.291
LHAR-RV2	-1.498	-1.795	**-3.529***	**-3.006***	-1.539	**-2.223***	**-2.362***	-1.536		-0.195
LHAR-RV-CJ	1.801	1.515	-1.861	-0.647	1.975	1.645	1.672	**2.026***	**2.904***	

SSEC	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-2.651***	**2.320***	**3.476***	1.249	1.940	**10.58***	-0.248	**-8.450***	-0.695
HAR-RV-J	**4.988***		**3.161***	**4.180***	**2.548***	**3.084***	**10.75***	1.884	**-5.685***	0.169
HAR-CJ	**-12.23***	**11.27***		**3.818***	-1.482	-1.119	**5.172***	**-2.309***	**-6.543***	**-4.882***
HAR-RV-CJ	**-11.36***	**10.36***	**-8.424***		**-2.627***	**-2.867***	**3.884***	**-3.433***	**-7.422***	**-8.300***
HAR-PS	**-2.876***	**-4.533***	**-13.17***	**-12.32***		0.959	**14.39***	-1.687	**-8.663***	-1.375
HAR-RSV-SJV	**5.931***	**4.798***	**-13.01***	**-11.73***	**7.515***		**6.767***	**-2.143***	**-7.465***	**-2.879***
HAR-RSV	**-3.476***	**-4.941***	**-13.29***	**-12.47***	**-2.172***	**-7.699***		**-11.75***	**-17.48***	**-7.654***
LHAR-RV1	0.777	**-2.178***	**-11.54***	**-10.70***	**3.969***	**-5.768***	**4.419***		**-10.15***	-0.656
LHAR-RV2	0.604	-1.228	**-10.63***	**-9.711***	**3.187***	**-4.902***	**4.079***	0.200		**3.989***
LHAR-RV-CJ	**9.722***	**8.852***	**-2.689***	-0.743	**11.78***	**9.313***	**12.28***	**9.961***	**10.81***	

SPX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-4.337***	**2.539***	**6.380***	-0.845	-1.319	**-6.576***	0.156	**2.205***	**6.061***
HAR-RV-J	**-3.944***		**3.347***	**7.186***	1.674	0.561	**-4.316***	**2.488***	**3.548***	**6.755***
HAR-CJ	**-8.112***	**7.396***		**7.647***	**-2.938***	**-3.771***	**-5.567***	**-2.417***	-1.332	**6.370***
HAR-RV-CJ	1.810	1.183	**-18.69***		**-6.979***	**-8.355***	**-9.458***	**-6.159***	**-5.222***	1.016
HAR-PS	1.323	-0.894	**-8.039***	-1.518		-0.884	**-8.691***	1.076	**2.777***	**6.761***
HAR-RSV-SJV	**-2.055***	0.196	**-8.634***	-1.309	1.194		**-4.979***	1.388	**3.022***	**7.794***
HAR-RSV	**-4.928***	**2.741***	**-7.095***	-0.275	**6.766***	**2.645***		**6.875***	**6.799***	**9.061***
LHAR-RV1	-0.910	-3.057*	**-8.203***	-1.997	**-2.426***	**-2.702***	**-5.995***		**2.637***	**6.281***
LHAR-RV2	**-3.549***	**-4.759***	**-9.643***	**-3.315***	**-4.455***	**-5.015***	**-7.168***	**-3.757***		**6.015***
LHAR-RV-CJ	1.604	1.015	**-13.91***	-0.371	1.350	1.119	0.158	1.855	**3.308***	

NSEI	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		-0.976	**3.543***	**3.861***	**2.151***	1.899	-0.275	1.054	-0.477	**-2.782***
HAR-RV-J	**5.204***		**3.667***	**4.004***	**2.360***	1.813	0.152	1.417	-0.186	**-2.921***
HAR-CJ	**-5.617***	**-6.233***		0.743	**-3.074***	**-2.429***	**-3.622***	**-3.225***	**-3.262***	-0.795
HAR-RV-CJ	**-2.056***	**-2.677***	**9.949***		**-3.393***	**-2.732***	**-3.889***	**-3.532***	**-3.559***	-1.750
HAR-PS	**-4.746***	**-6.790***	**4.770***	1.254		1.086	**-2.242***	-1.172	-1.460	**-2.422***
HAR-RSV-SJV	-1.207	**-2.141***	**4.950***	1.462	0.517		-1.844	-1.432	-1.762	1.774
HAR-RSV	**6.869***	**4.444***	**7.979***	**4.209***	**12.17***	**5.115***		1.080	-0.313	**-2.992***
LHAR-RV1	**-4.240***	**-6.433***	**4.861***	1.381	1.067	-0.192	**-9.716***		-1.087	**-2.623***
LHAR-RV2	**22.83***	**21.21***	**15.12***	**11.71***	**25.29***	**17.93***	**20.85***	**25.85***		**-3.222***
LHAR-RV-CJ	**6.015***	**5.387***	**20.88***	**19.11***	**6.935***	**6.557***	**4.414***	6.760*	**-4.811***	

N225	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-3.897***	1.936	**2.543***	-0.626	0.404	-1.487	0.313	1.681	**2.996***
HAR-RV-J	**4.645***		**3.147***	**3.807***	1.953	1.899	0.338	**2.740***	**3.182***	**4.028***
HAR-CJ	**-5.189***	**-6.194***		1.015	**-2.153***	-1.830	**-2.599***	-1.702	-0.660	1.898
HAR-RV-CJ	**-8.366***	**-9.192***	**-5.460***		**-2.733***	**-2.548***	**-3.103***	-2.245	-1.077	1.554
HAR-PS	-1.014	**-3.549***	**4.441***	**7.287***		0.783	-1.463	0.963	**2.049***	**3.378***
HAR-RSV-SJV	**-2.802***	**-4.460***	**3.198***	**6.630***	**-2.307***		-1.446	-0.300	0.832	**3.350***
HAR-RSV	-0.769	**-2.621***	**3.947***	**6.344***	-0.129	1.827		1.714	**3.330***	**4.187***
LHAR-RV1	-0.540	**-3.115***	**4.617***	**7.445***	0.651	**2.866***	0.495		1.809	**3.071***
LHAR-RV2	-1.947	**-3.714***	**3.416***	**5.894***	-1.185	1.178	-1.152	**-2.048***		**2.516***
LHAR-RV-CJ	**-8.087***	**-8.845***	**-4.964***	**-2.263***	**-8.019***	**-7.500***	**-8.233***	**-8.531***	**-9.012***	

BVSP	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**4.544***	**4.462***	**3.760***	**3.733***	**3.416***	**3.512***	0.648	1.736	**4.213***
HAR-RV-J	-1.010		**3.865***	**3.191***	1.673	**2.427***	**2.044***	-1.509	0.285	**3.688***
HAR-CJ	**-4.253***	**-4.280***		-1.218	**-3.369***	**-3.908***	**-2.914***	**-4.220***	**-3.554***	0.340
HAR-RV-CJ	**-7.068***	**-7.156***	**-6.581***		**-2.651***	**-2.669***	**-2.224***	**-3.511***	**-2.867***	1.997
HAR-PS	**-5.437***	**-5.171***	**2.666***	**5.580***		1.390	1.245	**-3.388***	-0.994	**3.298***
HAR-RSV-SJV	**-4.036***	**-4.108***	**3.057***	**7.195***	-1.129		-0.673	**-3.127***	-1.908	**3.415***
HAR-RSV	**-5.464***	**-5.301***	1.850	**4.726***	**-2.415***	-0.159		**-3.205***	-1.855	**3.010***
LHAR-RV1	-1.631	-1.461	**3.762***	**6.509***	**4.247***	**3.273***	**4.567***		1.720	**4.205***
LHAR-RV2	-1.915	-1.816	**3.284***	**5.939***	1.988	**2.326***	**3.849***	-1.146		**3.888***
LHAR-RV-CJ	**-6.887***	**-6.957***	**-5.393***	**-3.855***	**-5.673***	**-6.854***	**-5.108***	**-6.750***	**-6.759***	

DAX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		1.382	**15.11***	**12.80***	1.410	**2.089***	**3.643***	0.213	0.580	**9.001***
HAR-RV-J	**-2.836***		**14.68***	**12.54***	0.608	1.393	**2.895***	-0.313	0.210	**8.762***
HAR-CJ	**-39.45***	**-37.79***		-0.445	**-12.56***	**-13.38***	**-10.16***	**-11.67***	**-9.292***	-0.962
HAR-RV-CJ	**-37.72***	**-35.76***	**4.372***		**-10.97***	**-11.44***	**-8.710***	**-10.35***	**-8.444***	-0.824
HAR-PS	**-4.956***	**-3.562***	**34.42***	**32.76***		0.835	**4.048***	-1.123	-0.129	**9.109***
HAR-RSV-SJV	**-8.299***	**-6.714***	**35.62***	**33.59***	**-3.579***		1.659	-1.774	-0.677	**9.354***
HAR-RSV	**-8.168***	**-7.041***	**32.00***	**29.86***	**-7.420***	-0.708		**-3.492***	**-2.208***	**7.948***
LHAR-RV1	-0.869	-0.068	**31.86***	**30.19***	**3.409***	**5.906***	**6.915***		0.597	**10.32***
LHAR-RV2	**-2.782***	**-2.211***	**25.91***	**24.56***	-0.664	1.439	**2.190***	**-3.020***		**12.17***
LHAR-RV-CJ	**-28.76***	**-27.83***	0.639	-1.287	**-29.28***	**-28.73***	**-28.96***	**-32.44***	**-38.78***	

MXX	HAR-RV	HAR-RV-J	HAR-CJ	HAR-RV-CJ	HAR-PS	HAR-RSV-SJV	HAR-RSV	LHAR-RV1	LHAR-RV2	LHAR-RV-CJ
HAR-RV		**-2.340***	**-2.439***	-1.393	0.047	-1.427	1.039	0.531	1.593	-0.914
HAR-RV-J	0.935		**-2.022***	-1.012	0.938	-0.730	1.660	1.541	**2.213***	-0.580
HAR-CJ	1.593	**15.32***		**3.853***	**2.590***	**2.359***	**3.101***	**2.522***	**2.968***	**2.936***
HAR-RV-CJ	1.401	**15.15***	-1.593		1.487	0.879	**2.007***	1.497	1.982	0.924
HAR-PS	**-2.837***	**-3.074***	**-17.58***	**-17.43***		-1.597	1.450	0.441	1.537	-1.021
HAR-RSV-SJV	**6.949***	**6.664***	**-16.31***	**-16.24***	**9.964***		**2.156***	1.635	**2.300***	-0.263
HAR-RSV	-1.167	-1.341	**-17.55***	**-17.25***	0.994	**-7.154***		-0.805	0.388	-1.617
LHAR-RV1	**-2.502***	**-2.699***	**-15.93***	**-15.67***	1.055	**-7.957***	-0.111		1.622	-1.094
LHAR-RV2	**-4.667***	**-4.756***	**-16.81***	**-16.52***	**-2.274***	**-9.152***	**-2.783***	**-3.982***		-1.782
LHAR-RV-CJ	**-13.02***	**-12.83***	**-4.321***	**-3.897***	**15.26***	**12.93***	**15.81***	**14.01***	**16.22***	

Notes: This table presents the values of Diebold and Mariano's test of no difference in predicative accuracy for daily forecasting horizon under both rolling window and recursive forecasting approaches. The positive value indicates that the model in the row outperforms the model in the column. In each panel, in cell above the main diagonal this table reports the rolling window results; below the main diagonal, recursive forecasting results are presented. The value highlighted with * indicates rejection of the null hypothesis below the 5% significant level.

Table 3.17: the Model Confidence Set (MSC) test under rolling window approach

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-J	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-CJ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-PS	0.0000	0.0000	0.9788	0.0000	0.9998	0.0000	0.0000	0.0000
HAR-RSV-SJV	0.0000	0.0000	0.0000	0.0000	0.8780	0.0000	**1.0000***	0.0000
HAR-RSV	0.0000	0.0000	0.9930	0.0000	0.0000	0.0000	0.0000	0.0000
LHAR-RV1	0.0000	0.0000	**1.0000***	0.0000	**1.0000***	**1.0000***	0.0000	0.0000
LHAR-RV2	0.0000	0.0000	0.9960	0.0000	0.9646	0.0000	0.0000	0.0000
LHAR-RV-CJ	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9066	0.0000	**1.0000***
				H=5				
HAR-RV	0.0000	0.0000	0.8723	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-J	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-CJ	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
HAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	0.9842	0.0000	0.0000	0.0000
HAR-PS	0.0000	0.0000	0.8754	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RSV-SJV	0.0000	0.0000	0.0000	0.0000	0.9842	0.0000	**1.0000***	0.0000
HAR-RSV	0.0000	**1.0000***	**1.0000***	**1.0000***	0.0000	0.0000	0.0000	**1.0000***
LHAR-RV1	0.0000	0.0000	**1.0000***	0.0000	0.0000	**1.0000***	0.0000	0.0000
LHAR-RV2	**1.0000***	0.0000	**1.0000***	0.0000	0.9908	0.0000	0.0000	0.0000
LHAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
				H=22				
HAR-RV	0.0000	0.0000	0.9238	0.0000	0.9986	0.0000	0.8498	0.0000
HAR-RV-J	0.0000	0.0000	**1.0000***	0.0000	0.9960	0.0000	0.8388	0.0000
HAR-CJ	0.0000	0.0000	0.0000	**1.0000***	0.9814	0.0000	0.0000	0.0000
HAR-RV-CJ	0.0000	0.0000	**1.0000***	0.0000	0.9638	0.0000	0.0000	0.0000
HAR-PS	0.0000	0.0000	**1.0000***	0.9742	**1.0000***	0.0000	0.8388	0.0000
HAR-RSV-SJV	0.0000	0.0000	0.0000	0.8690	0.9816	0.0000	0.8116	0.0000
HAR-RSV	0.0000	**1.0000***	**1.0000***	0.9990	0.7834	0.0000	**1.0000***	0.0000
LHAR-RV1	0.0000	0.0000	**1.0000***	0.9990	**1.0000***	**1.0000***	**1.0000***	0.0000
LHAR-RV2	0.9932	0.0000	**1.0000***	**1.0000***	0.9748	0.0000	**1.0000***	**1.0000***
LHAR-RV-CJ	**1.0000***	0.0000	0.0000	0.9992	0.0000	0.0000	0.2784	0.0000

Note: This table reports the MSC test in term of MSE criterion under rolling window approach over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 3.18: the Model Confidence Set (MSC) test under recursive approach

	FTSE	SPX	N225	DAX	SSEC	NSEI	BVSP	MXX
				H=1				
HAR-RV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-J	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-CJ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-PS	0.0000	0.0000	0.9988	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RSV-SJV	0.0000	0.0000	0.0000	0.0000	0.8292	0.0000	**1.0000***	0.0000
HAR-RSV	0.0000	0.0000	0.9824	0.0000	0.0000	0.0000	0.0000	0.0000
LHAR-RV1	0.0000	0.0000	**1.0000***	0.0000	**1.0000***	0.0000	0.0000	0.0000
LHAR-RV2	0.0000	0.0000	**1.0000***	0.0000	0.9442	**1.0000***	0.0000	0.0000
LHAR-RV-CJ	**1.0000***	**1.0000***	0.9810	**1.0000***	**1.0000***	0.9332	0.0000	**1.0000***
				H=5				
HAR-RV	0.0000	0.0000	0.8246	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-J	0.0000	0.0000	0.9653	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-CJ	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
HAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	0.8936	0.0000	0.0000	0.0000
HAR-PS	0.0000	0.0000	0.6816	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RSV-SJV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RSV	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	**1.0000***	**1.0000***
LHAR-RV1	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000	0.0000
LHAR-RV2	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***	0.0000	0.0000
LHAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
				H=22				
HAR-RV	0.0000	0.0000	0.7250	0.0000	0.9382	0.0000	0.8188	0.0000
HAR-RV-J	0.0000	0.0000	**1.0000***	0.0000	**1.0000***	0.0000	0.7998	0.0000
HAR-CJ	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
HAR-RV-CJ	0.0000	0.0000	0.0000	0.0000	0.7681	0.0000	0.0000	0.0000
HAR-PS	0.0000	0.0000	0.3482	0.0000	**1.0000***	0.0000	0.6768	0.0000
HAR-RSV-SJV	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	1.0000*	0.0000
HAR-RSV	0.0000	0.0000	**1.0000***	0.0000	**1.0000***	0.0000	1.0000*	0.9520
LHAR-RV1	0.0000	0.0000	**1.0000***	0.0000	**1.0000***	0.0000	**1.0000***	0.0000
LHAR-RV2	0.0000	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
LHAR-RV-CJ	**1.0000***	0.0000	0.0000	0.0000	0.0000	0.0000	0.1582	0.0000

Note: This table reports the MSC test in term of MSE criterion under recursive approach over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Chapter 4 Lasso-Based Flexible Lags for Realised Volatility Forecasting

4.1 Introduction

Since Ding et al. (1993) and Bollerslev and Mikkelsen (1996) put forward that volatility is persistence, a partial history of volatility models captures the long memory feature of volatility to model and forecast volatility(Baillie et al., 1996 and Engle and Lee, 1993). The realised volatility (RV) is an unbiased estimator of past return volatility (Andersen et al., 2003). The current empirical works concentrate on modelling and forecasting RV. The HAR model (Corsi, 2009) becomes the tendency in forecasting models, as it simply exhibits the volatility persistence through aggregating daily, weekly and monthly RV. According to Corsi (2009) notes, the HAR model could be regarded as a restricted AR(22) model with an economic perspective. Hence, it raises whether there is an appropriate lag length or lag structure that could provide more efficient forecasting information.

An important contribution in model selection within linear models is the least absolute shrinkage and selection operator (Lasso) method proposed by Toshigami (1996). The Lasso produces parsimonious parameters in the linear model, making the selected parameter more efficient (Friedman et al., 2010). The Lasso method can test the validity of efficient variables, and it has been increasingly employed in the forecasting model. For example, Audrino and Knaus (2016) extend the Lasso method used in the AR model of Nardi and Rinaldo (2011) to the case of the HAR model. However, Audrino and Knaus (2016) find the Lasso approach limited improve accuracy over the HAR model in the out-of-sample prediction.

In light of this background, this chapter investigates whether the AR model with parsimonious lags could generate a more accurate RV than the HAR model with fixed lags. Therefore, this chapter uses the Lasso approach to obtain more sparse coefficients in the AR model and compares the forecasting performance with HAR models. This chapter extends and updates the work of Audrino and Knaus (2016) and Audrino et al. (2019) in terms of the different lag lengths in the AR model. I consider two AR models, which are the AR(22) model and the AR(100) model.[9] Then, the chapter also employs the Lasso improvements based on the different penalty functions. For example, the more flexible penalization used in adaptive Lasso (Zou, 2006), the grouped Lasso is suitable for strongly correlated variables (Yuan and Lin, 2006), and ordered Lasso (Toshigami and Suo, 2016) to capture the dynamic feature the time series model. In addition, this chapter access forecasting performance in management applications in calculating the Value at Risk.

The empirical results in this paper suggest three conclusions. First, the in-sample results show that the AR models with parsimonious lags have slightly improved the model fitness over the HAR models. In the analysis of coefficients, this chapter finds the forecasting information is concentrated in the first 22 lags, but the longer lags beyond 22 also provide some efficient information. Second, the out-of-sample results indicate that the AR models using the Lasso approach significantly outperform the HAR models. Especially, the AR(100) model with ordered Lasso performs the best at daily forecasting, and the AR(22) model with ordered Lasso dominates at the weekly and monthly prediction. Consistent results are found in the risk management application.

The rest of this chapter is shown as follows. Section 2 provides a review of relevant literature. Section 3 introduces the Lasso-based methods and HAR models considered in this chapter and the methodology of forecasting evaluation. The data and empirical findings of in-sample estimation and out-of-sample forecasting are introduced in sections 4 and 5. Section 6

[9] The lag length of AR(22) model is same as the basic HAR model(Coris, 2009). In the empirical work of Audrino et al. (2019), the AR(50) does not perform well using Lasso approaches and they conclude that some lags still have predictive information beyond the HAR model. The forecasting information is gradually decay in the time series data. While AR(100) is arbitrary in this chapter, it is selected partly on the basis that it will be 'too long' and therefore capture all information within the data. The Lasso approach sets lags to zero if they do not add explanatory power, so minimises any issues in regard of over-fitting.

applies the forecasts to risk management application. And the conclusion is provided in section 7.

4.2 Literature Review

For many years, the analysis of financial assets with time-varying behaviours has been a vital part of asset allocation and risk management. For modelling the volatility, it is well known that the volatility of asset returns is persistent, and it can be captured relatively well in the works of Ding et al. (1993) and Bollerslev and Mikkelsen (1996), in which the volatility persistence of asset returns could be described as long memory feature. It can be shown as highly autocorrelation for long lags, which exponentially decay with the number of lags increasing. Since then, the feature of long memory is widely used in the forecasting model. In the past years, there are some extension models of the GARCH model to indicate the high volatility persistence [10](e.g. Engle and Bollerslev, 1986; Baillie et al. 1996 and Engle and Lee 1993). In addition, Granger and Ding (1996) provide the generalized fractionally integrated processes for the non-GARCH setting.

Long Memory In Realised Volatility

Andersen and Bollerslev (1998) use the cumulative sum of squared intraday returns as a measurement of RV, displaying a long memory feature and high persistence. Over those decades, this integrated form has been used extensively to capture the persistence of the observed volatility sequence. For instance, Andersen et al. (2003), Lieberman and Philips (2008), and Martens et al. (2009) report the evidence of long memory in the high-frequency data, modelled by the fractionally integrated process. Since then, it has been found that RV can

[10] A more complete review can be found in Section 2.2.3.

improve the performance of the fitting and prediction process, and a long memory model can improve the prediction performance (Martens and Zein, 2004).

To capture the long-memory feature of RV, Andersen et al. (2004) suggest that the ARMA-type model is directly adapted to RV and performs well under actual forecasting conditions. Then the long-memory feature of RV has been accommodated by the ARFIMA process. Hol and Koopman (2002) find the relatively simple form of ARFIMA $(1, d, 0)$ model for RV to obtain more accurate predictions than the SV model and GARCH model. However, some researchers raise the question of the fractional integrated process. Poskitt (2006) and Wang and Hsiao (2012) show the ARFIMA (p, d, q) process can be well approximated by the AR(k) model using information criterion to determine the order. The empirical work of Wang et al. (2013) proves this AR-based method could provide better forecasting performance than the fractional integrated process.

Long Memory In HAR Model

As previous mentioned in section 3.2, to accommodate the long memory of RV, Corsi (2009) proposes the HAR model, which corresponds to different trading behaviours with daily, weekly and monthly horizons. Corsi (2009) emphasised that the standard HAR model can be regarded as a restricted AR(22) model. Following the work of Corsi (2009), the standard HAR model is extended along with different patterns in order to account for different stylized facts of volatility.[11] (E.g. Andersen et al., 2007; Narndorff-Nielsen et al., 2008; Patton and Sheppard, 2015; Corsi et al., 2012 and Patton and Sheppard, 2015).

Despite the HAR model having an economically meaningful fixed lag structure (1, 5, 22) for modelling RV, which represent the daily, weekly, and monthly time horizon and this fixed lag structure being widely used in recent research, some empirical works still doubt the

[11] More details can be found in Chapter3.

validity in the lag structure in a HAR-like structure with the fixed aggregation of the three scales. Craioveanu and Hillebrand (2012) extend the fixed lag structure of the HAR model to a flexible lag structure; however, they find there is no forecasting improvement in the in-sample and out-of-sample fit when employing the flexible lag structure in the HAR model. In addition, an enhanced AR model accommodated with structure breaks for forecasting a long memory process developed by Wand et al. (2013), the AR-based forecasting method outperforms ARFIMA-based methods in the out-of-sample evaluation. Their results of persistence change in memory parameter also provide an econometric explanation for the empirical success of the HAR model, which can be considered as a special structure of the AR method. Another lag structure justification of the HAR model is proposed by Hwang and Shin (2014). They extend the basic three lag structure HAR model to an infinite-order HAR model, namely HAR (∞), with infinite long memory conditions. The HAR (∞) model has exponentially decaying coefficients, but the result of Hwang and Shin's (2014) research shows the forecasting errors are mainly dominated by the finite-order HAR (p) instead of HAR (∞).

Lasso Method

As the empirical works discussed above, the lag structure of daily, weekly, and monthly time horizons might be perfect for forecasting volatility. However, the deficiency of those works is that they cannot test the forecasting information of each lag separately. To test the validity and improve the forecasting ability of every lag in the AR model, the model selection theory is employed in the estimation of lag structure coefficients, which can be restored with statistical means. Model selection plays an essential role in computational statistics perspective. Toshigami (1996) proposes the least absolute shrinkage and selection operator (Lasso), which gains great popularity and extensive application in model selection in linear models. To reduce the basis of each independent variable, the Lasso process shrinks estimators toward zero based on a tuning parameter. The ordinary least squared (OLS) method gives non-zero coefficients to every variable, Lasso is differed in terms of the parsimonious coefficients impose on the linear parameters.

The empirical researches based on the Lasso process are increasingly employed in different econometric models. Wand et al. (2007) extend the Lasso to the linear regression model and find it could obtain a satisfactorily finite sample performance for exogenous variables and lagged dependent variables. Hsu et al. (2008) derive the Lasso estimator under vector auto-regressive processes and find the Lasso method perform better than other subset selection method for the small sample under several loss functions. For the AR model, Nardi and Rinaldo (2011) derive the Lasso is model selection consistent, estimation consistent and prediction consistent under certain conditions.

Due to the Lasso approach provides parsimonious and efficient forecasting variables, it has been increasingly used in time series model forecasting research. For the empirical works of Li and Chen (2014), Roy et al. (2015) and Ziel (2016), works have proved the Lasso-based estimations exhibit superiority from other model approaches. Tian et al. (2015) and Nazemi and Fabozzi (2018) both show that the Lasso-selected models have a good predictive performance for financial assets. Corsi (2009) indicates that the HAR model is the restricted AR(22) with only three coefficients. Subsequently, motivated by Corsi (2009), Audrino and Knaus (2016) find the Lasso process of the AR model has the same forecasting performance as the HAR model in the out-of-sample prediction at the individual stock level. The fixed lag structure of the HAR model is hard to beat by the linear model of the selected lag structure. However, Lasso cannot accurately restore the HAR lag structure in its empirical application, which raises questions about whether the lag structure of the HAR model is suitable for modelling RV (Audrino and Knaus, 2016).

Adaptive Lasso

However, due to every estimator are penalized equally in the basic Lasso method, Fan and Lin (2001) indicate the estimator of Lasso is unbiased, which means the Lasso process maybe provide an inefficient and inconsistent model selection result. To deal with this drawback, Zou (2006) allows more flexible penalization to obtain estimators and proposes the adaptive Lasso, which uses the adaptive weight penalty to shrink the variable coefficients. Park and Sakaori (2013) and Audrino and Camponovo (2013) both indicate it provides more efficient estimators for the adaptive Lasso for time series model.

Empirically, in the robustness check of Audrino and Knaus' (2016) work, the Lasso and adaptive Lasso process have the same prediction ability in the out-of-sample analysis, which means the forecasting performance of Lasso and adaptive Lasso is indistinguishable. Subsequently, Audrino et al. (2019) consider the HAR model employing the adaptive Lasso method to test whether the lag structure of the flexible HAR model could recover that in the fixed HAR (1, 5, 22) model. Once more, they provide empirical evidence to show that only slightly outperformance in terms of flexible HAR model and it is insignificant outperformed in the out-of-sample forecasting test. However, Fang et al. (2020) find that using adaptive Lasso as variable selection can significantly improve the predictive ability of long-term volatility.

Group Lasso

The common drawback of Lasso and adaptive Lasso is that they penalize every estimator separately and not suitable to the strongly correlated variable. So, in the cases with correlated predictors, the reliable estimators would not be produced by Lasso and adaptive Lasso. However, in the many multifactor regressions, the variables are naturally grouped. Such as the cases of Corsi (2009), the lagged RV are categorized into different groups in terms of time horizon, and none of the coefficients should be omitted. Hillebrand and Medeiros (2010) also indicate that bagging lagged RV is reasonable, improving foresting accuracy. As for the Lasso method, Yuan and Lin (2006) propose the Group Lasso considering the problem of group model selection, which penalizes coefficients and selects estimators as a group instead of an individual variable.

To check the validity of lag structure further on the classic HAR model, Audrino et al. (2019) group the AR(50) model and estimated it by the Group Lasso method. However, this lag structure does not survive in the performance testing. Thus they conclude there are still some lags that have forecasting information beyond one month. There are two additional algorithms of Grouped Lasso. The first one is called the Cluster Group Lasso by Buhlmann et al. (2013). As for the variables that are strongly correlated or have a nearly linear relationship in the multifactor regression, different from the Group Lasso, the Cluster Group Lasso tend to choose only one variable from a group and neglect others. The second one is the Sparse Group Lasso (Friedman et al., 2010; Simon et al., 2013), in which the penalized parameter is employed at both group and individual levels.

Ordered Lasso

Another defect of the Lasso is that it cannot capture the dynamic feature of the time series model. The prediction ability of the AR process is decay gradually, but the Lasso allows a time series model with higher-order lags. Meanwhile, the lower order lag may be absent in the same model. The two measurements that both the lower order lags are considered, and higher-order lags are included are the Hierarchical Lasso (Bien et al. 2013) and the Ordered Lasso (Toshigami and Suo, 2016). Both of these two Lasso-based approaches focus on the selection of lower lagged coefficients before higher lagged coefficients. Moreover, the order Lasso forces the absolute value of lag effects not to increase.

For the empirical work of those extension approaches in the RV forecasting field, Wilms et al. (2016) indicate that the ordered Lasso has the best forecasting performance for RV between these two Lasso approaches. They also compare Lasso approaches with the HAR model and find the ordered slightly outperform HAR model. Moreover, Croux et al. (2018) employ the ordered Lasso method into the univariate and multivariate model and find better forecasting performance than the HAR model.

4.3 Methodology

The main idea of this chapter is to investigate the lag structure validity of the Lasso-based method and the HAR model. In this section, the standard HAR model and its extension will present first, then the considered Lasso-based approaches used in AR(22) and AR(100) models will be described follow, the penalization method for Lasso and model compression approaches are introduced in the end. The details of the alternative models are given as follow:

4.3.1 Empirical Models

HAR model

As mentioned in previous methodology of Section 3.3, the calculations of RV and HAR model are shown as follow:

$$\mathrm{RV_t} = \sum_{\mathrm{i=1}}^{\mathrm{N}} \left(\mathrm{r_{t,i}}\right)^2 \tag{4.1}$$

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t \tag{4.2}$$

where the weekly and monthly averages of RV are calculated as:

$$RV_{t-1:t-5} = \frac{1}{5}\sum_{i=1}^{5} RV_{t-i} \tag{4.3}$$

$$RV_{t-1:t-22} = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i} \tag{4.4}$$

Thus, the HAR model can be explained by the expected RV as a linear equation of yesterday's RV and the average RV over last week and last month. Emphasized by Corsi (2009), the standard HAR model also regards as a restricted AR(22) model, so it could also be written as:

$$RV_{t+1} = \theta_0 + \sum_{i=1}^{22} \theta_i RV_{t-i} + u_t \tag{4.5}$$

The additional restrictions of coefficients, θ_i, implied by the lag structure of HAR model, shown as follow:

$$\theta_i = \begin{cases} \beta_d + \frac{1}{5}\beta_w + \frac{1}{22}\beta_m & for\ i = 1; \\ \frac{1}{5}\beta_w + \frac{1}{22}\beta_m & for\ i = 2, \ldots, 5; \\ \frac{1}{22}\beta_m & for\ i = 6, \ldots, 22. \end{cases} \tag{4.6}$$

The simplification from 22 parameters of AR(22) to three regression parameters has been empirically proved that possess better fitness (Corsi, 2009). However, the information criteria between restricted HAR and unrestricted AR(22) model provide unclear results, so the HAR model may be uncertainly successful in capturing the real financial data.

HAR-free model

To incorporate the sudden unexpected change in the market, Bollerslev et al. (2018) propose another form of HAR model, namely HAR-free model, is shown as follow:

$$\begin{aligned} RV_t = \beta_0 + \beta_1 RV_{t-1} + \beta_2 RV_{t-2} + \beta_3 RV_{t-3} + \beta_4 RV_{t-4} + \beta_5 RV_{t-5} + \beta_6 RV_{t-6} \\ + \beta_m RV_{t-1:t-22} + u_t \end{aligned} \tag{4.7}$$

As an augmented HAR model, the first six daily lagged RV are freely estimated in the HAR-free model, and $\beta_m RV_{t-1:t-22}$ is computed as same as equation (4) above. Comparing with the standard HAR model, the HAR-free model is AR(6) with aggregated monthly RV. For comparison purpose, this chapter also includes this HAR-free model in which the model freely estimate the impact of the first six daily lagged RVs.

Lasso

Toshigami (1996) proposes the Lasso method, and it is frequently used in the statistical perspective of computer science. According to Friedman et al. (2010), the Lasso method could provide an efficient algorithm to select computationally affordable estimators. Recently, the lasso method and its extensions play a crucial role in econometrics and forecast financial asset performance (Tian et al., 2015; and Nazemi and Fabozzi, 2018).

The Lasso can be treated as a constrained least square regression, as the tested model is linear autoregressive in the regression. Let RV_t donates the realised variance, The Lasso estimator of AR(p) model:

$$RV_{t+1} = \theta_0 + \sum_{i=1}^{n} \theta_i RV_{t-i+1} + u_t \tag{4.8}$$

where u_t is independent and identically distributed (i.i.d.) innovations with zero mean. Consequently, the Lasso estimator can be defined as:

$$\hat{\beta}_{lasso} = argmin \left\{ \sum_{t=p}^{T} \left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p} \beta_i RV_{t-j+1} \right)^2 + \lambda \sum_{i=1}^{p} |\beta_i| \right\} \tag{4.9}$$

where the λ is the tuning parameter that controls the number of shrinkage estimators in terms of penalty strictness. The first part in equation (9) is the least square criterion and the second part is the penalty term on the regression parameters. It is clear that let $\lambda = 0$ will lead the Lasso estimators to coincide with OLS estimators so that the estimators will be set equal to zero. Increasing λ gradually causes more and more coefficients of Lasso, which is penalized exactly to zero and performs a stricter coefficient selection. The penalty term λ is ranging from zero to one, so when the penalty term tends to one, all coefficients will be close to 0.

Adaptive Lasso

According to the basic Lasso method introduced by Toshigami (1996), every estimator is penalized equally. To this drawback, Zou (2006) develops adaptive Lasso, which allows more flexible penalization to obtain estimators. The Adaptive Lasso is given as follow:

$$\hat{\beta}_{adoptive\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}\lambda_i|\beta_i|\right\} \qquad (4.10)$$

where λ_i are adaptive weights for each coefficient to reduce false positives. When every λ_i are equal to 1, the adaptive Lasso transforms into the original Lasso. Compared with the standard Lasso method, adaptive Lasso allows stricter penalty for zero coefficients and the lower penalty for non-zero coefficients, reducing the estimations bias and improving the efficiency and accuracy of variable selection. Following the literature (Audrino and Knaus, 2016), as a common choice for the adaptive weights, this chapter sets $\lambda_i = \left|1/\hat{\beta}_{lasso}\right|$, with the notation that in case a variable is excluded by the Lasso, this chapter also excludes is from the adaptive Lasso estimation.

Group Lasso

The common drawback of Lasso and adaptive Lasso is that they penalize every estimator separately and ignore the correlations between each estimator. They will select one of the correlated estimators and omit others in the penalizing process. Thus, besides the Lasso and adaptive Lasso approaches, the chapter employs the Group Lasso (Yuan and Lin, 2006) to offset this shortcoming. The grouped Lasso penalizes coefficients and selects estimators as a group instead of an individual variable. The Group lasso (Yuan and Lin, 2006) shown as follow:

$$\hat{\beta}_{group\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{k=1}^{K}\sqrt{p_k}\sqrt{\sum_{i\in I_k}{\beta_i}^2}\right\} \tag{4.11}$$

Audrino et al. (2019) group the AR(50) model as {1}, {2-5}, {6-22}, {23-50} and estimated by Group Lasso method, but this lag structure does not perform well, so they conclude that some lags still have predicting information beyond the lag structure HAR model. Taking their suggesting and follow the lag structure of standard HAR model, this chapter groups the lag length of AR(22) as {1}, {2-5}, {6-22} implied by the lag structure of Corsi (2009), the AR(100) also be grouped as {1}, {2-5}, {6-22}, {23-50}, {51-75}, {76-100}.

Ordered Lasso

The core idea of this approach is time-lagged regression, where the prediction at a particular time from the features at the previous time, so they naturally assume that the predictive information of coefficients is decay gradually. Toshigami and Suo, (2016) point the auto-regressive time series where natural feature exists should consider the Lasso approach with an additional monotone decreasing constraint, so the order-constrained coefficients of the Lasso approach are developed by Toshigami and Suo (2016), namely ordered Lasso. The ordered Lasso is proposed as follow:

$$\hat{\beta}_{ordered\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\left(\beta_j^+ + \beta_j^-\right)RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}\left(\beta_j^+ + \beta_j^-\right)\right\} \tag{4.12}$$

subject to $\beta_1^+ \geq \beta_2^+ \geq \cdots \geq \beta_p^+ \geq 0$ and $\beta_1^- \geq \beta_2^- \geq \cdots \geq \beta_p^- \geq 0$. The ordered Lasso method modifies the penalized parameters from the absolute value ($|\beta_j|$) of Lasso to positive and negative components ($\beta_j^+ + \beta_j^-$) that means some coefficients of β_j will be estimated as exactly zero. In addition, this ordered constraint penalty allows the higher lag order is estimated only when lower lag order lags are already included.

4.3.2 Cross-Validation

As the previous discussion shows, the most important process of the Lasso method is to determine the tuning parameter (λ), which determines the flexibility of the parameters' estimation by the number of non-zero coefficients. According to the common ways in previous empirical works, this chapter selects the optimal parameters using cross-validation (CV).

Specifically, this chapter chooses λ based on the K-folds Cross-Validation method as previous literature used (Nardi and Rinadlo, 2011; Audrino et al. 2017; Audrino et al. 2019). Toshigami (1996) firstly estimates the prediction error of the Lasso approach by K-folds cross-validation. Particularly, the whole sample observations are split into K groups, G_K stand for each group, the estimators are obtained on $K-1$ groups, and the test error is predicted on the remaining group. The process is repeated for $k = 1, 2, \cdots K$, and the results of test error are averaged so the procedure can be conducted for each value of the tuning parameter λ. Especially, this chapter set $k = 10$ for the CV method. The cross-validation error function is the mean square error (MSE) in this chapter, written as follow:

$$\text{CV}\ (\lambda) = \frac{1}{T}\sum_{k=1}^{K}\sum_{G_K}\left(y_t - \widehat{y_t^k}(x_t)\right)^2 \tag{4.13}$$

where $\widehat{y_t^k}(x_t)$ is the predictions of Kth-fold. The optimal λ is selected by the minimum error of CV (λ):

$$\widehat{\lambda_{CV}} = \arg\min_{\lambda} CV(\lambda) \tag{4.14}$$

Another common approach to select the tuning parameter is to use information criteria, including the Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). For example, Audrino and Knaus (2016) determine λ according to the minimization of the BIC, and Nardi and Rinaldo (2008) estimate the AR model use the AIC criterion. Taking the analysis of Audrino and Knaus (2016) one step further, Wilms et al. (2016) and Croux et al. (2018) produce the tuning parameter λ by the forecasting combination, which is combination weighted BIC. Empirically, the Lasso approaches using forecasting combination in Wilms et al. (2016) only slightly improve forecasting performance, but almost the same with the models without forecasting combination. The BIC to get the best offset between fitness and complexity in models, and the CV method is to select λ with the best out-of-sample prediction accuracy. However, the lasso models with either CV or BIC produces the same model fitting (Wand et al., 2007). As the works of Audrino and Knaus (2016) and Audrino et al. (2019) also note that even if the approach of selecting tuning parameter switched, the results of estimations would remain almost qualitatively identical.

4.3.3 Forecasting Evaluation

There are many ways to evaluate and compare the accuracy of the different forecasting models, but Patton (2011) indicates the Quasi-Likelihood (QLIKE) and Mean Squared Error (MSE) are the most robust loss function for heteroscedasticity. Thus, those two loss functions this chapter used are shown as follow:

$$\mathrm{MSE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}}\left(\mathrm{RV_t} - \widehat{\mathrm{RV_t}}\right)^2 \tag{4.15}$$

$$\mathrm{QLIKE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}}\left(\log\left(\widehat{\mathrm{RV_t}}\right) + \frac{\mathrm{RV_t}}{\widehat{\mathrm{RV_t}}}\right) \tag{4.16}$$

where the actual volatility denotes as $\mathrm{RV_t}$, and the volatility forecast obtained is indicated by $\widehat{\mathrm{RV}}_\mathrm{t}$.

Model Confidence Set

In addition to loss functions, this chapter also uses the Model Confidence Set (MSC) approach to remove the worst model sequentially according to the null hypothesis of equal predictive ability is rejected. [12]

4.4 Data

All of the RV data used are from the Oxford-Man Institute of Quantitative Finance. The chapter employs the 5 minutes RV of eight international stock indices[13], including four developed countries: the UK stock index (FTSE), Japanese stock index (N225), the USA stock index (SPX) and German stock index (DAX), and four developing countries: Chinese stock index (SSEC), Brazilian stock index (BSVP), Indian stock index (NSEI) and Mexican stock index (MXX), respectively. This chapter uses the RV in logarithmic form, log-RV. This chapter considers 15-year data, and the observation period is from 1st January 2003 to 31st December 2017. The first 5-year in-sample period is from 1st January 2003 to 31st December 2007, and the last 10 years (1st January 2008 to 31st December 2017) are out-of-sample periods.

Table 4.1 describes the summary statistics of log-RV for each index—all series of log-RV exhibit a non-normal distribution with excess kurtosis and right-skewed. The further supportable evidence of non-normal distribution is that the Jarque-Bera test statistic rejects the null hypothesis of normal distribution at the 1% level. All log-RV series are persistent, as can be seen from the first-order autocorrelation value. Figure 4.1 provides the time-series plots of

[12] For more detail about the MCS test see Section 3.3.2.
[13] The discussion of date sample frequency can be seen in Footnote 7.

the log-RV, respectively. For all indices, the log-RV occurs a rise around the year 2008 as the financial crisis happened. A standard approach to exam the long memory feature of time-series data is to use the exanimation of the sample autocorrelation function. Figure 4.2 shows the sample autocorrelation function of log-RV up to 100 lags. The sample autocorrelation function of all indices decreases slowly as the lag length increases, while the autocorrelations of BSVP around lag 95 are insignificant. Therefore, the evidence in those graphs is that the RV of all indices used is consistent with the long memory feature, and this pattern of exponentially decaying in autocorrelation function means the AR(p) models with long length lags in those graphs this chapter are accessible.

4.5 Empirical Results

4.5.1 In-Sample Results

This chapter estimates the RV of eight indices to employ the HAR model and HAR-free model with fixed lag structure model estimated by OLS method, and the flexible AR(22) and AR(100) models are used by the model selection method including Lasso, adaptive Lasso, group Lasso and ordered Lasso, respectively. Especially, the tuning parameter, λ, in the Lasso-method approach is the minimum error of the CV. The in-sample period used for estimation is the first five years from 1st January 2003 to 31st December 2007. Finally, the coefficient plots demonstrate that the estimators from all forecasting models, and the loss functions provide the fitness accuracy in the in-sample period.

The estimated coefficients are plotted as line graphs in Figure 4.3 (a) and (b), in which the coefficients are divided by three figures based on the model types for each index. The upper figure is the fixed coefficients of the standard HAR model and HAR-free model estimated by OLS method. The middle figure is the flexible AR(22) coefficients using Lasso, adaptive Lasso, group Lasso, and ordered Lasso approaches, respectively. And the bottom figure is the AR(100) estimated by Lasso-based approaches. As for the fixed coefficients of all indices, the value of the first six coefficients of the HAR model and HAR-free model are decreased, and the coefficients beyond lags seven are similar, which are above zero. Then, in terms of the flexible structure of AR(22), all four Lasso-based methods select coefficients from lag 1 to 5 with a declining trend. The Lasso and adaptive Lasso have a similar pattern, in which select several coefficients and set others as zero for the following lags. The grouped Lasso of AR(22) is grouped as {1}, {2-5}, {6-22}, which is the same as the HAR model. Thus the coefficients of grouped Lasso (the green line in the middle figure) for all RV indices have a similar tendency with the HAR model. The coefficients tendency of ordered Lasso (purple lines in the middle figure) is monotonically non-increasing and lag length. The bottom figure of all RV indices observes that the lags beyond lag 22 are much less flexible selected by all Lasso-based methods. In the longer lag length selected by grouped Lasso and ordered Lasso, their coefficients are much close to zero or exactly equal to zero. However, the Lasso (blue line) and adaptive Lasso (red line) still select some longer lags to have efficient forecasting information, and the coefficient number of adaptive Lasso is greater than Lasso.

Table 4.2 provides the MSE and QLIKE loss function in the in-sample period. [14] While the focus is on the out-of-sample forecasting, it is still worth to note whether the alternative models provide a better in-sample fit and whether the same model perform well in-sample and out-of-sample. In this table, the standard HAR model is set as the benchmark model to compare the loss function value, so all the loss function value is standardized by the HAR model to simplify the comparison. According to the value of MSE and QLIKE, the AR(100) using adaptive Lasso provides the best estimation among all indices, and the AR(22) with the Lasso

[14] The in-sample period is usually used to construct forecasting models, in-sample forecasting is the process of using observed data to evaluate the predictive capability of the forecasting models. In this chapter, the two differences between the Lasso-based forecasting models are the two lag lengths and the coefficients retained by the Lasso approaches, therefore, it is worth to generate in-sample forecasting to see how effective these models are in reproducing data.

method also provides a good performance. This indicates there is still some forecasting information in the lags beyond 22 in the RV series.

To sum up, the best model to estimate the RV for all indices is AR(100) estimated by adaptive Lasso according to the loss function of MSE, but all the flexible models only slightly improve fitness, this result is consistent with the work of Audrino and Knaus (2016) and Audrino et al. (2019). In the coefficient plots, the flexible AR models show that the first 22 lags are more frequently selected than the lags beyond 22, which means the first 22 lags contain almost relevant information to estimate forecasting models. The longer lags also provide relevant forecasting information selected by Lasso and adaptive Lasso method. There are no differences between developing countries and developed countries.

4.5.2 Out-Of-Sample Results

This chapter concentrates on the forecasting performance comparison of all models mentioned in section 3.2 and the out-of-sample period is ten years from 1st January 2008 to 31st December 2017. First, all forecasting models considered produce the RV forecasts over the daily, weekly, and monthly horizon. A simple approach for generating multi-step-ahead forecasts is to replace the data frequency of volatility model with long-term forecasting. To replace RV_{t+1} on the lift-head side in Equation (8) over the forecasting horizon h, say $RV_{t+h}^{h} = \frac{1}{h}\sum_{i=1}^{h} RV_{t-h+i}$, thus $h = 1, 5\ and\ 22$ in this chapter. Second, the rolling window and increasing window approaches are used in the out-of-sample period for the data generation process, and the window size is 1000 daily observations for rolling window forecasting. Third, the forecasting performance is measured by the MSE and QLIKE loss function, where the standard HAR model is regarded as a benchmark against another forecasting model as well. Last, the MSC test selects optimal models with the EPA for rolling windows and increasing window approaches.

Table 4.3 and Table 4.4 present the MSE value using rolling window and increasing window forecasting approaches over the daily, weekly, and monthly horizon (h= 1, 5 and 22). Generally, Table 4.3 and Table 4.4 show the same results. For the one-day-ahead forecast, the AR(100) with ordered Lasso perform best on three RV indices. According to the number of values greater than 1, the Lasso and adaptive Lasso both on the AR model with 22 lags and 100 lags perform poorly. The two fixed lag structure models, the HAR model, and the HAR-free model, have similar forecasting performance in daily forecasts. As for the forecasts of one-week-ahead, the Lasso-methods overwhelmingly outperform the HAR model and HAR-free model, especially, the ordered Lasso used in AR(22) dominates all stock indices. As for the one-month-ahead forecast, the HAR model and HAR-free model perform poorly, and the ordered Lasso performs best. In terms of the QLIKE comparison, Tables 4.5 and 4.6 show the results of the rolling window and increasing forecast over the daily, weekly, and monthly horizon. Identically, the QLIKE and MSE provide the same results over three horizons. The AR(100) with the ordered Lasso method generates the lowest value of the daily forecasting. For the weekly and monthly prediction, the ordered Lasso of AR(22) outperform other models.

The MCS test results of rolling window approaches are provided in Tables 4.7 and 4.8, in which the value 1 means the optimal model has been chosen, and the MSC test chooses a subset of models with EPA at a 75% confidence level. Tables 4.7 and 4.8 show the MCS test using MSE and QLIKE criterion. Generally, the Lasso-based method performs significantly better than the HAR model. Specifically, the AR(100) using the ordered Lasso method has the best performance for daily forecasts. As for weekly forecasting, there is overwhelming evidence of the superiority of the AR(22) using ordered Lasso specifications. Due to the volatility predictions obtained from monthly forecasting are smoother than daily and weekly forecasting, several models have EPA for the monthly forecast. Monthly results show the

AR(22) with Lasso, group Lasso and ordered Lasso seem to perform better jointly, while ordered Lasso account for more EPA.

On the other hand, Tables 4.9 and 4.10 present the MSC test to increase the window approach. To comparing with Tables 4.7 and 4.8, similar results are obtained in the increasing window approach. Again, the AR(100) using ordered Lasso is the best model for daily forecasting. The weekly results indicate the AR(22) using ordered Lasso is the best performing forecast method, and the AR(100) using ordered Lasso also performs well at the DAX and NSEI. Similarly, the AR(22) using Lasso, ordered Lasso, and AR(100) using Lasso perform equally in the monthly forecasting, while the AR(22) using ordered Lasso account for all indices.

To summarize, according to the lowest loss function value and MSC test based on MSE in rolling window and increasing window forecasting, the best forecasting model for the daily forecast is the AR(100) model with ordered Lasso, the AR(22) estimated by ordered Lasso has best forecasting ability for both weekly and monthly prediction. Corresponding with the estimation of coefficients, the AR(100) used adaptive Lasso has the best performance in model fitting, while it does not perform well in getting predictions. Identically, the differences between four developing countries and four developed countries are not observed for forecasting comparison in the out-of-sample results.

4.6 Risk Management Application

To further access the predictions obtained from the Lasso-based models and HAR model from the risk management aspect, this paper also uses the Value at Risk (VaR) measures. The VaR is a vital application to define and monitor the risk of specific financial assets as it illustrates the maximum loss occurring with a possibility over a specific period. The VaR of an asset is calculated as:

$$VaR = \mu_t + \sigma_t N(\alpha) \tag{4.21}$$

where μ_t is the mean of asset's log-return, σ_t is the predicted volatility, and $N(\alpha)$ defines the left αth quantile of the normal distribution.

To evaluate the accuracy of VaR forecasting, the first test is to compute the failure rate for daily return, which is the number of times daily returns exceed the forecasted VaR. Therefore, the failure is computed as the number of actual losses divided by the number of observations. Then, this chapter uses the Dynamic Quantile (DQ) test of Engle and Manganelli (2004) to examine if the present violations of the VaR measure are not correlated with the past violations. So, they define the hit sequence as follow:

$$Hit_t = I(r_t < -VaR_t) - a \tag{4.21}$$

this sequence assumes that value $(1 - a)$ whenever the actual returns are less than the VaR quantile and the value $(-a)$ otherwise. The expected value of Hit_t is zero, and the sequence is uncorrelated with past information. In this case, there will be no autocorrelation in the hit sequence and the fraction of exceptions will be correct. The DQ test statistic is calculated as:

$$DQ = \frac{\widehat{\beta}' X' X \hat{\beta}}{a(1-a)} \sim \chi^2(k) \tag{4.22}$$

which X is the explanation variables and $\hat{\beta}$ is OLS estimates. The DQ test follows χ^2 distribution with a degree of freedom equal to the number of parameters.

This chapter also computes the Weibull test, a duration-based test of Christoffersen and Pelletier (2004). The main idea of this duration-based test is that the duration between VaR violations should be independent and no cluster. Thus, the VaR violations should be memoryless and should follow an exponential distribution. Therefore, Christoffersen and Pelletier (2004) consider a Weibull distribution to be used for the duration variable. The Weibull distribution has the density function:

$$f_w(x, a, b) = a^b b x^{b-1} e^{-(ax)^b} \tag{4.23}$$

where the exponential distribution is the special case when $b = 1$, the null hypothesis of VaR violations are independent and memoryless corresponds to $b = 1$, so H_0: $b = 1$.

This chapter uses the same in-sample and out-of-sample period to produce the daily, weekly, and monthly VaR forecasts at both 1% and 5% VaR levels to examine which forecasting model provides more accurate VaR estimates. [15]The VaR results for the HAR model and Lasso-method forecasting are reported in Tables 4.11 and 4.12. In addition, the VaR forecasts are obtained from the rolling window and increasing window forecasting method, respectively.

The daily VaR forecasts for the rolling window are shown in Table 4.11 for the 1% and 5% VaR levels. The lowest average failure rate is the AR(100) with the ordered Lasso method, which is close to the specific level (i.e. 1% and 5%), and the Lasso and adaptive Lasso method used in AR(22) and AR(100) are both perform poorly[16]. In terms of the Weibull test and DQ test, the HAR model and HAR-free model perform best and only indicate the FTSE significance on the Weibull test at 1% VaR level. All models reject the null hypothesis of non-autocorrelation in the sequence of exceptions on the DQ test. For the weekly VaR forecasts, the AR(22) with the ordered Lasso method has the best performance, the HAR model and HAR-free model perform poorly, having the highest average failure rate across all models and having four markets significant at 1% level and all market significant at 5% on the Weibull test. The Lasso-method models improve the accuracy of the weekly VaR, comparing with the HAR model. Notably, the AR(22) with the ordered Lasso method outperforms the best among the models at the 5% VaR level. Examining the monthly VaR results makes volatility forecasts are

[15] This chapter directly uses the daily, weekly, and monthly predictions generated from forecasting models to compute VaR. In addition, this chapter employs the RV in the logarithmic form to generate forecasts, so the exponential RV is used when computing VaR.

[16] Although a higher failure rate is determined to be poor performance, a too low failure rate is also considered poor performance, because a too low failure rate represents that investors may miss out potentially higher returns when choosing low-risk and low-yield investments, it is related to opportunity cost. From an academic perspective, the common way to determine the best forecasting model is to find the violation rate or failure rate that is close to a specified level (i.e. 1% or 5%).

more smooth and flat, so the failure rate is lower than daily and weekly VaR results. The lowest average failure rate is the AR(22) with the ordered Lasso method at monthly horizon, which is close to the specific level. Moreover, all models do not reject the null hypothesis that VaR violations are independent and memoryless of the Weibull test and reject the null hypothesis of non-autocorrelation in the sequence of exceptions of the DQ test.

Table 4.12 provides the VaR results for the increasing forecasting method. Generally, Table 4.12 has similar results to Table 4.11. For the daily VaR forecasts, the AR(100) with the ordered Lasso method achieve the lowest average failure rate. The HAR model and HAR-free model show only one market significant on the Weibull test at a 1% VaR level. Examining the weekly VaR forecasts, the HAR model and HAR-free model also perform worse in terms of average failure rate and the Weibull test. For the monthly VaR results, identically with Table 4.11, the AR(22) with the ordered Lasso method outperforms the best among the models. All models do not reject the null hypothesis of the Weibull test and reject the null hypothesis of the DQ test.

4.7 Summary and Conclusion

The Lasso approach is originally noticed in the computational statistics perspective. The increasing availability of Lasso-based modelling has led to the developments of time series econometrics filed to deal with financial data. These developments have potential importance in selecting efficient parameters, generating models, obtaining predictions, and applying risk management. This chapter aims to investigate whether the AR model using the Lasso-based method could provide more accurate forecasts than the HAR model. Specifically, this chapter compares the RV forecasting ability between the flexible models of the Lasso method and fixed models of the HAR model. This chapter extends and updates the previous works of Audrino and Knaus (2016) and Croux et al. (2018). The four Lasso approaches and two different lag structures in the AR model are utilized to obtain volatility forecasts and VaR measures for daily, weekly and monthly forecasting horizons.

The in-sample analysis provides slight evidence that the flexible AR models outperform the HAR model. The best model for model fitness is the AR(100) with adaptive Lasso. All the flexible models only slightly improve fitness according to the loss function value. In the coefficients plots, the frequent selection of coefficients indicates most forecasting information is contained in the first 22 lags, but Lasso and adaptive Lasso of AR(100) indicate longer lags are still efficient and contain relevant information. The out-of-sample forecasting results present the ordered Lasso method performs best, in which the AR(100) model with ordered Lasso outperforms at the daily foresting horizon, and the AR(22) model with ordered Lasso performs significantly better than other models in the weekly and monthly prediction. The Lasso-based models overwhelmingly outperform the HAR model for the risk management application, especially the order Lasso approach performs best. In addition, this result finds no obvious differences between developing countries and developed countries.

The current paper shows that the HAR model dominants the forecasting models; its extensions have become the leading research trend for forecasting the RV. This chapter questions the fixed lags structure in the HAR model and follows Audrino and Knaus (2016) and Audrino et al. (2019) to generate flexible lags in the AR models using the four Lasso method. The results show that the Lasso approaches provide more accurate forecasts than the HAR model and also indicate that the longer lags contain relevant forecasting information. In addition, identical results are presented in the VaR application.

Figure 4.1: The plots of the time series of log-RV of eight market indices

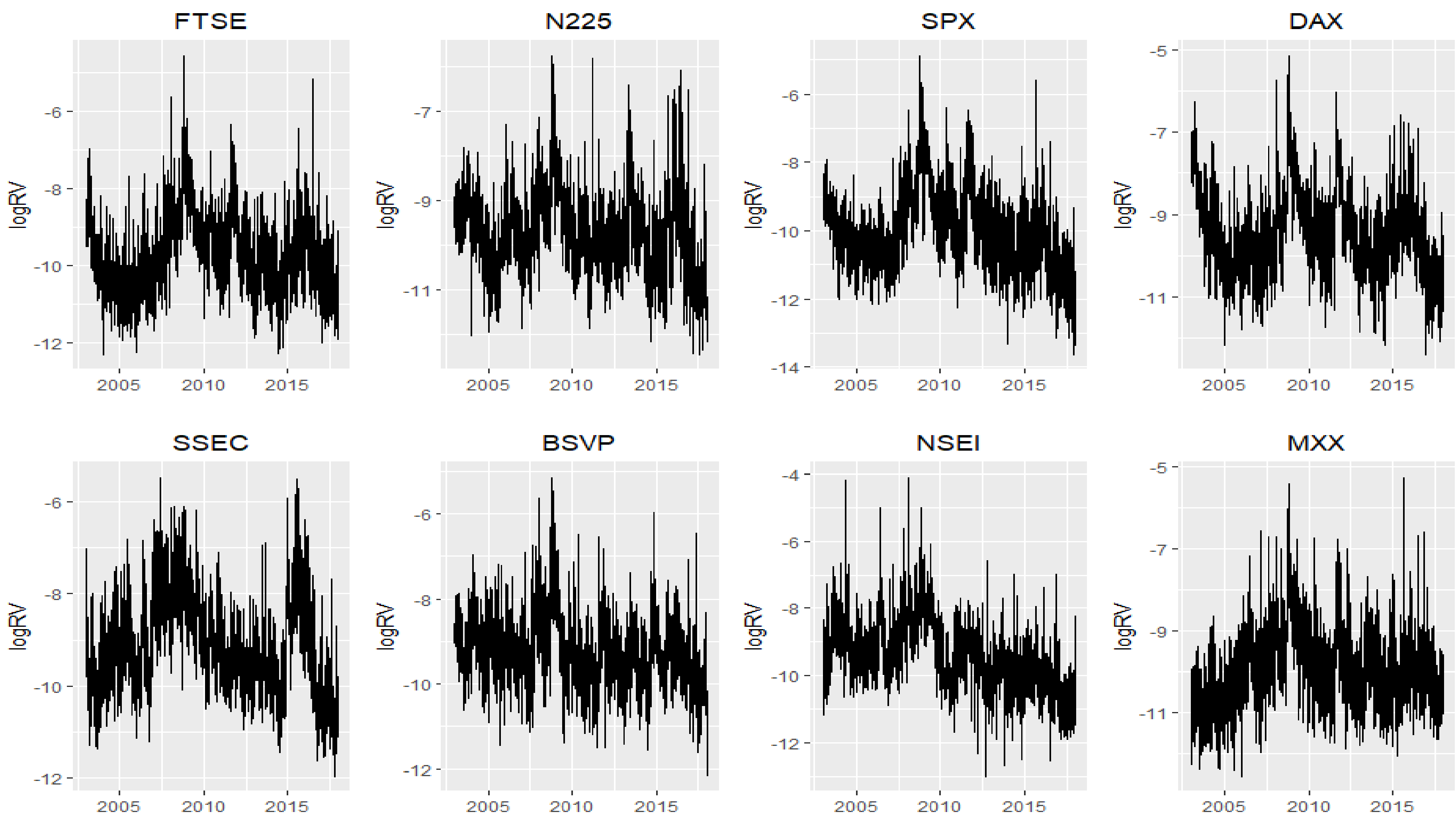


Figure 4.2: The Autocorrelation Function of eight stock indices

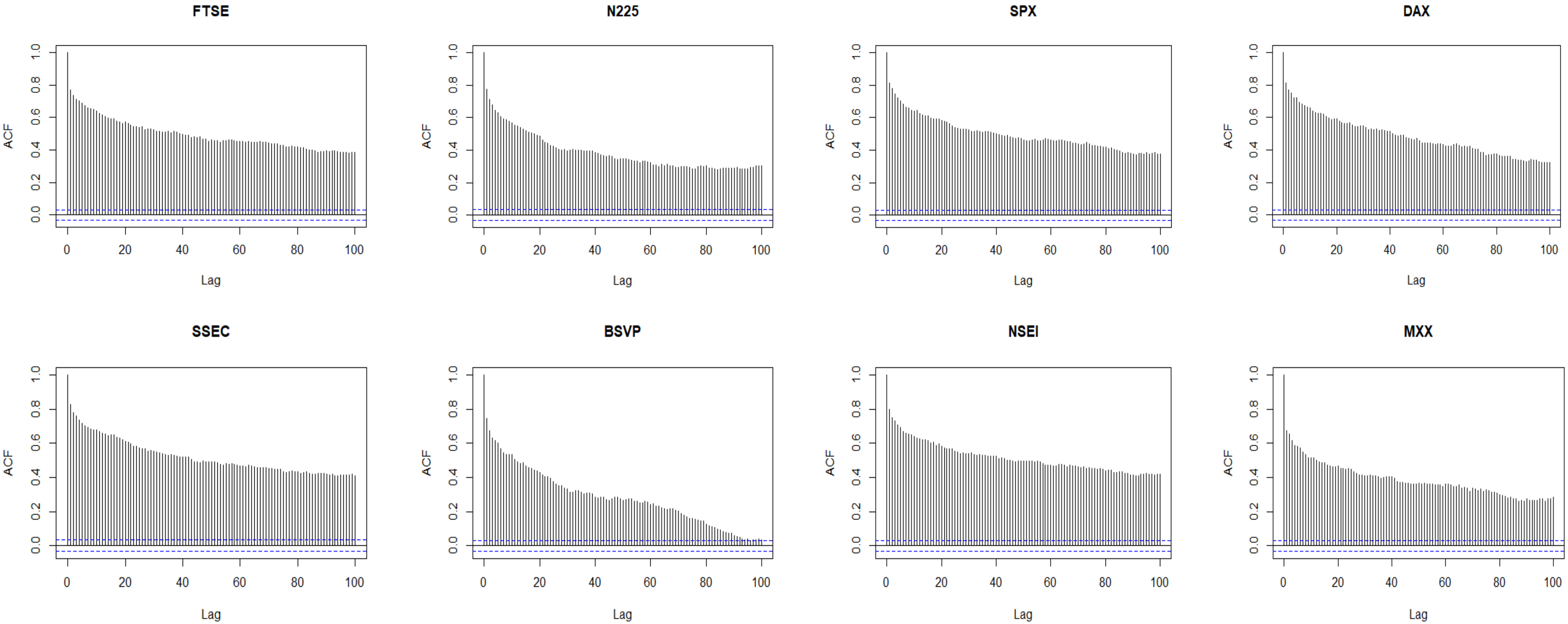
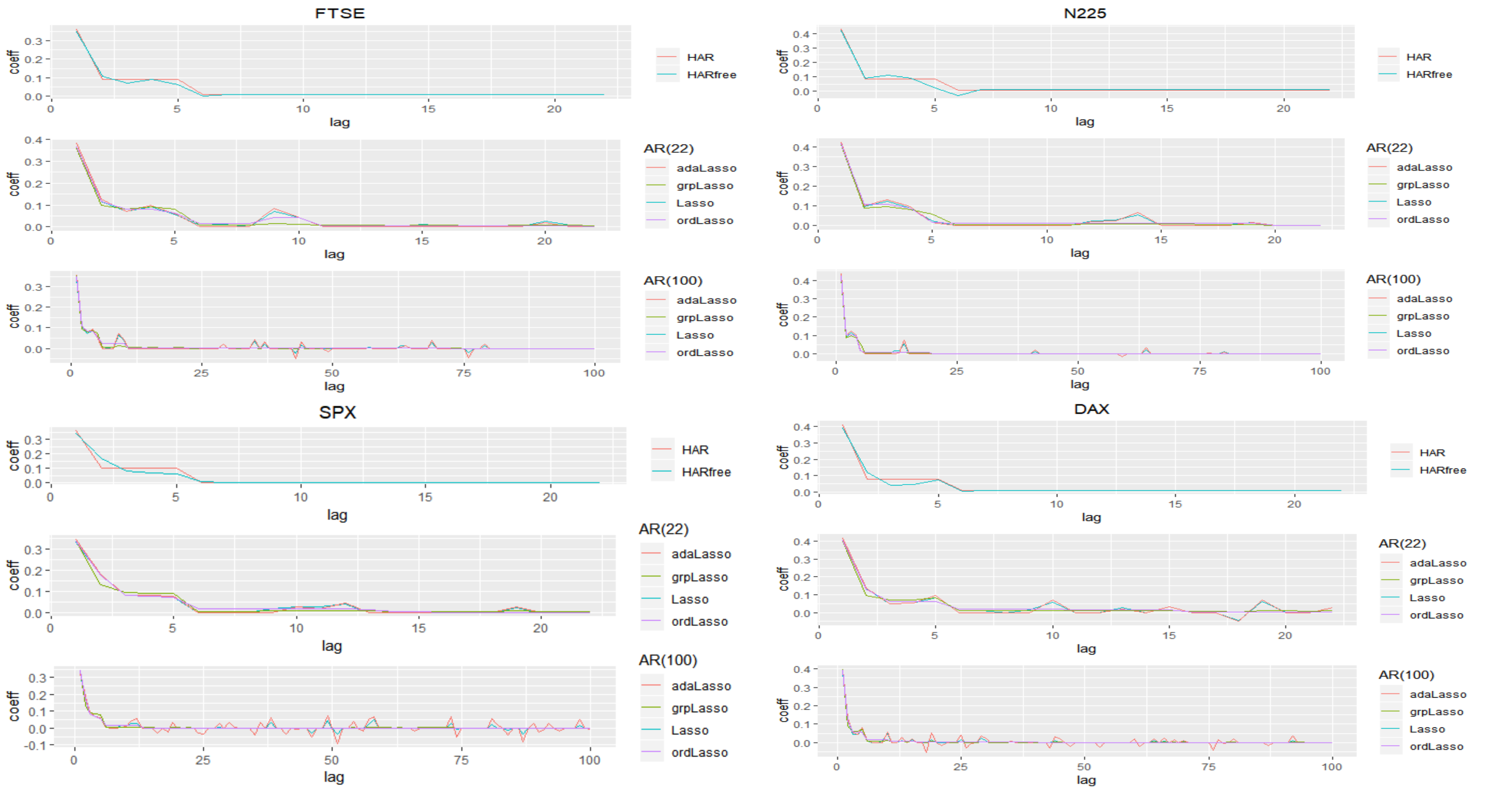
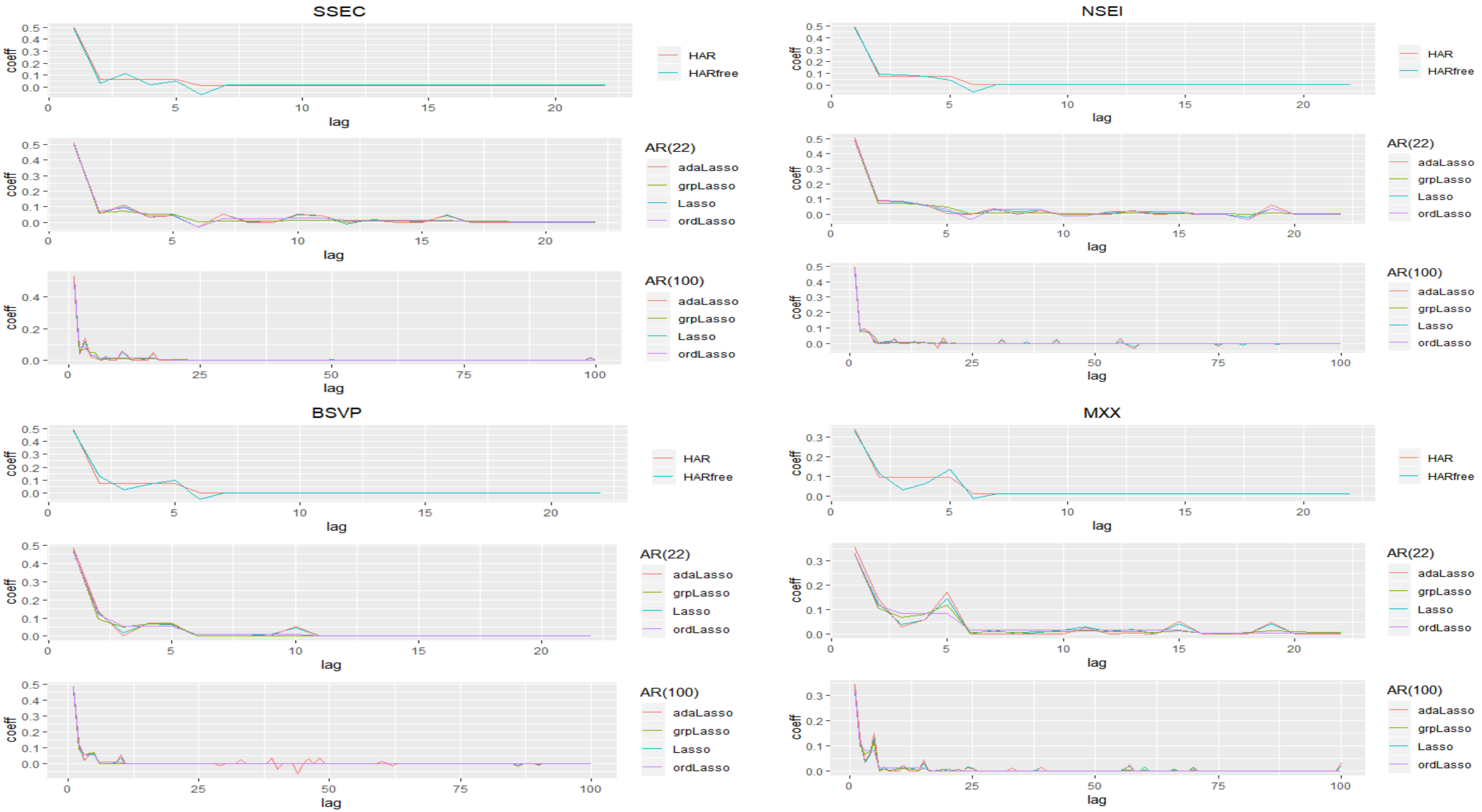


Figure 4.3: The plots of coefficients for eight stock indices in the in-sample period



- Continued on next page -

Figure 4.3: (continued)



Note: This figure shows the estimated coefficients for eight stock index in the in-sample period from 1st January 2003 to 31st December 2007. The upper figure is the fixed coefficients of the standard HAR model and HAR-free model. The middle figure is the flexible coefficients of AR(22) using Lasso-based method, and the bottom figure is the AR(100) estimated by Lasso-based method. Adalasso means the adaptive Lasso method, grpLasso means the grouped Lasso method, ordLasso means the ordered Lasso method, respectively.

Table 4.1: Statistics Description of Log-RV

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
Mean	-9.7402	-10.007	-9.7491	-9.4562	-9.1845	-9.4308	-9.2092	-10.002
Std.Dev.	1.0135	1.1337	0.9116	0.9930	1.0466	0.9983	0.8326	0.9455
Kurtosis	3.5395	3.7275	3.9222	3.4039	2.9057	3.7974	4.5988	3.8934
Skewness	0.6308	0.5336	0.4646	0.4171	0.4157	0.5109	0.6781	0.7380
Median	-9.8656	-10.124	-9.7761	-9.5317	-9.2967	-9.5095	-9.2636	-10.1405
25%-quantile	-10.472	-10.768	-10.339	-10.129	-9.9390	-10.123	-9.7399	-10.666
75%-quantile	-9.1138	-9.3334	-9.2109	-8.8467	-8.4997	-8.8174	-8.7582	-9.4284
AutoCorr$_{lag=1}$	0.7711	0.8128	0.7714	0.8121	0.8269	0.7957	0.7441	0.6752
Jarque-Bera	297.00***	262.06***	262.06***	136.40***	106.09***	260.79***	679.10***	468.11***
Obs.	3786	3771	3670	3811	3637	3726	3708	3774

Note: This table repots the summary statistics of log-RV of eight different stock index for the whole period from 1st January 2003 to 31st December 2017. *** indicate significant level at 1%.

Table 4.2: In-sample estimation error of MSE and QLIKE

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				MSE				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9991	0.9940	0.9946	0.9973	0.9924	0.9946	0.9951	0.9948
AR(22)-Lasso	0.9719	0.9685	**0.9714***	0.9571	**0.9847***	0.9808	0.9681	**0.9677***
AR(22)-adalasso	0.9743	0.9683	0.9715	0.9575	0.9859	0.9803	0.9676	0.9710
AR(22)-grpLasso	0.9837	0.9784	0.9827	0.9769	1.0038	0.9931	0.9770	0.9772
AR(22)-ordLasso	0.9785	0.9725	0.9758	0.9700	0.9888	0.9789	0.9717	0.9773
AR(100)-Lasso	0.9682	0.9481	0.9832	0.9767	0.9856	0.9788	0.9962	0.9745
AR(100)-adalasso	**0.9585***	**0.8981***	0.9801	**0.9432***	0.9889	**0.9721***	**0.9633***	0.9773
AR(100)-grpLasso	0.9941	0.9905	0.9995	0.9925	1.0025	1.0007	1.0017	0.9903
AR(100)-ordLasso	0.9910	0.9882	0.9913	0.9893	0.9953	0.9963	0.9964	0.9907
				QLIKE				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9993	0.9940	0.9950	0.9975	0.9898	0.9936	0.9950	0.9955
AR(22)-Lasso	0.9880	0.9804	**0.9754***	0.9793	**0.9723***	0.9728	0.9703	**0.9620***
AR(22)-adalasso	0.9900	0.9793	0.9756	0.9794	0.9736	0.9715	0.9698	0.9643
AR(22)-grpLasso	1.0006	0.9909	0.9859	1.0026	0.9924	0.9847	0.9792	0.9716
AR(22)-ordLasso	0.9939	0.9836	0.9796	0.9918	0.9766	**0.9698***	0.9739	0.9707
AR(100)-Lasso	0.9702	0.9486	0.9834	0.9774	0.9843	0.9814	0.9975	0.9758
AR(100)-adalasso	**0.9600***	**0.8960***	0.9803	**0.9447***	0.9864	0.9729	**0.9642***	0.9774
AR(100)-grpLasso	0.9955	0.9916	0.9990	0.9925	1.0017	1.0015	1.0024	0.9912
AR(100)-ordLasso	0.9916	0.9884	0.9915	0.9892	0.9942	0.9962	0.9971	0.9909

Note: This table reports the MSE and QLIKE value of eight RV indices for all forecasting models considered for the in-sample period from 1st January 2003 to 31st December 2007. The standard HAR model is regarded as benchmark against other forecasting models, the forecasting model with best performance is highlighted with *.

Table 4.3: Out-of-sample forecasting evaluation using MSE of rolling window forecasting

	FTSE	**SPX**	**N225**	**DAX**	**SSEC**	**NSEI**	**BSVP**	**MXX**
					h=1			
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	1.0021	0.9971	1.0034	1.0062	0.9977	0.9945	1.0048	0.9999
AR(22)-Lasso	1.2724	1.0729	1.0438	1.1079	1.1124	1.2088	1.0833	1.3920
AR(22)-adaLasso	1.0348	1.0352	0.9532	1.0771	1.0200	1.1500	1.0322	1.3291
AR(22)-grpLasso	0.8237	0.7726	0.6755	**0.6167***	0.6101	0.7803	0.7359	0.9786
AR(22)-ordLasso	0.8365	**0.4355***	0.8025	1.0188	0.9341	1.1335	**0.4972***	1.1906
AR(100)-Lasso	1.2627	1.0771	1.0322	1.0985	1.0972	1.2016	1.0804	1.4006
AR(100)-adalasso	1.1490	1.0447	1.0262	1.0805	**0.3150***	1.1694	0.8513	1.4000
AR(100)-grpLasso	0.8360	0.7735	0.6327	0.6631	0.6079	0.7379	0.6993	**0.9297***
AR(100)-ordLasso	**0.7678***	0.7094	**0.6246***	0.8337	0.7400	**0.7215***	0.9652	1.0031
					h=5			
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9810	0.9680	0.9946	0.9755	0.9852	0.9834	0.9782	0.9629
AR(22)-Lasso	0.3677	0.3493	0.3674	0.3840	0.3137	0.3607	**0.3722***	0.4209
AR(22)-adalasso	0.3798	0.3641	0.3969	0.4436	0.3667	0.4105	0.6261	0.4665
AR(22)-grpLasso	0.4137	0.3547	0.3757	0.4696	0.4402	0.3859	0.4000	0.4708
AR(22)-ordLasso	**0.3442***	**0.3282***	**0.3414***	**0.3620***	**0.2904***	**0.3431***	0.3764	**0.4016***
AR(100)-Lasso	0.3737	0.3513	0.3734	0.3853	0.3199	0.3686	0.3900	0.4377
AR(100)-adalasso	0.3992	0.3626	0.4982	0.4744	0.4162	0.4778	0.5475	0.4840
AR(100)-grpLasso	0.4138	0.3599	0.3980	0.4296	0.4970	0.4091	0.4247	0.5147
AR(100)-ordLasso	0.3539	0.3365	0.4342	0.3735	0.4651	0.3811	0.4271	0.5398
					h=22			
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9963	0.9976	0.9898	0.9994	0.9954	0.9970	0.9939	0.9958
AR(22)-Lasso	0.5175	0.5276	0.5501	0.5406	0.5489	0.5420	0.6595	0.4640
AR(22)-adalasso	0.5292	0.5441	0.6075	0.5403	0.5512	0.8091	0.7109	0.4629
AR(22)-grpLasso	0.5325	0.5332	0.5546	0.5797	0.6260	0.5493	0.6612	0.4760
AR(22)-ordLasso	**0.5083***	**0.5213***	0.5433	**0.5303***	**0.5359***	**0.5537***	**0.6467***	**0.4547***
AR(100)-Lasso	0.5276	0.5339	0.5410	0.5364	0.5448	0.5670	0.6518	0.4856
AR(100)-adalasso	0.5616	0.6370	0.6207	0.6062	0.5803	0.7718	0.6877	0.5338
AR(100)-grpLasso	0.5309	0.5284	0.5418	0.5414	0.6409	0.5733	0.6519	0.4955
AR(100)-ordLasso	0.5217	0.5426	**0.5258***	0.5341	0.5498	0.5765	0.6922	0.4755

Note: This table reports the forecasting evaluation (MSE) of eight RV indices for all forecasting models considered using rolling window approach (window size = 1000) over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The standard HAR model is regarded as benchmark and the forecasting model with the best performance is highlighted with *.

Table 4.4: Out-of-sample forecasting evaluation using MSE of increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9975	0.9938	1.0013	0.9967	0.9972	0.9946	1.0010	0.9924
AR(22)-Lasso	1.2744	1.0725	1.0462	1.1106	1.1194	1.1858	1.0771	1.3904
AR(22)-adalasso	0.9912	1.0244	0.9433	1.0751	1.0265	1.1240	1.0280	1.3124
AR(22)-grpLasso	0.8072	0.7350	0.6482	**0.6029***	0.6001	0.7561	0.7285	0.9524
AR(22)-ordLasso	1.0439	0.6534	0.9308	1.0698	1.0345	1.1559	**0.4838***	1.2890
AR(100)-Lasso	1.2657	1.0767	1.0346	1.1022	1.1065	1.1800	1.0751	1.4001
AR(100)-adalasso	1.1285	1.0375	1.0277	1.0788	**0.2403***	1.1511	0.8366	1.3915
AR(100)-grpLasso	0.8219	0.7352	0.6082	0.6501	0.5842	0.7141	0.6906	**0.9029***
AR(100)-ordLasso	**0.5660***	**0.4937***	**0.4401***	0.9755	0.9308	**0.4548***	0.7162	1.1842
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9796	0.9656	0.9950	0.9731	0.9825	0.9810	0.9776	0.9622
AR(22)-Lasso	0.3683	0.3318	0.3716	0.3763	0.3207	0.3487	0.3664	0.4042
AR(22)-adalasso	0.3855	0.3513	0.4048	0.4451	0.3785	0.3921	0.6226	0.4654
AR(22)-grpLasso	0.4145	0.3557	0.3859	0.4465	0.3929	0.3461	0.3861	0.4629
AR(22)-ordLasso	**0.3479***	**0.3207***	**0.3516***	0.3617	**0.3035***	0.3382	**0.3467***	**0.3929***
AR(100)-Lasso	0.3740	0.3354	0.3750	0.3795	0.3250	0.3546	0.3851	0.4196
AR(100)-adalasso	0.4046	0.3460	0.4975	0.4701	0.4209	0.4453	0.5361	0.4770
AR(100)-grpLasso	0.4174	0.3609	0.4105	0.4143	0.4308	0.3603	0.4083	0.5084
AR(100)-ordLasso	0.3537	0.3218	0.3691	**0.3506***	0.3324	**0.3228***	0.3659	0.4479
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9945	0.9958	0.9887	0.9971	0.9927	0.9945	0.9926	0.9939
AR(22)-Lasso	0.5088	0.5399	0.5560	0.5135	0.5376	0.5205	0.6454	0.4603
AR(22)-adalasso	0.5192	0.5538	0.6111	0.5151	0.5444	0.7618	0.6959	0.4597
AR(22)-grpLasso	0.5146	0.5418	0.5647	0.5261	0.5780	**0.5130***	0.6521	0.4628
AR(22)-ordLasso	**0.5046***	**0.5363***	0.5520	**0.5101***	**0.5288***	0.5139	**0.6319***	**0.4563***
AR(100)-Lasso	0.5185	0.5505	0.5496	0.5114	0.5417	0.5331	0.6417	0.4823
AR(100)-adalasso	0.5502	0.6706	0.6300	0.5807	0.5758	0.7163	0.6797	0.5424
AR(100)-grpLasso	0.5212	0.5499	0.5600	0.5191	0.5884	0.5227	0.6460	0.4905
AR(100)-ordLasso	0.5193	0.5603	**0.5353***	0.5111	0.5550	0.5345	0.6473	0.4792

Note: This table reports the forecasting evaluation (MSE) of eight RV indices for all forecasting models considered using increasing window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The standard HAR model is regarded as benchmark and the forecasting model with the best performance is highlighted with *.

Table 4.5: Out-of-sample forecasting evaluation using QLIKE of rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	1.0018	0.9980	1.0028	1.0064	0.9974	0.9950	1.0040	1.0004
AR(22)-Lasso	1.2850	1.0946	1.0550	1.0941	1.1251	1.2198	1.0902	1.4162
AR(22)-adaLasso	1.0324	1.0520	0.9608	1.0617	1.0244	1.1563	1.0357	1.3453
AR(22)-grpLasso	0.8219	0.7790	0.6799	**0.6076***	**0.6145***	0.7774	0.7372	0.9817
AR(22)-ordLasso	0.8322	0.4416	0.8088	1.0043	0.9416	1.1405	**0.5018***	1.2025
AR(100)-Lasso	1.2836	1.1085	1.0480	1.0919	1.1154	1.2215	1.0899	1.4292
AR(100)-adalasso	1.1570	1.0730	1.0411	1.0730	0.3322	1.1853	0.8550	1.4285
AR(100)-grpLasso	0.8409	0.7861	**0.6412***	0.6567	0.6185	0.7402	0.7029	**0.9346***
AR(100)-ordLasso	**0.7852***	**0.7430***	0.6456	0.8250	0.7491	**0.7253***	0.9912	1.0096
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9781	0.9695	0.9924	0.9780	0.9844	0.9860	0.9774	0.9645
AR(22)-Lasso	0.3660	0.3462	0.3776	0.3752	0.3179	0.3759	**0.3717***	0.4200
AR(22)-adalasso	0.3800	0.3653	0.4103	0.4332	0.3712	0.4172	0.6214	0.4661
AR(22)-grpLasso	0.4207	0.3604	0.3922	0.4725	0.4479	0.3920	0.4072	0.4760
AR(22)-ordLasso	**0.3443***	**0.3255***	**0.3517***	**0.3538***	**0.2930***	**0.3575***	0.3819	**0.4022***
AR(100)-Lasso	0.3757	0.3538	0.3864	0.3806	0.3273	0.3864	0.3952	0.4384
AR(100)-adalasso	0.4055	0.3684	0.5095	0.4752	0.4246	0.4881	0.5512	0.4818
AR(100)-grpLasso	0.4231	0.3698	0.4188	0.4341	0.5116	0.4165	0.4355	0.5220
AR(100)-ordLasso	0.3557	0.3406	0.4572	0.3699	0.4765	0.3877	0.4378	0.5485
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9958	0.9972	0.9884	0.9988	0.9961	0.9958	0.9943	0.9957
AR(22)-Lasso	0.5222	0.5294	0.5475	0.5378	0.5611	**0.5368***	0.6509	0.4637
AR(22)-adalasso	0.5367	0.5503	0.6061	0.5379	0.5625	0.7800	0.7071	0.4630
AR(22)-grpLasso	0.5437	0.5415	0.5549	0.5896	0.6430	0.5379	0.6578	0.4794
AR(22)-ordLasso	**0.5133***	**0.5243***	**0.5404***	**0.5282***	**0.5487***	0.5439	**0.6361***	**0.4547***
AR(100)-Lasso	0.5393	0.5454	0.5474	0.5423	0.5625	0.5590	0.6489	0.4888
AR(100)-adalasso	0.5734	0.6530	0.6305	0.6143	0.5996	0.7482	0.6841	0.5352
AR(100)-grpLasso	0.5499	0.5455	0.5512	0.5555	0.6595	0.5621	0.6552	0.5025
AR(100)-ordLasso	0.5381	0.5642	0.5304	0.5472	0.5699	0.5598	0.6994	0.4804

Note: This table reports the forecasting evaluation (QLIKE) of eight RV indices for all forecasting models considered using rolling window approach (window size = 1000) over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The standard HAR model is regarded as benchmark, the forecasting model with the best performance is highlighted with *.

Table 4.6: Out-of-sample forecasting evaluation using QLIKE of increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9975	0.9943	1.0012	0.9973	0.9971	0.9949	1.0004	0.9928
AR(22)-Lasso	1.2876	1.0936	1.0580	1.0968	1.1339	1.1987	1.0848	1.4134
AR(22)-adalasso	0.9912	1.0408	0.9514	1.0599	1.0328	1.1331	1.0325	1.3262
AR(22)-grpLasso	0.8066	0.7423	0.6531	**0.5940***	0.6040	0.7551	0.7298	0.9554
AR(22)-ordLasso	1.0404	0.6468	0.9383	1.0545	1.0438	1.1645	**0.4822***	1.3049
AR(100)-Lasso	1.2872	1.1075	1.0512	1.0959	1.1269	1.2017	1.0857	1.4273
AR(100)-adalasso	1.1366	1.0657	1.0433	1.0717	**0.2546***	1.1693	0.8422	1.4175
AR(100)-grpLasso	0.8276	0.7487	0.6169	0.6444	0.5934	0.7182	0.6944	**0.9079***
AR(100)-ordLasso	**0.5898***	**0.5307***	**0.4596***	0.9645	0.9417	**0.4738***	0.7476	1.1958
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9774	0.9669	0.9937	0.9752	0.9819	0.9837	0.9768	0.9636
AR(22)-Lasso	0.3674	0.3313	0.3831	0.3667	0.3245	0.3650	0.3673	0.4043
AR(22)-adalasso	0.3865	0.3557	0.4195	0.4329	0.3826	0.4014	0.6209	0.4653
AR(22)-grpLasso	0.4238	0.3668	0.4037	0.4457	0.3958	0.3594	0.3961	0.4706
AR(22)-ordLasso	**0.3476***	**0.3200***	**0.3626***	**0.3518***	**0.3067***	**0.3531***	**0.3517***	**0.3937***
AR(100)-Lasso	0.3767	0.3405	0.3899	0.3735	0.3318	0.3740	0.3922	0.4216
AR(100)-adalasso	0.4106	0.3549	0.5099	0.4662	0.4274	0.4612	0.5415	0.4750
AR(100)-grpLasso	0.4284	0.3758	0.4325	0.4154	0.4397	0.3764	0.4226	0.5193
AR(100)-ordLasso	0.3553	0.3265	0.3894	0.3442	0.3370	0.3376	0.3761	0.4579
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9942	0.9955	0.9878	0.9966	0.9931	0.9936	0.9931	0.9939
AR(22)-Lasso	0.5134	0.5405	0.5554	0.5081	0.5509	0.5208	0.6384	0.4591
AR(22)-adalasso	0.5269	0.5614	0.6129	0.5103	0.5570	0.7445	0.6945	0.4589
AR(22)-grpLasso	0.5264	0.5524	0.5676	0.5298	0.5919	**0.5112***	0.6514	0.4672
AR(22)-ordLasso	**0.5092***	**0.5378***	**0.5509***	**0.5046***	**0.5427***	0.5124	**0.6219***	**0.4551***
AR(100)-Lasso	0.5297	0.5619	0.5573	0.5130	0.5595	0.5337	0.6399	0.4840
AR(100)-adalasso	0.5609	0.6833	0.6420	0.5834	0.5945	0.7085	0.6780	0.5406
AR(100)-grpLasso	0.5397	0.5693	0.5708	0.5274	0.6032	0.5236	0.6516	0.4987
AR(100)-ordLasso	0.5309	0.5777	0.5425	0.5155	0.5771	0.5323	0.6520	0.4814

Note: This table reports the forecasting evaluation (QLIKE) of eight RV indices for all forecasting models considered using increasing window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st January 2008 to 31st December 2017. The standa rd HAR model is regarded as benchmark, the forecasting model with the best performance is highlighted with *.

Table 4.7: The Model Confidence Set test of MSE criterion for rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.4220	0.0000	0.0000	0.3382	0.0000	**1.0000***
AR(100)-ordLasso	**1.0000***	0.0000	**1.0000***	0.0000	0.0000	**1.0000***	0.0000	0.0000
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9748	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.0000	0.0000	0.3062	0.0000	0.0000	0.0000	0.0000
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	**1.0000***	0.8446	0.7626	0.5592	0.0000	**1.0000***	**1.0000***	**1.0000***
AR(22)-adalasso	0.0000	0.0000	0.0000	0.4852	0.0000	0.0000	0.0000	**1.0000***
AR(22)-grpLasso	0.7292	0.8700	0.7400	0.0000	0.0000	0.9818	**1.0000***	0.4914
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.8410	**1.0000***	**1.0000***
AR(100)-Lasso	0.7944	**1.0000***	0.9996	**1.0000***	**1.0000***	0.4626	**1.0000***	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2748	0.0000
AR(100)-grpLasso	0.9636	**1.0000***	**1.0000***	**1.0000***	0.0000	0.0000	**1.0000***	0.0000
AR(100)-ordLasso	**1.0000***	0.9998	**1.0000***	**1.0000***	0.8534	0.5212	0.6980	0.9754

Note: This table reports the MSC test in terms of MSE criterion for eight RV indices over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 4.8: The Model Confidence Set test of QLIKE criterion for rolling window forecasting

	FTSE	**SPX**	**N225**	**DAX**	**SSEC**	**NSEI**	**BSVP**	**MXX**
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0560
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0598
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.3184	0.0000	**1.0000***
AR(100)-ordLasso	**1.0000***	0.0000	0.8654	0.0000	0.0000	**1.0000***	0.0000	0.0000
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.9294	**1.0000***	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.3844	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.3896	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.6110	0.0000	0.0000
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	**1.0000***	**1.0000***	0.9996	**1.0000***	0.8272	**1.0000***	**1.0000***	**1.0000***
AR(22)-adalasso	0.5454	0.0000	0.0000	**1.0000***	0.4976	0.0000	0.0000	**1.0000***
AR(22)-grpLasso	0.4128	0.9042	0.6218	0.0000	0.0000	**1.0000***	**1.0000***	0.2638
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9950	**1.0000***	**1.0000***
AR(100)-Lasso	0.9998	**1.0000***	0.9290	**1.0000***	**1.0000***	0.9556	**1.0000***	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4628	0.0000
AR(100)-grpLasso	0.4506	**1.0000***	0.7836	0.5426	0.0000	0.6166	**1.0000***	0.0000
AR(100)-ordLasso	0.9980	0.5332	**1.0000***	0.9758	0.9798	0.5134	0.4462	0.7268

Note: This table reports the MSC test in terms of QLIKE criterion for eight RV indices over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 4.9: The Model Confidence Set test of MSE criterion for increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***
AR(100)-ordLasso	**1.0000***	**1.0000***	**1.0000***	0.0000	0.0000	**1.0000***	0.0000	0.0000
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	0.8316	**1.0000***	0.0000	**1.0000***	0.0000	**1.0000***	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	**1.0000***	0.0000	**1.0000***	0.0000	**1.0000***	0.0000	0.0000
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	**1.0000***	0.9994	0.9384	0.9996	**1.0000***	**1.0000***	0.9656	**1.0000***
AR(22)-adalasso	0.4268	0.4296	0.0000	0.9712	0.2510	0.0000	0.0000	**1.0000***
AR(22)-grpLasso	**1.0000***	**1.0000***	0.4736	0.9364	0.0000	**1.0000***	0.9534	**1.0000***
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
AR(100)-Lasso	0.9850	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.8360	**1.0000***	0.2800
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	**1.0000***	**1.0000***	0.8130	**1.0000***	0.0000	**1.0000***	**1.0000***	0.4392
AR(100)-ordLasso	0.9554	0.9672	**1.0000***	**1.0000***	0.5804	0.7936	0.9982	0.6746

Note: This table reports the MSC test in terms of MSE criterion for eight RV indices over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 4.10: The Model Confidence Set test of QLIKE criterion increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***
AR(100)-ordLasso	**1.0000***	**1.0000***	**1.0000***	0.0000	0.0000	**1.0000***	0.0000	0.0000
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.6630	0.0000	0.8950	0.0000	0.5874	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***	0.0000	**1.0000***	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.3592	0.0000	0.4018	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.3756	0.7478	**1.0000***	0.3962	**1.0000***	0.3280	0.0000
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9870	**1.0000***	**1.0000***
AR(22)-adalasso	0.5228	0.4144	0.0000	**1.0000***	0.7708	0.0000	0.0000	**1.0000***
AR(22)-grpLasso	0.9976	**1.0000***	0.3062	0.4004	0.0000	**1.0000***	0.4178	0.5058
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
AR(100)-Lasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9546	**1.0000***	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.5468	0.7796	0.5360	0.5926	0.0000	**1.0000***	0.6544	0.0000
AR(100)-ordLasso	0.9942	0.5448	**1.0000***	**1.0000***	0.3514	0.9916	0.8990	0.0000

Note: This table reports the MSC test in terms of QLIKE criterion for eight RV indices over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 4.11: Summary of 1% and 5% VaR failure rates of rolling window forecasting

	1%			5%		
	Ave. failure rate	Sig. Weibull test	Sig. DQ test	Ave. failure rate	Sig. Weibull test	Sig. DQ test
			h=1			
HAR model	5.3714%	FTSE	All	9.1453%	SPX, NSEI, N225, BSVP	All
HAR-free model	5.3713%	FTSE	All	9.0895%	SPX, NSEI, N225, BSVP	All
AR(22)-Lasso	5.5575%	FTSE, SPX, NSEI, BSVP, DAX	All	9.3298%	SSEC, SPX, NSEI, N225, MXX	All
AR(22)-Lasso	5.4286%	SSEC, SPX, NSEI, N225, BSVP, DAX	All	9.2454%	SSEC, SPX, NSEI, N225, MXX	All
AR(22)-grpLasso	4.7532%	SPX, BSVP, DAX	All	8.7333%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(22)-ordLasso	4.7745%	SSEC, SPX, NSEI, DAX	All	8.8177%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(100)-Lasso	5.6025%	SSEC, SPX, NSEI, BSVP, DAX	All	9.3873%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(100)-adaLasso	5.4000%	SPX, NSEI, BSVP, DAX	All	9.1458%	SPX, NSEI, N225, DAX, MXX	All
AR(100)-grpLasso	4.7948%	SPX, N225, BSVP, DAX	All	8.7888%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(100)-ordLasso	**4.4469%***	SPX, NSEI, BSVP, DAX	All	**8.5669%***	SSEC, SPX, NSEI, N225, DAX, MXX	All
			h=5			
HAR model	4.5038%	NSEI, N225, DAX, MXX	All	8.4064%	All	All
HAR-free model	4.5234%	NSEI, N225, DAX, MXX	All	8.3861%	All	All
AR(22)-Lasso	4.5122%	N225	All	8.3489%	FTSE, NSEI, N225, DAX	All
AR(22)-adaLasso	4.5038%	N225	All	8.3275%	FTSE, SSEC, N225, DAX	All
AR(22)-grpLasso	4.3012%	N225	All	8.0391%	FTSE, SSEC, N225, DAX	All
AR(22)-ordLasso	**4.2845%***	N225	All	**7.8476%***	FTSE, NSEI, N225	All
AR(100)-Lasso	4.3444%	N225	All	8.2072%	FTSE, SSEC, NSEI, N225, DAX, MXX	All
AR(100)-adaLasso	4.3444%	N225	All	8.1275%	FTSE, SSEC, NSEI, N225, DAX, MXX	All
AR(100)-grpLasso	4.3245%	N225	All	8.0857%	FTSE, SSEC, NSEI, N225, DAX	All
AR(100)-ordLasso	4.3452%	N225	All	7.9486%	FTSE, SSEC, N225	All
			h=22			
HAR model	3.4849%	None	All	7.3307%	None	All
HAR-free model	3.4826%	None	All	7.3307%	None	All
AR(22)-Lasso	3.4109%	None	All	7.1315%	None	All
AR(22)-adaLasso	3.4109%	None	All	7.0558%	None	All
AR(22)-grpLasso	3.3519%	None	All	6.8932%	None	All
AR(22)-ordLasso	**3.3379%***	None	All	**6.6159%***	None	All
AR(100)-Lasso	3.4502%	None	All	6.9558%	None	All
AR(100)-adaLasso	3.4226%	None	All	6.8127%	None	All
AR(100)-grpLasso	3.4488%	None	All	6.8127%	None	All
AR(100)-ordLasso	3.4249%	None	All	6.7307%	None	All

Notes: this table provides the VaR results of rolling window test at the 1% and 5% VaR level. The average failure rate for each model over each index. The series are significant in the Weibull test and DQ test are listed.

Table 4.12: Summary of 1% and 5% VaR test of increasing window forecasting

	1%			5%		
	Ave. failure rate	Sig. Weibull test	Sig. DQ test	Ave. failure rate	Sig. Weibull test	Sig. DQ test
			h=1			
HAR model	5.3532%	NSEI	All	9.1382%	SPX, NSEI, N225, BSVP	All
HAR-free model	5.3390%	FTSE	All	9.1173%	SPX, NSEI, N225, BSVP	All
AR(22)-Lasso	5.5421%	FTSE, SPX, NSEI, DAX	All	9.3158%	SSEC, SPX, NSEI, N225, MXX	All
AR(22)-adaLasso	5.3986%	SPX, NSEI, BSVP, DAX	All	9.2429%	SSEC, SPX, NSEI, N225, MXX, BSVP	All
AR(22)-grpLasso	4.7291%	SPX, BSVP	All	8.7509%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(22)-ordLasso	5.0889%	SPX, NSEI, DAX	All	9.1211%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(100)-Lasso	5.6850%	SSEC, SPX, NSEI, BSVP, DAX	All	9.3886%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(100)-adaLasso	5.3490%	FTSE, SPX, NSEI, BSVP, DAX	All	9.1603%	SPX, NSEI, N225, DAX, MXX	All
AR(100)-grpLasso	4.7988%	SPX, N225, BSVP, DAX	All	8.7822%	SSEC, SPX, NSEI, N225, DAX, MXX	All
AR(100)-ordLasso	**4.4332%***	FTSE, SSEC, SPX, DAX	All	**8.5790%***	SSEC, SPX, NSEI, N225, DAX, MXX	All
			h=5			
HAR model	4.5458%	NSEI, N225, DAX, MXX	All	8.6420%	All	All
HAR-Free model	4.5431%	NSEI, N225, DAX, MXX	All	8.5185%	All	All
AR(22)-Lasso	4.4451%	N225	All	8.4774%	FTSE, NSEI, N225, DAX	All
AR(22)-adaLasso	4.3444%	N225	All	8.1720%	FTSE, SSEC, N225, BSVP, DAX	All
AR(22)-grpLasso	4.3241%	N225	All	7.6132%	FTSE, SSEC, NSEI, N225	All
AR(22)-ordLasso	**4.3045%***	N225	All	**7.2774%***	FTSE, NSEI, N225	All
AR(100)-Lasso	4.5038%	N225	All	8.5597%	FTSE, NSEI, N225, DAX, MXX	All
AR(100)-adaLasso	4.4934%	N225	All	8.4362%	FTSE, SSEC, NSEI, N225, DAX, MXX	All
AR(100)-grpLasso	4.3491%	N225	All	7.8021%	FTSE, SSEC, NSEI, N225	All
AR(100)-ordLasso	4.3300%	N225	All	7.6601%	FTSE, N225	All
			h=22			
HAR model	3.6206%	None	All	7.4216%	None	All
HAR-free model	3.6127%	None	All	7.4097%	None	All
AR(22)-Lasso	3.5266%	None	All	7.0782%	None	All
AR(22)-adaLasso	3.5407%	None	All	7.0251%	None	All
AR(22)-grpLasso	3.4238%	None	All	6.5821%	None	All
AR(22)-ordLasso	**3.3417%***	None	All	**6.5044%***	None	All
AR(100)-Lasso	3.6502%	None	All	7.2074%	None	All
AR(100)-adaLasso	3.6431%	None	All	7.1809%	None	All
AR(100)-grpLasso	3.5069%	None	All	7.0016%	None	All
AR(100)-ordLasso	3.5106%	None	All	6.8104%	None	All

Notes: this table provides the VaR results of increasing window test at the 1% and 5% VaR level. The average failure rate for each model over each index. The series are significant in the Weibull test and DQ test are listed.

Chapter 5 Forecasting Realised Volatility Using Non-linear Threshold and Regime-Switching Approach

5.1 Introduction

Accurately modelling and forecasting financial volatility is of crucial importance for risk management, derivative allocation, and asset pricing. Andersen and Bollerslev (1998) propose the Realised Volatility (RV), which employs the sum of squared intraday returns as an alternative approach to measure financial volatility. Unlike low-frequency data, the RV is observable rather than latent. Subsequently, the RV has been increasingly used in current research and has overtaken the GARCH and stochastic volatility setup. The vital feature of RV is that it exhibits high persistence and has a long memory feature (Andersen et al., 2003 and Lieberman and Philips, 2008). The HAR model (Corsi, 2009) uses the aggregated daily, weekly, and monthly RV to simply exhibit the volatility persistence. Currently, the HAR model and its extensions have dominated modelling and forecasting volatility (Andersen et al., 2012; Buncic and Gisler, 2017; Patton and Sheppard, 2015; Horpestad et al., 2019).

One of the primary investigations of current papers is that the forecasting performance of the HAR model dominates among forecasting models, as it effectively captures the high persistence of RV. However, Granger and Ding (1996) and Longin (1997) provide evidence that the persistence of volatility is not always consistent, and suggest that the persistence is nonlinear. In fact, the volatility occasionally exhibits high and low regimes, the high regimes in the market are associated with extreme events, including financial crisis or sudden policy changes, and in which the high regimes are usually short-lived (Medeiros, 2008 and Cipollini et al. 2017). In addition, the low and high regimes are also linked to cyclical economic expansions and recessions (Hamilton, 1989).

Baillie and Kapetanios (2007) and Ohanissian et al. (2008) confirm that the nonlinearity and long memory also exist in RV. Thus, the nonlinear persistence of RV can lead to a better prediction (Raggi and Bordignon, 2012). In order to obtain more accurate forecasts, the forecasting models should require different ways to treat structural breaks and regime-switching. For such reasons, the HAR models are incorporated with nonlinear models in regime-switching frameworks, such as smooth transition (McAleer and Medeiros, 2008; Qu et al., 2016) and Markov-switching (Liu et al., 2012 and Ma et al., 2017), which consider the existence of two different volatility regimes.

Against this backdrop, there are inherent limits in the literature; thus far, they have refrained from performing a systematic forecasting performance assessment of the nonlinear frameworks combined with the HAR model. This chapter seeks to improve upon the previous literature for a range of international stock indices in three ways. First, this chapter explores the question of whether nonlinear models combined with the HAR model could improve the forecasting performance of RV over linear alternatives, and seeks to investigate which nonlinear models could accomplish this. Most of those models have been considered separately in the specific papers, but the wider range of nonlinear models and different forecasting horizons are novel. Therefore, this chapter begins with the linear AR model and HAR model, and then incorporates them with the smooth transition and Markov-switching approaches to compare the forecasting performance among the considered models. Second, to examine the time-variation in the regime-switching model, this chapter extends the Markov-switching model by considering the time-varying transition probability. In addition, this chapter also considers the variance shifts between the regimes in the Markov-switching model and implies the heteroscedasticity in the Markov-switching dynamics. Finally, this chapter evaluates the forecasting performance not only in the statistical aspect, which entails similar work to existing papers, but also in economic forecasting evaluation in terms of calculating the Value at Risk (VaR) and Expected Shortfall (ES).

To preview the results in this chapter, first, for in-sample results, the regime-switching HAR models are preferred over linear models. Additionally, the Markov-switching HAR models have a better goodness-of-fit than the smooth transition approaches. Second, for out-of-sample results, there is no clear evidence that the regime-switching HAR models improve forecasting performance on the daily forecasting horizon, though the linear HAR model also performs well. In contrast, the Markov-switching HAR model dominates forecasting performance for weekly and monthly horizons. Lastly, the regime-switching models hardly improve forecasting evaluation on the daily level for risk management application. The same applies for the smooth transition AR models at the daily level and the Markov-switching models with time-varying transition probability at the weekly and monthly levels.

The rest of this chapter is shown as follows. Section 2 provides a review of relevant literature. Section 3 introduces the smooth transition and Markov-switching models considered in this chapter, as well as the forecasting evaluation methodology. Sections 4 and 5 provide the empirical data and findings of in-sample estimation and out-of-sample exercise. Then, section 6 provides the results of risk management applications. The conclusion is provided in section 7.

5.2 Literature Review

The volatility of financial returns has been widely recognized as a latent proxy for obtaining accurate estimators, as well as risk management. It plays a crucial role in financial research. Thus far, in the field of volatility modelling, the most popular nonlinear model is the GARCH model proposed by Bollerslev (1986), A growing number of volatility models have been developed to incorporate the volatility features to extend and modify the GARCH model [17](e.g., Engle and Bollerslev, 1986; Engle and Lee, 1993 and Baillie et al., 1996; Zakoian, 1990; Nelson, 1991 and Glosten et al., 1993). The GARCH family models have made great empirically achievements

[17] A more complete literature can be seen in Section 2.2.

and obtained outstanding performance in forecasting conditional volatility (see Hansen and Lunde, 2005; Awartani and Corradi 2005 and Wei et al., 2010).

Additionally, the Stochastic Volatility (SV) model (Taylor, 1986) is considered to be an alternative to the GARCH model to estimate conditional volatility. Similar to the GARCH model, the various nonlinear extensions of SV models have also been widened. Robinson and Zaffaroni (1998) and Robinson (2001) capture the nonlinear long memory of the SV model, Yu (2005) incorporates the nonlinear SV model with leverage effect, and Elliott et al. (2012) add the impact of business cycles. However, none of these aforementioned models dominate the others. As noted by Carnero et al. (2004) and Malmsten and Teräsvirta (2010), these models hardly reproduce the stylized facts in typical financial volatility.

Nonlinearity in Realised Volatility

Due to the inappropriateness mentioned above of GARCH models and SV models, it is always important to find superior estimators. The estimators of these models are usually daily or lower-frequency data; therefore, intraday information is neglected. Hence, the empirical research is increasingly concentrated on the RV approach proposed by Andersen and Bollerslev (1998). The RV thus becomes a dominant estimator of volatility modelling and forecasting.[18]

Although the property of financial volatility is persistence, Granger and Ding (1996) note that the persistence of volatility is not constant over time. Login (1997) also provides evidence that the high volatility is less persistent than low volatility; thus, the persistence of market volatility is nonlinear. One interpretation of the nonlinear volatility refers to the business cycle (Hamilton, 1989), which is commonly related to economic expansions and recessions. Moreover, another explanation, which is more widely applied, indicates that the nonlinearity is caused by the financial breaks usually associated with important events such as financial crises or sudden

[18] For a more complete literature review, see Section 3.2.

changes in government policies. McAleer and Medeiros (2008) and Cipollini et al. (2017) indicate that the high regimes are short-lived, whereas the low regimes have longer persistence.

Threshold Models

The applications of nonlinearity in volatility models can be found in a wide range of empirical works. The asymmetric GARCH models capture the leverage effect by switching between two different parameters in which negative past returns have a greater impact on future volatility (Zakoian, 1990 and Glosten et al., 1993). Besides GARCH models, another widespread model used to set a threshold of regime-switching is the Threshold Autoregressive (TAR) model (Tong, 1978). Subsequent developments have extended the threshold model to adapt the abrupt regime-switching. Chen et al. (2008) developed the range-based threshold model to improve forecasting performance, Glodman et al. (2013) found that the threshold ARFIMA model with regime-switching outperforms the ARFIMA model, and Zhang et al. (2019) introduced the TAR models with a non-Gaussian error, which are well fitted the RV. In order to maintain the simplicity of estimation and inference, the number of lags in the TAR model are restricted to be small; the threshold depends on the state of the market and it is constant over time, which is an unreasonable assumption.

As the HAR model has multiple groups of lags, an available solution to incorporate the HAR model with a threshold is the moving average threshold HAR model proposed by Motegi et al. (2020). Their model combines the TAR and HAR models to generate the moving average threshold by time-varying parameters in each group of the HAR model. Empirically applying the moving average threshold HAR model, Salisu et al. (2020) show that this model improves forecasting accuracy over the basic HAR model and fixed threshold HAR model.

Smooth Transition Models

The threshold models that capture the regime-switching allow only two possible regimes: usually low and high-volatility regimes (Guidolin and Timmermann, 2005). The threshold model expresses an abrupt regime-switching behaviour depending on whether the variables are below or above a threshold value, while the smooth transition model allows for a gradual transition between regimes. Teräsvirta (1994) firstly considers the Smooth Transition Autoregressive model (STAR) model, which embeds the regime-dependent linear specification in the smooth transition nonlinear framework. Teräsvirta (1994) also introduces two types of STAR model based on the shape and location of the smooth threshold, namely the logistic (LSTAR) and exponential (ESTAR) STAR models. Those extensions of smooth transition models allow different types of market behaviours according to the nature of transition function.

Subsequently, the smooth transition methods have been incorporated with various models. Following the work of González-Rivera (1998), which proposes the smooth transition GARCH (ST-GARCH) model, Taylor (2004) presents the ST-GARCH model with time-varying parameters to adapt to changes over time. Khemiri (2011) and Cheikh et al. (2020) utilize the ST-GARCH in the international stock index and cryptocurrency markets, respectively.

Additionally, the smooth transition model has been combined with the HAR model. For instance, Qu et al. (2016) employ the logistic smooth transition in the HAR model and find that it can improve in-sample fit and out-of-sample forecasting performance. Izzeldin et al. (2020) use the exponential smooth transition HAR model to assess the impact of COVID-19 at both aggregate and sectoral levels. Moreover, although two regimes are commonly used, the smooth transition models can also accommodate multiple regimes transition. McAleer and Mediros (2008) introduce a multiple-regime smooth transition HAR model (HARST); however, in their forecasting results, the HARST model performs worse than the linear HAR model. The forecasting ability only improves when the HARST model and linear HAR model are combined.

Markov-switching Models

An alternative approach to smooth transition models is the family of nonlinear regime-switching models with unobserved transition variables, namely the Markov-switching model. The state-dependent process of the Markov-switching model only depends on the current state, not on the previous state. The number of optimal switching regimes is controversial, whereas it is generally recognized that volatility models combined with the Markov-switching model take into account the two different volatility regimes: high and low volatility regimes. Hamilton (1989) proposes using the Markov-switching model with the maximum likelihood methodology to estimate the unknown parameters of the transition probability matrices. Then, Hamilton and Susmel (1994) allow the parameters of the ARCH model to be governed by unobserved Markov Chain.

The numerous empirical works of the MS-GARCH model have been used to estimate and forecast various types of financial time series, including stock returns (Henry, 2009), exchange rate returns (Bohl et al., 2011), and cryptocurrency volatility (Ardia et al., 2018 and Caporale and Zekokh, 2019). All of this research indicates that the MS-GARCH models outperform single-regime specifications in generating forecasts, and are also effective in risk management. Specially, Gallo and Otranto (2016) combine Markov-switching and smooth transition in modelling realised kernel volatility, this method tradeoff the performance between these two approaches.

Markov-switching HAR Model and Its Extensions

Although the HAR-type models are linear models that exhibit RV's high persistence, persistence always undergoes structure breaks in the market. Raggi and Bordignon (2012) provide evidence of nonlinearities and indicate that both the persistence and nonlinearities are significant for to improve the description of RV. In their work, the persistence shifting between low and high-volatility regimes is simulated by the Markov-switching approach. The forecasting results show that nonlinearities can improve forecasting ability over several horizons. As the HAR models have dominated modelling and forecasting volatility recently, it is appropriate to use HAR models with regime-switching to describe RV dynamics.

Considering the nonlinearity and high persistence of RV, Ma et al. (2017) add the Markov-switching approach into the HAR models and find all of the HAR-type models with Markov-switching to yield more accurate forecasts. Luo et al. (2019) suggest that the HAR model with infinite hidden Markov-switching models has better prediction ability than the basic HAR model on the RV of agricultural commodity futures. Notably, Ma et al. (2019) also note that Markov-switching and volatility of volatility to HAR-type models could lead to better forecasting accuracy. Overall, the Markov-switching approach combined with HAR models can significantly improve forecasting accuracy.

The drawback of the Markov-switching model is that the transition probability between regimes is fixed and cannot model the time variation. As a result, Diebold et al. (1994) and Filardo (1998) provide an extension that the transition probability of the Markov-switching model could vary over time, dependent on explanatory variables. However, it is usually uncertain which variables or specifications should be employed to express time variation in the transition probability. Bazzi et al. (2017) propose a new Markov-switching model with time-varying transition probability (MS-TVTP) where the transition probability is driven by the mean of observations over time. After that, Wang et al. (2019) and Wang et al. (2020) incorporate the MS-TVTP model with the basic linear HAR model and find that the MS-HAR model with TVTP can obtain superior forecasting performance than the basic HAR model and the MS-HAR model with constant transition probability.

In addition, the variances in the Markov-switching model are state-independent. This means that the variance of each regime is the same; the switching only involves the predictive regression parameters. Kim and Nelson (1999) consider an alternative extension of the MS model, which involves the variance shifts in the Markov-switching model. Subsequently, Guidolin and Timmermann (2006) extend the Markov-switching model and imply the heteroscedasticity in the Markov-switching dynamics. In the empirical works of Guidolin et al. (2009) and Guidolin et al. (2013), the Markov-switching model with heteroscedasticity has good forecasting performance of stock and bond returns.

5.3 Methodology

This chapter compares the forecasting performance of regime-switching models combined with the HAR model. In this section, I introduce the models used in this study. This section starts with introducing two linear models as benchmarks, which are the AR1 and HAR models, and then describe the extensions of the smooth transition and Markov-switching models. The loss functions used in this chapter and MCS test are introduced in the end.

5.3.1 Empirical Models

As mentioned in previous methodology of Section 3.3, the calculation of RV is shown as follow:

$$RV_t = \sum_{i=1}^{N} \left(r_{t,i}\right)^2 \tag{5.1}$$

AR(1) Model

The first linear model used in this chapter is the autoregressive (AR) model. The AR model indicates that it is a regression of the variable against itself. In this chapter, the AR model restricts the autoregressive order to be one. Thus, the AR1 model is given as:

$$RV_t = \theta_0 + \theta_i RV_{t-1} + u_t \tag{5.2}$$

where $u_t \sim iid(0, \sigma^2)$.

HAR Model

As mentioned in previous methodology of Section 3.3, the standard HAR-RV model is given as:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t \tag{5.3}$$

where weekly and monthly averages of RV are calculated as:

$$RV_{t-1:t-5} = \frac{1}{5}\sum_{i=1}^{5} RV_{t-i} \tag{5.4}$$

$$RV_{t-1:t-22} = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i} \tag{5.5}$$

Smooth Transition Models

The financial volatility usually presents the existence of two regimes: the low volatility and high volatility regimes. Consequently, this chapter allows the AR and HAR models to depend on the smooth transition model in which the transition variable is observed. Different from the abrupt transition of the threshold model, the smooth transition model allows for a gradual transition between regimes. A two-regime smooth transition model is given as:

$$RV_t = X_t\alpha + G(s_t;\gamma,\psi)Z_t'\beta + \big(1 - G(s_t;\gamma,\psi)\big)Z_t'\delta + \varepsilon_t \tag{5.6}$$

where X_t denotes the regime invariant variables, Z_t denotes regime transition variables, G denotes the continuous transition function with a return value between 0 and 1, and ε_t is the stochastic error team. The smooth transition models allow different types of market behaviours according to the nature of transition function G.

First, the logistic smooth transition model depends on whether the transition variable is above or below the transition value, and the logistic transition function is shown as:

$$G(s_t; c, \gamma) = \frac{1}{1 + exp\big(-\gamma(s - c)\big)}, \gamma > 0 \tag{5.7}$$

where s is the smoothing parameter, c is the transition parameter to determine the point at which regimes are weighted equally, and γ is the slope value to control the speed and smoothness of the transition process.

Second, the exponential smooth transition function depends on the distance between the transition value and threshold value, and is given as:

$$G(s_t; c, \gamma) = 1 - exp(-\gamma(s - c)^2), \gamma > 0 \tag{5.8}$$

where s is the smoothing parameter, c is the transition parameter, and γ is the slope value.

This chapter employs both the LSTAR and ESTAR models, which allow for a logistic and exponential smooth transition in the autoregressive process. Following common practice (Guidolin et al., 2009), the LSTAR and ESTAR models restrict the number of delay parameter to be one. Moreover, this chapter also uses the smooth transition HAR model, which allows for a smooth transition between two regimes governed by both LST function and EST function, namely the LST-HAR model and EST-HAR model. Due to there are multiple parameters in the HAR model, the model selection of the smallest sum-of-squared residuals is used to determine the best regime transition variable, which is Z_t' in Eq(5.6).

Markov-switching Models

An alternative approach to smooth transition models is nonlinear regime-switching models in which the transition variable is unobserved. This chapter considers the regime-switching models that depend on a set of unobservable states, and which follow the Markov process, and then combines the Markov-switching method with the linear HAR model, namely the Markov-switching HAR (MS-HAR) model, given as:

$$RV_t = \beta_{0,S_t} + \beta_{d,S_t} RV_{t-1} + \beta_{w,S_t} RV_{t-1:t-5} + \beta_{m,S_t} RV_{t-1:t-22} + u_{t,S_t}$$
$$u_{t,S_t} \big| \zeta_t \sim N\left(0, v_{t,S_t}\right) \tag{5.9}$$

where the constant β_{0,S_t}, the HAR model coefficients $\beta_{d,S_t}, \beta_{w,S_t} and \beta_{m,S_t}$, and the variance v_{t,S_t}all depend on the unobservable states. In particular, this chapter imposes and estimates two-state predictive regression ($S_t = 1$ or $S_t = 2$), which presents the low volatility and high volatility regimes, respectively. The unobserved regime variable S_t is assumed to follow the first order Markov chain process with a constant transition probability matrix with the generic element P_{ji}, which is defined as:

$$Pr(S_t = i | S_{t-1} = j) = P_{ji} \quad for\ i, j = 1,2 \tag{5.10}$$

This is the probability of transition from regime *j* to regime i between $t-1$ and t. In matrix notation shown as:

$$P = \begin{bmatrix} P^{11} & P^{21} \\ P^{12} & P^{22} \end{bmatrix} = \begin{bmatrix} P^{11} & 1 - P^{11} \\ 1 - P^{22} & P^{22} \end{bmatrix} \tag{5.11}$$

Therefore, the RV at time t is formulated based on the two-regime Markov-switching model given as:

$$\begin{aligned} RV_t = {} & P^{11}\left(\beta_{0,S_1} + \beta_{d,S_1} RV_{t-1} + \beta_{w,S_1} RV_{t-1:t-5} + \beta_{m,S_1} RV_{t-1:t-22}\right) \\ & + P^{22}\left(\beta_{0,S_2} + \beta_{d,S_2} RV_{t-1} + \beta_{w,S_2} RV_{t-1:t-5} + \beta_{m,S_2} RV_{t-1:t-22}\right) + u_t \\ & u_t \big| \zeta_t \sim N\left(0, P^{11} v_{t,S_1} + P^{22} v_{t,S_2}\right) \end{aligned} \tag{5.12}$$

In order to gain more flexibility, the transition probability of the Markov-switching model can be assumed as a time-varying transition probability (MS-TVTP). According to Diebold et al. (1994), the transition probability follows an independent regime-switching process. The Eq(10) could be changed as:

$$P(S_t = i|S_{t-1} = i) = p_{ii} = \frac{exp(c_i + d_i\delta_{t-1})}{1 + exp(c_i + d_i\delta_{t-1})} \quad (5.13)$$

where c_i is a constant, d_i is the exogenous observable in regime i, and δ_{t-1} is coefficient, where d_i is the transition probabilities of MS-TVTP model become constant and coincide with MS model. Identically, the MS-TVTP method is also combined with the linear HAR model to model RV.

In addition, the variance of MS model is independent of the state $\left(v_{t,S_1} = v_{t,S_2}\right)$, which is the homoscedastic case. In addition, this chapter also follows the works of Guidolin et al. (2009) and Guidolin et al. (2013) to consider the heteroskedastic case, in which the variance is regime-specific $\left(v_{t,S_1} \neq v_{t,S_2}\right)$. Thus, this chapter makes the Markov-switching dynamics imply heteroscedasticity, namely MSH. The MSH model combined with HAR (MSH-HAR) model assumes Markov-switching probabilities with two regimes and constant transition probabilities.

5.3.2 Forecasting Evaluation

Loss Functions

As mentioned in previous Chapter3, Subsection 3.3.2, the loss functions in this chapter are given by:

$$QLIKE = \frac{1}{n}\sum_{t=1}^{n}\left(\log(\widehat{RV}_t) + \frac{RV_t}{\widehat{RV}_t}\right) \quad (5.14)$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(RV_t - \widehat{RV}_t\right)^2 \quad (5.15)$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|RV_t - \widehat{RV}_t\right| \quad (5.16)$$

$$RV_t = a_0 + a_1\widehat{RV}_t + \epsilon_t \quad (5.17)$$

where RV_t and $\widehat{RV}_t$ denote the actual volatility and forecast volatility, respectively.

The loss functions are used to determine the best model with the smallest forecasting error overall time. However, the forecasting performance of models may change over time, especially during unstable periods. To evaluate whether the forecasting performance changes over time, this chapter considers the cumulative forecast error of the MSE to display the sum of the forecasting error as it grows with time.

Model Confidence Set (MCS) Test

While the loss functions above could obtain an optimal value to determine a model with the best performance, this chapter also considers the Model Confidence Set (MSC) approach[19] to remove the worst model sequentially according to the null hypothesis of equal predictive ability is rejected.

5.4 Data

All of the RV data used are from the Oxford-Man Institute of Quantitative Finance. The chapter employs the 5 minutes RV of eight international stock indices[20], including the UK stock index (FTSE), Japanese stock index (N225), the USA stock index (SPX), German stock index (DAX), Chinese stock index (SSEC), Brazilian stock index (BSVP), Indian stock index (NSEI), and Mexican stock index (MXX), respectively. This chapter uses the RV in logarithmic form (log-RV) to produce results with more normal distribution. The data sample considered over fifteen years, from 1st November 2006 to 31st October 2020[21]. The first five years are set as the in-

[19] More details for the MCS test can be found in Section 3.3.2.
[20] The discussion of date sample frequency can be seen in Footnote 7.
[21] Due to the subsequent development of this chapter, the sample is updated and use the latest date at that time, so the data period is different from previous chapters.

sample period (i.e., 1st November 2006 to 31st October 2010), and the last ten years (i.e., 1st November 2010 to 31st October 2020) are used in the out-of-sample forecast generation.

Table 5.1 describes the summary statistics of log-RV for each index. All series of log-RV exhibit a non-normal distribution with excess kurtosis and is right-skewed. Further supportable evidence of non-normal distribution is that the Jarque-Bera test statistic rejects the null hypothesis of normal distribution at the 1% level. The first-order autocorrelation values indicate that all log-RV series are highly persistent and allow for further modelling analysis. Figure 5.1 shows the time-series plots of all log-RV, respectively. For all indices, the log-RV occurs around 2008 and 2020, when the financial crisis and global pandemic happened.

5.5 Empirical Results

5.5.1 In-Sample Results

This chapter starts with the linear models using the AR(1) and basic HAR models and the extensions of the smooth transition and Markov-switching model, which are combined with the AR(1) and HAR model. As noted above, this chapter considers the two-regime predictive regression in the smooth transition and Markov-switching models, which represents the low and high-regime volatility. The first five years of data samples are set as the in-sample period from 1st November 2006 to 31st October 2010[22].

Table 5.2 provides the three model selection criteria for each forecasting model, namely the Akaike information criteria (AIC), Bayesian information criterion (BIC), and log-likelihood (LL), respectively. Overall, as the three criteria show, the MSH-HAR model performs best among all forecasting models, while the MS-TVTP-HAR model performs well for N225 in AIC and BIC diagnostics. Comparing the linear AR1 model and HAR model, the nonlinear regime-

[22] As the recursive method are used to obtain forecasts in the chapter, the in-sample estimates will change each period continuously, so the final in-sample estimates are quite different from the initial ones. The in-sample results presented here are the initial in-sample estimates, it is included largely for illustrative purpose.

switching models of the smooth transition and Markov-switching are generally preferred in terms of more accurate criteria value. This result is consistent with the work of Raggi and Bordignon (2012). The regime-switching AR model performs poorly. Moreover, the Markov-switching models exhibit a better goodness-of-fit compare to the smooth transition model.

As the MSH-HAR model outperforms others in the model selection criteria all the time. The analysis of in-sample estimation results concentrates on it. Table 5.3 reports the parameter estimates for MSH-HAR models for each index. The models are allowed to switch between the two regimes ($S_t = 1$ and $S_t = 2$), which are the low and high volatility regimes. Due to the 2008 financial crisis being included in the in-sample period, the statistically significant coefficients in the high regimes are more than that of low regimes, besides SPX and MXX. In each regime, the MSH-HAR models have different standard deviation of the error, σ_t, so the regime-switching dynamics imply heteroscedasticity. Except for the low regimes for NSEI and MXX, the standard deviations of error are significant at the 1% level. The effects of the financial crisis are also manifested in the regime transition probability. For all RV indices, the transition probability of high regime, p_{22}, is greater than that of low regime, p_{11}, which means the high-volatility regimes exhibit more persistence than low-volatility regimes in the in-sample period.

Figure 5.2 plots the smoothed transition probability of the MSH-HAR model for all indices in the high and low-volatility regime. The upper charts are low-regime smoothed transition probability, showing that when the low-regime-switching occurs, and the lower charts are smoothed transition probability of high regime. The overall patterns of the transition probability are very sensitive, this is related to the unstable financial markets during the in-sample period. The transition probability of FTSE, SPX, N225, and DAX experienced a stable period in 2009, but it has been in a period of instability for SSEC, NSEI, BSVP, and MXX.

5.5.2 Out-Of-Sample Results

The forecasts are recursively generated [23]from all forecasting models in section 3.1 over the out-of-sample period from 1st November 2010 to 31st October 2020. All forecasting models considered produce the RV forecasts over the daily, weekly, and monthly horizons. For long-term forecasting, a simple approach for generating multi-step-ahead forecasts is to replace the data frequency of the volatility model. In other words, to replace RV_{t+1} on the lift-head side over the forecasting horizon h, say $RV_{t+h}^{h} = \frac{1}{h}\sum_{i=1}^{h} RV_{t-h+i}$, thus $h = 1, 5\ and\ 22$. Moreover, forecasting performance is measured by the loss functions of MSE, MAE, QLIKE, and adjusted R^2 of MZ regression, as well as the MSC tests of MSE and QLIKE criterion select the optimal models with EPA, respectively.

The out-of-sample results of four loss functions are reported over daily, weekly, and monthly horizon from Tables 5.4 to 5.7. Overall, the AR1 model and smooth transition AR models perform worse than regime transition HAR models. For the smooth transition models of AR and HAR models, the forecasting results of the logistic smooth transition and exponential smooth transition are very close. Especially, the ESTAR and LSTAR have almost identical forecasts. Table 5.4 presents the MSE loss functions over the daily, weekly, and monthly horizons (h= 1, 5 and 22). For one-day-ahead forecasts, the MS-TVTP-HAR model performs the best, whilst the EST-HAR model and MSH-HAR have good performance for two series, respectively. For the one-week-ahead forecasts, the MS-HAR model provides the best forecasting performance among all indices. Additionally, the nonlinear regime-switching models outperform the linear AR1 and HAR models. Specifically, the Markov-switching models can provide more accurate forecasts than smooth transition models. Again, the MS-HAR model

[23] The Markov-switching models are estimated by the maximum likelihood, it is very time-consuming to generate 10-year out-of-sample predictions. Therefore, different from Chapter 3 and 4, this chapter only uses the recursive approach to generate forecasts.

performs best for the one-month-ahead forecasts, and the linear AR1 and HAR models perform poorly.

Table 5.5 shows the MAE comparison results, consistent with Table 5.3, the AR1 and its smooth transition extensions do not have good forecasts. Specifically, the linear HAR model has the best forecasting performance at the one-day horizon, and the MSH-HAR model has competitive forecasting performance for three series as well. The MS-HAR model dominates all stock indices except FTSE at the one-week-ahead forecast horizon. The MS-HAR model also has the best forecasting ability at the monthly horizon, and the MSH-HAR model performs well for the DAX and SSEC.

For the QLIKE comparison in Table 5.6, it shows similar results to those in Table 5.5. The HAR model performs best at the one-day horizon, and the MS-TVTP-HAR and MS-HAR models also have good performance. At the weekly and monthly horizons, the MS-HAR model dominates. In terms of the adjusted R^2 of MZ regression in Table 5.7, the MS-TVTP-HAR and MSH-HAR models have the best performance for three series at the daily horizon, respectively. Again, the MS-HAR model outperforms others.

Moreover, to further strengthen the out-of-sample results and observe the forecasting error of all considered models over time, the cumulative forecasting error of MSE loss function over the daily, weekly, and monthly horizons are plotted in Figures 5.3, 5.4, and 5.5. The daily forecasting error in Figure 5.3 shows that the AR1, ESTAR, and LSTAR models perform worse, while there is no observable difference of daily forecasting error among the HAR model, smooth transition HAR models, and Markov-switching HAR models for all indices. In Figure 5.4, it can be observed that the MS-HAR model (blue lines) generates the lowest cumulative MSE all the time. This reveals that the MS-HAR model consistently maintains the most accurate weekly forecasting performance over time. Besides, the three types of Markov-switching HAR models perform better than smooth transition HAR models. The cumulative monthly forecasting error in Figure 5.5 presents similar results to Figure 5.4. The MS-HAR model produces the lowest forecasting error, except for DAX, the MS-HAR model is the best model for DAX.

The MSC test results for the MSE and QLIKE criteria are provided in Tables 5.8 and 5.9. The value 1 in the table means that the optimal model is chosen, and the MSC test chooses a subset of models with EPA at 75% confidence level. For the MSC test of MSE criterion in Table 5.8, the AR1 model and its smooth transition models perform worse. All of them are eliminated from EPA selection. For daily forecasts, the MS-TVTP-HAR model has the best performance for DAX, BSVP and MXX, whilst the EST-HAR model has good performance for N225 and NSEI, and the MSH-HAR model for FTSE and SPX, respectively. For weekly and monthly forecasting horizon, there is overwhelming evidence of the superiority of the MS-HAR model.

Table 5.9 presents the MSC test of QLIKE criterion. Compared to Table 5.8, similar results are obtained. The linear HAR model and MSH-HAR model outperform for daily forecasting, while the MS-TVTP-HAR model has good forecasting ability. Specifically, the linear HAR model provides the best forecasting performance for DAX, SSEC, and MXX, and the MSH-HAR model is the best model for FTSE, SPX, and DAX. The MS-TVTP-HAR model also yields good forecasts for N225 and BSVP. Again, the weekly results indicate that the MS-HAR model is the best-performing forecast model, and the MS-TVTP-HAR model also performs well at the NSEI and MXX. The MS-HAR dominates at the monthly horizon.

In summary, according to the lowest loss function value and the MSC test, the linear HAR model, MS-TVTP-HAR model, and MSH-HAR model have better predictive ability at daily forecasting horizon, while no model clearly outperforms each other. For both weekly and monthly horizons, the MS-HAR model has the best forecasting performance. Markov-switching models can provide more accurate forecasts than smooth transition models. Compared with present works, the results of this article are completely different from Wang et al. (2019) and Wang et al. (2020), which show that the Markov-switching model time varying transition probability has better performance than the Markov-switching model. Notably, the differences between four developed countries and four emerging countries are not clearly observed for forecasting comparison in the out-of-sample results.

5.6 Risk Management Application

Since one more typical forecasting performance application is the adoption of financial risk management, this chapter also considers a risk management backtesting and loss function, which is based upon the calculations of the Value at Risk (VaR) and Expected Shortfall (ES). The VaR is calculated to measure the maximum amount of loss for financial assets under a certain confidence level, and ES is designed to measure the expected loss value when a VaR violation has occurred. The VaR of an asset is calculated as:

$$VaR = \mu_t + \sigma_t N(\alpha) \tag{5.22}$$

where μ_t is the mean of asset's log-return, σ_t is the predicted volatility, and $N(\alpha)$ defines the left αth quantile of the normal distribution. Then, the calculation of ES given as:

$$ES = \mu_t + \sigma_t \frac{f\big(N(\alpha)\big)}{1-\alpha} \tag{5.23}$$

where μ_t is the mean of asset's log-return, σ_t is the predicted volatility, and f($N(\alpha)$) is the density function of a standard normal distribution at the left αth quantile.

To examine the accuracy of VaR forecasts, the first test is to compute the failure rate for daily return, which is the number of daily returns exceed the forecasted VaR divided by the total forecasted observations.

Second, the VaR backtesting is examined using both the Kupiec (Kupiec, 1995) and Christoffersen (Christoffersen, 1998) tests, which are the unconditional and conditional coverage tests for the correct number of exceedances. Specifically, the Kupiec test is an unconditional coverage test with the null hypothesis that the observed violation rate is statically equal to the expected violation rate. Kupiec (1995) notes that this test ignores all observations after the first value, which leads the value to be oversized. Christoffersen's conditional coverage test examines the null hypothesis that the failure rate occurred independently at every point in time and against the alternative hypothesis that the failure rate is stylized in volatility clustering. Both tests are carried out in the likelihood ratio (LR) framework. The LR for each test is shown as:

1. LR statistic for the test of correct unconditional coverage:

$$LR_{UC} = 2\, log((1-\pi_0)^{T-N}\pi_0^N) - 2\log((1-\alpha_0)^{T-N}\alpha_0^N) \sim \chi_1^2 \qquad (5.24)$$

where π_0 is the observed violation rate and calculated by the number of the days N when violations occurred divide by forecasting period T.

2. LR statistic for the test of correct conditional coverage:

$$\begin{aligned} LR_{CC} &= 2\, log\big((1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}\big) \\ &- 2\log\big((1-\pi_0)^{n_{00}+n_{10}}\pi_0^{n_{01}+n_{11}}\big) \sim \chi_1^2 \end{aligned} \qquad (5.25)$$

where the $n_{i,j}$ is the number of i followed by j (for $i,j = 0{,}1$), $\pi_{i,j}$ means the probability when the i occurs at time t followed by the j occurs at time $t-1$.

Third, this chapter uses the Dynamic Quantile (DQ) test of Engle and Manganelli (2004) to examine if the present violations of the VaR measure are not correlated with the past violations. The DQ test defines the hit sequence as follows:

$$Hit_t = I(r_t < -VaR_t) - a \qquad (5.26)$$

This sequence assumes that value $(1-a)$ whenever the actual returns are less than the VaR quantile and the value $(-a)$ otherwise. The expected value of Hit_t is zero and the sequence is uncorrelated with past information. In this case, there will be no autocorrelation in the hit sequence and the fraction of exceptions will be correct. The DQ test statistic is calculated as:

$$DQ = \frac{\widehat{\beta'}X'X\hat{\beta}}{a(1-a)} \sim \chi^2(k) \tag{5.27}$$

which X is the explanatory variables and $\hat{\beta}$ is the OLS estimates. The DQ test also follows χ^2 distribution with degree of freedom equal to the number of parameters.

Lastly, the VaR backtesting chooses more than one model. In order to choose the best performing model, this chapter uses the Quantile Loss (QL) (Koenker and Bassett, 1978) to rank the model with best VaR performance. So, the QL for predicted VaR of confidence level α at time t given as:

$$QL_t(\alpha) = (\alpha - d_t)\big(r_t - VaR_t(\alpha)\big) \tag{5.28}$$

QL is an asymmetric loss function. As the weight $(1-\alpha)$ increases, the penalties will be heavy, for the returns exceed the VaR. Models with lower average QL are preferred.

To date, there is no specific loss function for ES (see Bellini and Bignozzi, 2015, and Ziegel, 2016). The loss function introduced by Fissler and Ziegel (2016) (FZL) shows that VaR and ES can be assessed jointly. For predicted VaR and ES at risk level α for time t, the joint VaR and ES loss function of FZL given as:

$$FZL_t(\alpha) = \frac{1}{\alpha ES_t(\alpha)} d_t\big(r_t - VaR_t(\alpha)\big) + \frac{VaR_t(\alpha)}{ES_t(\alpha)} + log\big(-ES_t(\alpha)\big) - 1 \tag{5.29}$$

where $ES_t(\alpha) \ll VaR_t(\alpha) < 0$. The losses of FZL are averaged over the forecasting period and the model with the lowest average is preferred.

This chapter uses the same in-sample and out-of-sample period to produce the daily, weekly, and monthly VaR and ES forecasts at both 1% and 5% levels to examine which forecasting model provides more accurate VaR and ES estimates. [24]

[24] For more detail about the weekly and monthly risk management setting see Footnote 15.

The results of VaR and ES forecasts for the 1% level are presented in Table 5.10. For the daily forecasting horizon, the LSTAR model is preferred as the lowest average failure value is provided. In terms of the Kupiec, Christoffersen, and DQ tests, all forecasting models are statistically significant across all series. The null hypotheses of the Kupiec and Christoffersen tests are rejected. This means that all considered models produce the unexpected proportion of VaR violations and those VaR violations occur dependently. The null hypothesis of the DQ test is also rejected; thus, the VaR violations are correlated. The average QL and FZL show that the ESTAR model performs best for daily VaR and ES, while the EST-HAR model provides the worst forecasts. The LSTAR model also performs well, and it is close to the ESTAR model. For weekly forecasts, the ESTAR model has the lowest average failure rate. Again, all forecasting models are statically significant in the Kupiec, Christoffersen, and DQ tests. The MS-TVTP-HAR model is preferred in terms of the lowest QL and FZL value, but the ESTAR and LSTAR models perform worst for the monthly horizon. Examining monthly forecasts, which is consistent with weekly forecasts, the ESTAR model has the best performance in terms of failure rate, all three null hypotheses are rejected and the MS-TVTP-HAR model performs best for QL and FZL.

Table 5.11 presents the VaR and ES results at the 5% level. The results are broadly similar to Table 5.10. For the Kupiec, Christoffersen, and DQ tests across daily, weekly, and monthly forecasting horizons, all models do reject the null hypothesis of expected VaR violations (i.e., Kupiec test). VaR violations occurred independently (i.e., Christoffersen test) and VaR violations were not correlated (i.e., DQ test). For the daily forecasts, the ESTAR models achieve the lowest average failure rate, and the ESTAR model has the best performance for QL and FZL. The results for weekly and monthly results are consistent with that at the 1% level. The LSTAR model has the lowest average failure rate, and the MS-TVTP-HAR model is preferred in terms of the lowest QL and FZL value.

To sum up, all models do reject the null hypothesis of the Kupiec, Christoffersen, and DQ tests across daily, weekly, and monthly forecasting horizons, which means all forecasting models provide the VaR violation is unexpected, dependent, and correlated. There is no clear evidence that the nonlinear regime-switching model could improve the forecasting ability at the daily horizon. The smooth transition AR models have good performance. However, for weekly and monthly horizons, the ESTAR model, LSTAR model and the MS-TVTP-HAR model have better performance.

5.7 Conclusion and Discussion

Previous papers have shown that the nonlinear persistent dynamics of the RV can improve predictive performance. In order to systematically assess the predictive power of the nonlinear regime-switching models in combination with the HAR model, this chapter comprehensively integrates and compares the predictive performance of the linear and nonlinear regime-switching models for eight RV indicators. In addition to the linear AR and HAR models as benchmarks, the chapter considers smooth transition and Markov-switching methods for different prediction time horizons. In addition, this chapter also considers the basic Markov-switching model, the Markov-switching model with time-varying transition probability and the heteroscedasticity in the Markov-switching dynamics. Following most of the work, this chapter employs two regimes in the regime-switching model, presenting low and high volatility regimes respectively. Finally, all forecasts are used in risk management in terms of calculating Value at Risk (VaR) and Expected Shortfall (ES).

The in-sample results support that the nonlinear regime-switching models exhibit better goodness-of-fit than linear models, the Markov-switching model that allows for regime switching in the variance is better than the Markov-switching models that don't. (i.e., MSH-HAR model is preferred). For the out-of-sample results, there is no evidence that nonlinear regime-switching models outperform the HAR model for the daily forecasting horizon, while for both weekly and monthly horizons, the Markov-switching HAR model dominates forecasting performance. Moreover, Markov-switching models can provide more accurate forecasts than smooth transition models. For risk management applications, the daily forecasting ability of the smooth transition AR model is non-negligible and performs best. Markov-switching models with time-varying transition probabilities are preferred in the weekly and monthly ranges. Notably, no differences between the four developed and four emerging countries were clearly observed in the comparison of forecasts for out-of-sample results.

In contrast to previous work, the HAR model is a simple linear model to predict RV; however, RV also exhibits nonlinear persistence. This chapter systematically evaluates the forecasting performance of the nonlinear framework and concludes that combining nonlinear HAR models improves forecasting accuracy, with the Markov-switching HAR model performing best among the regime-switching models. In addition, all regime switching models are applied to risk management.

This chapter presents the same results as the previous papers, which is the nonlinear models have stronger forecasting performance. This chapter estimate the simple nonlinear frameworks[25], which outperform all linear models in terms of predictive performance. The two nonlinear regime-switching models are quite different in nature; the smooth transition model allows for a gradual transition between two regimes, whereas the transition in the Markov switching model is abrupt and unobservable. Empirically, the abrupt transition technique is better suited to real financial market changes and therefore outperforms smoothed changes in terms of forecasting performance.

[25] The "simple" means that the models are considered by two regimes. e.g., $S_t = 1$ or $S_t = 2$ in the Markov-switching models, and there is an imposed threshold in the LST and EST models.

Figure 5.1: The plots of the whole period of log-RV of eight market index

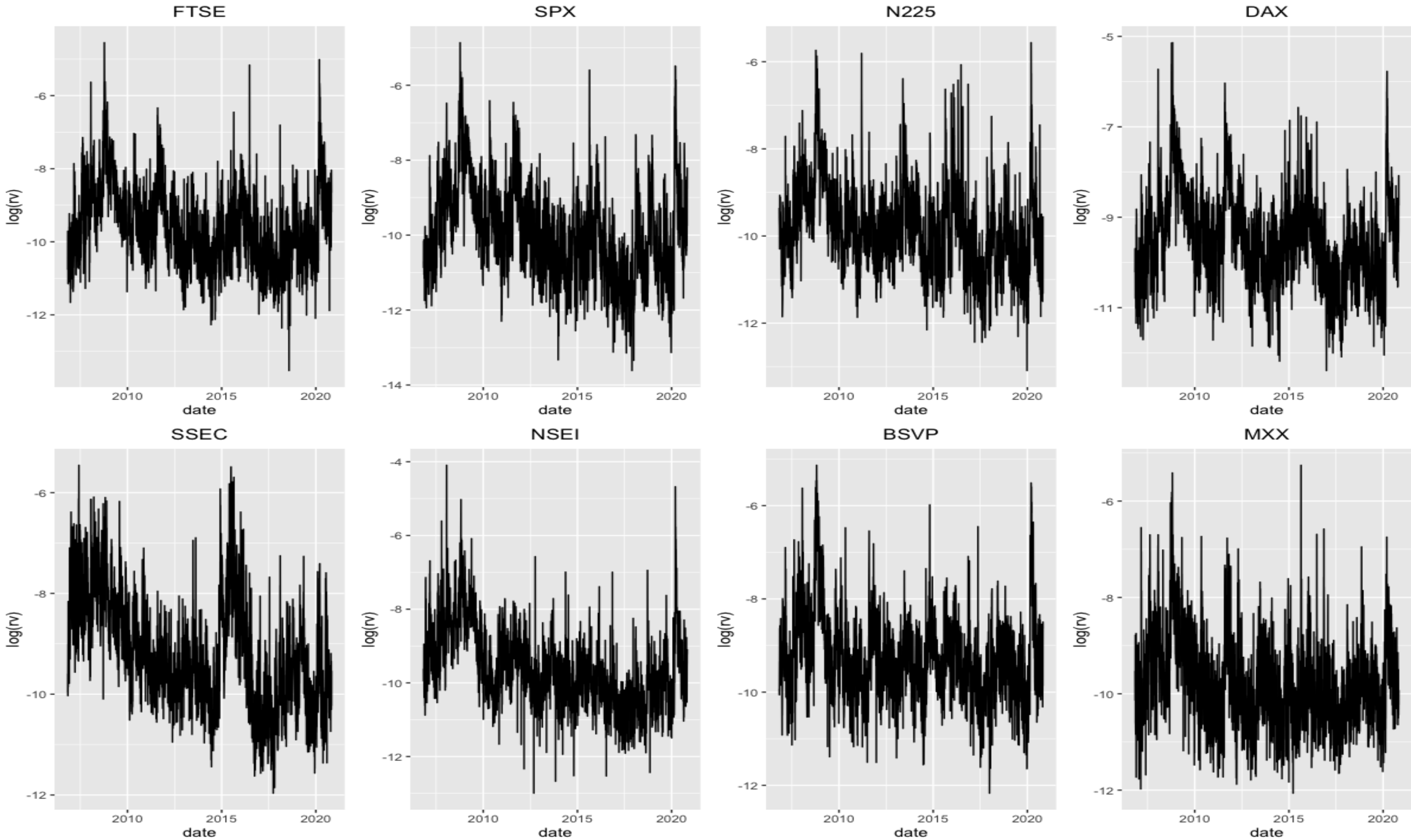


Figure 5.2: The smoothed transition probability of MSH-HAR model

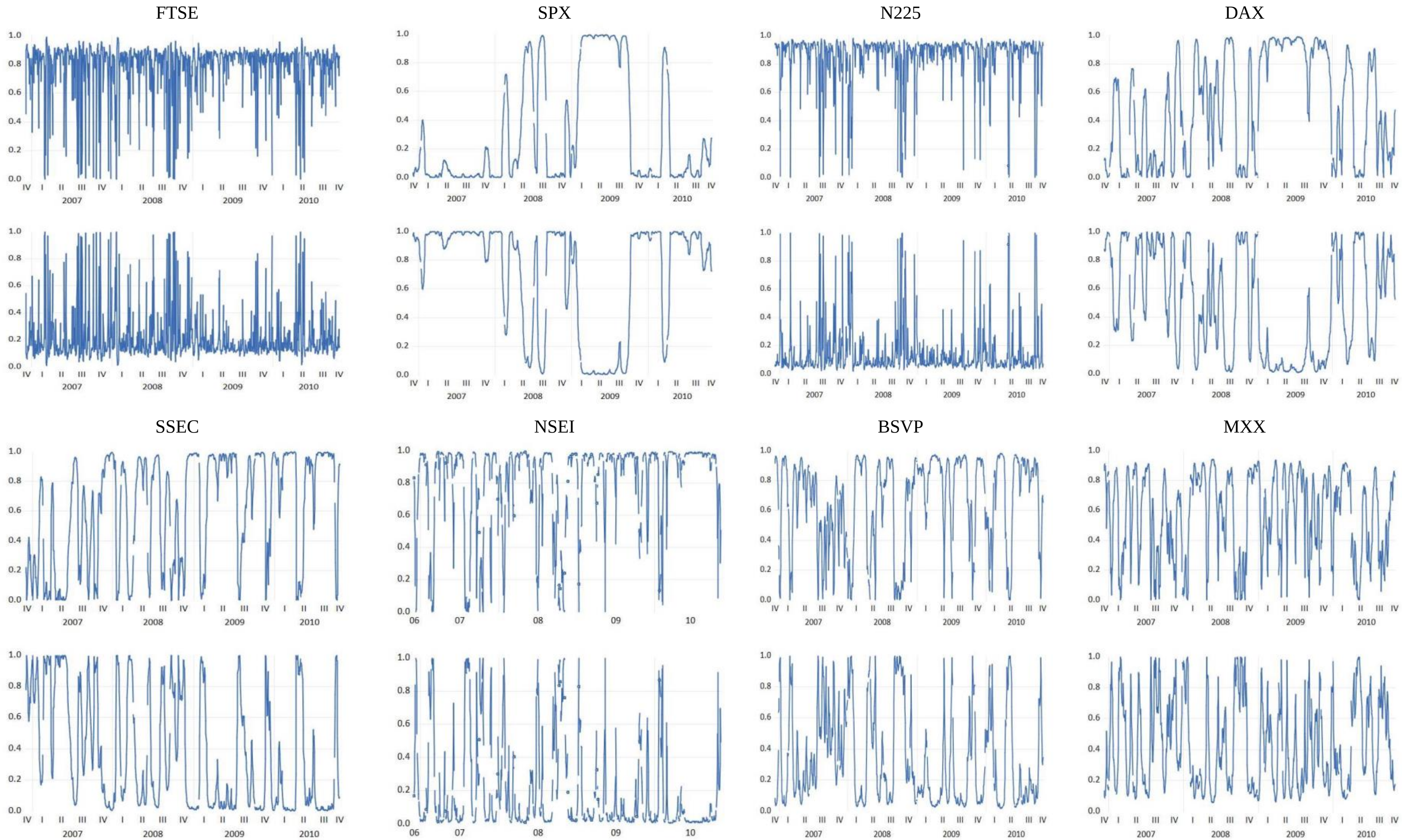


Figure 5.3: The sum of cumulative MSE forecasting error for one-day-ahead forecast

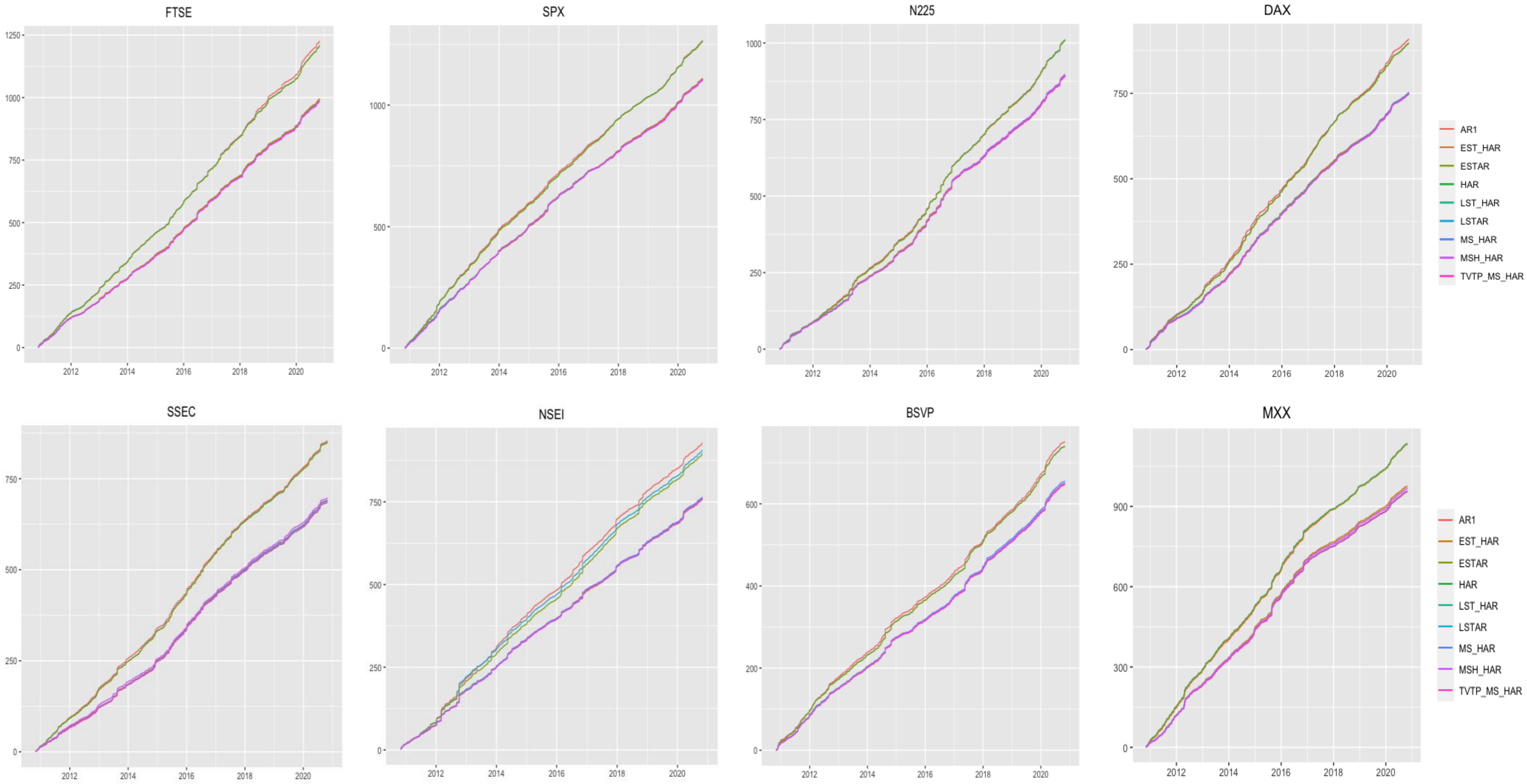


Figure 5.4: The sum of cumulative MSE forecasting error for one-week-ahead forecast

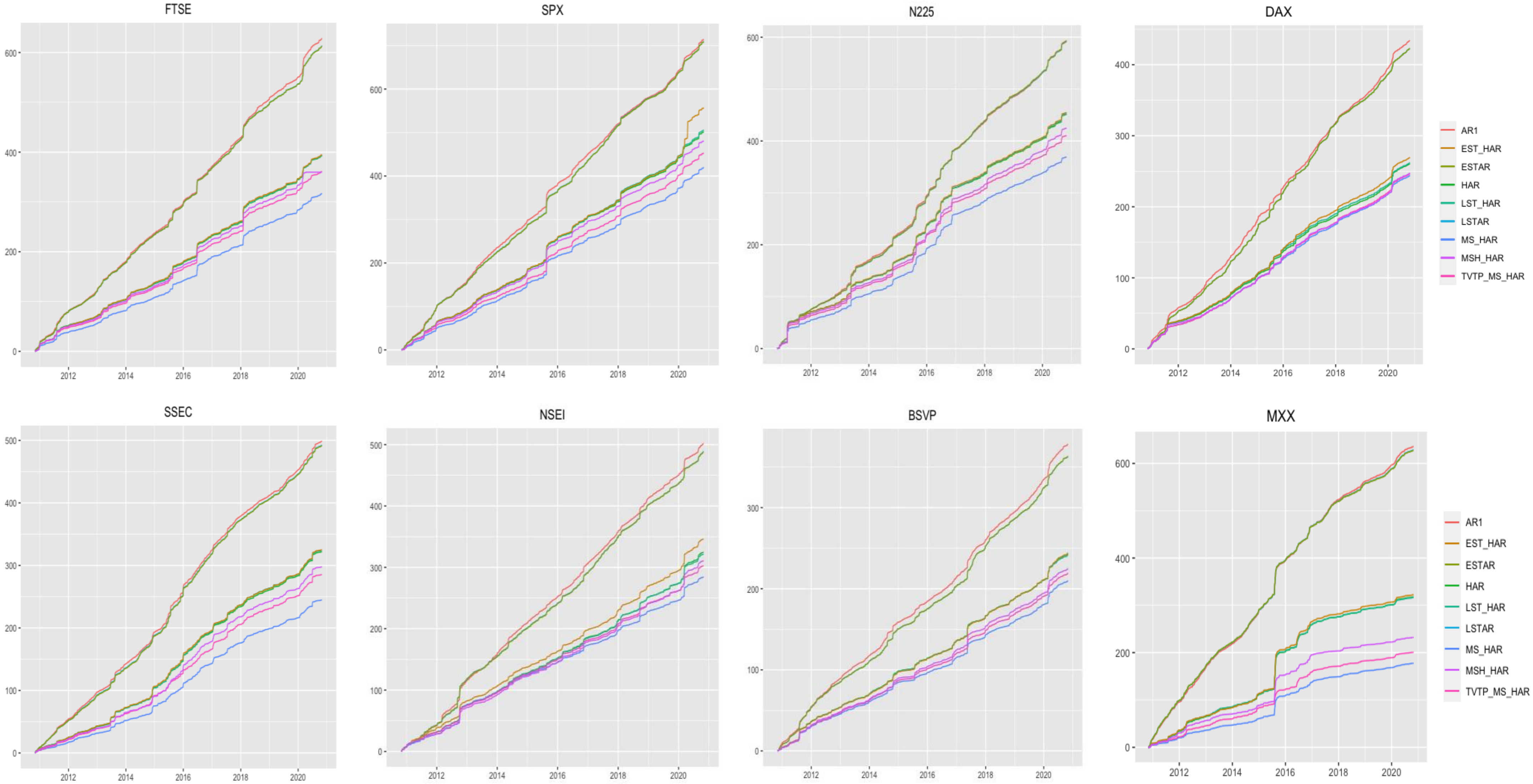


Figure 5.5: The sum of cumulative MSE forecasting error for one-month-ahead forecast

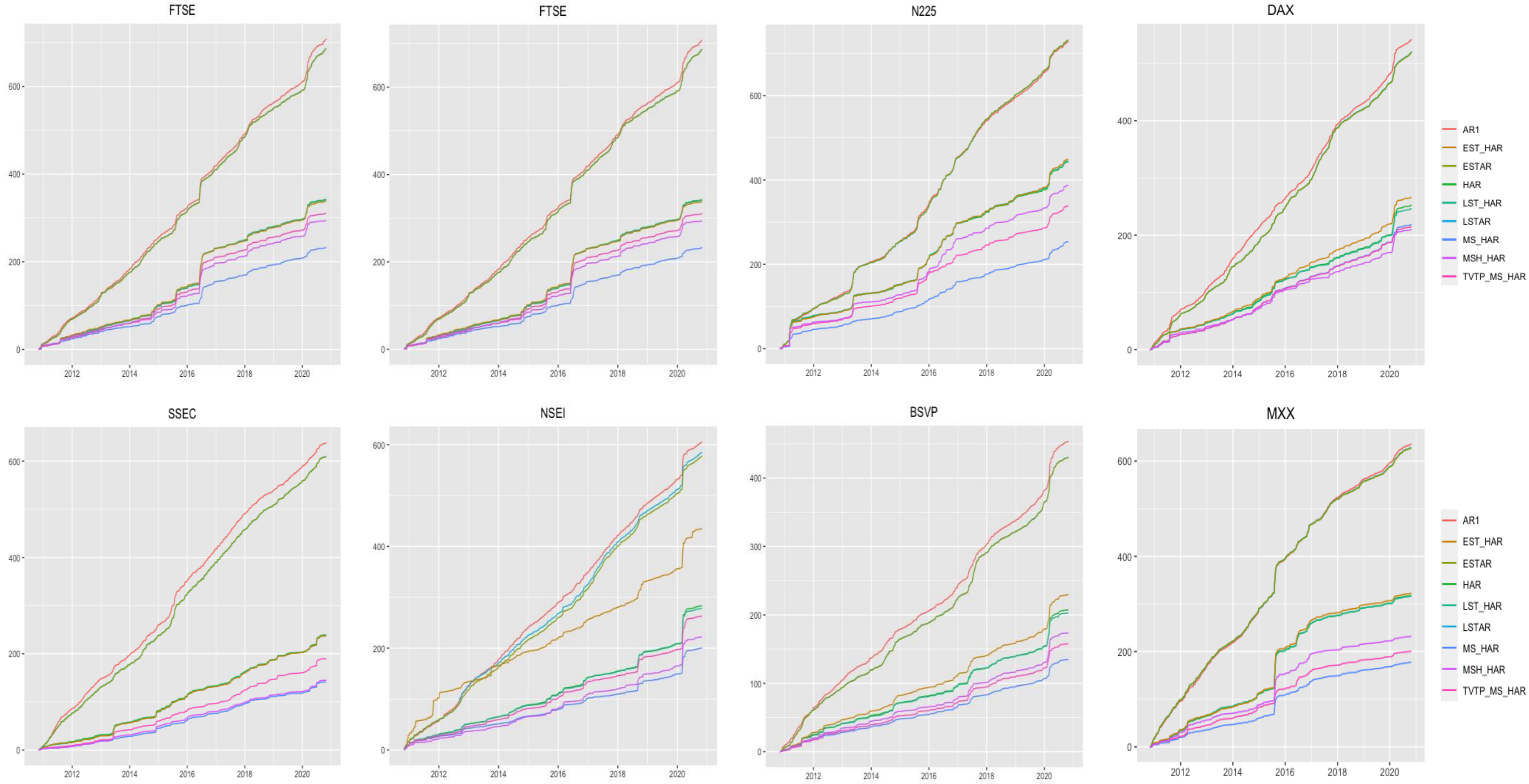


Table 5.1: Descriptive statistic of log-RV

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
Mean	-9.6253	-9.9753	-9.8512	-9.4991	-9.2238	-9.6553	-9.2901	-9.8426
St.Dev.	1.0289	1.2418	0.9759	0.9824	1.0784	1.0239	0.8881	0.8965
Kurtosis	0.7981	0.3428	0.9749	0.6954	-0.1832	0.9356	1.8284	1.0329
Skewness	0.6497	0.4930	0.6467	0.4802	0.4387	0.7223	0.8595	0.7881
Median	-9.7546	-10.088	-9.9397	-9.5673	-9.3426	-9.8042	-9.3765	-9.9778
25%-quantile	-10.355	-10.841	-10.506	-10.159	-9.986	-10.359	-9.849	-10.490
75%-quantile	-9.021	-9.216	-9.319	-8.910	-8.438	-9.063	-8.836	-9.316
AutoCorr	0.7600	0.8324	0.7748	0.8021	0.8383	0.8269	0.7671	0.6407
Jarque-Bera	343.41***	159.93***	374.53***	208.22***	113.91***	106.09***	906.68***	520.49***
Obs.	3537	3515	3419	3545	3402	3465	3448	3511

Note: This table reports the summary statistics of log-RV of eight stock index for the whole period from 1st November 2006 to 31st October 2020. *** indicate the significant level at 1%.

Table 5.2: In-sample diagnostics

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				AIC				
AR1	1.7959	1.9126	1.6333	1.6606	1.7391	1.7994	1.8577	2.2080
HAR	1.6245	1.7517	1.4782	1.4991	1.6370	1.6526	1.7290	2.0299
LSTAR	1.7996	1.9093	1.6246	1.6351	1.7329	1.7838	1.8392	2.1973
ESTAR	1.7996	1.9093	1.6292	1.6351	1.7295	1.6337	1.8396	2.0262
LST-HAR	1.6313	1.7507	1.4811	1.4930	1.6428	1.6500	1.7321	2.0337
EST-HAR	1.6266	1.7319	1.4685	1.4917	1.6361	1.6337	1.7273	2.0262
MS-HAR	1.5865	1.7398	1.4374	1.4548	1.5985	1.5829	1.7102	2.0156
MS-TVTP-HAR	1.5856	1.7399	**1.4364***	1.4569	1.5983	1.5787	1.7084	2.0099
MSH-HAR	**1.5779***	**1.7089***	1.4367	**1.4453***	**1.5515***	**1.5488***	**1.6899***	**2.0082***
				BIC				
AR1	1.8056	1.9224	1.6433	1.6703	1.7491	1.8093	1.8676	2.2178
HAR	1.6443	1.7716	1.4986	1.5189	1.6575	1.6727	1.7492	2.0499
LSTAR	1.8288	1.9386	1.6546	1.6642	1.7631	1.8135	1.8691	2.2268
ESTAR	1.8288	1.9387	1.6592	1.6643	1.7597	1.6842	1.8695	2.0761
LST-HAR	1.6808	1.8004	1.5319	1.5424	1.6939	1.7005	1.7828	2.0836
EST-HAR	1.6762	1.7816	1.5194	1.5411	1.6872	1.6842	1.7779	2.0761
MS-HAR	1.6410	1.7945	1.4933	1.5092	1.6547	1.6384	1.7658	2.0705
MS-TVTP-HAR	1.6400	1.7946	**1.4924***	1.5211	1.6545	1.6342	1.7641	**2.0648***
MSH-HAR	**1.6374***	**1.7686***	1.4977	**1.5046***	**1.6128***	**1.6093***	**1.7506***	2.0681
				LL				
AR1	-904.92	-959.07	-795.86	-839.08	-842.31	-886.01	-911.05	-1102.01
HAR	-799.30	-857.82	-702.60	-739.58	-773.58	-794.19	-827.64	-989.66
LSTAR	-902.78	-953.41	-787.63	-822.18	-835.34	-874.30	-897.97	-1092.66
ESTAR	-902.79	-953.44	-789.86	-822.19	-833.69	-779.10	-898.18	-981.84
LST-HAR	-796.69	-851.36	-697.94	-730.52	-770.32	-786.96	-823.16	-985.48
EST-HAR	-794.38	-842.09	-691.95	-729.87	-767.13	-779.10	-820.81	-981.84
MS-HAR	-773.52	-844.98	-676.07	-710.60	-748.26	-753.55	-811.59	-975.63
MS-TVTP-HAR	-773.07	-845.03	-675.60	-709.62	-748.18	-751.53	-810.75	-972.84
MSH-HAR	**-768.29***	**-828.78***	**-674.72***	**-704.87***	**-724.96***	**-736.07***	**-800.82***	**-971.02***

Note: This table reports the AIC, BIC and Log Likelihood (LL) of eight RV indices for all forecasting models considered for the in-sample period from 1st November 2006 to 31st October 2010. The forecasting model with best performance is highlighted with *.

Table 5.3: Maximum Likelihood Estimates of MSH-HAR models in in-sample period

	FTSE		**SPX**		**N225**		**DAX**	
	Low Regime	High Regime	Low Regime	High Regime	Low Regime	High Regime	Low Regime	High Regime
β_{0,S_t}	-0.7878	-0.7916**	-0.6419***	-0.4431	-0.8865	-0.6529	-0.9142***	-0.1247
	(1.7850)	(0.3933)	(0.2384)	(0.3844)	(3.4328)	(0.4959)	(0.3497)	(0.3129)
β_{d,S_t}	0.1950	0.3507***	0.4440***	0.0503	0.1177	0.3781***	0.4352***	0.3679***
	(0.2194)	(0.0610)	(0.0537)	(0.1040)	(0.9360)	(0.1301)	(0.0664)	(0.0671)
β_{w,S_t}	1.1838***	0.2459***	0.3587***	0.7040***	1.0340	0.2651***	0.3818***	0.4114***
	(0.3529)	(0.0737)	(0.0729)	(0.2500)	(0.6886)	(0.0699)	(0.0870)	(0.0874)
β_{m,S_t}	-0.4880	0.3310***	0.1352***	0.2058	-0.2728	0.2989	0.0890	0.2134***
	(0.3229)	(0.0721)	(0.0611)	(0.2309)	(0.6590)	(0.1923)	(0.0844)	(0.0690)
$\log(\sigma_t)$	-0.2992***	-0.8497***	-0.4421***	-0.9898***	-0.2762***	-0.8723***	-0.4628***	-1.0427***
	(0.0819)	(0.0879)	(0.0441)	(0.0683)	(0.0988)	(0.0744)	(0.0682)	(0.0624)
p_{11}	0.3570		0.9869		0.4590		0.9522	
p_{22}	0.8215		0.9689		0.9066		0.9528	

	SSEC		**NSEI**		**BSVP**		**MXX**	
	Low Regime	High Regime	Low Regime	High Regime	Low Regime	High Regime	Low Regime	High Regime
β_{0,S_t}	-2.6935***	-0.7418*	-1.1445	-0.9534***	-0.7171	-0.6548**	-1.4089	-1.2463
	(0.7064)	(0.3937)	(0.9835)	(0.1871)	(0.4395)	(0.2850)	(1.7059)	(1.1348)
β_{d,S_t}	0.2982***	0.4470***	0.2920**	0.3694***	0.5472***	0.2649**	0.3258**	0.1148
	(0.0883)	(0.0624)	(0.1421)	(0.0536)	(0.0875)	(0.1055)	(0.1192)	(0.3298)
β_{w,S_t}	0.1616	0.0948	0.1944	0.3268**	0.1030	0.4804***	0.3537	0.5195
	(0.1214)	(0.1488)	(0.3627)	(0.0758)	(0.1672)	(0.1057)	(1.2370)	(1.1310)
β_{m,S_t}	0.1920*	0.3916**	0.3485	0.2120***	0.2614	0.1977**	0.1708	0.2517
	(0.1118)	(0.1508)	(0.3467)	(0.0559)	(0.1618)	(0.0888)	(1.4299)	(1.3648)
$\log(\sigma_t)$	-0.3956***	-0.9238***	-0.1927	-0.8623***	-0.3039***	-0.8105***	-0.1900	-0.6985***
	(0.1191)	(0.0682)	(0.1320)	(0.0820)	(0.0986)	(0.0630)	(0.2234)	(0.2257)
p_{11}	0.9226		0.8155		0.8851		0.8754	
p_{22}	0.9529		0.9543		0.9419		0.9071	

Note: This table reports two-regime MSH-HAR model of each RV index considered for the in-sample period from 1st November 2006 to 31st October 2010. σ is the standard deviation of error in each regime. $p_{11}$and p_{22} are the transition probability of being in the low and high regime, respectively. Standard errors are in parentheses. ***, ** and * refer the significant at 99%, 95% and 90% confidence level, respectively.

Table 5.4: Out-of-sample forecasting performances of MSE

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				H = 1				
AR1	0.4854	0.5045	0.4137	0.3591	0.3515	0.3759	0.3050	0.4515
HAR	0.3915	0.4408	0.3655	0.2959	0.2820	0.3086	0.2635	0.3808
LSTAR	0.4784	0.5036	0.4136	0.3546	0.3498	0.3672	0.3005	0.4521
ESTAR	0.4778	0.5038	0.4145	0.3547	0.3495	0.3628	0.3006	0.4514
LST-HAR	0.3934	0.4424	0.3672	0.2974	0.2843	0.3073	0.2636	0.3808
EST-HAR	0.3943	0.4426	**0.3651***	0.2973	0.2842	**0.3071***	0.2638	0.3887
MS-HAR	0.3908	0.4403	0.3667	0.2971	0.2842	0.3097	0.2664	0.3813
MS-TVTP-HAR	0.3913	0.4411	**0.3651***	**0.2959***	**0.2825***	0.3084	**0.2629***	**0.3812***
MSH-HAR	**0.3900***	**0.4397***	0.3682	0.2961	0.2868	0.3077	0.2644	0.3854
				H = 5				
AR1	0.2488	0.2849	0.2432	0.1714	0.2053	0.2034	0.1536	0.2251
HAR	0.1556	0.2002	0.1850	0.1032	0.1324	0.1316	0.0989	0.1502
LSTAR	0.2428	0.2829	0.2424	0.1671	0.2026	0.1979	0.1474	0.2255
ESTAR	0.2427	0.2828	0.2427	0.1670	0.2022	0.1978	0.1475	0.2255
LST-HAR	0.1560	0.2018	0.1857	0.1036	0.1333	0.1303	0.0983	0.1512
EST-HAR	0.1565	0.2222	0.1865	0.1065	0.1337	0.1402	0.0992	0.1505
MS-HAR	**0.1255***	**0.1673***	**0.1514***	**0.0965***	**0.1007***	**0.1153***	**0.0853***	**0.1274***
MS-TVTP-HAR	0.1431	0.1807	0.1683	0.0975	0.1174	0.1226	0.0891	0.1275
MSH-HAR	0.1424	0.1918	0.1742	0.0978	0.1225	0.1259	0.0914	0.1424
				H = 22				
AR1	0.2803	0.3706	0.2984	0.2144	0.2628	0.2453	0.1843	0.2532
HAR	0.1356	0.2009	0.1814	0.0999	0.0984	0.1147	0.0842	0.1266
LSTAR	0.2721	0.3635	0.2997	0.2059	0.2511	0.2372	0.1748	0.2503
ESTAR	0.2719	0.3636	0.2999	0.2055	0.2509	0.2341	0.1747	0.2499
LST-HAR	0.1351	0.1977	0.1822	0.0975	0.0976	0.1128	0.0823	0.1261
EST-HAR	0.1338	0.1996	0.1839	0.1052	0.0976	0.1762	0.0932	0.1284
MS-HAR	**0.0921***	**0.1314***	**0.1039***	0.0865	**0.0582***	**0.0810***	**0.0548***	**0.0708***
MS-TVTP-HAR	0.1230	0.1640	0.1385	0.0850	0.0781	0.1066	0.0641	0.0800
MSH-HAR	0.1164	0.1669	0.1586	**0.0831***	0.0596	0.0899	0.0704	0.0925

Note: This table reports the forecasting evaluation (MSE) of eight RV indices for all forecasting models considered using recursive method over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st November 2010 to 31st October 2020. The forecasting model with the best performance is highlighted with *.

Table 5.5: Out-of-sample forecasting performances of MAE

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				H = 1				
AR1	0.5379	0.5616	0.4893	0.4663	0.4646	0.4633	0.4266	0.5245
HAR	0.4761	0.5232	0.4525	**0.4229***	**0.4051***	0.4163	**0.3905***	**0.4725***
LSTAR	0.5333	0.5628	0.4904	0.4658	0.4633	0.4578	0.4220	0.5247
ESTAR	0.5331	0.5631	0.4899	0.4657	0.4632	0.4574	0.4220	0.5242
LST-HAR	0.4780	0.5247	0.4539	0.4242	0.4067	**0.4153***	0.3897	0.4729
EST-HAR	0.4778	0.5248	**0.4512***	0.4254	0.4066	0.4158	0.3896	0.4819
MS-HAR	0.4757	0.5227	0.4531	0.4244	0.4077	0.4169	0.3908	0.4728
MS-TVTP-HAR	0.4760	0.5231	0.4521	0.4239	0.4054	0.4162	0.3892	0.4726
MSH-HAR	**0.4753***	**0.5222***	0.4541	**0.4229***	0.4091	0.4164	0.3906	0.4749
				H = 5				
AR1	0.3797	0.4143	0.3635	0.3259	0.3542	0.3495	0.3053	0.3651
HAR	0.2772	0.3222	0.2901	0.2388	0.2641	0.2618	0.2276	0.2732
LSTAR	0.3758	0.4143	0.3628	0.3206	0.3518	0.3441	0.2975	0.3664
ESTAR	0.3757	0.4144	0.3633	0.3206	0.3513	0.3443	0.2976	0.3662
LST-HAR	0.2793	0.3271	0.2911	0.2401	0.2648	0.2605	0.2267	0.2738
EST-HAR	0.2790	0.3298	0.2906	0.2446	0.2647	0.2758	0.2288	0.2746
MS-HAR	0.2538	**0.2884***	**0.2681***	**0.2347***	**0.2242***	**0.2431***	**0.2056***	**0.2524***
MS-TVTP-HAR	0.2676	0.3043	0.2782	0.2351	0.2500	0.2539	0.2154	0.2555
MSH-HAR	**0.2532***	0.3086	0.2774	0.2269	0.2471	0.2500	0.2121	0.2615
				H = 22				
AR1	0.4117	0.4798	0.4257	0.3636	0.4114	0.3914	0.3361	0.3975
HAR	0.2548	0.3118	0.2977	0.2264	0.2311	0.2356	0.2066	0.2474
LSTAR	0.4049	0.4761	0.4255	0.3567	0.3998	0.3884	0.3274	0.3955
ESTAR	0.4049	0.4760	0.4263	0.3561	0.3992	0.3880	0.3273	0.3954
LST-HAR	0.2547	0.3110	0.2970	0.2240	0.2280	0.2317	0.2051	0.2468
EST-HAR	0.2528	0.3120	0.3017	0.2402	0.2317	0.2935	0.2276	0.2481
MS-HAR	**0.2185***	**0.2607***	**0.2306***	0.2196	0.1748	**0.1960***	**0.1707***	**0.1881***
MS-TVTP-HAR	0.2435	0.2904	0.2604	0.2181	0.2020	0.2271	0.1811	0.2045
MSH-HAR	0.2216	0.2660	0.2565	**0.1927***	**0.1738***	0.1967	0.1775	0.2029

Note: This table reports the forecasting evaluation (MAE) of eight RV indices for all forecasting models considered using recursive method over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st November 2010 to 31st October 2020. The forecasting model with the best performance is highlighted with *.

Table 5.6: Out-of-sample forecasting performances of QLIKE

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				H = 1				
AR1	0.002701	0.002592	0.002225	0.001991	0.002112	0.002019	0.001794	0.002411
HAR	0.002166	0.002244	0.001971	**0.001638***	**0.001685***	0.001649	0.001547	**0.002030***
LSTAR	0.002671	0.002582	0.002231	0.001975	0.002102	0.002005	0.001768	0.002419
ESTAR	0.002667	0.002584	0.002238	0.001975	0.002102	0.001966	0.001769	0.002413
LST-HAR	0.002175	0.002252	0.001981	0.001650	0.001699	0.001646	0.001545	0.002032
EST-HAR	0.002188	0.002256	0.001978	0.001646	0.001699	**0.001641***	0.001545	0.002079
MS-HAR	0.002162	0.002242	0.001981	0.001643	0.001699	0.001658	0.001570	0.002034
MS-TVTP-HAR	0.002164	0.002246	**0.001970***	0.001639	0.001689	0.001651	**0.001542***	0.002035
MSH-HAR	**0.002157***	**0.002239***	0.001988	0.001639	0.001713	0.001647	0.001551	0.002058
				H = 5				
AR1	0.001432	0.001506	0.001345	0.000981	0.001251	0.001110	0.000927	0.001231
HAR	0.000893	0.001054	0.001036	0.000593	0.000795	0.000723	0.000606	0.000828
LSTAR	0.001407	0.001493	0.001348	0.000964	0.001235	0.001108	0.000890	0.001235
ESTAR	0.001406	0.001493	0.001350	0.000964	0.001233	0.001104	0.000890	0.001235
LST-HAR	0.000895	0.001059	0.001039	0.000596	0.000803	0.000716	0.000601	0.000835
EST-HAR	0.000897	0.001451	0.001045	0.000612	0.000807	0.000776	0.000606	0.000831
MS-HAR	**0.000726***	**0.000884***	**0.000857***	**0.000544***	**0.000604***	0.000679	**0.000522***	0.000707
MS-TVTP-HAR	0.000812	0.000943	0.000941	0.000551	0.000698	**0.000669***	0.000539	**0.000697***
MSH-HAR	0.000814	0.001012	0.000977	0.000561	0.000741	0.000697	0.000560	0.000789
				H = 22				
AR1	0.001634	0.001986	0.001662	0.001251	0.001635	0.001391	0.001141	0.001387
HAR	0.000803	0.001089	0.001021	0.000595	0.000593	0.000675	0.000529	0.000691
LSTAR	0.001595	0.001946	0.001684	0.001208	0.001568	0.001366	0.001083	0.001376
ESTAR	0.001595	0.001944	0.001687	0.001206	0.001567	0.001339	0.001083	0.001372
LST-HAR	0.000798	0.001061	0.001026	0.000580	0.000589	0.000661	0.000513	0.000688
EST-HAR	0.000788	0.001079	0.001037	0.000626	0.000588	0.001130	0.000586	0.000702
MS-HAR	**0.000545***	**0.000721***	**0.000593***	**0.000504***	**0.000346***	**0.000486***	**0.000345***	**0.000391***
MS-TVTP-HAR	0.000724	0.000874	0.000777	0.000494	0.000460	0.000636	0.000393	0.000434
MSH-HAR	0.000689	0.000905	0.000904	0.000497	0.000356	0.000538	0.000444	0.000514

Note: This table reports the forecasting evaluation (QLIKE) of eight RV indices for all forecasting models considered using recursive method over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st November 2010 to 31st October 2020. The forecasting model with the best performance is highlighted with *.

Table 5.7: Out-of-sample forecasting performances of MZ regression adjusted R^2

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				H = 1				
AR1	0.4691	0.6206	0.5284	0.5868	0.6296	0.4827	0.4826	0.2937
HAR	0.5601	0.6605	0.5771	0.6540	**0.6881***	0.5477	0.5426	**0.3815***
LSTAR	0.4752	0.6227	0.5295	0.5906	0.6306	0.4891	0.4845	0.2892
ESTAR	0.4754	0.6226	0.5286	0.5905	0.6310	0.4337	0.4844	0.2902
LST-HAR	0.5580	0.6599	0.5753	0.6522	0.6857	0.5491	0.5416	0.3795
EST-HAR	0.5568	0.6599	0.5769	0.6522	0.6856	0.5489	0.5414	0.3794
MS-HAR	0.5606	0.6610	0.5760	0.6535	0.6864	0.5463	0.5367	0.3804
MS-TVTP-HAR	0.5602	0.6603	**0.5775***	**0.6550***	0.6877	0.5477	**0.5431***	0.3799
MSH-HAR	**0.5617***	**0.6612***	0.5749	0.6536	0.6836	**0.5490***	0.5408	0.3758
				H = 5				
AR1	0.6585	0.7450	0.6671	0.7551	0.7590	0.6590	0.6643	0.5339
HAR	0.7792	0.8146	0.7426	0.8488	0.8330	0.7554	0.7732	0.6694
LSTAR	0.6649	0.7491	0.6690	0.7603	0.7610	0.6621	0.6712	0.5298
ESTAR	0.6651	0.7491	0.6690	0.7604	0.7613	0.6621	0.6712	0.5302
LST-HAR	0.7785	0.8134	0.7414	0.8481	0.8318	0.7567	0.7737	0.6669
EST-HAR	0.7779	0.7971	0.7402	0.8447	0.8310	0.7489	0.7721	0.6687
MS-HAR	**0.8224***	**0.8454***	**0.7916***	**0.8592***	**0.8740***	**0.7853***	**0.8033***	**0.7188***
MS-TVTP-HAR	0.7971	0.8322	0.7661	0.8575	0.8515	0.7707	0.7940	0.7171
MSH-HAR	0.7982	0.8229	0.7585	0.8569	0.8468	0.7677	0.7903	0.6870
				H = 22				
AR1	0.5617	0.6146	0.5260	0.6462	0.6802	0.5672	0.5147	0.3672
HAR	0.7760	0.7785	0.7040	0.8266	0.8625	0.7556	0.7568	0.6439
LSTAR	0.5728	0.6297	0.5255	0.6607	0.6867	0.5759	0.5261	0.3666
ESTAR	0.5730	0.6295	0.5260	0.6613	0.6868	0.5790	0.5263	0.3668
LST-HAR	0.7769	0.7830	0.7026	0.9307	0.8617	0.7593	0.7609	0.6447
EST-HAR	0.7789	0.7799	0.7006	0.8237	0.8618	0.6834	0.7493	0.6403
MS-HAR	**0.8483***	**0.8569***	**0.8373***	0.8523	**0.9189***	**0.8213***	**0.8393***	**0.7987***
MS-TVTP-HAR	0.7951	0.8195	0.7739	0.8546	0.8880	0.7711	0.8116	0.7710
MSH-HAR	0.8078	0.8152	0.7416	**0.8557***	0.9171	0.8086	0.7967	0.7398

Note: This table reports the Mincer-Zarnowitz regression adjusted R^2 of eight RV indices for all forecasting models considered using recursive method over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st November 2010 to 31st October 2020. The forecasting model with the highest adjusted R^2 is highlighted with *.

Table 5.8: The Model confidence set test of MSE criterion

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				H = 1				
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	0.0000	0.7168	0.2858	0.9992	**1.0000***	0.5386	0.8680	0.9998
LSTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ESTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LST-HAR	0.0000	0.9216	0.0000	0.9102	0.2106	0.9987	0.5636	0.9998
EST-HAR	0.0000	0.9416	**1.0000***	0.8226	0.5702	**1.0000***	0.6332	0.0000
MS-HAR	0.0000	0.9982	0.0000	0.2578	0.0000	0.0000	0.0000	0.9736
MS-TVTP-HAR	0.0000	0.7470	0.9162	**1.0000***	0.7868	0.7228	**1.0000***	**1.0000***
MSH-HAR	**1.0000***	**1.0000***	0.0000	0.9992	0.0000	0.9996	0.0374	0.0000
				H = 5				
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LSTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ESTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
EST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MS-HAR	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
MS-TVTP-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6778
MSH-HAR	0.0000	0.0000	0.0000	0.8212	0.0000	0.0000	0.0000	0.0000
				H = 22				
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LSTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ESTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
EST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MS-HAR	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
MS-TVTP-HAR	0.0000	0.0000	0.0000	0.4698	0.0000	0.0000	0.0000	0.0000
MSH-HAR	0.0000	0.0000	0.0000	0.0000	0.3678	0.0000	0.0000	0.0000

Note: This table reports the MSC test in term of MSE criterion for eight RV indices over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 5.9: The Model confidence set test of QLIKE criterion

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				H = 1				
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	0.0000	0.5932	0.9878	**1.0000***	**1.0000***	0.8224	0.9684	**1.0000***
LSTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ESTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LST-HAR	0.0000	0.8848	0.6200	0.5978	0.2676	0.8804	0.9138	0.9920
EST-HAR	0.0000	0.7996	0.9950	0.9020	0.7042	**1.0000***	0.9822	0.0000
MS-HAR	0.0000	0.9874	0.1236	0.9410	0.0010	0.0000	0.0000	0.9238
MS-TVTP-HAR	0.0000	0.4314	**1.0000***	0.9960	0.4658	0.5988	**1.0000***	0.5946
MSH-HAR	**1.0000***	**1.0000***	0.0430	**1.0000***	0.0000	0.8944	0.0000	0.0000
				H = 5				
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0254	0.0000	0.0000
LSTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ESTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0176	0.0000	0.0000
EST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MS-HAR	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9900	**1.0000***	0.5928
MS-TVTP-HAR	0.0000	0.0000	0.0000	0.3786	0.0000	**1.0000***	0.0000	**1.0000***
MSH-HAR	0.0000	0.0000	0.0000	0.5782	0.0000	0.3712	0.0000	0.0000
				H = 22				
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LSTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ESTAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
EST-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MS-HAR	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
MS-TVTP-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MSH-HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: This table reports the MSC test in term of QLIKE criterion for eight RV indices over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level are highlighted in table. The value 1 in the table means that the optimal model is chosen, the value 0 means the model is eliminated.

Table 5.10: The backtesting and model comparison summary of 1% VaR and ES

	Ave. failure rate	**Sig. Kupiec test**	**Sig. Christoffersen test**	**Sig. DQ test**	**Ave. QL**$*10^{-4}$	**Ave. FZL**
			H=1			
AR1	2.7487%	ALL	ALL	ALL	3.6110	-3.2553
HAR	2.9665%	ALL	ALL	ALL	3.6568	-3.2289
LSTAR	**2.6748%***	ALL	ALL	ALL	3.5960	-3.2792
ESTAR	2.6921%	ALL	ALL	ALL	**3.5943***	**-3.2793***
LST-HAR	2.9098%	ALL	ALL	ALL	3.6602	-3.2366
EST-HAR	2.9437%	ALL	ALL	ALL	3.6597	-3.2388
MS-HAR	2.9495%	ALL	ALL	ALL	3.6517	-3.2331
MS-TVTP-HAR	2.9327%	ALL	ALL	ALL	3.6507	-3.2344
MSH-HAR	2.9496%	ALL	ALL	ALL	3.6562	-3.2338
			H=5			
AR1	2.1302%	ALL	ALL	ALL	2.0394	-3.8271
HAR	2.1251%	ALL	ALL	ALL	1.9921	-3.8581
LSTAR	2.0567%	ALL	ALL	ALL	2.0398	-3.8249
ESTAR	**2.0508%***	ALL	ALL	ALL	2.0396	-3.8250
LST-HAR	2.1135%	ALL	ALL	ALL	2.0033	-3.8520
EST-HAR	2.0506%	ALL	ALL	ALL	1.9926	-3.8565
MS-HAR	2.0388%	ALL	ALL	ALL	1.9863	-3.8642
MS-TVTP-HAR	2.1602%	ALL	ALL	ALL	**1.9819***	**-3.8671***
MSH-HAR	2.0612%	ALL	ALL	ALL	1.9949	-3.8574
			H=22			
AR1	1.6153%	ALL	ALL	ALL	2.0937	-3.7824
HAR	1.7251%	ALL	ALL	ALL	2.0149	-3.8312
LSTAR	1.5873%	ALL	ALL	ALL	2.0943	-3.7787
ESTAR	**1.5870%***	ALL	ALL	ALL	2.0946	-3.7787
LST-HAR	1.6844%	ALL	ALL	ALL	2.0386	-3.8205
EST-HAR	1.6961%	ALL	ALL	ALL	2.0151	-3.8301
MS-HAR	1.8357%	ALL	ALL	ALL	2.0202	-3.8332
MS-TVTP-HAR	1.7539%	ALL	ALL	ALL	**2.0067***	**-3.8364***
MSH-HAR	1.8014%	ALL	ALL	ALL	2.0145	-3.8338

Notes: this table provides the VaR and ES results at the 1% level. The average failure rate for each model over each index. The series are significant in the Kupiec test, Christoffersen test and DQ test are listed. The average asymmetric Quantile Loss function (QL) and the Average Fissler and Ziegel (2016) Loss function (FZL) for each model over each index. * highlights the forecasting model with best performance.

Table 5.11: The backtesting and model comparison summary of 5% VaR and ES

	Ave. failure rate	**Sig. Kupiec test**	**Sig. Christoffersen test**	**Sig. DQ test**	**Ave. QL**$*10^{-4}$	**Ave. FZL**
				H=1		
AR1	8.1517%	ALL	ALL	ALL	11.052	-3.7392
HAR	8.5259%	ALL	ALL	ALL	11.010	-3.7375
LSTAR	8.0768%	ALL	ALL	ALL	11.051	**-3.7466***
ESTAR	**8.0572%***	ALL	ALL	ALL	11.051	-3.7465
LST-HAR	8.4459%	ALL	ALL	ALL	11.013	-3.7386
EST-HAR	8.4667%	ALL	ALL	ALL	11.011	-3.7401
MS-HAR	8.5065%	ALL	ALL	ALL	**11.001***	-3.7394
MS-TVTP-HAR	8.4934%	ALL	ALL	ALL	11.007	-3.7395
MSH-HAR	8.5594%	ALL	ALL	ALL	11.016	-3.7378
				H=5		
AR1	6.5752%	ALL	ALL	ALL	7.1793	-4.0744
HAR	6.8636%	ALL	ALL	ALL	7.0046	-4.1046
LSTAR	**6.5209%***	ALL	ALL	ALL	7.1840	-4.0718
ESTAR	6.5477%	ALL	ALL	ALL	7.1829	-4.0718
LST-HAR	6.8104%	ALL	ALL	ALL	7.0439	-4.1001
EST-HAR	6.8303%	ALL	ALL	ALL	7.0066	-4.1034
MS-HAR	6.9231%	ALL	ALL	ALL	7.0041	-4.1078
MS-TVTP-HAR	6.9500%	ALL	ALL	ALL	**6.9833***	**-4.1104***
MSH-HAR	6.8099%	ALL	ALL	ALL	7.0239	-4.1029
				H=22		
AR1	5.8658%	ALL	ALL	ALL	7.4019	-4.0208
HAR	6.2953%	ALL	ALL	ALL	7.1232	-4.0697
LSTAR	**5.8461%***	ALL	ALL	ALL	7.4039	-4.0171
ESTAR	5.8796%	ALL	ALL	ALL	7.4051	-4.0172
LST-HAR	6.2488%	ALL	ALL	ALL	7.2069	-4.0591
EST-HAR	6.3368%	ALL	ALL	ALL	7.1241	-4.0685
MS-HAR	6.3671%	ALL	ALL	ALL	7.1420	-4.0716
MS-TVTP-HAR	6.4227%	ALL	ALL	ALL	**7.0943***	**-4.0749***
MSH-HAR	6.2075%	ALL	ALL	ALL	7.1219	-4.0723

Notes: this table provides the VaR and ES results at the 5% level. The average failure rate for each model over each index. The series are significant in the Kupiec test, Christoffersen test and DQ test are listed. The average asymmetric Quantile Loss function (QL) and the Average Fissler and Ziegel (2016) Loss function (FZL) for each model over each index. * highlights the forecasting model with best performance.

Chapter 6 Summary and Conclusion

6.1 Summary

Since Andersen and Bollerslev (1998) indicated the daily squared returns is a very noisy proxy of volatility, the frontier research of volatility modelling is mainly on RV. Subsequently, the current literature suggests that the HAR model (Corsi, 2009) is the preferred approach for modelling and forecasting RV. In light of the basic HAR model setting, this thesis consists of three empirical chapters entering the ongoing debate on refining the current forecasting models and identifying the most accurate model for forecasting RV.

This thesis starts with a broad literature review on forecasting volatility, including previous forecasting models, volatility measurements and features of volatility. Then, in the literature review of each empirical chapter, the underlying theory and current research frontiers provide the foundation for the empirical research in this thesis. To improve the forecasting power of existing volatility models, this thesis considers a number of components in the modelling process of volatility (Chapter 3), considers parsimonious lags (Chapter 4), and finally a regime-switching approach is also considered (Chapter 5) to achieve better volatility predictions. A systematic approach is followed in the modelling and forecasting process in each chapter and several models are tested. The results are also evaluated within a risk management setting.

Chapter 3 evaluates the predictive ability of three volatility components in the HAR-RV setting, including volatility jumps, realised semi-variance, and leverage effect. So far, the literature assessed these components individually withing the modelling process (Andersen et al., 2007; Patton and Sheppard, 2015 and Corsi et al., 2012). This chapter extends the existing work with three different model settings for each component. Several forecast evaluation techniques are used in assessing the forecasts such as symmetric loss functions, asymmetric loss functions, pairwise comparisons, and equal predictive ability tests. In addition, forecasts are generated by two forecasting approaches, namely the rolling window method and the recursive method.

The results in Chapter 3 find that accommodating the leverage effect into the HAR model provides the best forecasting performance over daily, weekly and monthly horizons. According to the forecasting evaluation, all three volatility components improve the predictive accuracy of RV, however, the sophisticated HAR models with volatility jumps perform the worst, barely outperforming the basic HAR model, followed by the HAR models incorporate realised semi-variance. This chapter enables the RV to be predicted more accurately in the HAR model settings and it is beneficial to everyone involved in the financial market. More specifically, this results confirm the superiority of the leverage effect, showing that the inclusion of negative returns is the most accurate in the forecasting process. Policymakers can judge future market conditions based on volatility information and make full use of short- and long-term volatility to formulate macro-policies to reduce the risk of financial markets. If it predicts that the volatility of the stock market will increase in the future, the government can proactively implement macroeconomic policies to stabilize the economy, which enhances the foresight of policy. Investors can also adjust investment strategies timely based on predicted values to reduce investment risks.

Chapter 4 examines the predictive ability of the AR models with parsimonious lags generated by the Lasso-based methods, which follows the works of Audrino and Knaus (2016) and Audrino et al. (2019). Specifically, this chapter expands previous works in two aspects. First, apart from the AR(22) model, which has the same lags as the HAR model, the AR(100) with longer lags is also considered. Second, this chapter employs the more recent types of Lasso approaches with different parameter penalizations to obtain parsimonious lags, including the Lasso(Toshigami, 1996), the adaptive Lasso (Zou, 2006), the grouped Lasso (Yuan and

Lin, 2006), and the ordered Lasso (Toshigami and Suo, 2016). In addition, this employs the forecasting results into Value at Risk setting.

The in-sample analysis indicates the AR models with flexible lags can improve the model fitness over the HAR models. When plotting the coefficients, I find the forecasting information is concentrated in the first 22 lags, meanwhile, the longer lags also contain efficient information. In the out-of-sample results, the flexible AR models do significantly improve the forecasting ability, in which the ordered Lasso dominates the forecasting performance. Specifically, the AR(22) with ordered Lasso performs the best at daily horizon, AR(100) with ordered Lasso are preferred at weekly and monthly level. All forecasting results from the rolling window and recursive method are confirmed within the VaR setting over daily, weekly and monthly horizons.

The findings in Chapter 4 add a further dimension to the ongoing debate about the forecasting ability of a more flexible lag structure in the AR model. Although the Lasso approaches are originally used in the computational statistics, this chapter promotes Lasso approaches in a financial forecasting setting and show that more flexible lags do improve forecasting performance over HAR models. Overall, for the view of the practitioners and investors, the current HAR-type models with restricted lag structure might not as good as the flexible lag structure in the field of forecasting volatility, the parsimonious lags may contain more efficient predictive information. The flexible lags in the AR models give a new direction to predictive modelling, the Lasso is an important approach worth considering in future volatility forecasting exercises.

According to the finding that the nonlinear persistence of volatility can improve prediction (McAleer and Mediros, 2008 and Raggi and Bordignon, 2012), Chapter 5 evaluates the performance of nonlinear regime-switching frameworks incorporate the HAR model in forecasting RV. Thence, this chapter combines the AR model and HAR model with the smooth transition and Markov switching approaches to generate forecasts. To symmetrically extend the regime-switching framework, this chapter also extends the Markov-switching in two directions. One is considering the time-varying transition probability, and another one is implying the variance shift between regimes. Besides the statistical aspect, this chapter also employs the forecasts in the economic evaluation in terms of VaR and ES.

In the results of Chapter 5, generally, the regime-switching models are preferred over linear models in the in-sample analysis, and the Markov-switching models have a better goodness-of-fit than the smooth transition approaches. For out-of-sample exercise, although the regime-switching models have limited forecasting ability over the daily horizon, these models do outperform the linear HAR model over weekly and monthly horizons, where the Markov-switching HAR model is the best forecasting model and consistently exhibits the most accurate forecasts over time. The identical results are verified within the VaR and ES setting over daily, weekly and monthly horizons.

The current empirical works are preferred the linear HAR model to model and forecast volatility, Chapter 5 takes into account the existence of two different volatility regimes, namely high- and low-volatility regime. These findings suggest that different volatility levels due to sudden changes in the market should be considered. Therefore, the nonlinear regime-switching HAR models used in this chapter maintain the same structure of the HAR model and add the possibility of sudden changes in the market. From the view of financial practitioners, regime-switching models are crucial for many aspects of the real financial market. The actual financial markets are not consistent all the time, the different market regimes with abrupt transition techniques are more suitable for real market changes. Investors could use the appropriate market timing to allocate their assets, and policymakers can propose more suitable policies through different market regimes.

6.2 Conclusions

This thesis adds knowledge to the literature on RV forecasting, the findings in this thesis are crucial to provide a good understanding of finance across international stock markets. Currently, the frontier research is mainly concentrate on the HAR model (Corsi, 2009), and the HAR model dominates the forecasting model. The empirical chapters in this thesis improve the accuracy of forecasting performance from three aspects. First, from the view of volatility components, this thesis indicates the leverage effect is a superior component in the HAR model setting, which means negative returns have major importance for future volatility. Second, the flexible lags in AR models improve forecasting ability over the HAR model, these results suggest that the Lasso method with an appropriate penalty function can lead to developments in forecasting volatility. Third, the nonlinear regime-switching models exhibit better forecasts than linear models, with the Markov-switching model consistently are preferred. Smooth transition approaches allow for a more gradual transition between regimes, therefore, the abrupt transition of Markov-switching is more suitable than smooth transition in real financial markets.

In addition to the three main contributions mentioned above, the data sample in all empirical chapters is selected from developed and emerging countries to identify any regular pattern. It is worth noting that in all the results, there is no obvious difference between the developed and emerging markets, and the results are consistent for all markets. This enables the results in this thesis can be used by a wider range of market participants.[26]

Throughout the thesis, the RV is used in empirical investigations. The measurement of intraday data is an important factor in volatility forecasting, the alternative approaches for presenting true volatility domain with more recent studies. One aspect worth exploring further

[26] Due to data limitation, only four emerging markets are available from the Oxford-Man Institute of Quantitative Finance database. This thesis also selects four developed markets for comparison purposes. The consistency of the most appropriate forecasting models between developed and emerging markets is a contribution of this thesis. Other important factors such as financial liberalisation and sensitivity of information flow also need to be accounted for when considering emerging markets.

is the impact of alternative approaches on the accuracy of volatility forecasting to answer the following question: “Do other intraday measures obtain better forecasts than RV”? Moreover, the HAR model is a restricted AR model with 22 lags, noticing in Chapter 4, this thesis addresses the question of longer lags could improve accuracy. Based on the underlying principles and methodology followed in this thesis a further question is raised: “Are there any optimal lag structures perform better than the HAR model”? It can be seen that the topic of volatility prediction is a subject with continuous research potential.

Appendix

Appendix 1: Models specifications in Chapter 3

Model Name	Model specifications	Eq. number
HAR-RV	$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t$	Eq. (3.2)
HAR-RV-J	$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + \beta_j J_{t-1} + u_t$	Eq. (3.11)
HAR-RV-CJ	$RV_t = \beta_0 + \beta_{cd} C_{t-1} + \beta_{cw} C_{t-1:t-5} + \beta_{cm} C_{t-1:t-22} + \beta_{jd} J_{t-1} + \beta_{jw} J_{t-1:t-5} + \beta_{jm} J_{t-1:t-22} + u_t$	Eq. (3.12)
HAR-CJ	$RV_t = \beta_0 + \beta_{cd} C_{t-1} + \beta_{cw} C_{t-1:t-5} + \beta_{cm} C_{t-1:t-22} + \beta_{jd} J_{t-1} + u_t$	Eq. (3.13)
HAR-PS	$RV_t = \beta_0 + {\beta_d}^+ {RSV_{t-1}}^+ + {\beta_d}^- {RSV_{t-1}}^- + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t$	Eq. (3.20)
HAR-RV-SJV	$RV_t = \beta_0 + {\beta_j}^+ {SJV_{t-1}}^+ + {\beta_j}^- {SJV_{t-1}}^- + \beta_c C_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t$	Eq. (3.21)
HAR-RSV	$RV_t = \beta_0 + {\beta_d}^+ {RSV_{t-1}}^+ + {\beta_d}^- {RSV_{t-1}}^- + {\beta_w}^+ {RSV_{t-1:t-5}}^+ + {\beta_w}^- {RSV_{t-1:t-5}}^- + {\beta_m}^+ {RSV_{t-1:t-22}}^+ + {\beta_m}^- {RSV_{t-1:t-22}}^- + u_t$	Eq. (3.22)
LHAR-RV1	$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + \beta_{ld} {r_{t-1}}^- + u_t$	Eq. (3.23)
LHAR-RV2	$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + \beta_{ld} {r_{t-1}}^- + \beta_{lw} {r_{t-1,t-5}}^- + \beta_{lm} {r_{t-1,t-22}}^- + u_t$	Eq. (3.24)
LHAR-RV-CJ	$RV_t = \beta_0 + \beta_{cd} C_{t-1} + \beta_{cw} C_{t-1:t-5} + \beta_{cm} C_{t-1:t-22} + \beta_{jd} J_{t-1} + \beta_{jw} J_{t-1:t-5} + \beta_{jm} J_{t-1:t-5} + \beta_{ld} {r_{t-1}}^- + \beta_{lw} {r_{t-1,t-5}}^- + \beta_{lm} {r_{t-1,t-22}}^- + u_t$	Eq. (3.25)

Note: This table lists all model used in Chapter 3.

Appendix 2: Models specifications in Chapter 4

Model Name	Model specifications	Eq. number
HAR-RV	$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t$	Eq. (4.2)
HAR-free	$RV_t = \beta_0 + \beta_1 RV_{t-1} + \beta_2 RV_{t-2} + \beta_3 RV_{t-3} + \beta_4 RV_{t-4} + \beta_5 RV_{t-5} + \beta_6 RV_{t-6} + \beta_m RV_{t-1:t-22} + u_t$	Eq. (4.7)
AR(p)	$RV_{t+1} = \theta_0 + \sum_{i=1}^{n} \theta_i RV_{t-i+1} + u_t$	Eq. (4.8)
Lasso	$\hat{\beta}_{lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}\lvert\beta_i\rvert\right\}$	Eq. (4.9)
Adaptive Lasso	$\hat{\beta}_{adoptive\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}\lambda_i\lvert\beta_i\rvert\right\}$	Eq. (4.10)
Group Lasso	$\hat{\beta}_{group\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{k=1}^{K}\sqrt{p_k}\sqrt{\sum_{i\in I_k}{\beta_i}^2}\right\}$	Eq. (4.11)
Ordered Lasso	$\hat{\beta}_{ordered\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\left(\beta_j^+ + \beta_j^-\right)RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}\left(\beta_j^+ + \beta_j^-\right)\right\}$	Eq. (4.12)

Appendix 3: Models specifications in Chapter 5

Model Name	Model specifications	Eq. number
AR(1)	$RV_t = \theta_0 + \theta_i RV_{t-1} + u_t$	Eq. (5.2)
HAR	$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t$	Eq. (5.3)
ST model	$RV_t = X_t\alpha + G(s_t;\gamma,\psi)Z_t'\beta_1 + \big(1 - G(s_t;\gamma,\psi)\big)Z_t'\beta_2 + \varepsilon_t$	Eq. (5.6)
LST function	$G(s_t;c,\gamma) = \dfrac{1}{1+exp\big(-\gamma(s-c)\big)}, \gamma > 0$	Eq. (5.7)
EST function	$G(s_t;c,\gamma) = 1 - exp(-\gamma(s-c)^2), \gamma > 0$	Eq. (5.8)
Two-regime MS-HAR model	$RV_t = P^{11}\big(\beta_{0,S_1} + \beta_{d,S_1}RV_{t-1} + \beta_{w,S_1}RV_{t-1:t-5} + \beta_{m,S_1}RV_{t-1:t-22}\big) + P^{22}\big(\beta_{0,S_2} + \beta_{d,S_2}RV_{t-1} + \beta_{w,S_2}RV_{t-1:t-5} + \beta_{m,S_2}RV_{t-1:t-22}\big) + u_t$	Eq. (5.12)
MS-HAR	$u_t\big\vert\zeta_t \sim N\big(0, P^{11}v_{t,S_1} + P^{22}v_{t,S_2}\big)$ where $\big(v_{t,S_1} = v_{t,S_2}\big)$	Eq. (5.12)
MS-TVTP-HAR	$P(S_t = i\vert S_{t-1} = i) = p_{ii} = \dfrac{exp(c_i + d_i\delta_{t-1})}{1 + exp(c_i + d_i\delta_{t-1})}$	Eq. (5.13)
MSH-HAR	$u_t\big\vert\zeta_t \sim N\big(0, P^{11}v_{t,S_1} + P^{22}v_{t,S_2}\big)$ where $\big(v_{t,S_1} \neq v_{t,S_2}\big)$	

Appendix 4

Forecasting Realised Volatility: Does the LASSO approach outperform HAR?

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